MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If sum of first 4 terms of an A.P is 6 and sum of first 6 terms is 4, then sum of first 12 terms of an A.P is

(1) –22 (2) –21

(3) –23 (4) –24

Answer (1)

Sol. $S_4 = \frac{4}{2}[2a + 3d] = 6$

$\Rightarrow 2a + 3d = 3$ ...(i)

$S_6 = \frac{6}{2}[2a + 5d] = 4$

$\Rightarrow 2a + 5d = \frac{4}{3}$ ...(ii)

Solving (i) & (ii)

$a = \frac{11}{4}$

$d = \frac{-5}{6}$

$S_{12} = 6\left[2 \times \frac{11}{4} + 11 \times \left(-\frac{5}{6}\right)\right]$

$= -22$

2. The coefficient of x^{48} in the expansion of

$1\cdot(1 + x) + 2\cdot(1 + x)^2 + 3\cdot(1 + x)^3 + \ldots\ldots. + 100\cdot(1 + x)^{100}$ is

(1) $({}^{101}C_{46}) - 100$

(2) $100({}^{101}C_{49}) - {}^{101}C_{50}$

(3) $100({}^{101}C_{46}) - {}^{101}C_{47}$

(4) ${}^{101}C_{47} - {}^{101}C_{46}$

Answer (2)

Sol. Let $y = 1\cdot(1 + x) + 2\cdot(1 + x)^2 + 3\cdot(1 + x)^3 + \ldots\ldots. + 100\cdot(1 + x)^{100}$

$\therefore (1 + x)y = (1 + x)^2 + 2\cdot(1 + x)^3 + \ldots\ldots\ldots + 100(1 + x)^{101}$

$-x\cdot y = (1 + x) + (1 + x)^2 + (1 + x)^3 + \ldots\ldots.. + (1 + x)^{100} - 100(1 + x)^{101}$

$$y = \frac{1}{x}\cdot\left\{100(1+x)^{101} - \frac{(1+x)\left((1+x)^{100} - 1\right)}{1 + x - 1}\right\}$$

$$y = \frac{1}{x}\cdot\left\{100\cdot(1+x)^{101} - \frac{1}{x}\left((1+x)^{101} - (1+x)\right)\right\}$$

$\therefore$ Coeff. of $x^{48} = 100\cdot{}^{101}C_{49} - {}^{101}C_{50}$

3. If the domain of the function

$$\frac{1}{\ln(10 - x)} + \sin^{-1}\left(\frac{x+2}{2x+3}\right)$$

is $(-\infty, -a] \cup (-1, b) \cup (b, c)$, then $(b + c + 3a)$ is equal to

(1) 22

(2) 24

(3) 23

(4) 21

Answer (2)

Sol. Domain of $\frac{1}{\ln(10 - x)}$

$\Rightarrow \ln(10 - x) \neq 0$

$\Rightarrow 10 - x \neq 1$ $x \neq 9$

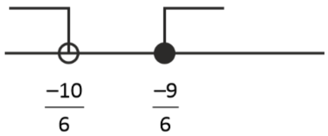
$10 - x > 0$ $x < 10$

Domain of $\sin^{-1}\left(\frac{x+2}{2x+3}\right) \Rightarrow -1 \leq \frac{x+2}{2x+3} \leq 1$

$\Rightarrow \frac{-x-1}{2x+3} \leq 0 \Rightarrow \frac{x+1}{2x+3} \geq 0$

$$\frac{3x+5}{2x+3} \ge 0$$

and



$$\Rightarrow \ x \in \left(\left(-\infty, \frac{-3}{2}\right) \cup [-1, \infty)\right) \cap \left(\left(-\infty, \frac{-10}{6}\right] \cup \left(\frac{-9}{6}, \infty\right)\right)$$

$$\Rightarrow \ x \in \left(-\infty, \frac{-10}{6}\right) \cup \left[\frac{-6}{6}, \infty\right)$$

$$x \in \left(-\infty, \frac{-5}{3}\right) \cup [-1, \infty)$$

$$\Rightarrow \text{ domain of } \frac{1}{\ln(10-x)} + \sin^{-1}\left(\frac{x+2}{2x+3}\right)$$

$$x \in \left(-\infty, \frac{-5}{3}\right] \cup [-1, 10) - \{9\}$$

$$x \in \left(-\infty, \frac{-5}{3}\right] \cup [-1, 9) \cup (9, 10)$$

4. Let $M = \{1, 2, 3, \ldots, 16\}$ and R be a relation on M defined by xRy if and only if $4y = 5x - 3$. Then, the number of elements required to added in R to make it symmetric is

(1) 2 (2) 3

(3) 4 (4) 5

Answer (1)

Sol. $R = \{(3, 3), (7, 8), (11, 13)\}$ Number of pairs to be added to R to made it symmetric are = 2. Which are (8, 7), (13, 11)

5. The solution of the differential equation $xdy - ydx = \sqrt{x^2+y^2}\,dx$ is (where c is integration constant)

(1) $\sqrt{x^2+y^2} = cx^2 - y$ (2) $\sqrt{x^2+y^2} = cx^2 + y$

(3) $\sqrt{x^2+y^2} = cx - y$ (4) $\sqrt{x^2+y^2} = cx + y$

Answer (1)

Sol. $xdy - ydx = \sqrt{x^2+y^2}$

divide by x^2 both sides

$$\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right) = \sqrt{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}dx$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \frac{1}{x}dx$$

$$\Rightarrow \log\left(\left(\frac{y}{x}\right) + \sqrt{1+\left(\frac{y}{x}\right)^2}\right) = \ln(x) + c$$

$$= \ln x + \ln k$$

$$\frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} = xk$$

$$\Rightarrow y + \sqrt{x^2+y^2} = x^2 k$$

6. The number of values of x satisfying $\tan^{-1}(4x) + \tan^{-1}(6x) = \frac{\pi}{6}$ and $x \in \left[-\frac{1}{2\sqrt{6}}, \frac{1}{2\sqrt{6}}\right]$ is

(1) 1 (2) 0

(3) 2 (4) 3

Answer (1)

Sol. $\tan^{-1}\left[\dfrac{4x+6x}{1-24x^2}\right]=\dfrac{\pi}{6}$

$$\frac{10x}{1-24x^2}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow 24x^2+10\sqrt{3x}-1=0$$

$$x=\frac{-10\sqrt{3}\pm\sqrt{300+96}}{48}$$

$$=\frac{-10\sqrt{3}\pm\sqrt{396}}{48}$$

$$=\frac{-10\sqrt{3}\pm 2\sqrt{99}}{48}$$

$$=\frac{-5\sqrt{3}\pm\sqrt{99}}{24}$$

$\dfrac{-5\sqrt{3}-\sqrt{99}}{24}$ (rejected) as $x\in\left[-\dfrac{1}{2\sqrt{6}},\dfrac{1}{2\sqrt{6}}\right]$

$$\Rightarrow x=\frac{-5\sqrt{3}+\sqrt{99}}{24}$$

7. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{[x]+4}dx$ is

Where [.] denotes greatest integer function.

(1) $\dfrac{\pi}{20}+\dfrac{7}{20}$

(2) $\dfrac{7\pi}{20}-\dfrac{7}{60}$

(3) $\dfrac{7\pi}{20}-\dfrac{1}{60}$

(4) $\dfrac{7\pi}{20}+\dfrac{1}{60}$

Answer (2)

Sol. $\displaystyle\int_{-\frac{\pi}{2}}^{-1}\frac{1}{2}dx+\int_{-1}^{0}\frac{1}{3}dx+\int_{-0}^{1}\frac{1}{4}dx+\int_{1}^{\frac{\pi}{2}}\frac{1}{5}dx$

$$\frac{1}{2}x\Big]_{-\frac{\pi}{2}}^{-1}+\frac{1}{3}x\Big]_{-1}^{0}+\frac{1}{4}x\Big]_{0}^{1}+\frac{1}{5}x\Big]_{1}^{\frac{\pi}{2}}$$

$$\frac{1}{2}\left[-1+\frac{\pi}{2}\right]+\frac{1}{2}\left(0-(-1)\right)+\frac{1}{4}(1-0)+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-\frac{1}{2}+\frac{\pi}{4}+\frac{1}{3}+\frac{1}{4}+\frac{\pi}{10}-\frac{1}{5}$$

$$\frac{7\pi}{20}-\frac{7}{60}$$

8. If $\dfrac{\cos^2 48^\circ-\sin^2 12^\circ}{\sin^2 24^\circ-\sin^2 6^\circ}=\dfrac{\alpha+\sqrt{5}\beta}{2}$. Then, the value of $(\alpha+\beta)$ is

(1) 3

(2) 2

(3) 4

(4) 1

Answer (3)

Sol. $\dfrac{\cos^2 48^\circ-\sin^2 12^\circ}{\sin^2 24^\circ-\sin^2 6^\circ}=\dfrac{\cos(48^\circ+12^\circ)\cos(48^\circ-12^\circ)}{\sin(24^\circ+6^\circ)\sin(24^\circ-6^\circ)}$

$$=\frac{\cos 60^\circ\cos(36^\circ)}{\sin 30^\circ\sin 18^\circ}=\frac{\cos 36^\circ}{\sin 18^\circ}$$

$$=\frac{\frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}-1}{4}}=\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$=\frac{5+1+2\sqrt{5}}{4}=\frac{6+2\sqrt{5}}{4}$$

$$=\frac{3+\sqrt{5}}{2}$$

α = 3, β = 1

$\alpha+\beta$ = 4

9. If $\int (\cos x)^{-5/2}(\sin x)^{-11/2}\,dx$

$$=\frac{p_1}{q_1}(\cot x)^{9/2}+\frac{p_2}{q_2}(\cot x)^{5/2}+\frac{p_3}{q_3}(\cot x)^{1/2}-\frac{p_4}{q_4}(\cot x)^{-3/2}+c$$

(where c is constant of integration), then value of $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ is

(1) 14

(2) 16

(3) 10

(4) 9

Answer (2)

Sol. $\int (\cos x)^{-5/2}(\sin x)^{-11/2}dx$

$$=\int \frac{1}{(\cos x)^{5/2}(\sin x)^{-1/2}}dx$$

$$=\int \frac{1}{(\cot x)^{5/2}\sin^8 x}dx$$

$$=\int \frac{\cos ec^2 x}{(\sin^6 x)(\cot x)^{5/2}}dx$$

= Let cotx = $t \Rightarrow -\text{cosec}^2\,dx = dt$

= 1 + $\cot^2 x$ = $\text{cosec}^2 x$ = $(1 + t^2)$

$$\Rightarrow \int \frac{(1+t^2)^3(-dt)}{t^{5/2}}$$

$$\Rightarrow \int \frac{(t^6+3t^4+3t^2+1)(-dt)}{t^{5/2}}$$

$$\Rightarrow -\int \left(t^{7/2}+3t^{3/2}+3t^{-1/2}+t^{-5/2}\right)dt$$

$$\Rightarrow -\frac{t^{9/2}}{\frac{9}{2}}-\frac{3t^{5/2}}{\frac{5}{2}}-\frac{3t^{1/2}}{\frac{1}{2}}-\frac{t^{-3/2}}{-\frac{3}{2}}+c$$

$$=-\frac{2}{9}\cot x^{9/2}-\frac{9}{5}\cot x^{5/2}-6\cot x^{1/2}+\frac{2}{3}(\cot x)^{-3/2}+c$$

$$\Rightarrow 15\left(-\frac{2}{9}\times-\frac{6}{5}\times 6\times\frac{2}{3}\right)=16$$

10. The value(s) of α for which the line $\alpha x + 2y = 1$ never touches the hyperbola $\frac{x^2}{9}-\frac{y^2}{1}=1$ is/are

(1) $R-\left\{-\frac{\sqrt{5}}{2},\sqrt{\frac{5}{2}}\right\}$

(2) $R-\left\{-\sqrt{5},\sqrt{5}\right\}$

(3) $R-\left\{-\frac{\sqrt{5}}{3},\frac{\sqrt{5}}{3}\right\}$

(4) R

Answer (3)

Sol. $\frac{x^2}{9}-\frac{y^2}{1}=1$

Line $\alpha x+2y=1$

$T: y = mx \pm \sqrt{9m^2-1}$

$T: 2y = -\alpha x - 1$

$$\frac{1}{2}=-\frac{m}{\alpha}=\frac{\sqrt{9m^2-1}}{-1}$$

$$\frac{1}{4}=9m^2-1$$

$$\frac{5}{36}=m^2 \Rightarrow m=\pm\frac{\sqrt{5}}{6}$$

$-2m = \alpha$

$$\boxed{\alpha=\pm\frac{\sqrt{5}}{3}}$$

For $\alpha=\pm\frac{\sqrt{5}}{3}$, the $\alpha x + 2y = 1$ will become tangent.

$\therefore$ The value of α for which line will not become tangent is $R-\left\{\pm\frac{\sqrt{5}}{3}\right\}$.

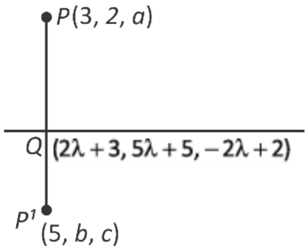
11. If the image of the point $P(3, 2, a)$ reflected about the line $\frac{x-3}{2}=\frac{y-5}{5}=\frac{z-2}{-2}$ is $(5, b, c)$. Then the value of $a^2 + b^2 + c^2$ is

(1) $\frac{4849}{8}$ (2) $\frac{4245}{4}$

(3) $\frac{3947}{8}$ (4) $\frac{2429}{4}$

Answer (1)

Sol.



Q is the mid-point of PP'

$\therefore \ \frac{5+3}{2}=2\lambda+3 \Rightarrow \lambda\frac{1}{2}$

$\frac{b+2}{2}=5\lambda+5=\frac{5}{2}+5=\frac{15}{2}$

$\Rightarrow b = 3$

$\frac{a+c}{2}=-2\lambda+2=1$

$\Rightarrow a + c = 2$ ...(i)

Now,

$2(5 - 3) + 5(b - 2) + (-2)(c - a) = 0$

$\Rightarrow c - a = \frac{59}{2}$ ...(ii)

From (i) & (ii)

$c=\frac{63}{4}, a=\frac{-55}{4}$

$\therefore \ a^2 + b^2 + c^2 = \frac{4849}{8}$

12. If probability distribution is given by

x	0	1	2		3	4	5	6	7
$P(x)$	k	$2k^2$	$6k^2$		$2k^2+k$	$4k$	k	k	k

Then, the value of $P(3 < x \le 6)$ is

(1) 0.6 (2) 0.8

(3) 0.4 (4) 0.2

Answer (1)

Sol. $10k^2 + 9k = 1$

$10k^2 + 9k - 1 = 0$

$10k^2 + 10k - k - 1 = 0$

$10k(k + 1) - 1(k + 1) = 0$

$k=\frac{1}{10}, \ \boxed{k=-1} \rightarrow$ Not possible

$\Rightarrow \boxed{k=\frac{1}{10}}$

$P(3 < x \le 6) = P(x = 4) + P(x = 5) + P(x = 6)$

$=\frac{4}{10}+\frac{1}{10}+\frac{1}{10}$

$=\frac{6}{10}=\frac{3}{5}$

$= 0.6$

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $A=\begin{bmatrix}2 & 3\\ 3 & 5\end{bmatrix}$, then the value of $\left|A^{2025}-3A^{2024}+A^{2023}\right|$ is

Answer (16)

Sol. $|A|^{2023}\left|A^2-3A+I\right|$

$|A| = 1$

Now,

$|(A^2 - 3A + I)|$

$$\Rightarrow \left|\begin{bmatrix}13 & 21\\21 & 34\end{bmatrix}-3\begin{bmatrix}2 & 3\\3 & 5\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right|$$

$$\Rightarrow \left|\begin{bmatrix}8 & 12\\12 & 20\end{bmatrix}\right|$$

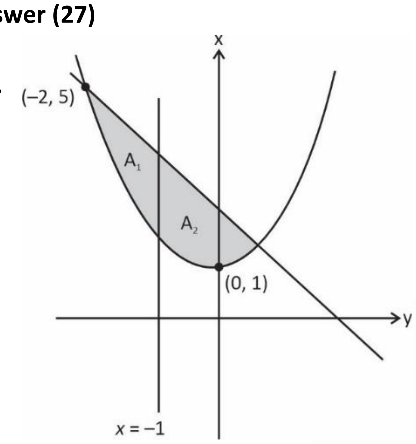
$= 8 \times 20 - 12 \times 12$

$= 16$

22. If the area of the region $\{(x, y): x^2 + 1 \le y \le 3 - x\}$ is divided by the line $x = -1$ in the ratio $m : n$ (where m and n are coprime natural numbers). Then, the value of $m + n$ is

Answer (27)

Sol.



$y = 3 - x$

$y = x^2 + 1$

$3 - x = x^2 + 1$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$\Rightarrow x = -2, 1$

$$A_1=\int_{-2}^{-1}\left((3-x)-(x^2+1)\right)dx$$

$$A_1=\int_{-2}^{-1}\left(3-x-x^2-1\right)dx$$

$$\boxed{A_1=\frac{7}{6}}$$

$$A_2=\int_{-1}^{1}\left((3-x)-\left(x^2+1\right)\right)dx$$

$$\boxed{A_2=\frac{10}{3}} \Rightarrow A_1 : A_2 = \frac{\frac{7}{6}}{\frac{10}{3}}=\frac{7}{20}$$

$\Rightarrow m + n = 27$

23. The number of solution(s) of equation $x|x + 4| + 3|x + 2| + 3 = 0$ is/are equal to

Answer (3.00)

Sol. If $x \ge -2$

Then $x(x + 4) + 3(x + 2) + 3 = x^2 + 7x + 9$

$$\Rightarrow x=\frac{-7\pm\sqrt{49-36}}{2}$$

$$=\frac{-7\pm\sqrt{13}}{2}\Rightarrow x=\frac{-7+\sqrt{13}}{2}$$

If $-4 \le x < -2$

$x(x + 4) + 3(-x - 2) + 3 = x^2 + x - 3 = 0$

$$\Rightarrow x=\frac{-1\pm\sqrt{1+12}}{2}=\frac{-1\pm\sqrt{13}}{2}$$

in $x \in (-4, -2)$, $x=\dfrac{-1-\sqrt{13}}{2}$

If $x \in (-\infty, -4)$

$x(-x - 4) + 3(-x - 2) + 3 = -x^2 - 7x - 3$

$$= -(x^2 + 7x + 3) = x = \frac{-7\pm\sqrt{37}}{2}$$

$$\Rightarrow x=\frac{-7-\sqrt{37}}{2}$$

$\Rightarrow 3$

24.

25.

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