Date: 04/06/2023

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Time:3 hrs. Answers \& Solutions
Max. Marks: 180

## for <br> JEE (Advanced)-2023 (Paper-1)

## PART-I : PHYSICS

## SECTION 1 (Maximum Marks : 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| Full Marks | $:$ | +4 | ONLY if (all) the correct option(s) is(are) chosen; |
| :--- | :--- | :--- | :--- | :--- |
| Partial Marks | $:$ | +3 | If all the four options are correct but ONLY three options are chosen; |
| Partial Marks | $:$ | +2 | If three or more options are correct but ONLY two options are chosen, both of which |
| are correct; |  |  |  |$\quad$| Partial Marks | $:$ | +1 | If two or more options are correct but ONLY one option is chosen and it is a correct <br> option; |
| :--- | :--- | :--- | :--- |
| Zero Marks | $:$ | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| Negative Marks | $:$ | -2 | In all other cases. |



1. A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height 3 h from the ground, as shown in the figure. A spherical ball of mass $m$ is released on the slide from rest at a height $h$ from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_{0}=u_{0} \hat{x}$ and falls on the ground at a distance $d$ from the building making an angle $\theta$ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height $h_{1}$. The acceleration due to gravity is $g$ and the coefficient of restitution of the ground is $1 / \sqrt{3}$. Which of the following statement(s) is(are) correct?

(A) $\vec{u}_{0}=\sqrt{2 g h} \hat{x}$
(B) $\vec{v}=\sqrt{2 g h}(\hat{x}-\hat{z})$
(C) $\theta=60^{\circ}$
(D) $d / h_{1}=2 \sqrt{3}$

Answer (A, C, D)
Sol.

$\stackrel{V_{1}}{\hookrightarrow} u_{0}$
(After)
$u_{0}=\sqrt{2 g h}$


2022


2021

$v_{z}=\sqrt{2 g(3 h)}$
$\tan \theta=\frac{v_{z}}{u}=\sqrt{3}$
$\theta=60^{\circ}$
$d=u_{0} T=u_{0} \sqrt{2\left(\frac{3 h}{g}\right)}=\sqrt{(2 g h)} \sqrt{(2)\left(\frac{3 h}{g}\right)}$
Velocity after collision, only velocity along z-direction change
$v_{1}=e v_{z}=\sqrt{2 g h}$
$\vec{v}=v_{1} \hat{k}+u_{0} \hat{i}$
$=\sqrt{2 g h}[\hat{i}+\hat{k}]$
$h_{1}=\frac{v_{1}^{2}}{2 g}=h$
Finally, $u_{0}=\sqrt{2 g h}, \theta=60^{\circ}, \frac{d}{h}=2 \sqrt{3}$
2. A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta=60^{\circ}$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{\min }=30^{\circ}$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?


Figure-1


Figure-2
(A) The blue light is polarized in the plane of incidence.
(B) The angle of the prism is $45^{\circ}$.
(C) The refractive index of the material of the prism for red light is $\sqrt{2}$.
(D) The angle of refraction for blue light in air at the exit plane of the prism is $60^{\circ}$.

Answer (A, C, D)

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Sol. For no reflection


$$
\begin{aligned}
\tan i & =\sqrt{3} \\
i & =60
\end{aligned}
$$

$\frac{\sin i}{\sin r_{1}}=\sqrt{3}, r_{1}=30^{\circ}$
$\delta=i+e-r_{1}-r_{2}=60^{\circ}$
$e=60^{\circ} r_{2}=30^{\circ}$
$A=60^{\circ}$
For red light,
$\mu=\frac{\sin \left(\frac{A+\delta_{m}}{2}\right)}{\sin \left(\frac{A}{2}\right)}=\sqrt{2}$
3. In a circuit shown in the figure, the capacitor $C$ is initially uncharged and the key $K$ is open. In this condition, a current of 1 A flows through the $1 \Omega$ resistor. The key is closed at time $t=t_{0}$. Which of the following statement(s) is(are) correct?
[Given : $\left.e^{-1}=0.36\right]$

(A) The value of the resistance $R$ is $3 \Omega$.
(B) For $t<t_{0}$, the value of current $I_{1}$ is 2 A .
(C) At $t=t_{0}+7.2 \mu \mathrm{~s}$, the current in the capacitor is 0.6 A .
(D) For $t \rightarrow \infty$, the charge on the capacitor is $12 \mu \mathrm{C}$.

Answer (A, B, C, D)


2022


2021


Sol. $15-I R=6$

$$
I_{1}[3]=6
$$

$I_{1}=2 \mathrm{~A}$
$I=I_{1}+1=3$
$15-3 R=6$
$\Rightarrow \quad R=3 \Omega$
Eq. circuit is
$\frac{1}{R_{\text {eq }}}=\frac{1}{3}+\frac{1}{3}+1$

$$
R_{e q}=\frac{3}{5} \Omega
$$

$E_{\text {eq }}=5+5+0=10$
$E_{\text {eq }}=10 \times \frac{3}{5}=6 \mathrm{~V}$
Current in circuit is $\frac{6}{\left(\frac{3}{5}+3\right)} e^{-t / C R}$
$=\frac{6 \times 5}{18} e^{-\frac{7.2 \times 10^{-6}}{2 \times 10^{-6} \times 3.6}}$
$=\frac{30}{18} \times e^{-1}=\frac{30}{18} \times 0.36$
$=0.6 \mathrm{~A}$
At steady state, voltage across capacitor $=6 \mathrm{~V}$
$Q=6 \times 2=12 \mu \mathrm{C}$

## SECTION 2 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

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4. A bar of mass $M=1.00 \mathrm{~kg}$ and length $L=0.20 \mathrm{~m}$ is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass $m=0.10 \mathrm{~kg}$ is moving on the same horizontal surface with $5.00 \mathrm{~ms}^{-1}$ speed on a path perpendicular to the bar. It hits the bar at a distance $L / 2$ from the pivoted end and returns back on the same path with speed $v$. After this elastic collision, the bar rotates with an angular velocity $\omega$. Which of the following statement is correct?
(A) $\omega=6.98 \mathrm{rad} \mathrm{s}^{-1}$ and $v=4.30 \mathrm{~ms}^{-1}$
(B) $\omega=3.75 \mathrm{rad} \mathrm{s}^{-1}$ and $v=4.30 \mathrm{~ms}^{-1}$
(C) $\omega=3.75 \mathrm{rad} \mathrm{s}^{-1}$ and $v=10.0 \mathrm{~ms}^{-1}$
(D) $\omega=6.80 \mathrm{rad} \mathrm{s}^{-1}$ and $v=4.10 \mathrm{~ms}^{-1}$

## Answer (A)

Sol. C.O.A.M. about point $O$
$m v_{0} \frac{L}{2}=\frac{M L^{2}}{3} \omega-\frac{m v L}{2}$
$e=1$
$\Rightarrow \quad v_{0}=v+\frac{L \omega}{2}$

Solve equation (i) and (ii)
$m=0.1 \mathrm{~kg}, M=1 \mathrm{~kg}, L=0.20 \mathrm{~m}$
$v_{0}=5 \mathrm{~m} / \mathrm{s}$


Solve (i) and (ii)

We get, $\omega=6.98 \mathrm{rad} / \mathrm{s}$
and $v=4.3 \mathrm{~m} / \mathrm{s}$

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5. A container has a base of $50 \mathrm{~cm} \times 5 \mathrm{~cm}$ and height 50 cm , as shown in the figure. It has two parallel electrically conducting walls each of area $50 \mathrm{~cm} \times 50 \mathrm{~cm}$. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of $250 \mathrm{~cm}^{3} \mathrm{~s}^{-1}$. What is the value of the capacitance of the container after 10 seconds?
[Given: Permittivity of free space $\varepsilon_{0}=9 \times 10^{-12} \mathrm{C}^{2} \mathrm{~N}^{-1} \mathrm{~m}^{-2}$, the effects of the non-conducting walls on the capacitance are negligible]

(A) 27 pF
(B) 63 pF
(C) 81 pF
(D) 135 pF

Answer (B)
Sol. $h=\frac{250 \times 10}{50 \times 5}=10 \mathrm{~cm}$

$$
\begin{aligned}
C_{1} & =\frac{(0.40 \times 0.50) \times 9 \times 10^{-12}}{5 \times 10^{-2}} \\
& =0.36 \times 10^{-10} \mathrm{~F} \\
C_{2} & =\frac{3 \times 0.10 \times 0.5 \times 9 \times 10^{-12}}{5 \times 10^{-2}} \\
C_{2} & =0.27 \times 10^{-10} \mathrm{~F} \\
C & =C_{1}+C_{2} \\
& =63 \mathrm{pF}
\end{aligned}
$$


6. One mole of an ideal gas expands adiabatically from an initial state ( $T_{A}, V_{0}$ ) to final state ( $T_{f} 5 V_{0}$ ). Another mole of the same gas expands isothermally from a different initial state ( $T_{B}, V_{0}$ ) to the same final state ( $T_{f} 5 V_{0}$ ). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is $\gamma$. What is the ratio $T_{A} / T_{B}$ ?
(A) $5^{\gamma-1}$
(B) $5^{1-\gamma}$
(C) $5^{\gamma}$
(D) $5^{1+\gamma}$

## Answer (A)





Sol. For Adiabatic process

$$
\begin{equation*}
T V^{\prime-1}=C \tag{i}
\end{equation*}
$$

$\Rightarrow T_{A} V_{0}^{\gamma-1}=T_{f}\left(5 V_{0}\right)^{\gamma-1}$
For Isothermal process
$T_{B}=T_{f}$
Equation (i) : equation (ii)

$\Rightarrow \frac{T_{A}}{T_{B}}=5^{\gamma-1}$
7. Two satellites $P$ and $Q$ are moving in different circular orbits around the Earth (radius $R$ ). The heights of $P$ and $Q$ from the Earth surface are $h_{P}$ and $h_{Q}$, respectively, where $h_{P}=\frac{R}{3}$. The accelerations of $P$ and $Q$ due to Earth's gravity are $g_{P}$ and $g_{Q}$. respectively. If $\frac{g_{P}}{g_{Q}}=\frac{36}{25}$, what is the value of $h_{Q}$ ?
(A) $\frac{3 R}{5}$
(B) $\frac{R}{6}$
(C) $\frac{6 R}{5}$
(D) $\frac{5 R}{5}$

## Answer (A)

Sol. Given $h_{P}=\frac{R}{3}$
$h_{Q}=$ ?
gravitational acceleration at height

$$
g_{h t}=\frac{G M}{(R+h)^{2}}
$$

$\frac{g_{P}}{g_{Q}}=\frac{36}{25}=\frac{\frac{G M}{\left(R+h_{P}\right)^{2}}}{\frac{G M}{\left(R+h_{Q}\right)^{2}}}$
Put $h_{P}=\frac{R}{3}$ solving
$h_{Q}=\frac{3 R}{5}$

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## SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
$\begin{array}{lrrl}\text { Full Marks } & : & +4 & \text { If ONLY the correct integer is entered; } \\ \text { Zero Marks } & : & 0 & \end{array}$
Zero Marks : $0 \quad$ In all other cases.

8. A Hydrogen-like atom has atomic number $Z$. Photons emitted in the electronic transitions from level $n=4$ to level $n=3$ in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1.95 eV . If the photoelectric threshold wavelength for the target metal is 310 nm , the value of $Z$ is $\qquad$ .
[Given $h c=1240 \mathrm{eV}$-nm and $R h c=13.6 \mathrm{eV}$, where $R$ is the Rydberg constant, $h$ is the Planck's constant and $c$ is the speed of light in vacuum]
Answer (3)
Sol. $\Delta E_{4 \text { to } 3}=1.95 \mathrm{eV}+\frac{1240}{310} \mathrm{eV}$
$13.6 Z^{2}\left(\frac{1}{3^{2}}-\frac{1}{4^{2}}\right)=5.95$
$13.6 Z^{2} \frac{7}{9 \times 16}=5.95$
$z^{2}=\frac{5.95 \times 9 \times 16}{13.6}$
Solving $Z=3$
9. An optical arrangement consists of two concave mirrors $M_{1}$ and $M_{2}$, and a convex lens $L$ with a common principal axis, as shown in the figure. The focal length of $L$ is 10 cm . The radii of curvature of $M_{1}$ and $M_{2}$ are 20 cm and 24 cm , respectively. The distance between $L$ and $M_{2}$ is 20 cm . A point object $S$ is placed at the mid-point between $L$ and $M_{2}$ on the axis. When the distance between $L$ and $M_{1}$ is $\frac{n}{7} \mathrm{~cm}$, one of the images coincides with $S$. The value of $n$ is $\qquad$ .


Answer (80 or 150 or 220 )

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For reflection from $M_{2}$
$\frac{1}{v}+\frac{1}{(-10)}=\frac{1}{(-12)}$
$\frac{1}{v}=\frac{1}{10}-\frac{1}{12}$
$v=+60 \mathrm{~cm}\left(\right.$ for $l_{1}$ )
For refraction from $L$
$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$
$\frac{1}{v}-\frac{1}{(-80)}=\frac{1}{10}$
$v=+\frac{80}{7}\left(\right.$ For $\left.I_{2}\right)$
This image should be at focus of $M_{1}$
$\therefore \quad \frac{20}{2}+\frac{80}{7}=\frac{n}{7}$
$n=150$
Also,
If $I_{2}$ is formed at pole of $M_{1}$
then $\frac{n}{7}=\frac{80}{7}$
$n=80$

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And further if $I_{2}$ is formed at centre of curvature of $M_{1}$ then
$\frac{n}{7}=\frac{80}{7}+20$
$\therefore \quad n=220$
10. In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is $10 \pm 0.1 \mathrm{~cm}$ and the distance of its real image from the lens is $20 \pm 0.2 \mathrm{~cm}$. The error in the determination of focal length of the lens is $n \%$. The value of $n$ is $\qquad$ -.

## Answer (1)

Sol. Object distance $=10 \pm 0.1 \mathrm{~cm}$
Image distance $=20 \pm 0.2 \mathrm{~cm}$
Applying lens formula
$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$
$\Rightarrow \frac{1}{20}-\frac{1}{(-10)}=\frac{1}{f}$
$\Rightarrow \quad f=\frac{20}{3} \mathrm{~cm}$
Differentiate equation (i)
$-\frac{1}{v^{2}} d v+\frac{1}{u^{2}} d u=\frac{-1}{f^{2}} d f$
For calculating error
$\frac{1}{f^{2}} d f=+\frac{1}{v^{2}} d v+\frac{1}{u^{2}} d u$
$\left(\frac{d f}{f}\right) \times 100=\left(\frac{0.2}{20^{2}}+\frac{0.1}{10^{2}}\right) \frac{20}{3} \times 100$
$=\left(\frac{0.2}{4}+\frac{0.1}{1}\right) \frac{20}{3}=1$
$\therefore \quad \frac{d f}{f} \times 100=1 \%$
11. A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas ( $\gamma=5 / 3$ ) and one mole of an ideal diatomic gas $(\gamma=7 / 5)$. Here, $\gamma$ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is $\qquad$ Joule.

Answer (121)

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Sol. $\Delta u=n_{1} C_{1} \Delta T+n_{2} C_{2} \Delta T$

$$
\begin{equation*}
=\left(n_{1} C_{1}+n_{2} C_{2}\right) \Delta T \tag{i}
\end{equation*}
$$

Work done $=P \Delta v$

$$
\begin{equation*}
=\left(n_{1}+n_{2}\right) R \Delta T \tag{ii}
\end{equation*}
$$

Divide (i) by (ii)

$$
\begin{aligned}
& \frac{\Delta u}{W}=\frac{\left(n_{1} C_{1}+n_{2} C_{2}\right) \Delta T}{\left(n_{1}+n_{2}\right) R \Delta T} \\
& \Delta u=\frac{W}{R}\left(\frac{n_{1} C_{1}+n_{2} C_{2}}{n_{1}+n_{2}}\right) \\
& =\frac{66}{R} \frac{\left[\frac{3 R}{2} \times 2+\frac{5 R}{2} \times 1\right]}{2+1} \\
& \quad=121 \mathrm{~J}
\end{aligned}
$$

12. A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is $60 \mathrm{~cm} \mathrm{~s}^{-1}$, the speed of the tip of the person's shadow on the ground with respect to the person is $\qquad$ $\mathrm{cm} \mathrm{s}^{-1}$.

## Answer (40)

Sol. Given that $\frac{d x_{1}}{d t}=$ speed of person $=60 \mathrm{~cm} / \mathrm{s}$



Also $\frac{d x_{2}}{d t}=$ speed of tip of person's shadow
Applying similar triangle rule in $\triangle A B E$ \& $\triangle D C E$
$\frac{4}{x_{2}}=\frac{1.6}{x_{2}-x_{1}}$
$4 x_{2}-4 x_{1}=1.6 x_{2}$
$2.4 x_{2}=4 x_{1}$
Differentiate both sides w.r.t. $t$
$2.4 \frac{d x_{2}}{d t}=4 \frac{d x_{1}}{d t}$

$$
\begin{aligned}
\frac{d x_{2}}{d t} & =\frac{4}{2.4}(60) \\
& =100 \mathrm{~cm} / \mathrm{s} \\
\vec{v}_{S P} & =\vec{v}_{S G}-\vec{v}_{P G} \\
v_{S P} & =100 \mathrm{~cm} \mathrm{~s}^{-1}-60 \mathrm{~cm} \mathrm{~s}^{-1} \\
& =40 \mathrm{~cm} \mathrm{~s}^{-1}
\end{aligned}
$$

13. Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm . This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is $1.2 \times 10^{-8} \mathrm{~N} \mathrm{~m} \mathrm{rad}^{-1}$. The angular frequency of the oscillations in $n \times 10^{-3} \mathrm{rad} \mathrm{s}^{-1}$. The value of $n$ is
$\qquad$ _.


## Answer (10)

Sol. $m_{1}=30 \mathrm{gm}$


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- 13 -

Moment of inertia about the axis of rotation is
$I=m_{1} r_{1}^{2}+m_{2} r_{2}^{2}$
Clearly $r_{1}=4 \mathrm{~cm}$
And $r_{2}=6 \mathrm{~cm}$
$\therefore \quad I=\left(30 \times 10^{-3} \times 16 \times 10^{-4}\right)+\left(20 \times 10^{-3} \times 36 \times 10^{-4}\right)$
$\Rightarrow \quad I=1200 \times 10^{-7} \mathrm{~kg} \mathrm{~m}^{2}$
If the system is rotated by small angle ' $\theta$ ', the restoring torque is $\tau_{(R)}=-k \theta$
And $\frac{d^{2} \theta}{d t^{2}}=\frac{-k}{l} \cdot \theta=-\omega^{2} \theta=\frac{-1.2 \times 10^{-8}}{1200 \times 10^{-7}} \cdot \theta$
$\therefore \quad \omega^{2}=10^{-4}$
So, $\omega=\frac{1}{100} \mathrm{rad} / \mathrm{s}$
$\Rightarrow \omega=10 \times 10^{-3} \mathrm{rad} / \mathrm{s}$

## SECTION 4 (Maximum Marks : 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
14. List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

## List-I

(P) ${ }_{92}^{238} U \rightarrow{ }_{91}^{234} \mathrm{~Pa}$
(Q) ${ }_{82}^{214} \mathrm{~Pb} \rightarrow{ }_{82}^{210} \mathrm{~Pb}$
(R) ${ }_{81}^{210} \mathrm{TI} \rightarrow{ }_{82}^{206} \mathrm{~Pb}$
(S) ${ }_{91}^{228} \mathrm{~Pa} \rightarrow{ }_{88}^{224} \mathrm{Ra}$
(A) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 2, \mathrm{~S} \rightarrow 1$
(C) $\mathrm{P} \rightarrow 5, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 1, \mathrm{~S} \rightarrow 4$

Answer (A)


## List-II

(1) one $\alpha$ particle and one $\beta^{+}$particle
(2) three $\beta^{-}$particles and one $\alpha$ particle
(3) two $\beta^{-}$particles and one $\alpha$ particle
(4) one $\alpha$ particle and one $\beta^{-}$particle
(5) one $\alpha$ particle and two $\beta^{+}$particles
(B) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 1, \mathrm{R} \rightarrow 2, \mathrm{~S} \rightarrow 5$
(D) $\mathrm{P} \rightarrow 5, \mathrm{Q} \rightarrow 1, \mathrm{R} \rightarrow 3, \mathrm{~S} \rightarrow 2$

Sol. Option (A) is correct answer.

- In $\alpha$ decay mass number decreases by 4 unit and atomic number decreases by 2 unit.
- In $\beta^{-}$decay mass number does not change but atomic number increases by 1 unit.
- In $\beta^{+}$decay mass number does not change but atomic number decreases by 1 unit.

15. Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option.
[Given: Wien's constant as $2.9 \times 10^{-3} \mathrm{~m}-\mathrm{K}$ and $\frac{h c}{e}=1.24 \times 10^{-6} \mathrm{~V}-\mathrm{m}$ ]

## List-I

(P) 2000 K
(Q) 3000 K
(R) 5000 K
(S) 10000 K
(A) $\mathrm{P} \rightarrow 3, \mathrm{Q} \rightarrow 5, \mathrm{R} \rightarrow 2, \mathrm{~S} \rightarrow 3$
(B) $\mathrm{P} \rightarrow 3, \mathrm{Q} \rightarrow 2, \mathrm{R} \rightarrow 4, \mathrm{~S} \rightarrow 1$
(D) $P \rightarrow 1, Q \rightarrow 2, R \rightarrow 5, S \rightarrow 3$

## Answer (C)

Sol. (P) 2000 K

$$
\lambda_{m} T=b
$$

$\lambda_{m}=\frac{b}{T}=\frac{2.9 \times 10^{-3}}{2000}=1.45 \times 10^{-6} \mathrm{~m}=1450 \mathrm{~nm}$
(Q) 3000 K
(R) 5000 K
(S) 10000 K
$\lambda_{m} T=b \Rightarrow \lambda_{m}=\frac{2.9 \times 10^{-3}}{3000}=966.66 \mathrm{~nm}$
$\lambda_{m} T=b \Rightarrow \lambda_{m}=\frac{2.9 \times 10^{-3}}{5000}=580 \mathrm{~nm}$
$\lambda_{m} T=b \Rightarrow \lambda_{m}=\frac{2.9 \times 10^{-3}}{10,000}=290 \mathrm{~nm}$


- 15 -


## List-II

(1) $\lambda^{\text {th }}=\frac{h c}{\phi}=\frac{1.24 \times 10^{-6}}{4}=0.31 \times 10^{-6} \mathrm{~m}=310 \mathrm{~nm}$

$$
\text { As, } \lambda \leq \lambda_{\mathrm{th}} \quad S \rightarrow 1
$$

(2) $400<\lambda<700 \mathrm{~nm}$

$$
R \rightarrow 2
$$

(3) Central maxima is widest for maximum wavelength

$$
\Rightarrow \quad P \rightarrow 3
$$

(4) $\sigma A\left(T_{1}\right)^{4}=\frac{1}{16} \sigma A T_{2}^{4} \Rightarrow T_{1}=\frac{1}{2} T_{2}=3000 \mathrm{~K} \quad Q \rightarrow 4$
(5) For imaging bones $X$-rays are used ( $1-10 \mathrm{~nm}$ )

None of the options in List-II
16. A series LCR circuit is connected to a $45 \sin (\omega t)$ Volt source. The resonant angular frequency of the circuit is $10^{5} \mathrm{rad} \mathrm{s}^{-1}$ and current amplitude at resonance is $I_{0}$. When the angular frequency of the source is $\omega=8 \times 10^{4}$ $\mathrm{rad} \mathrm{s}^{-1}$, the current amplitude in the circuit is $0.05 I_{0}$. If $L=50 \mathrm{mH}$, match each entry in List-I with an appropriate value from List-II and choose the correct option.

| List-I | List-II |
| :--- | :--- |
| (P) $\quad I_{0}$ in mA | (1) 44.4 |
| (Q) $\quad$ The quality factor of the circuit | (2) 18 |
| (R) The bandwidth of the circuit in rad s ${ }^{-1}$ | (3) 400 |
| (S) $\quad$ The peak power dissipated at resonance in Watt | (4) 2250 |
|  | (5) 500 |

(A) $\mathrm{P} \rightarrow 2, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 5, \mathrm{~S} \rightarrow 1$
(B) $\mathrm{P} \rightarrow 3, \mathrm{Q} \rightarrow 1, \mathrm{R} \rightarrow 4, \mathrm{~S} \rightarrow 2$
(C) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 5, \mathrm{R} \rightarrow 3, \mathrm{~S} \rightarrow 1$
(D) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 2, \mathrm{R} \rightarrow 1, \mathrm{~S} \rightarrow 5$

## Answer (B)

Sol. As per the given information :
$\frac{1}{\sqrt{L C}}=10^{5}$
$I_{0}=\frac{45}{R}$
$0.05 I_{0}=\frac{45}{\sqrt{R^{2}+\left(0.8 X_{L_{0}}-\frac{5}{4} X_{C_{0}}\right)^{2}}}$


- 16 -

Where $X_{L_{0}}=X_{C_{0}}$ are at resonant frequencies
On solving, $R \simeq \frac{450 \Omega}{4} \Rightarrow I_{0} \simeq 400 \mathrm{~mA}$
Quality factor $Q=\frac{1}{R} \sqrt{\frac{L}{C}} \simeq 44.44$
$Q=\frac{\omega_{0}}{\Delta \omega} \Rightarrow \Delta \omega \simeq 2250 \mathrm{rad} / \mathrm{s}$
Peak power $=45 \times \frac{400}{1000} \mathrm{~W}$
$=18$
$\Rightarrow$ Correct match is option (B)
17. A thin conducting rod MN of mass 20 gm , length 25 cm and resistance $10 \Omega$ is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field $B_{0}=4 \mathrm{~T}$ directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time $t=0$ and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option.
[Given: The acceleration due to gravity $g=10 \mathrm{~m} \mathrm{~s}^{-2}$ and $\left.e^{-1}=0.4\right]$


| List-I | List-II |
| :--- | :--- |
| (P) At $t=0.2 \mathrm{~s}$, the magnitude of the induced emf in Volt | (1) 0.07 |
| (Q) At $t=0.2 \mathrm{~s}$, the magnitude of the magnetic force in Newton | (2) 0.14 |
| (R) At $t=0.2 \mathrm{~s}$, the power dissipated as heat in Watt | (3) 1.20 |
| (S) The magnitude of terminal velocity of the rod in $\mathrm{m} \mathrm{s}^{-1}$ | (4) 0.12 |
|  | (5) 2.00 |

(A) $\mathrm{P} \rightarrow 5, \mathrm{Q} \rightarrow 2, \mathrm{R} \rightarrow 3, \mathrm{~S} \rightarrow 1$
(B) $\mathrm{P} \rightarrow 3, \mathrm{Q} \rightarrow 1, \mathrm{R} \rightarrow 4, \mathrm{~S} \rightarrow 5$
(C) $\mathrm{P} \rightarrow 4, \mathrm{Q} \rightarrow 3, \mathrm{R} \rightarrow 1, \mathrm{~S} \rightarrow 2$
(D) $\mathrm{P} \rightarrow 3, \mathrm{Q} \rightarrow 4, \mathrm{R} \rightarrow 2, \mathrm{~S} \rightarrow 5$


2022


2021


Answer (D)

Sol. Induced emf $\varepsilon=B \ell v$
$\Rightarrow$ Induced current $i=\frac{\varepsilon}{R}=\frac{B \ell V}{R}$
$\Rightarrow m g-i \ell B=m a$
$\Rightarrow m g-\frac{B^{2} \ell^{2} v}{R}=m \frac{d v}{d t}$
$\Rightarrow \frac{d v}{m g-\frac{B^{2} \ell^{2} v}{R}}=\frac{d t}{m} \quad \Rightarrow \quad \frac{\ln \left[m g-\frac{B^{2} \ell^{2} v}{R}\right]_{0}^{v}}{\frac{-B^{2} \ell^{2}}{R}}=\frac{t}{m}$
$\Rightarrow \frac{m g-\frac{B^{2} l^{2} v}{R}}{m g}=e^{\frac{-B^{2} l^{2}}{m R} t}$
$\Rightarrow \quad v=2\left[1-e^{-5 t}\right]$
$\Rightarrow$ At $t=0.2 \mathrm{~s}, v=2\left[1-\frac{1}{e}\right]$
$\Rightarrow \quad \varepsilon=B \ell \times 2\left[1-\frac{1}{e}\right]=1.2$ volts
and magnetic force $=i \ell B=0.12 \mathrm{~N}$
and power dissipated $=0.144 \mathrm{~W}$
also, Terminal velocity $=2 \mathrm{~m} / \mathrm{s}$
$\Rightarrow$ Correct match is (D)

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## PART-II : CHIDMISTRY

SECTION 1 (Maximum Marks : 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| Full Marks | $:$ | +4 | ONLY if (all) the correct option(s) is(are) chosen; |
| :--- | :--- | :--- | :--- |
| Partial Marks | $:$ | +3 If all the four options are correct but ONLY three options are chosen; |  |
| Partial Marks | $:$ | +2 If three or more options are correct but ONLY two options are chosen, both of which |  |
| are correct; |  |  |  | |  |  |  |  |
| :--- | :--- | :--- | :--- |
| Partial Marks | $:$ | +1 | If two or more options are correct but ONLY one option is chosen and it is a correct |
| option; |  |  |  |

1. The correct statement(s) related to processes involved in the extraction of metals is(are)
(A) Roasting of Malachite produces Cuprite
(B) Calcination of Calamine produces Zincite
(C) Copper pyrites is heated with silica in a reverberatory furnace to remove iron
(D) Impure silver is treated with aqueous KCN in the presence of oxygen followed by reduction with zinc metal

Answer (B, C, D)
Sol. $\mathrm{Cu}(\mathrm{OH})_{2} \cdot \mathrm{CuCO}_{3} \longrightarrow \mathrm{CuO}+\mathrm{H}_{2} \mathrm{O}+\mathrm{CO}_{2}$

$\mathrm{CuFeS}_{2}+\mathrm{O}_{2} \longrightarrow \mathrm{Cu}_{2} \mathrm{~S}+\mathrm{FeS}+\mathrm{SO}_{2}$
$\mathrm{FeS}+\mathrm{O}_{2} \longrightarrow \mathrm{FeO}+\mathrm{SO}_{2}$

$4 \mathrm{Ag}+8 \mathrm{CN}^{-}+2 \mathrm{H}_{2} \mathrm{O}+\mathrm{O}_{2} \longrightarrow 4\left[\mathrm{Ag}(\mathrm{CN})_{2}\right]^{-}+4 \mathrm{OH}^{-}$
$2\left[\mathrm{Ag}(\mathrm{CN})_{2}\right]^{-}+\mathrm{Zn}(\mathrm{s}) \longrightarrow\left[\mathrm{Zn}(\mathrm{CN})_{4}\right]^{2-}+2 \mathrm{Ag}$

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 Champions in JEE Advanced

2022


2. In the following reactions, $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ are the major products.



(i) PhMgBr , then $\mathrm{H}_{2} \mathrm{O}$
$\mathrm{PhCH}_{2} \mathrm{CHO}$
(ii) $\mathrm{CrO}_{3}$, dil. $\mathrm{H}_{2} \mathrm{SO}_{4}$

The correct statement(s) about $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ is(are)
(A) Both $\mathbf{P}$ and $\mathbf{Q}$ have asymmetric carbon(s)
(B) Both $\mathbf{Q}$ and $\mathbf{R}$ have asymmetric carbon(s)
(C) Both $\mathbf{P}$ and $\mathbf{R}$ have asymmetric carbon(s)
(D) $\mathbf{P}$ has asymmetric carbon(s), $\mathbf{S}$ does not have any asymmetric carbon

Answer (C, D)

Sol.

(P)

(Q)

(R)

(S)


2022


3. Consider the following reaction scheme and choose the correct option(s) for the major products $\mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$.

$$
\text { Styrene } \xrightarrow[\text { (ii) } \mathrm{NaOH}, \mathrm{H}_{2} \mathrm{O}_{2}, \mathrm{H}_{2} \mathrm{O}]{\text { (i) } \mathrm{B}_{2} \mathrm{H}_{6}} \quad \text { P } \xrightarrow[\begin{array}{l}
\text { (ii) } \mathrm{Cl}_{2}, \text { Red phosphorus } \\
\text { (iii) } \mathrm{H}_{2} \mathrm{O}
\end{array}]{\text { (i) } \mathrm{CrO}_{3}, \mathrm{H}_{2} \mathrm{SO}_{4}} \text { Q }
$$


(iii) $\mathrm{H}_{3} \mathrm{O}^{+}, \Delta$
(A)

Q

R

S
(C)

Q
(B)
Q


R

S


R

S
(D)

Q

R

S

## Answer (B)

Sol.


(Q)


2022


SECTION 2 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
4. In the scheme given below, $\mathbf{X}$ and $\mathbf{Y}$, respectively, are

(A) $\mathrm{CrO}_{4}^{2-}$ and $\mathrm{Br}_{2}$
(B) $\mathrm{MnO}_{4}^{2-}$ and $\mathrm{Cl}_{2}$
(C) $\mathrm{MnO}_{4}^{-}$and $\mathrm{Cl}_{2}$
(D) $\mathrm{MnSO}_{4}$ and HOCl


2022



- 22 -

Answer (C)
Sol. $\mathrm{MnCl}_{2} \xrightarrow{\text { aq } \mathrm{NaOH}} \mathrm{Mn}(\mathrm{OH})_{2}+\mathrm{NaCl}$
(P) $\quad(Q)$
$\underset{\mathrm{P}}{\mathrm{Mn}(\mathrm{OH})_{2}} \xrightarrow[\substack{\mathrm{H}_{2} \mathrm{SO}_{4}}]{\mathrm{PbO}_{(X)}} \mathrm{MnO}_{4}^{-}$
$\mathrm{NaCl} \xrightarrow[\substack{\text { Conc. } \mathrm{H}_{2} \mathrm{SO}_{4} \\ \Delta}]{\mathrm{MnO}(\mathrm{OH})_{2}} \underset{\text { (Y) }}{\mathrm{Cl}_{2}} \xrightarrow[\text { starch }]{\mathrm{KI}}$ blue
5. Plotting $1 / \Lambda_{\mathrm{m}}$ against $\mathrm{c} \Lambda_{\mathrm{m}}$ for aqueous solutions of a monobasic weak acid $(H X)$ resulted in a straight line with $y$-axis intercept of $P$ and slope of $S$. The ratio $P / S$ is
[ $\Lambda_{m}=$ molar conductivity
$\Lambda_{\mathrm{m}}^{\circ}=$ limiting molar conductivity
$\mathrm{c}=$ molar concentration
$\mathrm{K}_{\mathrm{a}}=$ dissociation constant of HX ]
(A) $\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}$
(C) $2 \mathrm{~K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}$
(B) $\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ} / 2$
(D) $1 /\left(\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}\right)$

## Answer (A)

Sol. $\alpha=\frac{\Lambda_{\mathrm{m}}}{\Lambda_{\mathrm{m}}^{\circ}}$
$\mathrm{K}_{\mathrm{a}}=\frac{\mathrm{c} \alpha^{2}}{1-\alpha}$
$\mathrm{K}_{\mathrm{a}}=\frac{\mathrm{c}\left(\Lambda_{\mathrm{m}} / \Lambda_{\mathrm{m}}^{\circ}\right)^{2}}{1-\left(\Lambda_{\mathrm{m}} / \Lambda_{\mathrm{m}}^{\circ}\right)}$
$\mathrm{K}_{\mathrm{a}}=\frac{\mathrm{c} \Lambda_{\mathrm{m}}^{2}}{\Lambda_{\mathrm{m}}^{\circ}\left(\Lambda_{\mathrm{m}}^{\circ}-\Lambda_{\mathrm{m}}\right)}$
$\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}{ }^{2} \quad-\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ} \Lambda_{\mathrm{m}}=\mathrm{c} \Lambda_{\mathrm{m}}^{2}$

## Aakashians Shine as



2022


2021


$$
\frac{\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}{ }^{2}}{\Lambda_{\mathrm{m}}}-\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}=\mathrm{c} \Lambda_{\mathrm{m}}
$$

$$
\frac{\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}}{\Lambda_{\mathrm{m}}}=\mathrm{c} \Lambda_{\mathrm{m}}+\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}
$$

$$
\frac{1}{\Lambda_{\mathrm{m}}}=\left(\frac{\mathrm{c} \Lambda_{\mathrm{m}}}{\mathrm{~K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ 2}}\right)+\frac{1}{\Lambda_{\mathrm{m}}^{\circ}}
$$

$$
\mathrm{P}=\frac{1}{\Lambda_{\mathrm{m}}^{\circ}}
$$

$$
\mathrm{S}=\frac{1}{\mathrm{~K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}{ }^{2}}
$$

$$
\frac{\mathrm{P}}{\mathrm{~S}}=\left(\frac{\frac{1}{\Lambda_{\mathrm{m}}^{\circ}}}{\frac{1}{\mathrm{~K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ 2}}}\right)=\mathrm{K}_{\mathrm{a}} \Lambda_{\mathrm{m}}^{\circ}
$$

6. On decreasing the pH from 7 to 2 , the solubility of a sparingly soluble salt $(\mathrm{MX})$ of a weak acid $(H X)$ increased from $10^{-4} \mathrm{~mol} \mathrm{~L}^{-1}$ to $10^{-3} \mathrm{~mol} \mathrm{~L}^{-1}$. The $\mathrm{pKa}_{\mathrm{a}}$ of HX is
(A) 3
(B) 4
(C) 5
(D) 2

## Answer (B)

Sol. $\mathrm{MX} \rightleftharpoons \mathrm{M}^{\oplus}+\mathrm{X}^{\ominus}$

$$
\begin{align*}
& X^{\ominus}+H^{\oplus} \rightleftharpoons H X \\
& S=\sqrt{k_{s p}\left(1+\frac{H^{\oplus}}{\mathrm{k}_{\mathrm{a}}}\right)} \\
& 10^{-4}=\sqrt{\mathrm{k}_{\mathrm{sp}}\left(1+\frac{10^{-7}}{\mathrm{k}_{\mathrm{a}}}\right)}  \tag{1}\\
& 10^{-3}=\sqrt{\mathrm{k}_{\mathrm{sp}}\left(1+\frac{10^{-2}}{\mathrm{k}_{\mathrm{a}}}\right)} \tag{2}
\end{align*}
$$

## Aakashians Shine as



2022


2021


Equation (1)/(2) gives
$10^{-2}=\frac{\left(1+\frac{10^{-7}}{\mathrm{k}_{\mathrm{a}}}\right)}{\left(1+\frac{10^{-2}}{\mathrm{k}_{\mathrm{a}}}\right)}$
$10^{-2}+\frac{10^{-4}}{k_{a}}=1+\frac{10^{-7}}{k_{a}}$
$\frac{10^{-4}-10^{-7}}{\mathrm{k}_{\mathrm{a}}}=0.99$
$\frac{10^{-4}}{\mathrm{k}_{\mathrm{a}}}=0.99$
$\mathrm{k}_{\mathrm{a}}=\frac{10^{-4}}{0.99}=\frac{1}{99} \times 10^{-2}$
$\mathrm{pK}_{\mathrm{a}}=2+\log 99 \simeq 4$
7. In the given reaction scheme, $\mathbf{P}$ is a phenyl alkyl ether, $\mathbf{Q}$ is an aromatic compound; $\mathbf{R}$ and $\mathbf{S}$ are the major products.
(i) NaOH
$\mathbf{P} \xrightarrow{\mathrm{HI}} \mathbf{Q} \xrightarrow[\text { (iii) } \mathrm{H}_{3} \mathrm{O}^{+}]{\text {(ii) } \mathrm{CO}_{2}} \mathbf{R} \xrightarrow[\text { (ii) } \mathrm{H}_{3} \mathrm{O}^{+}]{\text {(i) }\left(\mathrm{CH}_{3} \mathrm{CO}\right)_{2} \mathrm{O}} \mathbf{s}$

The correct statement about $\mathbf{S}$ is
(A) It primarily inhibits noradrenaline degrading enzymes
(B) It inhibits the synthesis of prostaglandin
(C) It is a narcotic drug
(D) It is ortho-acetylbenzoic acid

Answer (B)

Sol.


(iii) $\mathrm{H}_{3} \mathrm{O}^{+}$


(S)
$S$ is ortho-acetoxybenzoic acid, it inhibits the synthesis of prostaglandin.


## SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : $0 \quad$ In all other cases.
8. The stoichiometric reaction of 516 g of dimethyldichlorosilane with water results in a tetrameric cyclic product $\mathbf{X}$ in $75 \%$ yield. The weight (in g ) of $\mathbf{X}$ obtained is $\qquad$ _.
[Use, molar mass $\left(\mathrm{g} \mathrm{mol}^{-1}\right): \mathrm{H}=1, \mathrm{C}=12, \mathrm{O}=16, \mathrm{Si}=28, \mathrm{Cl}=35.5$ ]

## Answer (222)

Sol. Dimethyldichlorosilane -

mol. mass - $129 \mathrm{~g} / \mathrm{mol}$
Number of moles of dimethyldichlorosilane initially taken $=\frac{516}{129}=4$ moles


Applying POAC on Si atom
Moles of tetrameric cyclic product formed $=\frac{4}{4} \times \frac{75}{100}=0.75$ moles
Molar mass of product formed $=296 \mathrm{~g} /$ moles
The mass of product formed $=296 \times 0.75=222 \mathrm{~g}$

## Aakashians Shine as

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2022


2021


- 26 -

9. A gas has a compressibility factor of 0.5 and a molar volume of $0.4 \mathrm{dm}^{3} \mathrm{~mol}^{-1}$ at a temperature of 800 K and pressure $\mathbf{x}$ atm. If it shows ideal gas behaviour at the same temperature and pressure, the molar volume will be $y \mathrm{dm}^{3} \mathrm{~mol}^{-1}$. The value of $\mathrm{x} / \mathrm{y}$ is $\qquad$ .
[Use: Gas constant, $\mathrm{R}=8 \times 10^{-2} \mathrm{~L} \mathrm{~atm} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$ ]

## Answer (100)

Sol. Compressibility factor $(Z)=\frac{V_{\text {real }}}{V_{\text {ideal }}}=0.5$
$V_{\text {real }}=0.4 \mathrm{dm}^{3} \mathrm{~mol}^{-1}=0.4 \mathrm{~L} / \mathrm{mol}$
$\therefore \quad V_{\text {ideal }}=\frac{0.4}{0.5}=0.8 \mathrm{~L} / \mathrm{mol}$
$\therefore \mathrm{y}=0.8 \mathrm{~L} / \mathrm{mol}$
Using ideal gas equation : $\mathrm{PV}=\mathrm{nRT}$
$P=\frac{1 \times 8 \times 10^{-2} \times 800}{0.8}$
$x=80 \mathrm{~atm}$
$\therefore \quad \frac{x}{y}=\frac{80}{0.8}=100$
10. The plot of $\log \mathrm{k}_{\mathrm{f}}$ versus $\frac{1}{\mathrm{~T}}$ for a reversible reaction $\mathrm{A}(\mathrm{g}) \rightleftharpoons \mathrm{P}(\mathrm{g})$ is shown.



2022



Pre-exponential factors for the forward and backward reactions are $10^{15} \mathrm{~s}^{-1}$ and $10^{11} \mathrm{~s}^{-1}$, respectively. If the value of $\log \mathrm{K}$ for the reaction at 500 K is 6 , the value of $\left|\log _{b}\right|$ at 250 K is $\qquad$ .
[ $K=$ equilibrium constant of the reaction, $k_{f}=$ rate constant of forward reaction, $k_{b}=$ rate constant of backward reaction]

## Answer (5)

Sol. From the question
$A_{f}=10^{15}, A_{b}=10^{11}$,
$\log K$ at $500 K=6$
$\mathrm{f}=$ Forward reaction
b = Backward reaction
$\log \mathrm{k}_{\mathrm{f}}$ at $500 \mathrm{~K}=9$ (from graph)
$\log _{\mathrm{k}}$ at 500 K :

$$
\log K=\log \left(\frac{k_{f}}{k_{b}}\right) \quad \text { since } \Rightarrow K=\frac{k_{f}}{k_{b}}
$$

$6=\log k_{f}-\log k_{b}$
$6=9-\log \mathrm{k}_{\mathrm{b}}$
$\log \mathrm{k}_{\mathrm{b}}=3$ at 500 K
$\log \frac{k_{2}}{k_{1}}=\frac{-E_{a}}{R}\left(\frac{1}{T_{2}}-\frac{1}{T_{1}}\right)$
$k_{b}=A_{b} e^{\frac{-E_{a b}}{R T}}$
$\operatorname{lnk}_{b}=\ln \left(A_{b} e^{\frac{-E_{a b}}{R T}}\right)$
$\operatorname{lnk}_{b}=\ln A_{b}-\frac{E_{a b}}{R T}$
$2.303 \log \mathrm{k}_{\mathrm{b}}=2.303 \log \mathrm{~A}_{\mathrm{b}}-\frac{\mathrm{E}_{a b}}{500 \mathrm{R}}$
$\frac{E_{a}}{500 R}=2.303\left(\log A_{b}-\log k_{b}\right)$
$\frac{E_{a}}{500 R}=2.303\left(\log 10^{11}-3\right)$
$\frac{E_{a}}{500 R}=2.303(11-3)=2.303 \times 8$


2022


2021

$\mathrm{E}_{\mathrm{a}}=2.303 \times 8 \times 500 \mathrm{R}$
$\ln \left(\frac{\mathrm{k}_{2}}{\mathrm{k}_{1}}\right)=\frac{-\mathrm{E}_{\mathrm{a}}}{\mathrm{R}}\left(\frac{1}{\mathrm{~T}_{2}}-\frac{1}{\mathrm{~T}_{1}}\right)$
$\ln \left(\frac{\mathrm{k}_{250 \mathrm{~K}}}{\mathrm{k}_{500 \mathrm{~K}}}\right)=\frac{-\mathrm{E}_{\mathrm{a}}}{\mathrm{R}}\left(\frac{1}{250}-\frac{1}{500}\right)$
$\ln \left(\frac{\mathrm{k}_{250 \mathrm{~K}}}{\mathrm{k}_{500 \mathrm{~K}}}\right)=\frac{-2.303 \times 8 \times 500 \mathrm{R}}{\mathrm{R}}\left(\frac{1}{500}\right)$
$2.303\left(\log \mathrm{k}_{250} \mathrm{k}-\log \mathrm{k}_{500} \mathrm{k}\right)=-2.303 \times 8$
$\log \mathrm{k}_{250} \mathrm{k}-3=-8$
$\log \mathrm{k}_{250} \mathrm{~K}=-5$
| $\log \mathrm{k}_{250} \mathrm{k}$ | = 5
11. One mole of an ideal monoatomic gas undergoes two reversible processes $(A \rightarrow B$ and $B \rightarrow C$ ) as shown in the given figure:

$A \rightarrow B$ is an adiabatic process. If the total heat absorbed in the entire process ( $A \rightarrow B$ and $B \rightarrow C$ ) is $R T_{2} \ln 10$, the value of $2 \log _{3}$ is $\qquad$ .
[Use, molar heat capacity of the gas at constant pressure, $C_{p, m}=\frac{5}{2} R$ ]

## Answer (7)

Sol. $q_{A \rightarrow C}=R T_{2} \ln 10$
$q_{A \rightarrow B}=0$
( $\because$ adiabatic)
$q_{A \rightarrow C}=q_{A \rightarrow B}+q_{B \rightarrow C}$

## Aakashians Shine as



2022


2021

$\mathrm{q}_{\mathrm{A} \rightarrow \mathrm{C}}=\mathrm{q}_{\mathrm{B} \rightarrow \mathrm{C}}$
$\mathrm{q}_{\mathrm{A} \rightarrow \mathrm{c}}=\mathrm{nR} \mathrm{T}_{2} \ln \left(\frac{\mathrm{~V}_{3}}{\mathrm{~V}_{2}}\right)$
For $\mathrm{B} \rightarrow \mathrm{C}$
$\Delta E=q+w$
$\Delta \mathrm{E}=0$
$q=-w$

$$
=-\left(-n R T_{2} \ln \frac{V_{3}}{V_{2}}\right)
$$

$$
=n R T_{2} \ln \left(\frac{V_{3}}{V_{2}}\right)
$$

$\underset{B \rightarrow C}{q}=n R T_{2} \ln \left(\frac{V_{3}}{V_{2}}\right)$
$\mathrm{q}_{\mathrm{B} \rightarrow \mathrm{C}}=R \mathrm{~T}_{2} \ln \left(\frac{\mathrm{~V}_{3}}{\mathrm{~V}_{2}}\right)$
From $A \rightarrow B$
$\mathrm{T}_{1} \mathrm{~V}_{1}^{\gamma-1}=\mathrm{T}_{2} \mathrm{~V}_{2}^{\gamma-1}$
$600 \mathrm{~V}_{1}^{\gamma-1}=60 \mathrm{~V}_{2}^{\gamma-1}$
$10 \times 10^{\frac{5}{3}-1}=\mathrm{V}_{2}^{\gamma-1}$
$10^{5 / 3}=V_{2}^{\frac{5}{3}-1}$
$10^{5 / 3}=V_{2}^{2 / 3}$
$V_{2}=10^{\frac{5}{3} \times \frac{3}{2}}=10^{\frac{5}{2}}$
$V_{2}=10^{\frac{5}{2}}$
From equation (1)
$\mathrm{q}_{\mathrm{A} \rightarrow \mathrm{C}}=\mathrm{nR} T_{2} \ln \left(\frac{\mathrm{~V}_{3}}{\mathrm{~V}_{2}}\right)$
Given, $\mathrm{q}_{\mathrm{A} \rightarrow \mathrm{C}}=\mathrm{RT}_{2} \ln 10$
(since isothermic)

## [Since $\mathrm{n}=1$ ]

$\mathrm{RT}_{2} \ln 10=\mathrm{RT}_{2} \ln \left(\frac{\mathrm{~V}_{3}}{\mathrm{~V}_{2}}\right)$
$\ln 10=\ln \left(\frac{V_{3}}{V_{2}}\right)$
$\ln 10=\ln \left(\frac{V_{3}}{10^{\frac{5}{2}}}\right)$
$10=\frac{V_{3}}{10^{\frac{5}{2}}}$
$V_{3}=10^{1+\frac{5}{2}}=10^{\frac{7}{2}}$
$2 \log V_{3}=2 \log 10^{7 / 2}$
$=7$
12. In a one-litre flask, 6 moles of $A$ undergoes the reaction $A(g) \rightleftharpoons P(g)$. The progress of product formation at two temperatures (in Kelvin), $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$, is shown in the figure:


If $T_{1}=2 T_{2}$ and $\left(\Delta G_{2}^{\ominus}-\Delta G_{1}^{\ominus}\right)=R T_{2} \ln x$, then the value of $x$ is $\qquad$ .
[ $\Delta G_{1}^{\ominus}$ and $\Delta G_{2}^{\ominus}$ are standard Gibb's free energy change for the reaction at temperatures $T_{1}$ and $T_{2}$, respectively.]
Answer (8)
Sol.

$$
\mathrm{A}(\mathrm{~g}) \rightleftharpoons \mathrm{P}(\mathrm{~g})
$$

| Initially | 6 | 0 |
| :--- | :--- | :--- |
| At equilibrium | $6-a$ | $a$ |

at $\mathrm{T}_{1} \quad \mathrm{a}=4$
$\therefore\left(\mathrm{K}_{\text {eq }}\right)_{1}=\frac{4}{2}=2$

$$
\text { at } \mathbf{T}_{2} \quad a=2
$$

$$
\therefore\left(\mathrm{K}_{\text {eq }}\right)_{2}=\frac{2}{4}=\frac{1}{2}
$$

## Aakashians Shine as



2022


2021


+ BByuus
$\Delta G_{1}^{\ominus}=-R T_{1} \ln \left(K_{\text {eq }}\right)_{1}$
$\Delta G_{1}^{\ominus}=-2 R T_{2} \ln \left(K_{\text {eq }}\right)_{1}$
[Given: $\mathrm{T}_{1}=2 \mathrm{~T}_{2}$ ]
$\Delta G_{2}^{\ominus}=-R T_{2} \ln \left(K_{\text {eq }}\right)_{2}$
$\therefore \Delta G_{2}^{\ominus}-\Delta G_{1}^{\ominus}=\mathrm{RT}_{2} \ln \frac{\left(\mathrm{~K}_{e \mathrm{eq}_{1}}\right)^{2}}{\left(\mathrm{~K}_{\mathrm{eq}_{2}}\right)}$

$$
=R T_{2} \ln \frac{2^{2}}{\frac{1}{2}}=R T_{2} \ln 8
$$

$\therefore \Delta \mathrm{G}^{\ominus}=\mathrm{RT} \ln \mathrm{x}$ has $\mathrm{x}=8$
13. The total number of $s p^{2}$ hybridised carbon atoms in the major product $\mathbf{P}$ (a non-heterocyclic compound) of the following reaction is $\qquad$ -.


## Answer (28)

Sol.


All marked C-atoms are $s p^{2}$ hybridised.

## Aakashians Shine as

 Champions in JEE Advanced

2022


2021


## SECTION 4 (Maximum Marks : 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

| Full Marks | $:$ | +3 | ONLY if the option corresponding to the correct combination is chosen; |
| :--- | :--- | :--- | :--- |
| Zero Marks | $:$ | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| Negative Marks | $:$ | -1 | In all other cases. |

14. Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in ListII and choose the correct option.

|  | List-I |  | List-II |
| :--- | :--- | ---: | :--- |
| $(\mathrm{P})$ | $\mathrm{P}_{2} \mathrm{O}_{3}+3 \mathrm{H}_{2} \mathrm{O} \rightarrow$ | $(1)$ | $\mathrm{P}(\mathrm{O})\left(\mathrm{OCH}_{3}\right) \mathrm{Cl}_{2}$ |
| $(\mathrm{Q})$ | $\mathrm{P}_{4}+3 \mathrm{NaOH}+3 \mathrm{H}_{2} \mathrm{O} \rightarrow$ | $(2)$ | $\mathrm{H}_{3} \mathrm{PO}_{3}$ |
| $(\mathrm{R})$ | $\mathrm{PCl}_{5}+\mathrm{CH}_{3} \mathrm{COOH} \rightarrow$ | $(3)$ | $\mathrm{PH}_{3}$ |
| $(\mathrm{~S})$ | $\mathrm{H}_{3} \mathrm{PO}_{2}+2 \mathrm{H}_{2} \mathrm{O}+4 \mathrm{AgNO}_{3} \rightarrow$ | $(4)$ | $\mathrm{POCl}_{3}$ |
|  |  | $(5)$ | $\mathrm{H}_{3} \mathrm{PO}_{4}$ |

(A) $\mathrm{P} \rightarrow 2 ; \mathrm{Q} \rightarrow 3 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 5$
(B) $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 5 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 2$
(C) $\mathrm{P} \rightarrow 5 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 3$
(D) $\mathrm{P} \rightarrow 2 ; \mathrm{Q} \rightarrow 3 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 5$

Answer (D)
Sol. $\mathrm{P}_{2} \mathrm{O}_{3}+\mathrm{H}_{2} \mathrm{O} \rightarrow \mathrm{H}_{3} \mathrm{PO}_{3}$
$\mathrm{P}_{4}+\mathrm{NaOH}+\mathrm{H}_{2} \mathrm{O} \rightarrow \mathrm{PH}_{3}+\mathrm{NaH}_{2} \mathrm{PO}_{2}$
$\mathrm{PCl}_{5}+\mathrm{CH}_{3} \mathrm{COOH} \rightarrow \mathrm{CH}_{3} \mathrm{C}-\mathrm{Cl}+\mathrm{POCl}_{3}+\mathrm{HCl}$
$\mathrm{H}_{3} \mathrm{PO}_{2}+\mathrm{H}_{2} \mathrm{O}+\mathrm{AgNO}_{3} \rightarrow \mathrm{H}_{3} \mathrm{PO}_{4}+\mathrm{HNO}_{3}+\mathrm{Ag} \downarrow$
Hence :-
$P \rightarrow 2$
$Q \rightarrow 3$
$\mathrm{R} \rightarrow 4$
$S \rightarrow 5$

15. Match the electronic configurations in List-I with appropriate metal complex ions in List-II and choose the correct option.
[Atomic Number: $\mathrm{Fe}=26, \mathrm{Mn}=25, \mathrm{Co}=27$ ]

|  | List-I |  | List-II |
| :--- | :--- | :--- | :--- |
| $(P)$ | $\mathrm{t}_{29}^{6} \mathrm{e}_{9}^{0}$ | $(1)$ | $\left[\mathrm{Fe}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$ |
| (Q) | $\mathrm{t}_{29}^{3} \mathrm{e}_{9}^{2}$ | (2) | $\left[\mathrm{Mn}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$ |
| (R) | $\mathrm{e}_{2}^{2} \mathrm{t}_{2}^{3}$ | (3) | $\left[\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6}\right]^{3+}$ |
| $(\mathrm{S})$ | $\mathrm{t}_{29}^{4} \mathrm{e}_{9}^{2}$ | (4) | $\left[\mathrm{FeCl}_{4}\right]^{-}$ |
|  |  | (5) | $\left[\mathrm{CoCl}_{4}\right]^{2-}$ |

(A) $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 4 ; \mathrm{R} \rightarrow 2 ; \mathrm{S} \rightarrow 3$
(B) $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 5$
(C) $\mathrm{P} \rightarrow 3$; $\mathrm{Q} \rightarrow 2$; $\mathrm{R} \rightarrow 5$; $\mathrm{S} \rightarrow 1$
(D) $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 1$

## Answer (D)

Sol. (1) $\left[\mathrm{Fe}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+} \rightarrow 3 d^{6}$
$\mathrm{H}_{2} \mathrm{O} \rightarrow$ weak ligand

(2) $\left[\mathrm{Mn}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+} \rightarrow 3 d^{5}$
$\mathrm{H}_{2} \mathrm{O} \rightarrow$ weak ligand



2022
(3) $\left[\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6}\right]^{3+} \rightarrow 3 d^{6}$
$\mathrm{NH}_{3} \rightarrow$ strong ligand

(4) $\left[\mathrm{FeCl}_{4}\right]^{-} \rightarrow \mathrm{Fe}^{+3} \rightarrow 3 d^{5}$
$\mathrm{Cl}^{-} \rightarrow$ weak ligand

(5) $\left[\mathrm{CoCl}_{4}\right]^{2-} \rightarrow \mathrm{Co}^{+2} \rightarrow 3 d^{7}$
$\mathrm{Cl}^{-} \rightarrow$ weak ligand

$\therefore \mathrm{P} \rightarrow 3, \mathrm{Q} \rightarrow 2, \mathrm{R} \rightarrow 4, \mathrm{~S} \rightarrow 1$
16. Match the reactions in List-I with the features of their products in List-II and choose the correct option.

|  | List-I |  | List-II |
| :---: | :---: | :---: | :---: |
| (P) | $\text { (-)-1-Bromo-2-ethylpentane } \underset{\text { (single enantiomer) }}{\substack{\text { aq. NaOH }}}$ | (1) | Inversion of configuration |
| (Q) | $(-)-2-\underset{\text { (single enantiomer) }}{\text { Bromopentane }} \xrightarrow[s_{\mathrm{N}} \text { reaction }]{\text { aq. NaoH }}$ | (2) | Retention of configuration |
| (R) | $(-)-3$-Bromo-3-methylhexane $\xrightarrow[\text { (single enantiomer) }]{\substack{\text { aq. } \mathrm{NaOH}}}$ | (3) | Mixture of enantiomers |
| (S) | $\overbrace{\substack{\mathrm{Me} \\ \text { (single enantiomer) }}}^{\text {Men }}$ | (4) | Mixture of structural isomers |
|  |  | (5) | Mixture of diastereomers |

(A) $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 5$; $\mathrm{S} \rightarrow 3$
(B) $\mathrm{P} \rightarrow 2 ; \mathrm{Q} \rightarrow 1 ; \mathrm{R} \rightarrow 3 ; \mathrm{S} \rightarrow 5$
(C) $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 5$; $\mathrm{S} \rightarrow 4$
(D) $\mathrm{P} \rightarrow 2 ; \mathrm{Q} \rightarrow 4 ; \mathrm{R} \rightarrow 3 ; \mathrm{S} \rightarrow 5$

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## Answer (B)

Sol. (P) Configuration at chiral carbon is same.
$\mathrm{P} \rightarrow 2$ [reaction does not occur at chiral carbon]
(Q) Configuration at chiral carbon changes.
$Q \rightarrow 1$
$(R) S_{N} 1 \rightarrow$ Mixture of enantiomers formed.

$$
\mathrm{R} \rightarrow 3
$$

(S)

$\therefore$ So mixture of diastereomers are formed.
$S \rightarrow 5$
17. The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option.

|  | List-I |  | List-II |
| :---: | :---: | :---: | :---: |
| (P) | Etard reaction | (1) | $\text { Acetophenone } \xrightarrow{\mathrm{Zn}-\mathrm{Hg}, \mathrm{HCl}}$ |
| (Q) | Gattermann reaction | (2) | Toluene $\xrightarrow[\text { (i) } \mathrm{SOCl}_{2}]{\text { (i) } \mathrm{KMnO}_{4}, \mathrm{KOH}, \Delta}$ |
| (R) | Gattermann-Koch reaction | (3) | $\text { Benzene } \xrightarrow[\text { anhyd. } \mathrm{AlCl}_{3}]{\mathrm{CH}_{3} \mathrm{Cl}}$ |
| (S) | Rosenmund reduction | (4) | Aniline $\xrightarrow[273-278 \mathrm{~K}]{\mathrm{NaNO}_{2} / \mathrm{HCl}}$ |
|  |  | (5) | Phenol $\xrightarrow{\mathrm{Zn}, \Delta}$ |

(A) $\mathrm{P} \rightarrow 2 ; \mathrm{Q} \rightarrow 4 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 3$
(B) $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 3 ; \mathrm{R} \rightarrow 5 ; \mathrm{S} \rightarrow 2$
(C) $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 4$
(D) $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 4 ; \mathrm{R} \rightarrow 5 ; \mathrm{S} \rightarrow 2$


2022



Answer (D)
Sol. (P) $\rightarrow$ (3)


(Q) $\rightarrow$ (4)


$(\mathrm{R}) \rightarrow(5)$

(S) $\rightarrow$ (2)



2022

## PART-III : MATHDMATICS

## SECTION 1 (Maximum Marks : 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : $0 \quad$ If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -2 In all other cases.

1. Let $S=(0,1) \cup(1,2) \cup(3,4)$ and $T=\{0,1,2,3\}$. Then which of the following statements is(are) true?
(A) There are infinitely many functions from $S$ to $T$
(B) There are infinitely many strictly increasing functions from $S$ to $T$
(C) The number of continuous functions from $S$ to $T$ is at most 120
(D) Every continuous function from $S$ to $T$ is differentiable

Answer (A, C, D)
Sol. $S=(0,1) \cup(1,2) \cup(3,4)$ and $T=\{0,1,2,3\}$.
Let domain and co-domain of a function $y=f(x)$ are $S$ and $T$ respectively.
(A) There are infinitely many elements in domain and four elements in co-domain.
$\Rightarrow \quad$ There are infinitely many functions from $S$ to $T$.
$\Rightarrow$ Option (A) is correct
(B) If number of elements in domain is greater than number of elements in co-domain, then number of strictly increasing function is zero.
$\Rightarrow$ Option (B) is incorrect
(C) Maximum number of continuous functions $=4 \times 4 \times 4=64$
(Every subset ( 0,1 ), $(1,2),(3,4)$ has four choices)
$\because \quad 64<120 \Rightarrow$ option (C) is correct.
(D) For every point at which $f(x)$ is continuous, $f(x)=0$
$\Rightarrow \quad$ Every continuous function from $S$ to $T$ is differentiable.



2021


2020


Option (D) is correct.
2. Let $T_{1}$ and $T_{2}$ be two distinct common tangents to the ellipse $E: \frac{x^{2}}{6}+\frac{y^{2}}{3}=1$ and the parabola $P: y^{2}=12 x$. Suppose that the tangent $T_{1}$ touches $P$ and $E$ at the points $A_{1}$ and $A_{2}$, respectively and the tangent $T_{2}$ touches $P$ and $E$ at the points $A_{4}$ and $A_{3}$, respectively. Then which of the following statements is(are) true?
(A) The area of the quadrilateral $A_{1} A_{2} A_{3} A_{4}$ is 35 square units
(B) The area of the quadrilateral $A_{1} A_{2} A_{3} A_{4}$ is 36 square units
(C) The tangents $T_{1}$ and $T_{2}$ meet the $x$-axis at the point $(-3,0)$
(D) The tangents $T_{1}$ and $T_{2}$ meet the $x$-axis at the point $(-6,0)$

Answer (A, C)
Sol. $E: \frac{x^{2}}{6}+\frac{y^{2}}{3}=1$, Tangent : $y=m_{1} x \pm \sqrt{6 m_{1}^{2}+3}$
$P: y^{2}=12 x$, Tangent: $y=m_{2} x+\frac{3}{m_{2}}$
For common tangent
$m=m_{1}=m_{2}, \pm \sqrt{6 m_{1}^{2}+3}=\frac{3}{m_{2}}$
$\Rightarrow m= \pm 1$
$\Rightarrow$ equation of common tangents $y=x+3$ and $y=-x-3$ point of contact for parabola is $\left(\frac{a}{m^{2}}, \frac{2 a}{m}\right)$
$\Rightarrow \quad A_{1} \equiv(3,6), \quad A_{4}(3-6)$
Let $A_{2}\left(x_{1}, y_{1}\right) \Rightarrow$ tangent to $E$ is $\frac{x x_{1}}{6}+\frac{y y_{1}}{3}=1$
$A_{3}$ is mirror image of $A_{2}$ in $x$-axis $\Rightarrow A_{3}(-2,-1)$


Intersection point of $T_{1}=0$ and $T_{2}=0$ is $(-3,0)$
Area of quadrilateral $A_{1} A_{2} A_{3} A_{4}=\frac{1}{2}(12+2) \times 5=35$ square units

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2022


2021


- 39 -

3. Let $f:[0,1] \rightarrow[0,1]$ be the function defined by $f(x)=\frac{x^{3}}{3}-x^{2}+\frac{5}{9} x+\frac{17}{36}$. Consider the square region $S=[0,1] \times[0,1]$. Let $G=\{(x, y) \in S: y>f(x)\}$ be called the green region and $R=\{(x, y) \in S: y<f(x)\}$ be called the red region. Let $L_{h}=\{(x, h) \in S: x \in[0,1]\}$ be the horizontal line drawn at a height $h \in[0,1]$. Then which of the following statements is(are) true?
(A) There exists an $h \in\left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line $L_{h}$ equals the area of the green region below the line $L_{h}$
(B) There exists an $h \in\left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line $L_{h}$ equals the area of the red region below the line $L_{h}$
(C) There exists an $h \in\left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line $L_{h}$ equals the area of the red region below the line $L_{h}$
(D) There exists an $h \in\left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line $L_{h}$ equals the area of the green region below the line $L_{h}$
Answer (B, C, D)
Sol.


$$
f(x)=\frac{x^{3}}{3}-x^{2}+\frac{5}{9} x+\frac{17}{36}, f^{\prime}(x)=x^{2}-2 x+\frac{5}{9}
$$

For maxima/minima, $f^{\prime}(x)=0 \Rightarrow x=\frac{1}{3}$

$$
A_{R}=\int_{0}^{1} f(x) d x=\frac{1}{2} \Rightarrow A_{G}=\frac{1}{2}
$$



2022

- 40 -
(A) $1-h=h-\frac{1}{2} \Rightarrow h=\frac{3}{4}, \frac{3}{4}>\frac{2}{3}$ option (A) is incorrect
(B) $h=\frac{1}{2}-h \Rightarrow h=\frac{1}{4} \Rightarrow$ option (B) is correct.
(C) $\int_{0}^{1} f(x) d x=\frac{1}{2}, \int_{0}^{1} \frac{1}{2} d x=\frac{1}{2} \Rightarrow \int_{0}^{1}\left(f(x)-\frac{1}{2}\right) d x=0$
$\Rightarrow \quad h=\frac{1}{2} \Rightarrow$ option (C) is correct.
(D) $\because$ Option (C) is correct $\Rightarrow$ option (D) is also correct.


## SECTION 2 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
4. Let $f:(0,1) \rightarrow \mathbb{R}$ be the function defined as $f(x)=\sqrt{n}$ if $x \in\left[\frac{1}{n+1}, \frac{1}{n}\right)$ where $n \in \mathbb{N}$. Let $g:(0,1) \rightarrow \mathbb{R}$ be a function such that $\int_{x^{2}}^{x} \sqrt{\frac{1-t}{t}} d t<g(x)<2 \sqrt{x}$ for all $x \in(0,1)$. Then $\lim _{x \rightarrow 0} f(x) g(x)$
(A) Does NOT exist
(B) Is equal to 1
(C) Is equal to 2
(D) Is equal to 3

Answer (C)
Sol. We need to solve 1 sided limit here to get some answer, otherwise $\lim _{x \rightarrow 0^{-}}$doesn't exist here (not in domain)

$$
\begin{aligned}
& f(x)=\sqrt{\left(\frac{1}{x}\right)-1} \quad \text { where }(\cdot)=\text { least integer function } \\
& \lim _{x \rightarrow 0^{+}} \int_{x^{2}}^{x} \sqrt{\frac{1-t}{t}} d t \cdot \sqrt{\left(\frac{1}{x}\right)-1} \leq \lim _{x \rightarrow 0^{+}} f(x) \cdot g(x) \leq \lim _{x \rightarrow 0^{+}} \sqrt{\left(\frac{1}{x}\right)-1} \times 2 \sqrt{x}
\end{aligned}
$$

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2022


2021


2020


Now $\lim _{x \rightarrow 0^{+}} \sqrt{\left(\frac{1}{x}\right)-1} \times 2 \sqrt{x}=\lim _{x \rightarrow 0^{+}} 2 \sqrt{x} \sqrt{\left[\frac{1}{x}\right]} \quad\left(\frac{1}{x} \notin Z\right)$
$=\lim _{x \rightarrow 0^{+}} 2 \sqrt{x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}\right)}=2$
$=\lim _{x \rightarrow 0^{+}} 2 \sqrt{x\left(\frac{1}{x}\right)}=2 ;\left(\frac{1}{x} \notin Z\right)$
$\lim _{x \rightarrow 0^{+}} \int_{x^{2}}^{x} \sqrt{\frac{1-t}{t}} d t \cdot \sqrt{\frac{1}{x}-\left\{\frac{1}{x}\right\}}=\frac{\int_{x^{2}}^{x} \sqrt{\frac{1-t}{t}} d t \cdot \sqrt{1-x\left\{\frac{1}{x}\right\}}}{\sqrt{x}}$
$\lim _{x \rightarrow 0^{+}} \frac{\int_{x^{2}}^{x} \sqrt{\frac{1-t}{t}} d t}{\sqrt{x}}=\lim _{x \rightarrow 0^{+}} \frac{\sqrt{\frac{1-x}{x}}-2 x \sqrt{\frac{1-x^{2}}{x^{2}}}}{\frac{1}{2 \sqrt{x}}}$
$\lim _{x \rightarrow 0^{+}} 2 \sqrt{1-x}-4 \sqrt{x} \cdot \sqrt{1-x^{2}}=2$
Similarly for $\frac{1}{x} \in Z$ is equal to 2 .
5. Let $Q$ be the cube with the set of vertices $\left\{\left(x_{1}, x_{2}, x_{3}\right) \in \mathbb{R}^{3}: x_{1}, x_{2}, x_{3} \in\{0,1\}\right\}$. Let $F$ be the set of all twelve lines containing the diagonals of the six faces of the cube $Q$. Let $S$ be the set of all four lines containing the main diagonals of the cube $Q$; for instance, the line passing through the vertices $(0,0,0)$ and $(1,1,1)$ is in $S$. For lines $\ell_{1}$ and $\ell_{2}$, let $d\left(\ell_{1}, \ell_{2}\right)$ denote the shortest distance between them. Then the maximum value of $d\left(\ell_{1}, \ell_{2}\right)$, as $\ell_{1}$ varies over $F$ and $\ell_{2}$ varies over $S$, is
(A) $\frac{1}{\sqrt{6}}$
(B) $\frac{1}{\sqrt{8}}$
(C) $\frac{1}{\sqrt{3}}$
(D) $\frac{1}{\sqrt{12}}$


Answer (A)

Sol.


Equation of $O D$ line is
$\vec{r}=\overrightarrow{0}+\lambda(\hat{i}+\hat{j})$
Equation of diagonal $B E$ is
$\vec{r}_{1}=\hat{j}+\alpha(\hat{i}-\hat{j}+\hat{k})$
$S . D=\left|\frac{\hat{j} \cdot(\hat{i}-\hat{j}-2 \hat{k})}{\sqrt{6}}\right|=\frac{1}{\sqrt{6}}$
In other case S.D is zero.
6. Let $X=\left\{(x, y) \in \mathbb{Z} \times \mathbb{Z}: \frac{x^{2}}{8}+\frac{y^{2}}{20}<1\right.$ and $\left.y^{2}<5 x\right\}$. Three distinct point $P, Q$ and $R$ are randomly chosen from $X$.

Then the probability that $P, Q$ and $R$ form a triangle whose area is a positive integer, is
(A) $\frac{71}{220}$
(B) $\frac{73}{220}$
(C) $\frac{79}{220}$
(D) $\frac{83}{220}$

Answer (B)


Sol. The given region are as


The points inside region are $\{(2,1),(2,-1),(2,2),(2,-2),(2,3),(2,-3),(2,0),(1,1),(1,-1),(1,2),(1,-2)$, $(1,0)\}$.
Total number of ways to select three points $={ }^{12} \mathrm{C}_{3}=220$
Required number of triangle $=4 \times{ }^{7} \mathrm{C}_{1}+9 \times{ }^{5} \mathrm{C}_{1}=73$
Points are taken such a way that distance between two points are multiple of 2 .
7. Let $P$ be a point on the parabola $y^{2}=4 a x$, where $a>0$. The normal to the parabola at $P$ meets the $x$-axis at a point $Q$. The area of the triangle $P F Q$, where $F$ is the focus of the parabola, is 120 . If the slope $m$ of the normal and $a$ are both positive integers, then the pair $(a, m)$ is
(A) $(2,3)$
(B) $(1,3)$
(C) $(2,4)$
(D) $(3,4)$

Answer (A)

Sol.


Equation of normal at $P\left(a m^{2},-2 a m\right)$ is $y=m x-2 a m-a m^{3}$
$\Rightarrow$ Area of $\triangle P F Q=\frac{1}{2}\left(a+a m^{2}\right) \times 2 a m=120$
$a^{2} m\left(1+m^{2}\right)=120$
Pair $(a, m) \equiv(2,3)$ satisfies above equation


2022


2021


- 44 -


## SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;
Zero Marks : $0 \quad$ In all other cases.
8. Let $\tan ^{-1}(x) \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in \mathbb{R}$. Then the number of real solutions of the equation $\sqrt{1+\cos (2 x)}=\sqrt{2} \tan ^{-1}(\tan x)$ in the set $\left(-\frac{3 \pi}{2},-\frac{\pi}{2}\right) \cup\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup\left(\frac{\pi}{2}, \frac{3 \pi}{2}\right)$ is equal to

## Answer (3)

Sol. $\sqrt{1+\cos 2 x}=\sqrt{2} \tan ^{-1}(\tan x)$

$$
\Rightarrow \quad|\cos x|=\tan ^{-1}(\tan x)
$$



Number of solutions $=$ Number of intersection points $=3$


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2021


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9. Let $n \geq 2$ be a natural number and $f:[0,1] \rightarrow \mathbb{R}$ be the function defined by

$$
f(x)= \begin{cases}n(1-2 n x) & \text { if } 0 \leq x \leq \frac{1}{2 n} \\ 2 n(2 n x-1) & \text { if } \frac{1}{2 n} \leq x \leq \frac{3}{4 n} \\ 4 n(1-n x) & \text { if } \frac{3}{4 n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(n x-1) & \text { if } \frac{1}{n} \leq x \leq 1\end{cases}
$$

If $n$ is such that the area of the region bounded by the curves $x=0, x=1, y=0$ and $y=f(x)$ is 4 , then the maximum value of the function $f$ is

## Answer (8)

$\int n(1-2 n x), 0 \leq x<\frac{1}{2 n}$
Sol. $f(x)=\left\{\begin{array}{l}2 n(2 n x-1), \frac{1}{2 n} \leq x<\frac{3}{4 n} \\ 4 n(1-n x), \frac{3}{4 n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(n x-1), \frac{1}{n} \leq x \leq 1\end{array}\right.$
$x \in[0,1]$
$f(x)$ is decreasing in $\left[0, \frac{1}{2 n}\right]$
increasing in $\left[\frac{1}{2 n}, \frac{3}{4 n}\right]$
decreasing in $\left[\frac{3}{4 n}, \frac{1}{n}\right]$
increasing in $\left[\frac{1}{n}, 1\right]$
Graph

$f(x) \in[0, n]$
Area $=4 \Rightarrow n=8=$ and $f(x)_{\max }=n=8$


2022


2021


- 46 -

10. Let $7 \overbrace{5 \ldots 57}^{r}$ denote the $(r+2)$ digit number where the first and the last digits are 7 and the remaining $r$ digits are 5. Consider the sum $S=77+757+7557+\ldots+7 \overbrace{5 \ldots 57}^{98}$. If $S=\frac{\overbrace{75 \ldots 57}^{99}}{n}+m$, where $m$ and $n$ are natural numbers less than 3000, then the value of $m+n$ is
Answer (1219)
Sol. $S=77+757+7557+\ldots+7 \overbrace{5 \ldots 57}^{98}$

$$
\begin{aligned}
& =7\left(10+10^{2}+\ldots+10^{09}\right)+50(1+11+\ldots+\overbrace{111 \ldots 1}^{98})+7 \times 99 \\
& =70\left(\frac{10^{99}-1}{9}\right)+\frac{50}{9}\left[(10-1)+\left(10^{2}-1\right)+\ldots+\left(10^{98}-1\right)\right]+7 \times 99 \\
& =70\left(\frac{10^{99}-1}{9}\right)+\frac{50}{9}\left[10\left(\frac{10^{98}-1}{9}\right)-98\right]+7 \times 99 \\
& =\frac{7 \times 10^{100}}{9}-\frac{70}{9}+\frac{50}{9}\left[\frac{10^{99}-1-9}{9}-98\right]+7 \times 99 \\
& =\frac{7 \times 10^{100}}{9}-\frac{70}{9}+\frac{50}{9}[\overbrace{111 \ldots 1}^{99}-99]+7 \times 99 \\
& =\frac{7 \times 10^{100}-70+\overbrace{555}^{99} \ldots 50}{9}-550+693 \\
& =\frac{7 \overbrace{555 \ldots 5}^{99}-70+143 \times 9}{9} \\
& =\frac{7 \overbrace{55 \ldots 57+1210}^{99}}{m+n=1219}
\end{aligned}
$$

11. Let $A=\left\{\frac{1967+1686 i \sin \theta}{7-3 i \cos \theta}: \theta \in R\right\}$. If $A$ contains exactly one positive integer $n$, then the value of $n$ is

## Answer (281)

Sol. $z=\frac{1967+1686 i \sin \theta}{7-3 i \cos \theta}$ is a positive integer.


2022


2021


$$
\begin{aligned}
& z=\frac{(1967+1686 i \sin \theta)(7+3 i \cos \theta)}{(7-3 i \cos \theta)(7+3 i \cos \theta)} \\
& 1967=281 \times 7 ; 1686=281 \times 6 \\
& z=\frac{1967 \times 7-1686 \times 3 \sin \theta \cos \theta+i(1686 \times 7 \sin \theta+1967 \times 3 \cos \theta)}{49+9 \cos ^{2} \theta}
\end{aligned}
$$

$$
(281 \times 6) \times 7 \sin \theta+(281 \times 7) \times 3 \cos \theta=0
$$

$$
\tan \theta=-\frac{1}{2}
$$

$$
\Rightarrow \cos ^{2} \theta=\frac{4}{5} ; \sin \theta \cos \theta=-\frac{2}{5}
$$

$$
z=\frac{(281 \times 7 \times 7)-(281 \times 6) \times 3 \times\left(-\frac{2}{5}\right)}{49+9 \times \frac{4}{5}}=\frac{281\left(49+\frac{36}{5}\right)}{\left(49+\frac{36}{5}\right)}=281
$$

12. Let $P$ be the plane $\sqrt{3} x+2 y+3 z=16$ and let
$S=\left\{\alpha \hat{i}+\beta \hat{j}+\gamma \hat{k}: \alpha^{2}+\beta^{2}+\gamma^{2}=1\right.$ and the distance of $(\alpha, \beta, \gamma)$ from the plane $P$ is $\left.\frac{7}{2}\right\}$.
Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in $S$ such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let $V$ be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}} V$ is

## Answer (45)

Sol. $P: \sqrt{3} x+2 y+3 z=16$
$S=\alpha \hat{i}+\beta \hat{j}+\gamma \hat{k}: \alpha^{2}+\beta^{2}+\gamma^{2}=1$
$d(\alpha, \beta, \gamma)$ from $P=\frac{7}{2}$
$|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|-$
$V$ : volume of parallelepiped by vectors $\vec{u}, \vec{v}, \vec{w}$
$\frac{80}{\sqrt{3}} V=?$

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2022


2021


2020


- 48 -
$d(\alpha, \beta, \gamma)$ from $\beta=\frac{7}{2}$ (Given)
$\Rightarrow \frac{|\sqrt{3} \alpha+2 \beta+3 \gamma-16|}{\sqrt{3+4+9}}=\frac{7}{2}$
$=\frac{|\sqrt{3} \alpha+2 \beta+3 \gamma-16|}{4}=\frac{7}{2}$
$|\sqrt{3} \alpha+2 \beta+3 \gamma-16|=14$
$\alpha^{2}+\beta^{2}+\gamma^{2}=1$
Volume of parallelepiped by vector $\vec{u}, \vec{v}, \vec{w}$

$$
\begin{align*}
V & =[\vec{u} \vec{v} \vec{w}] \\
& =\vec{u} \cdot(v \times w) \tag{iii}
\end{align*}
$$

$|\vec{u}|=|\vec{v}|=|\vec{w}|=1$ (Given)
$|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$-(Given)
$\Rightarrow|\vec{u}-\vec{v}|^{2}=|\vec{v}-\vec{w}|^{2}=|\vec{w}-\vec{u}|^{2}$
$\Rightarrow u^{2}+v^{2}-2 \vec{u} \cdot \vec{v}=v^{2}+w^{2}-2 \vec{v} \cdot \vec{w}$
(A)
(B)
$=w^{2}+u^{2}-2 \vec{w} \cdot \vec{u}$
(C)
(A) and (B)
$\Rightarrow u^{2}+v^{2}-2 \vec{u} \cdot \vec{v}=v^{2}+w^{2}-2 \vec{v} \cdot \vec{w}$
$\Rightarrow u^{2}-w^{2}=2 \vec{u} \cdot \vec{v}-2 \vec{v} \cdot \vec{w}$
$\Rightarrow \vec{u} \cdot \vec{v}=\vec{v} \cdot \vec{w}$
Hence, by using (B) and (C) also, we will get
$\vec{u} \cdot \vec{v}=\vec{v} \cdot \vec{w}=\vec{w} \cdot \vec{u}=m$ (say)
$\Rightarrow \vec{u}, \vec{v}, \vec{w}$ are the vectors of an equilateral triangle (say $\triangle A B C$ )



2022


2021

$$
d(O, P)=\frac{16}{\sqrt{3+4+9}}
$$

$$
=\frac{16}{4}
$$

$$
=4 \text { units }
$$

$\overrightarrow{O A}=\vec{u}, \overrightarrow{O B}=\vec{v}, \overrightarrow{O C}=\vec{w}$
$|\overrightarrow{O A}|=|\overrightarrow{O B}|=|\overrightarrow{O C}|=1$ (Given)
In an equilateral triangle, circumcentre, orthrocentre and centroid coincide.
Let $D$ be the circumcentre of $\triangle A B C$, then
$\angle A D B=120^{\circ}$
Given $=\frac{D A^{2}+D B^{2}-A B^{2}}{2(D A) \cdot(D B)}$
$O E=O D+D E$
$=O D+A F$
$\Rightarrow 4=O D+\frac{7}{2}$
$\Rightarrow O D=4-\frac{7}{2}=\frac{1}{2}$
$\Rightarrow D A=\sqrt{O A^{2}-O D^{2}}$
$=\sqrt{1-\frac{1}{4}}$
$D A=\frac{\sqrt{3}}{2}$
$\Rightarrow \quad D A=D B=\frac{\sqrt{3}}{2}$
From (vi) and (vii),
$-\frac{1}{2}=\frac{\frac{3}{4}+\frac{3}{4}-A B^{2}}{2 \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}}$

## Aakashians Shine as

Champions in JEE Advanced


2022


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$$
\begin{align*}
& -\frac{1}{2}=\frac{\frac{3}{2}-A B^{2}}{\frac{3}{2}} \\
& \Rightarrow \quad-\frac{1}{2} \times \frac{3}{2}=\frac{3}{2}-A B^{2} \\
& \Rightarrow A B^{2}=\frac{3}{2}+\frac{3}{4} \\
& \Rightarrow A B^{2}=\frac{9}{4} \\
& \Rightarrow A B-\frac{3}{2}=|\vec{u}-\vec{v}| \\
& \Rightarrow A B^{2}=\frac{9}{4}=u^{2}+v^{2}-2 \vec{u}-\vec{v} \\
& \Rightarrow \quad \frac{9}{4}=1+1-2 m \\
& \Rightarrow \quad 2 m=2-\frac{9}{4}=-\frac{1}{4} \\
& \Rightarrow \quad m=-\frac{1}{8} \tag{viii}
\end{align*}
$$

Volume of parallelepiped,
$V=\left\lvert\,\left[\begin{array}{lll}\vec{u} & \vec{v} & \vec{w}\end{array}\right]\right.$
$|\vec{u} \vec{v} \vec{w}|^{2}=\left|\begin{array}{ccc}1 & \vec{u} \cdot \vec{v} & \vec{u} \cdot \vec{w} \\ \vec{u} \cdot \vec{v} & 1 & \vec{v} \cdot \vec{w} \\ \vec{w} \cdot \vec{u} & \vec{w} \cdot \vec{v} & 1\end{array}\right|$

$$
\begin{aligned}
& =\left|\begin{array}{ccc}
1 & m & m \\
m & 1 & m \\
m & m & 1
\end{array}\right| \\
& =1\left(1-m^{2}\right)-m\left(m-m^{2}\right)+m\left(m^{2}-m\right) \\
& =1-m^{2}-m^{2}+m^{3}+m^{3}-m^{2} \\
& =1-3 m^{2}+2 m^{3}
\end{aligned}
$$

$$
|\vec{u} \vec{v} \quad \vec{w}|^{2}=2 m^{3}-3 m^{2}+1
$$

$$
=(m-1)\left[2 m^{2}-m-1\right]
$$



$$
\begin{aligned}
& =(m-1)\left[2 m^{2}-2 m+m-1\right] \\
& =(m-1)(m-1)(2 m+1) \\
& =(m-1)^{2}(2 m+1)
\end{aligned}
$$

$\left.\Rightarrow \quad \left\lvert\, \begin{array}{lll}\vec{u} & \vec{v} & \vec{w}\end{array}\right.\right]=(m-1) \sqrt{(2 m+1)}=v$
$\left|\left(-\frac{1}{8}-1\right) \sqrt{2 \times-\frac{1}{8}+1}\right|$
$V=\frac{9}{8} \times \frac{\sqrt{3}}{2}$

$$
\begin{aligned}
\frac{80}{\sqrt{3}} v & =\frac{80}{\sqrt{3}} \times \frac{9}{8} \times \frac{\sqrt{3}}{2} \\
& =45
\end{aligned}
$$

13. Let $a$ and $b$ be two nonzero real numbers. If the coefficient of $x^{5}$ in the expansion of $\left(a x^{2}+\frac{70}{27 b x}\right)^{4}$ is equal to the coefficient of $x^{-5}$ in the expansion of $\left(a x-\frac{1}{b x^{2}}\right)^{7}$, then the value of $2 b$ is

## Answer (3)

Sol. $T_{r+1}={ }^{4} C_{r}\left(a x^{2}\right)^{4-r}\left(\frac{70}{27 b x}\right)^{r}$
For coefficient of $x^{5}, 8-2 r-r=5 \Rightarrow r=1$
$\Rightarrow$ Coefficient of $x^{5}={ }^{4} C_{1} a^{3}\left(\frac{70}{27 b}\right)$
$t_{r+1}={ }^{7} C_{r}(a x)^{7-r}\left(-\frac{1}{b x^{2}}\right)^{r}$
For coefficient of $x^{-5}, 7-r-2 r=-5 \Rightarrow r=4$
$\Rightarrow$ coefficient of $x^{-5}={ }^{7} C_{4} a^{3} \frac{1}{b^{4}}$
$\Rightarrow{ }^{4} C_{1} a^{3}\left(\frac{70}{27 b}\right)={ }^{7} C_{4} a^{3} \frac{1}{b^{4}} \Rightarrow 2 b=3$


2022


2021


2020


## SECTION 4 (Maximum Marks: 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.
14. Let $\alpha, \beta$ and $\gamma$ be real numbers. Consider the following system of linear equations
$x+2 y+z=7$
$x+\alpha z=11$
$2 x-3 y+\beta z=\gamma$
Match each entry in List-I to the correct entries in List-II.

| List-I | List-II |
| :--- | :--- |
| (P) If $\beta=\frac{1}{2}(7 \alpha-3)$ and $\gamma=28$, then the system has | (1) a unique solution |
| (Q) If $\beta=\frac{1}{2}(7 \alpha-3)$ and $\gamma \neq 28$, then the system has | (2) no solution |
| (R) If $\beta \neq \frac{1}{2}(7 \alpha-3)$ where $\alpha=1$ and $\gamma \neq 28$, then the | (3) infinitely many solutions |
| system has | (4) $x=11, y=-2$ and $z=0$ as a solution |
| (S) If $\beta \neq \frac{1}{2}(7 \alpha-3)$ where $\alpha=1$ and $\gamma=28$, then the | system has |

The correct option is:
(A) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (4)
(B) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (5), (S) $\rightarrow$ (4)
(C) (P) $\rightarrow(2),(\mathrm{Q}) \rightarrow(1),(\mathrm{R}) \rightarrow(4),(\mathrm{S}) \rightarrow(5)$
(D) (P) $\rightarrow(2),(Q) \rightarrow(1),(R) \rightarrow(1),(S) \rightarrow(3)$

## Answer (A)

Sol. $x+2 y+z=7$

$$
x+\alpha z=11
$$

$2 x-3 y+\beta z=\gamma$
$\Delta=\left|\begin{array}{ccc}1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta\end{array}\right|=0$
$3 \alpha-2(\beta-2 \alpha)-3=0$
$7 \alpha-2 \beta=3$
$\Rightarrow \beta=\frac{1}{2}(7 \alpha-3)$
$\Delta_{1}=\left|\begin{array}{ccc}7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta\end{array}\right|, \Delta_{2}=\left|\begin{array}{ccc}1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta\end{array}\right|, \Delta_{3}=\left|\begin{array}{ccc}1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma\end{array}\right|$
$\Delta_{3}=0$
$\Rightarrow 33-2(\gamma-22)+7(-3)=0$
$\gamma=28$
$\Delta_{1}=21 \alpha-2(11 \beta-\alpha \gamma)-33$

$$
=21 \alpha-22 \beta+2 \alpha \gamma-33
$$

$$
\Delta_{2}=11 \beta-\alpha \gamma-7(\beta-2 \alpha)+\gamma-22
$$

$$
=14 \alpha+4 \beta+\gamma-\alpha \gamma-22
$$

(P) If $\beta=\frac{1}{2}(7 \alpha-3)$ and $\gamma=28$
$\Delta=0, \Delta_{1}=0, \Delta_{2}=0, \Delta_{3}=0$
Infinitely many solutions
$x=11, y=-2$ and $z=0$ will satisfy all the three given equations, so it is a solution.
(Q) If $\beta=\frac{1}{2}(7 \alpha-3)$ and $\gamma \neq 28$ then
$\Delta=0$, but $\Delta 3 \neq 0$ so no solution
(R) If $\beta \neq \frac{1}{2}(7 \alpha-3), \alpha=1$ and $\gamma \neq 28$
$\Delta \neq 0, \Delta 3 \neq 0$ so a unique solution
(S) If $\beta \neq \frac{1}{2}(7 \alpha-3), \alpha=1, \gamma=28$
$\Delta \neq 0, \Delta_{3}=0, \Delta_{1} \neq 0, \Delta_{2} \neq 0$, so a unique solution
$x=11, y=-2$ and $z=0$ will satisfy all the three equations
Option A is correct.

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2022


2021

15. Consider the given data with frequency distribution

| $x_{i}$ | 3 | 8 | 11 | 10 | 5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f_{i}$ | 5 | 2 | 3 | 2 | 4 | 4 |

Match each entry in List-I to the correct entries in List-II.

| List-I | List-II |
| :--- | :--- | :--- |
| (P) The mean of the above data is | (1) 2.5 |
| (Q) The median of the above data is | (2) 5 |
| (R)The mean deviation about the <br> mean of the above data is | (3) 6 |
| (S)The mean deviation about the <br> median of the above data is | (4) 2.7 |
|  | (5) 2.4 |

The correct option is
(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)
(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)
(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (4) (S) $\rightarrow$ (1)
(D) $(P) \rightarrow(3)(Q) \rightarrow(3)(R) \rightarrow(5)(S) \rightarrow(5)$

## Answer (A)

Sol.

$$
\left.\begin{array}{rllllllll}
x & \ldots & 3 & 4 & 5 & 8 & 10 & 11 & \text { (ascending order) } \\
f & \ldots & 5 & 4 & 4 & 2 & 2 & 3
\end{array}\right] \begin{aligned}
\text { Mean } & =\frac{3 \times 5+8 \times 2+11 \times 3+10 \times 2+5 \times 4+4 \times 4}{5+2+3+2+4+4} \\
& =\frac{15+16+33+20+20+16}{20}=\frac{120}{20}=6
\end{aligned}
$$

Median $=\frac{1}{2}\left(10^{\text {th }}+11^{\text {th }}\right.$ observation $)$

$$
=\frac{1}{2}(5+5)=5
$$

Mean deviation about mean
$=\frac{3 \times 5+2 \times 4+1 \times 4+2 \times 2+4 \times 2+5 \times 3}{20}$
$=\frac{54}{20}=2.7$
Mean deviation about median


2022


- 55 -

$$
=\frac{2 \times 5+1 \times 4+0+3 \times 2+5 \times 2+6 \times 3}{20}
$$

$$
=\frac{4.8}{20}=2.4
$$

$\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 5$
$\therefore$ Option A is correct.
16. Let $\ell_{1}$ and $\ell_{2}$ be the lines $\vec{r}_{1}=\lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_{2}=(\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively, Let $X$ be the set of all the planes $H$ that contain the line $\ell_{1}$. For a plane $H$, let $d(H)$ denote the smallest possible distance between the points of $\ell_{2}$ and $H$. Let $H_{0}$ be a plane in $X$ for which $d\left(H_{0}\right)$ is the maximum value of $d(H)$ as $H$ varies over all planes in $X$.

Match each entry in List-I to the correct entries in List-II.

| List-I | List-II |
| :--- | :--- |
| (P) The value of $d\left(H_{0}\right)$ is | (1) $\sqrt{3}$ |
| (Q) The distance of the point $(0,1,2)$ from $H_{0}$ is $\frac{1}{\sqrt{3}}$ |  |
| (R) The distance of origin from $H_{0}$ is | (3) 0 |
| (S)The distance of origin from the point of <br> intersection of planes $y=z, x=1$ and $H_{0}$ is | (4) $\sqrt{2}$ |
|  | (5) $\frac{1}{\sqrt{2}}$ |

The correct option is
$(\mathrm{A})(\mathrm{P}) \rightarrow(2)(\mathrm{Q}) \rightarrow(4)(\mathrm{R}) \rightarrow(5)(\mathrm{S}) \rightarrow(1)$
$(B)(P) \rightarrow(5)(Q) \rightarrow(4)(R) \rightarrow(3)(S) \rightarrow(1)$
(C) $(P) \rightarrow(2)(Q) \rightarrow(1)(R) \rightarrow(3)(S) \rightarrow(2)$
$(\mathrm{D})(\mathrm{P}) \rightarrow(5)(\mathrm{Q}) \rightarrow(1)(\mathrm{R}) \rightarrow(4)(\mathrm{S}) \rightarrow(2)$
Answer (B)

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2022


- 56 -

Sol. $H_{0}$ will be the plane containing the line $\ell_{1}$ and parallel to $\ell_{2}$.
$\begin{aligned} & \therefore \text { Normal vector of plane parallel } \ell_{1} \text { and } \ell_{2} \text { is }\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1\end{array}\right|=\hat{j}(1)-\hat{j}(1-1)+\hat{k}(-1) \\ &=\hat{i}-\hat{k}\end{aligned}$
$\therefore H_{0}: x-z=c \mid(0,0,0)$
$\Rightarrow C=0$
$\therefore H_{0}: x-z=0$
(P) $d\left(H_{0}\right)=1$ distance of point $(0,1,-1)$ from $H$.

$$
d=\left|\frac{0-(-1)}{\sqrt{2}}\right|=\frac{1}{\sqrt{2}} \therefore P \rightarrow 5
$$

(Q) $d=\left|\frac{0-2}{\sqrt{2}}\right|=\sqrt{2} \therefore Q \rightarrow 4$
(R) $d=\left|\frac{0}{\sqrt{2}}\right|=0 \therefore R \rightarrow 3$
$(S)$ Point of intersection will be $(1,1,1) \therefore S \rightarrow 1$

$$
d=\sqrt{1+1+1}=\sqrt{3}
$$

$\therefore$ Option (B) is correct.
17. Let $z$ be a complex number satisfying $|z|^{3}+2 z^{2}+4 \bar{z}-8=0$, where $\bar{z}$ denotes the complex conjugate of $z$. Let the imaginary part of $z$ be nonzero.
Match each entry in List-I to the correct entries in List-II.

| List-I | List-II |
| :--- | :--- |
| (P) $\|z\|^{2}$ is equal to | (1) 12 |
| (Q) $\|z-\bar{z}\|^{2}$ is equal to | (2) 4 |
| (R) $\|z\|^{2}+\|z+\bar{z}\|^{2}$ is equal to | (3) 8 |
| (S) $\|z+1\|^{2}$ is equal to | (4) 10 |
|  | (5) 7 |

## Aakashians Shine as

 Champions in JEE Advanced

2022



The correct option is
(A) (P) $\rightarrow$ (1) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (5) (S) $\rightarrow$ (4)
(B) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (3) (S) $\rightarrow$ (5)
(C) $(P) \rightarrow(2)(Q) \rightarrow(4)(R) \rightarrow(5)(S) \rightarrow(1)$
(D) (P) $\rightarrow(2)(\mathrm{Q}) \rightarrow(3)(\mathrm{R}) \rightarrow(5)(\mathrm{S}) \rightarrow(4)$

## Answer (B)

$|z|^{3}+2 z^{2}+4 \bar{z}-8=0$
Sol. $|z|^{3}+2 \bar{z}^{2}+4 z-8=0$
$2\left(z^{2}-\bar{z}^{2}\right)+4(\bar{z}-z)=0$
$(z-\bar{z})[2(z+\bar{z})-4]=0$
$\because z=\bar{z}$ (not possible) or $4 x=4 \Rightarrow x=1$.
$z=1+\lambda i \Rightarrow|z|=\sqrt{1+\lambda^{2}} \Rightarrow \bar{z}=1-\lambda i$
$\left(1+\lambda^{2}\right)^{3 / 2}+2\left(1-\lambda^{2}+2 \lambda i\right)+4(1-\lambda i)-8=0$
$\Rightarrow\left(1+\lambda^{2}\right)^{3 / 2}+2\left(1-\lambda^{2}\right)=4$
$\left(1+\lambda^{2}\right)^{3 / 2}=2\left(1+\lambda^{2}\right)$
$\left(1+\lambda^{2}\right)\left[\sqrt{1+\lambda^{2}}-2\right]=0$
$\Rightarrow \lambda^{2}=3$
Now
(P) $|z|^{2}=1+\lambda^{2}=1+3=4$
(Q) $|z-\bar{z}|^{2}=|1+\lambda i-(1-\lambda i)|^{2}=|2 \lambda i|^{2}=4 \lambda^{2}=12$
(R) $|z|^{2}+|z+\bar{z}|^{2}=4+|(1+\lambda i)+(1-\lambda i)|^{2}=4+4=8$
(S) $|z+1|^{2}=|1+\lambda i+1|^{2}=4+\lambda^{2}=4+3=7$
$\therefore \mathrm{P} \rightarrow(2), \mathrm{Q} \rightarrow(1), \mathrm{R} \rightarrow(3), \mathrm{S} \rightarrow(5)$


2022


2021


