28/01/2026
Evening

Aakash
Medical | IIT-JEE | Foundations

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# Memory Based
# Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

# JEE (Main)-2026 (Online) Phase-1

## (Physics, Chemistry and Mathematics)

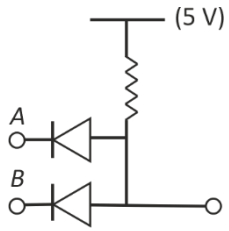
# PHYSICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. For the circuit given below, identify the logic gate.




(1) AND

(2) OR

(3) NAND

(4) NOR

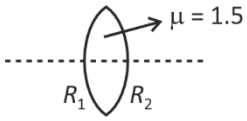
**Answer (1)**

**Sol.** Truth table

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Clearly AND Gate.

2. Object is placed at distance 30 cm from lens given below, then distance of image from lens is ($R_1$ = 10 cm, $R_2$ = 20 cm)




(1) 36 cm (2) 24 cm

(3) 20 cm (4) 30 cm

**Answer (2)**

**Sol.** $\frac{1}{f}=\frac{1}{2}\left(\frac{1}{10}+\frac{1}{20}\right)=\frac{3}{40}$

$\frac{1}{v}-\frac{1}{-30}=\frac{3}{40}$

$\Rightarrow \frac{1}{v}=\frac{5}{120}$

$v$ = 24 cm

3. The position vector is given as $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and if its signs reversed then which of the following physical quantity remains unaffected?

(1) Acceleration

(2) Velocity

(3) Displacement

(4) Torque

**Answer (4)**

**Sol.** $\vec{r}\times\vec{F}\equiv-\vec{r}\times-\vec{F}\equiv\vec{\tau}$

4. If the mass number of nucleus is $\alpha$, its radius is $R_\alpha$. And another mass number of $\beta$ then its radius is $R_\beta$ ; then $\frac{R_\alpha}{R_\beta} = ?$ [Given $\beta = 8\alpha$]

(1) 8 (2) $\frac{1}{2}$

(3) 1 (4) 2

**Answer (2)**

**Sol.** We know $R = R_0(A)^{1/3}$

So, $R_\alpha = R_0(\alpha)^{1/3}$

$R_\beta = R_0(\beta)^{1/3}$

So, $\frac{R_\alpha}{R_\beta} = \left[\frac{\alpha}{\beta}\right]^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

5. Which of following physical quantity is not measurable?

(1) Displacement (2) Voltage

(3) Voltage difference (4) Acceleration

**Answer (2)**

**Sol.** Voltage difference or EMF difference, $\Delta\varepsilon = -\int_1^2 \vec{E}\cdot\vec{dl}$ can be measured. But to define voltage we need a reference point, but as being non-conservative field that will give inconsistent result.

6. Two light sources of 450 nm and 550 nm are used for YDSE with slit distance 2.25 mm and distance between the slits and screen is 1.5 m. Then the distance from central maxima at which minima of both wavelength coincide?

(1) 1.65 mm (2) 1.20 mm

(3) 1.30 mm (4) 1.40 mm

**Answer (1)**

**Sol.** $\frac{\lambda_1}{\lambda_2} = \frac{2m-1}{2n-1} = \frac{9}{11}$

5th maximum of 550 nm coincides will 11th maxima of 450 nm.

$y = \frac{9}{2}\frac{\lambda D}{d} = \frac{9}{2}\times\frac{550\times10^{-9}}{2.25\times10^{-3}}\times\frac{3}{2} = 1.650$

7. A beam of power 2 $\mu$W is hitting a metal surface beam contains photons of wavelength 662 nm. Find number of photons striking per second.

(1) $2 \times 10^{14}$ (2) $6.67 \times 10^{12}$

(3) $4 \times 10^{11}$ (4) $3.2 \times 10^{13}$

**Answer (2)**

**Sol.** $n = \frac{P}{\text{energy of one photons}}$

$= \frac{2\times10^{-6}}{hC/\lambda}$

$= \frac{2\times10^{-6}\times662\times10^{-3}}{6.63\times10^{-34}\times3\times10^{8}}$

$N = 6.67 \times 10^{12}$

8. Mean free path of gas particles of diameter 5 Å at temperature and pressure of 41°C and $1.38 \times 10^5$ Pa.

(1) 14.14 nm (2) 20 nm

(3) 28.28 nm (4) 10 nm

**Answer (3)**

**Sol.** $\lambda = \frac{1kT}{\sqrt{2}\pi d^2 P}$ and $P = nkT$

$\lambda = \frac{1.38\times10^{-23}\times3.14}{1.414\times3.14\times25\times10^{-20}\times1.38\times10^{5}}$

$= \frac{\sqrt{2}}{2\times25}\times10^{-21+20-5}$

9. Find the percentage error in *K*, where $T = 2\pi\sqrt{\frac{m}{k}}$. Given that 60 oscillations completes in 50 second. Time resolution is 2 second, $m$ = 10 g and $\Delta m = \pm 10mg$.

(1) 8% (2) 9 %

(3) 9.1% (4) 8.1 %

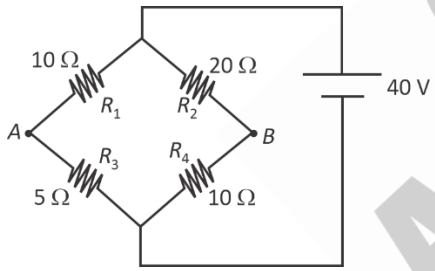
**Answer (4)**

**Sol.** $\frac{\Delta T}{T} = \frac{2}{50}$ and $\frac{\Delta m}{m} = 10^{-3}$

$\Rightarrow \frac{2\Delta T}{T} + \frac{\Delta m}{m} = \frac{\Delta k}{k}$

$\Rightarrow \left(\frac{4}{50} + 10^{-3}\right) \times 100 = 8.1\%$

10. In a balanced wheatstone bridge $R_2R_3 : R_1R_4$. Because of heating $R_3$ increases by 20%. Then potential difference across *A* & *B* becomes



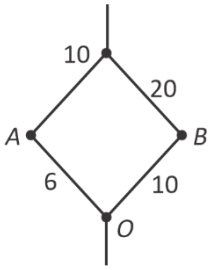

(1) 1.50 V (2) 2.40 V

(3) 1.67 V (4) 3.60 V

**Answer (3)**

**Sol.** $V_A = \frac{40}{16} \times 6$

$V_B = \frac{40}{3}$




$\Delta V = 40 \times \frac{3}{8} - \frac{40}{3}$

$= 40\left\{\frac{3}{8} - \frac{1}{3}\right\}$

$= \frac{5}{3}$

11. Match the two columns and choose the correct option.

| | **Column-I** | | **Column-II** |
|---|---|---|---|
| (a) | Coefficient of viscosity | (p) | $[ML^0T^{-2}]$ |
| (b) | Surface tension | (q) | $[ML^{-1}T^{-2}]$ |
| (c) | Pressure | (r) | $[ML^2T^{-2}]$ |
| (d) | Work | (s) | $[ML^{-1}T^{-1}]$ |

(1) (a)-(p), (b)-(q), (c)-(r), (d)-(s)

(2) (a)-(s), (b)-(p), (c)-(q), (d)-(r)

(3) (a)-(q), (b)-(s), (c)-(p), (d)-(r)

(4) (a)-(p), (b)-(q), (c)-(s), (d)-(r)

**Answer (2)**

**Sol.** For (a) $F = -\eta A\frac{dv}{dx}$

(b) $T = \frac{F}{L}$

(c) $P = \frac{F}{A}$

(d) $W = \vec{F} \cdot \vec{r}$

12. Given that $v = \sqrt{\frac{Y}{P}}$. Find the maximum % error in $v$.

Given that $\frac{\Delta Y}{Y}\times 100 = 1\%$ and $\frac{\Delta P}{P}\times 100 = 0.5\%$

(1) $\frac{3}{2}\%$

(2) $\frac{3}{4}\%$

(3) 1%

(4) $\frac{1}{2}\%$

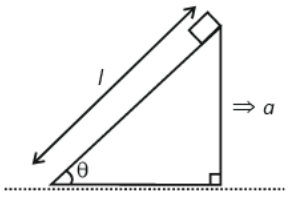
**Answer (2)**

**Sol.** $\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta Y}{Y} + \frac{1}{2}\frac{\Delta P}{P}$

So $\left(\frac{\Delta v}{v}\times 100\right) = \frac{1}{2}\left(\frac{\Delta Y}{Y}\right)\times 100 + \frac{1}{2}\left(\frac{\Delta P}{P}\times 100\right)$

$\Rightarrow \left(\frac{\Delta V}{V}\times 100\right) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}\%$

13. Find the time taken by block to reach the ground when released from a top of a wedge of inclination $\theta$ and length $l$, maintained at a constant acceleration '$a$' to the right as shown. (all contacts case smooth and block doesn't loose contact with wedge).




(1) $t = \sqrt{\frac{l}{g\cos^2\theta + a\sin\theta}}$

(2) $t = \sqrt{\frac{l}{g\sin\theta + a\cos\theta}}$

(3) $t = \sqrt{\frac{2l}{g\sin\theta + a\cos\theta}}$

(4) $t = \sqrt{\frac{l}{2g\cos\theta + a\sin\theta}}$

**Answer (3)**

**Sol.** Applying pseudo force on block and resolving forces along incline we get $A = g\sin\theta + a\cos\theta$.

$\Rightarrow \frac{1}{2}At^2 = l$

$\Rightarrow t = \sqrt{\frac{2l}{g\sin\theta + a\cos\theta}}$

14.

15.

16.

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21.

22.

23.

24.

25.

# CHEMISTRY

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. The plot of $\log_{10}k$ vs $\frac{1}{T}$ gives a straight line. The intercept and slope respectively are

(1) c = logA

$$m = -\frac{E_a}{2.303R}$$

(2) $c = -\frac{E_a}{2.303R}$

m = logA

(3) c = –logA

$$m = -\frac{E_a}{2.303R}$$

(4) c = logA

$$m = \frac{E_a}{2.303R}$$

**Answer (1)**

**Sol.** $k = Ae^{-\frac{E_a}{RT}}$

$$\log k = \log A - \frac{E_a}{2.303RT}$$

y = c + mx

c = intercept = logA

m = slope = $-\frac{E_a}{2.303R}$

2. Consider the following electromagnetic waves :

Wavelength of A = 400 nm

Frequency of B = $10^{16}$ $s^{-1}$

Wave number of C = $10^4$ $cm^{-1}$

Order of energies is

(1) A > B > C (2) B > A > C

(3) B > C > A (4) C > A > B

**Answer (2)**

**Sol.** $\lambda_A = 400 \times 10^{-9}$ m = $4 \times 10^{-7}$ m

$\lambda_B = 3 \times 10^{-8}$ m

$\lambda_C = 10^{-6}$ m

$\lambda_C > \lambda_A > \lambda_B$

Energy order will be opposite.

3. Which of the following order is correct.

(1) HF > HI > HBr > HCl (Boiling point)

(2) HF > HI > HBr > HCl (Melting point)

(3) HI > HF > HBr > HCl (Boiling point)

(4) HI > HBr > HF > HCl (Melting point)

**Answer (1)**

**Sol.**

| | HF | HCl | HBr | HI |
|---|---|---|---|---|
| B.P. (K) | 293 | 189 | 206 | 238 |
| MP (K) | 190 | 159 | 185 | 222 |

4. Match the isostructural species

| | Column-I | | Column-II |
|---|---|---|---|
| (a) | $XeO_3$ | (p) | $BrF_5$ |
| (b) | $XeF_2$ | (q) | $NH_3$ |
| (c) | $XeO_2F_2$ | (r) | $I_3^-$ |
| (d) | $XeOF_4$ | (s) | $SF_4$ |

(1) a – q, b – r, c – s, d – p

(2) a – p, b – q, c – s, d – p

(3) a – q, b – r, c – p, d – s

(4) a – p, b – q, c – r, d – s

**Answer (1)**

**Sol.** $XeO_3$ and $NH_3$ are trigonal pyramidal

$XeF_2$ and $I_3^-$ are linear

$XeO_2F_2$ and $SF_4$ are see-saw

$XeOF_4$ and $BrF_5$ are square pyramidal

5. Consider a reaction $A \rightleftharpoons B$. At 'T' K, the equilibrium concentration of A and B are 0.3 M and 0.315 M. Now, 0.1 mol of A is added to the flask of 1 L, then equilibrium constant and equilibrium concentration of B are

(1) 1.05, 0.35 M (2) 0.95, 0.37 M

(3) 1.05, 0.37 M (4) 0.95, 0.35 M

**Answer (3)**

**Sol.**

| | A | $\rightleftharpoons$ | B |
|---|---|---|---|
| $t_{eq}$ | 0.3 M | | 0.315 M |

$$K_{eq} = \frac{0.315}{0.3}$$

Now 0.1 mole of A is added,

| | A | $\rightleftharpoons$ | B |
|---|---|---|---|
| $t = 0$ | $0.3 + 0.1 = 0.4$ | | |
| $t = t_{eq}$ | $0.4 - x$ | | $0.315 + x$ |

$$1.05 = \frac{0.315 + x}{0.4 - x}$$

$1.05 \times 0.4 - 1.05x = 0.315 + x$

$0.42 - 0.315 = 2.05x$

$0.105 = 2.05x$

$x = 0.051$

[B] = 0.366 M

6. Diamagnetic species among the following complexes is

(1) $[MnBr_4]^{2-}$

(2) $[Cu(H_2O)_6]^{2+}$

(3) $[Ni(CN)_4]^{2-}$

(4) $[Ni(H_2O)_6]^{2+}$

**Answer (3)**

**Sol.** $[MnBr_4]^{2-} \Rightarrow Mn^{2+} \Rightarrow 3d^5$

$\Rightarrow sp^3$ hybridised

$\Rightarrow$ paramagnetic (n = 5)

$[Cu(H_2O)_6]^{2+} \Rightarrow Cu^{2+} \Rightarrow 3d^9$

$\Rightarrow sp^3d^2$ hybridised

$\Rightarrow$ paramagnetic (n = 1)

$[Ni(CN)_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow 3d^8$

$\Rightarrow dsp^2$ hybridised

$\Rightarrow$ diamagnetic (n = 0)

$[Ni(H_2O)_6]^{2+} \Rightarrow Ni^{2+} \Rightarrow 3d^8$

$\Rightarrow sp^3d^2$ hybridised

$\Rightarrow$ paramagnetic (n = 2)

7. Correct statement about $-NO_2$ group is

(A) Ring deactivating group in electrophilic substitution

(B) Ring activating group in electrophilic substitution

(C) Activating for aromatic nucleophilic substitution in aryl halides

(D) Deactivating for aromatic nucleophilic substitution in aryl halides.

(1) A, C are correct statement

(2) B, D are correct

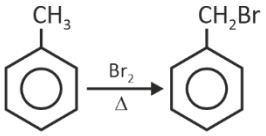
(3) A, D are correct

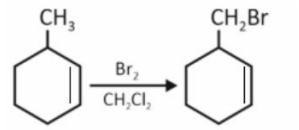
(4) B, C are correct

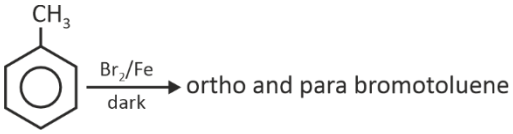
**Answer (1)**

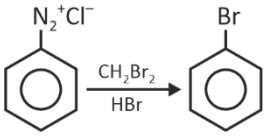
**Sol.** It shows –R and –I effect.

8. The major organic product of which of the following reaction is incorrectly represented?

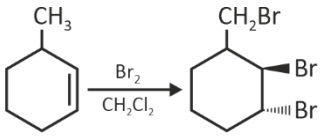
(1) 

(2) 

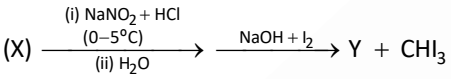
(3) 


(4) 

**Answer (2)**

**Sol.** 

9. Consider the following reaction sequence



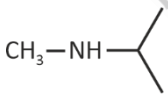

X has %C = 65.75

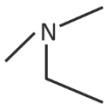
%H = 15.25

%N = 19

Identify (X)

(1) 

(2) 

(3) 

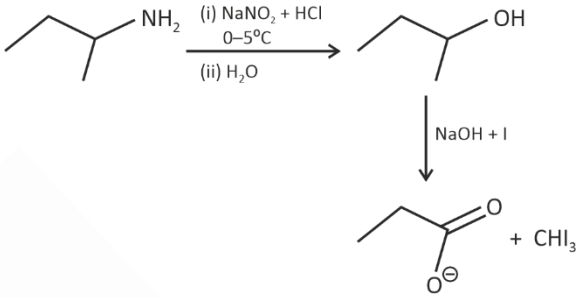
(4) 

**Answer (4)**

**Sol.**

| | mole | | molar ratio |
|---|---|---|---|
| C = 65.75 $\Rightarrow$ | $\frac{65.75}{12} = 5.5$ | = | 4 |
| H = 15.25 $\Rightarrow$ | $\frac{15.25}{1} = 15.25$ | = | 11 |
| N = 19 $\Rightarrow$ | $\frac{19}{14} = 1.35$ | = | 1 |

MF = $C_4H_{11}N$




10. Which of the following compounds on reacting with Heinsberg reagent form an alkali insoluble product.

(A) Ethanamine

(B) N-methylaniline

(C) N-Ethyl-N-Methylaniline

(D) N-Methylethanamine

(E) N-Phenylaniline

(F) Aniline

(1) A, C, D, E only

(2) B, C, D & E only

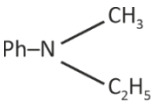
(3) B, D, and E only

(4) A, C, F only

**Answer (3)**

**Sol.** (A) Ethanamine $\Rightarrow CH_3CH_2NH_2$ (1°)

(B) N-Methylaniline $\Rightarrow$ Ph–NH–$CH_3$ (2°)

(C) N-Ethyl-N-methylaniline $\Rightarrow$  (3°)

(D) N-Methyethanamine $\Rightarrow CH_3CH_2NHCH_3$ (2°)

(E) N-Phenylaniline $\Rightarrow$ Ph–NH–Ph (2°)

(F) Aniline $\Rightarrow$ Ph–$NH_2$ (1°)

2° Amines react with Heinsberg reagent to form an alkali insoluble product.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The sum of valence $e^-$ in element with most and least metallic character among the following is :

Na, P, Cl, S, O and F

**Answer (8)**

**Sol.** Element with most metallic character $\Rightarrow$ Na

Element with least metallic character $\Rightarrow$ F

Sum of valence $e^-$ in Na and F = 1 + 7 = 8

22. In 'S' estimation, 0.314 g of organic compound gave 0.4813 g of barium sulphate. What is % of 'S' in organic compound?

(Report to nearest integer).

**Answer (21)**

**Sol.** $$\% \text{ of } S = \frac{\frac{0.4813}{233} \times 32}{0.314} \times 100$$

$$= 21.05$$

23. Among $Sc^{3+}$, $Cr^{2+}$, $Mn^{3+}$, $Co^{3+}$ number of isoelectronic species are 'n'.

'n' moles of AgCl is obtained upon reaction with excess of $AgNO_3$ with 1 mol of $Co(en)_2NH_3Cl_3$. Number of $t_{2g}$ electrons in the complex are

**Answer (6)**

**Sol.** $Sc^{3+} \Rightarrow 3d^0$

$Cr^{2+} \Rightarrow 3d^4$

$Mn^{3+} \Rightarrow 3d^4$

$Co^{3+} \Rightarrow 3d^6$

n = 2

2 mol of AgCl is precipitated.

Hence, complex should be $[Co(en)_2(NH_3)Cl]Cl_2$

complex is inner orbital octahedral complex with hybridisation $d^2sp^3$

$Co^{3+} \Rightarrow 3d^6$

$t_{2g}^6 e_g^0$

24. An alpha particle and proton are accelerated in a discharge tube under same potential difference of 200 KeV. The de Broglie wavelength of proton is $x\sqrt{2}$ times of de Broglie wavelength of $\alpha$-particle. The value of x is

**Answer (2)**

**Sol.** $$\lambda = \frac{h}{\sqrt{2m(qV)}}$$

$$\frac{\lambda_P}{\lambda_\alpha} = \left[\frac{\frac{h}{\sqrt{2\times1\times1\times200\times1000}}}{\frac{h}{\sqrt{2\times4\times2\times200\times1000}}}\right]$$

$$\frac{\lambda_P}{\lambda_\alpha} = \sqrt{8} = 2\sqrt{2}$$

25.

# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The sum of coefficients of $x^{499}$ and $x^{500}$ in the binomial expansion of $(1+x)^{1000} + (1+x)^{999}(x) + x^2(1+x)^{998} + \ldots + x^{1000}$ is

(1) ${}^{1002}C_{501}$ (2) ${}^{1002}C_{500}$

(3) ${}^{1001}C_{500}$ (4) ${}^{1001}C_{501}$

**Answer (2)**

**Sol.** $S = (1+x)^{1000} + x(1+x)^{949} + x^2(1+x)^{898} + \ldots + x^{1000}$

$$\Rightarrow 8 = (1+x)^{1000}\frac{\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{1-\left(\frac{x}{1+x}\right)}$$

$$= (1+x)^{1001}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]$$

$$= (1+x)^{1001} - x^{1001}$$

Coefficient's sum of $x^{499}$ and $x^{500}$

$= {}^{1001}C_{499} + {}^{1001}C_{500} = {}^{1002}C_{500}$

2. If $\sum_{r=1}^{25}\frac{r}{r^4+r^2+1} = \frac{p}{q}$, where $p$ and $q$ are coprime positive integer, then $p+q$ is equal to

(1) 841 (2) 976

(3) 984 (4) 890

**Answer (2)**

**Sol.** $$\sum_{r=1}^{25}\frac{r}{(r^2+1)^2-r^2} = \sum_{r=1}^{25}\frac{r}{(r^2+1+r)(r^2+1-r)}$$

$$= \frac{1}{2}\sum_{r=1}^{25}\left(\frac{1}{r^2+1-r} - \frac{1}{r^2+1+r}\right)$$

$$= \frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\ldots\left(\frac{1}{601}-\frac{1}{657}\right)\right]$$

$$= \frac{1}{2}\left[1-\frac{1}{651}\right]$$

$$= \frac{1}{2}\left[\frac{650}{651}\right]$$

$$= \frac{325}{657}$$

$\therefore p+q = 976$

3. $\frac{6}{3^{26}} + \frac{10}{3^{25}} + \frac{10.2}{3^{24}} + \frac{10.2^2}{3^{23}} + \ldots + \frac{10.2^{24}}{3}$ is equal to

(1) $2^{26}$ (2) $2^{25}$

(3) $3^{26}$ (4) $3^{25}$

**Answer (1)**

**Sol.** $$\frac{6}{3^{26}} + \frac{10.1}{3^{25}} + \frac{10.2}{3^{24}} + \ldots + \frac{10.2^{24}}{3}$$

Let $S = 10\left[\frac{1}{3^{25}} + \frac{2}{3^{24}} + \frac{2^2}{3^{23}} + \ldots + \frac{2^{24}}{3}\right]$

$$S = 10\frac{[6^{25}-1]}{5\times 3^{25}}$$

$$= \frac{2}{3^{25}}[6^{25}-1]$$

Now sum $= \frac{6}{3^{26}} + \frac{2}{3^{25}}[6^{25}-1]$

$$= \frac{6}{3^{26}} + \frac{6}{3^{26}}[6^{25}-1]$$

$$= \frac{6^{26}}{3^{26}} = 2^{26}$$

4. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12(3+[x])}{3+[\sin x]+[\cos x]}dx$, where [ ] represents greatest integer function, is equal to

(1) $3 + 10\pi$ (2) $11\pi + 4$

(3) $10\pi + 2$ (4) $11\pi + 2$

**Answer (4)**

**Sol.** $\int_{-\frac{\pi}{2}}^{-1} \frac{12(3-2)}{3-1+0}dx + \int_{-1}^{0} \frac{12(3-1)}{3-1+0}dx +$

$$\int_{0}^{1} \frac{12(3+0)}{3+0+0}dx + \int_{1}^{\frac{\pi}{2}} \frac{12(3+1)}{3+0+0}dx$$

$$= \int_{-\frac{\pi}{2}}^{-1} 6dx + \int_{-1}^{0} 12dx + \int_{0}^{1} 12dx + \int_{1}^{\frac{\pi}{2}} 16dx$$

$$= 6\left(-1+\frac{\pi}{2}\right) + 12(1) + 12(1) + 16\left(\frac{\pi}{2}-1\right)$$

$= -6 + 3\pi + 12 + 12 + 8\pi - 16$

$= 2 + 11\pi$

5. By the principal of inverse trigonometric function, the value of $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)-2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to

(1) $\frac{33}{56}$ (2) $\frac{31}{55}$

(3) $\frac{32}{59}$ (4) $\frac{38}{55}$

**Answer (1)**

**Sol.** $\because \tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)-2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$

$$= \tan\left(2\tan^{-1}\left(\frac{2}{3}\right)-2\tan^{-1}\left(\frac{1}{3}\right)\right)$$

$$= \tan\left(2\tan^{-1}\left(\frac{\frac{2}{3}-\frac{1}{3}}{1+\frac{2}{9}}\right)\right)$$

$$= \tan\left(2\tan^{-1}\left(\frac{3}{11}\right)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\frac{6}{11}}{1-\frac{9}{14}}\right)\right)$$

$$= \frac{66}{112} = \frac{33}{56}$$

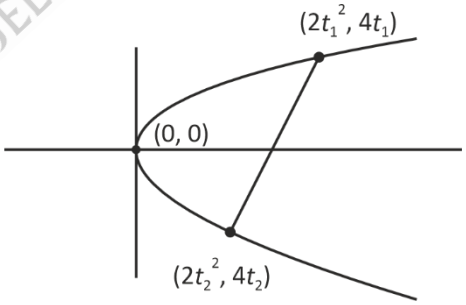
6. Let a triangle $ABC$ such that $A \equiv (0, 0)$ and vertices B and C lie on the parabola $y^2 = 8x$ such that $\left(\frac{7}{3}, \frac{4}{3}\right)$ is the centroid of the $\Delta ABC$ then $(BC)^2$ is equal to

(1) 90 (2) 120

(3) 150 (4) 110

**Answer (2)**

**Sol.**




$$\Rightarrow \frac{2\left(t_1^2+t_2^2\right)}{3} = \frac{7}{3} \Rightarrow t_1^2 + t_2^2 = \frac{7}{2}$$

$$\frac{4(t_1+t_2)}{3}=\frac{4}{3}\Rightarrow t_1+t_2=1$$

$$\Rightarrow (t_1-t_2)^2=(t_1+t_2)^2-4t_1t_2$$

also, $(t_1+t_2)^2=\frac{7}{2}+2t_1t_2=1$

$$\Rightarrow t_1t_2=\frac{-5}{4}$$

$$\Rightarrow (t_1-t_2)^2=1-4\left(\frac{-5}{4}\right)=6$$

Now, $BC=\sqrt{\left(2t_1^2-2t_2^2\right)^2+\left(4(t_1+t_2)\right)^2}$

$$BC=2|(t_1-t_2)|\sqrt{(t_1+t_2)^2+4}$$

$$\Rightarrow BC=2\sqrt{6}(\sqrt{5})$$

$$\Rightarrow BC^2=4\times6\times5=120$$

7. Let = $A\{Z\in C : |Z-2|\le 4\}$ and $B\{Z\in C : |Z-2|+|Z+2|\le 4\}$ then max $\{Z_1-Z_2\} : Z_1\in A$ and $Z_2\in B\}$ is equal to

(1) 6 (2) 8

(3) 4 (4) 5

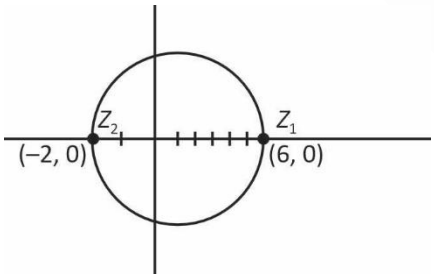
**Answer (2)**

**Sol.** $A\Rightarrow (x-2)^2+y^2+\le 16$

$B\Rightarrow$ Line segment

$$\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}\le 4$$

$\Rightarrow y=0, x\in(-2, 2)$




The max $|z-z_2|$ = diameter or circle

= 8

8. If the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{5}{16}$ and $a$, 4, $\alpha$, $\beta$ are in increasing A.P. then both the roots of the equation $ax^2-ax+2(\alpha-2\beta)=0$ lie between

(1) (–3, 0)

(2) (–2, 3)

(3) (0, 3)

(4) (–3, 1)

**Answer (2)**

**Sol.** Let $4-d$, 4, $4+d$, $4+2d$ are in A.P.

$$\Rightarrow \frac{1}{2}\left(\frac{1}{4-d}+\frac{1}{4+2d}\right)=\frac{5}{10}$$

$$\Rightarrow 5d^2-6d-8=0$$

$$\Rightarrow d=2, \frac{-4}{5}$$

$\Rightarrow d=2$

$a=2, \beta=8, \alpha=6$

$6x^2-2x+2(6-16)=0$

$\Rightarrow 3x^2-x-10=0$

$\Rightarrow x=2, \frac{-5}{3}$

9. The range of $f(x)$ = sgn(sin$x$)+ sgn(cos$x$) + sgn(tan$x$) + sgn(cot$x$), $x\ne\frac{n\pi}{2}, n\in I$,

where $\text{sgn}(t)=\begin{cases}1, & t>0\\ -1, & t<0\\ 0, & t=0\end{cases}$

(1) {4, – 4, 2, – 2} (2) {–2, 0, 4}

(3) {4, –4, 0, –2} (4) {2, – 2, 0, 4, –4}

**Answer (2)**

**Sol.** $f(x)$ = sgn(sin$x$)+ sgn(cos$x$) + sgn(tan$x$) + sgn(cot$x$),

$x\notin\frac{n\pi}{2}, n\in I$,

The value of

$$f(x)=\begin{cases}1+1+1+1, & for \quad x\in\left(0,\frac{\pi}{2}\right)\\ 1-1-1-1, & for \quad x\in\left(\frac{\pi}{2},\pi\right)\\ -1-1+1+1, & for \quad x\in\left(\pi,\frac{3\pi}{2}\right)\\ -1+1-1-1, & for \quad x\in\left(\frac{3\pi}{2},2\pi\right)\end{cases}$$

Hence range of $f(x)$ = {–2, 0, 4?

10. **Statement I:** $25^{13}+20^{13}+8^{13}+3^{13}$ is divisible by 7.

**Statement II:** The integral value of $\left(7+4\sqrt{3}\right)^{25}$ is an odd number

(1) Neither statements are correct

(2) Only statement I is correct

(3) Only statement II is correct

(4) Both the statements are correct

**Answer (4)**

**Sol.** **Statement 1 :**

$(25^{13}+3^{13})+(20^{13}+8^{13})$ is divisible by 28 as

$(a+b)|\ a^n+b^n$ for $n\in$ odd natural numbers

or using congruence modular arithmetic,

$25^{13}+3^{13}+20^{13}+8^{13}$

$\equiv\left(4^{13}+3^{13}+(-1)^{13}+1^{13}\right)\pmod 7$

$\equiv\left((-3)^{13}+3^{13}\right)\pmod 7$

$\equiv 0 \pmod 7$ $\Rightarrow$ 7 divides the sum

**Statement 2**

Let $I+f_1=(7+4\sqrt{3})^{25}$

$f_2=(7-4\sqrt{3})^{25}$ as $0<7-4\sqrt{3}<1$

$\Rightarrow I+f_1+f_2=$

$2\left({}^{25}C_1(4\sqrt{3})^{24}7^1+{}^{25}C_3(4\sqrt{3})^{22}7^3+\ldots+{}^{25}C_{25}(4\sqrt{3})^0 7^{25}\right)$

= $2k$ for some integer $k$

$\Rightarrow$ Since $f_1+f_2\in(0,2)$

$\Rightarrow I+f_1+f_2=2k\Rightarrow I=2k-1$

as only integer in

(0, 2) is 1 $\Rightarrow f_1+f_2=1\Rightarrow I$ is odd.

11. Let $y=y(x)$ be the solution of the differential equation $x\frac{dy}{dx}-y=x^2\cot x$, $x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right)-8y\left(\frac{\pi}{4}\right)$ is

(1) $2\pi$ (2) $-3\pi$

(3) $-\pi$ (4) $\pi$

**Answer (3)**

**Sol.** $x\frac{dy}{dx}-y=x^2\cot x$

$\frac{dy}{dx}-\frac{y}{x}=x\cot x$

$e^{-\int\frac{1}{x}dx}=e^{-\ln x}=\frac{1}{x}$

$\frac{y}{x}=\int\cot x\,dx$

$\frac{y}{x}=\int\ln|\sin x|+c$

$y=x\ln|\sin x|+cx$

$y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$

$\Rightarrow c = 1$

$\Rightarrow y = x\ln|\sin x| + x$

$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$

$= 6\left[\frac{\pi}{6}\ln\left(\frac{1}{2}\right)+\frac{\pi}{6}\right]-8\left[\frac{\pi}{4}\ln\left(\frac{1}{\sqrt{2}}\right)+\frac{\pi}{4}\right]$

$\pi - 2\pi = -\pi$

12. Let $f(x)=\lim\limits_{\theta\to 0}\dfrac{\cos\pi x-(x^{2/\theta})\sin(x-1)}{1+(x^{2/\theta})\sin(x-1)}, x\in R$. Then which of the following is correct.

(1) $f$ is continuous at $x = 1$ and $f(1) = -1$

(2) $f$ is discontinuous at $x = -1$ and $f(1) = -1$

(3) $f$ is continuous at $x = 1$ and $f(1) = 1$

(4) $f$ is discontinuous at $x = 1$ and $f(1) = 1$

**Answer (1)**

**Sol.** $f(x)=\lim\limits_{\theta\to 0}\dfrac{\cos\pi x-(x^2)^{1/\theta}\sin(x-1)}{1+(x^2)^{1/\theta}(x-1)}, x\in R$

$$f(x)=\begin{cases}\dfrac{-\sin(x-1)}{(x-1)}, & x>1\\ -1, & x=1\\ \cos(\pi x), & x<1\end{cases}$$

$\Rightarrow \lim\limits_{x\to 1} f(x) = -1$

13. **Statement I:** The function $F$ defined from $R \to R$ $F(x)=\dfrac{x}{1+|x|}$ is one-one

**Statement II:** The function $F$ defined from $R \to R$ $F(x)=\dfrac{x^2+4x-30}{x^2-8x+18}$ is may-are

(1) Statement I is correct but statement II is not correct

(2) Statement I and statement II both are correct

(3) Statement I is incorrect but statement II is correct

(4) Both statement are incorrect

**Answer (2)**

**Sol.** **Statement 1:** $F(x)=\dfrac{x}{1+|x|}$

$$F(x)=\begin{cases}\dfrac{x}{1-x} & x<0\\ \dfrac{x}{1+x} & x\ge 0\end{cases}$$

$$F'(x)=\begin{cases}\dfrac{1}{(1-x)^2} & x<0\\ \dfrac{1}{(1+x)^2} & x\ge 0\end{cases}$$

$\Rightarrow F(x)$ is increasing in $R$ and $f(x)$ is continuous $\forall x\in R$

$\Rightarrow F(x)$ is one-one

**Statement 2:** $F(x)=\dfrac{x^2+4x-30}{x^2-8x+18}$

$$F'(x)=\frac{(x^2-8x+18)(2x+4)-(x^2+4x-30)(2x-8)}{(x^2-8x+18)^2}$$

$$=\frac{(2x^3-16x^2+36x+4x^2-32x+72)-(2x^3+8x^2-60x-8x^2-32x+240)}{(x^2-8x+18)^2}$$

$$=-\frac{12x^2+96x-72}{(x^2-8x+18)^2}$$

$$=-\frac{12(x^2-8x+6)}{(x^2-8x+18)^2}$$

$F'(x)$ will become 0 at 2 distinct points and also $F(x)$ is continuous.

$\Rightarrow F(x)$ is many-one function.

14. Consider the data:

| $x$: | $4k$ | $\frac{30}{7}k$ | $\frac{32}{7}k$ | $\frac{34}{7}k$ | $\frac{36}{7}k$ | $\frac{38}{7}k$ | $\frac{40}{7}k$ | $6k$ |
|---|---|---|---|---|---|---|---|---|
| $p(x)$: | $\frac{2}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{15}$ |

If $E(x)=\frac{263}{15}$, then $P(x < 20)$ is equal to

(1) $\frac{1}{15}$ (2) $\frac{8}{15}$

(3) $\frac{4}{15}$ (4) $\frac{14}{15}$

**Answer (4)**

**Sol.** $E(x) = \frac{263}{15}$

$\Rightarrow \sum xP(x)=\frac{263}{15}$

$\Rightarrow \frac{526}{105}k=\frac{263}{15}$

$\Rightarrow k=\frac{7}{2}$

$P(x < 20) = 1 - P(6k)$

$= 1 - P(21)$

$= 1=\frac{1}{15}$

$= \frac{14}{15}$

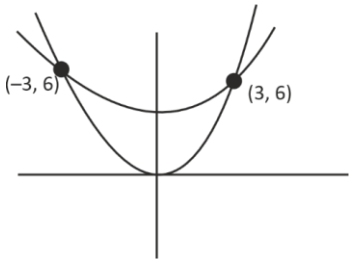
15. Let $P_1 : y=4x^2$ and $P_2 : x^2+27$ be two parabolas. If the area bounded by region enclosed by $P_1$ and $P_2$ is 6 times the area bounded by $P_1$ and $y=\alpha x, \alpha > 0$, then $\alpha$ is equal to

(1) 8 (2) 12

(3) 13 (4) 6

**Answer (2)**

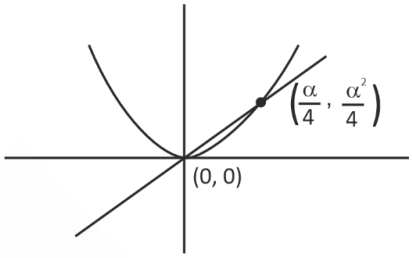
**Sol.**




$A_1$ = Area between $P_1$ and $P_2$

$\Rightarrow \int_{-3}^{3}\left[(x^2+27)-4x^2\right]dx=108$

$\Rightarrow$ Area between $P_1$ and $y = \alpha x$ is




$\Rightarrow A_2 = \int_0^{\alpha/4}\left(\alpha x-4x^2\right)dx$

$=\frac{\alpha x^2}{2}-\frac{\alpha x^3}{3}\Big|_0^{\alpha/4}$

$=\frac{\alpha^3}{32}\frac{-4}{3}\times\frac{\alpha^3}{64}=\frac{108}{6}=18$

$=\frac{\alpha^3}{32}\left(1-\frac{2}{3}\right)=18$

$\Rightarrow \alpha^3=18\times32\times3=27\times64$

$\Rightarrow \alpha=3\times4=12$

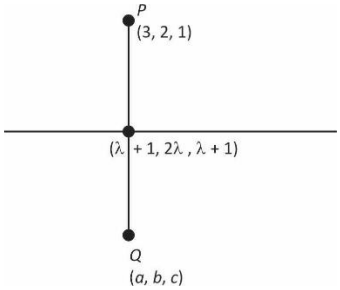
16. Let $Q$ be the image of the point $P(3, 2, 1)$ in the line $\frac{x-1}{1}=\frac{y}{2}=\frac{z-1}{1}$, then the distance of $Q$ from the line $\frac{x-9}{3}=\frac{y-9}{2}=\frac{3-5}{-2}$ is

(1) 3 (2) 4

(3) 5 (4) 7

**Answer (4)**

**Sol.**




$PQ \perp$ line

$\Rightarrow \quad (\lambda+1-3)(1)+(2\lambda-2)(2)+(\lambda+1-1)(1)=0$

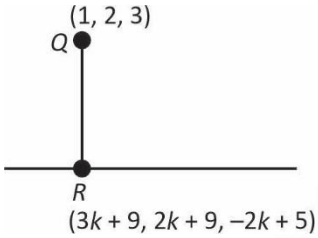
$\Rightarrow (\lambda-2)+4(\lambda-1)+\lambda=0$

$6\lambda=6 \Rightarrow \lambda=1$

$\Rightarrow$ Image point is $Q(1,2,3)$.

The Distance of Point $Q$ from a Second Line

$$\frac{x-9}{3}=\frac{y-9}{2}=\frac{z-5}{-2}$$




$(3k+9-1)(3)+(2k+9-2)(2)+(-2k+5-3)(-2)=0$

$\Rightarrow k=-2$

$R \equiv (3k+9, 2k+, -2k+5)$, for $k = -2$

$\Rightarrow \quad R \equiv (3,5,9)$

$QR=\sqrt{2^2+3^2+6^2}=7$

17.

18.

19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B$ be a $2 \times 2$ matrix such that $A^{100} = 1008 + I$, then sum of all elements of $B^{100}$ is

**Answer (0)**

**Sol.** $A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$$A^2=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 3.3-4 & -3.4+1.4 \\ 1.3-1 & -4+1 \end{bmatrix}=\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 2.2+1 & -(4.2) \\ 1.2 & -(2.2-1) \end{bmatrix}$$

$A^2=\begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$, $A^3=\begin{bmatrix} 11 & -10 \\ 5 & -9 \end{bmatrix}$, ....

$$\therefore \quad A^n=\begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$$

$$\Rightarrow A^{100}=\begin{bmatrix} 201 & -400 \\ 100 & -199 \end{bmatrix}=100B+\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 200 & -400 \\ 100 & -200 \end{bmatrix}=100B$$

$$\Rightarrow \quad B=\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$$

$$B^2=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow B^{100}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Sum of elements of $B = 0$

22. Ellipse $\frac{x^2}{144}+\frac{y^2}{169}=1$ and hyperbola $\frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$ have same focus and $e$ and $L$ denotes the eccentricity and length of latus rectum of hyperbola then $24(e + L)$ is

**Answer (296)**

**Sol.** $\frac{x^2}{144}+\frac{y^2}{169}=1$

$e^2=1-\frac{144}{169}$

$=\frac{25}{169}\Rightarrow e=\frac{5}{13}$

$F_{\text{ellipse}}=\left(0,\pm\frac{5}{13}\times 13\right)$

$=(0,\pm 5)$

$\frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$

$e_H^2=1+\frac{16}{\lambda^2}$

$F\equiv\left(0,\pm\lambda\sqrt{1+\frac{16}{\lambda^2}}\right)$

$\Rightarrow\quad \lambda\sqrt{1+\frac{16}{\lambda^2}}=5$

$\lambda^2 + 16 = 25$

$\lambda^2 = 9$

Now $e_H=\sqrt{1+\frac{16}{9}}=\sqrt{\frac{25}{9}}=\frac{5}{3}$

$L(\text{LR}) = \frac{2a^2}{b}=\frac{2\times 16}{3}=\frac{32}{3}$

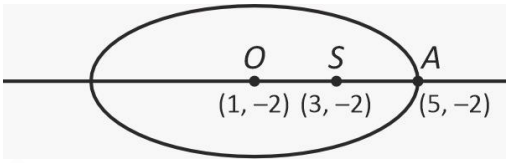
$24(e+L) = 24\left(\frac{5}{3}+\frac{32}{3}\right)=24\left(\frac{37}{3}\right)$

$= 8 \times 37 = 296$

23. An ellipse has centre at (1, –2) and one of the focus at (3, –2) and one vertex at (5, –2), then the length of its latus rectum is

**Answer (6)**

**Sol.**




$\therefore$ ae = 2 and a = 4

$\therefore\ e=\frac{1}{2}$

$b^2 = a^2(1 - e^2)$

$b^2=16\left(1-\frac{1}{4}\right)=12$

$\therefore$ Length of lotus rectum $=\frac{2b^2}{a}=\frac{2\times 12}{4}$

= 6

24. $F(x)=\int\frac{dx}{x^{2/3}+2x^{1/2}}$ be such that $F(0)=-26+24\ln 2$. If $F(1)=a+b\ln 3$, then $a + b$ is equal to

**Answer (13)**

**Sol.** $\int\frac{dx}{x^{2/3}+2x^{1/2}}$

Let $x = t^6$

$dx = 6t^5dt$

$=6\int\frac{t^5dt}{t^4+2t^3}$

$= 6\int \frac{t^2}{t+2}dt$

$= 6\int \frac{t^2-4+4}{t+2}dt$

$= 6\int t+2\frac{+4}{t+2}dt$

$= 6\left[\frac{t^2}{2}+2t+4\ln|t+2|\right]+c$

$F(x)=6\left[\frac{x^{1/3}}{2}+2x^{1/3}+4\ln|x^{1/6}+2|\right]+c$

$F(0)=24\ln 2+c=-26+24\ln 2$

$c=-26$

$F(1)=\left[\frac{1}{2}+2+4\ln 3\right]-26\equiv a+b\ln 3$

$-11+24\ln 3=a+b\ln 3$

$a + b = 24 - 11 = 13$

25.

❑ ❑ ❑