MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of cosec10° – $\sqrt{3}$ sec10°

(1) 1 (2) 2

(3) 4 (4) 5

Answer (3)

Sol. $\dfrac{1}{\sin 10} - \dfrac{\sqrt{3}}{\cos 10}$

$$= \frac{1}{2}\left[\frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\frac{1}{2}\sin 10^\circ \cos 10^\circ}\right]$$

$$= 2 \times 2\frac{[\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ]}{\sin 20^\circ} = 4$$

2. If $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ \beta & 1 \end{bmatrix}$ and $A^2 - 4A + 2I = 0$, $B^2 - 2B + I = 0$, then $|\text{adj}\,(A^3 - B^3)|$ is equal to

(1) 11

(2) 7

(3) –11

(4) 121

Answer (1)

Sol. $\begin{vmatrix} \alpha - \lambda & 2 \\ 1 & 2 - \lambda \end{vmatrix} = 0$

$(\lambda - \alpha)(\lambda - 2) - 2 = 0$

$\lambda^2 + \lambda(-2 - \alpha) + 2\alpha - 2 = 0$

$-2 - \alpha = -4 \Rightarrow \alpha = 2$

Similarly,

$\begin{vmatrix} 1 - \mu & 1 \\ \beta & 1 - \mu \end{vmatrix} = 0$

$\mu^2 - 2\mu + 1 - \beta = 0$

$1 - \beta = 1 \Rightarrow \beta = 0$

$\Rightarrow A^3 = 4A^2 - 2A$

$= 4(4A - 2I) - 2A$

$= 14A - 8I$

$B^3 = 2B^2 - B$

$= 2(2B - I) - B$

$= 3B - 2I$

$A^3 - B^3 = 14A - 3B - 6I$

$= 14\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 6\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$= \begin{vmatrix} 19 & 25 \\ 14 & 19 \end{vmatrix} \Rightarrow \det(A^3 - B^3) = 11$

$= |\text{adj}(A^3 - B^3)| = |A^3 - B^3| = 11$

3. If $y = y(x)$ and $(1 + x^2)dy + (1 - \tan^{-1}x)dx = 0$ and $y(0) = 1$ then $y(1)$ is

(1) $\dfrac{\pi^2}{32} + \dfrac{\pi}{4} + 1$

(2) $\dfrac{\pi^2}{32} - \dfrac{\pi}{4} + 1$

(3) $\dfrac{\pi^2}{32} - \dfrac{\pi}{2} - 1$

(4) $\dfrac{\pi^2}{32} - \dfrac{\pi}{2} + 1$

Answer (2)

Sol. $(1 + x^2)dy + (1 - \tan^{-1}x)dx = 0$

$$dy + \frac{1 - \tan^{-1}x}{1 + x^2}dx = 0$$

$$\int dy + \int \frac{dx}{1+x^2} - \int \frac{\tan^{-1} x}{1+x^2} dx = 0$$

Or $y + \tan^{-1}x - I = 0$, where

$$I = \int \frac{\tan^{-1} x}{1+x^2} dx$$

Let $\tan^{-1}x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$\int t ds \Rightarrow \frac{t^2}{2} = \frac{(\tan^{-1} x)^2}{2}$$

$$\Rightarrow \quad y + \tan^{-1} x - \frac{(\tan^{-1} x)^2}{2} + c = 0$$

$\therefore\ y(0) = 1 \qquad 1 + c = 0 \qquad \Rightarrow c = -1$

$$\Rightarrow \quad y = \frac{(\tan^{-1} x)^2}{2} - \tan^{-1} x + 1$$

Now $y(1) = \frac{1}{2}\left(\frac{\pi}{4}\right)^2 - \frac{\pi}{4} + 1$

4. The value of

$$\int_0^{\pi/2} |\sin x + \sin 2x + \sin 3x| dx \text{ is}$$

(1) $\frac{8}{3}$

(2) $\frac{7}{3}$

(3) $\frac{2}{3}$

(4) 3

Answer (2)

Sol. $\because\ \sin x + \sin 3x = \sin(2x - x) + \sin(2x + x)$

$= 2\sin 2x \cos x$

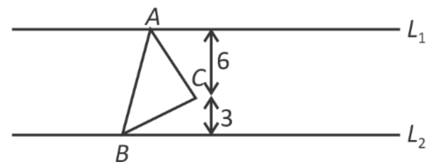
$\therefore\ \sin 3x + \sin 2x + \sin x = \sin 2x\,(2\cos x + 1)$

$$\therefore \int_0^{\pi/2} |\sin 2x(2\cos x + 1)| dx$$

$$= \int_0^{\pi/2} (\sin 2x) dx + \int_0^{\pi/2} 4\cos^2 x \sin x dx$$

$$= \frac{-\cos 2x}{2}\Bigg|_0^{\pi/2} + \left(\frac{-4}{3}(\cos^3 x)\right)\Bigg|_0^{\pi/2} = \frac{7}{3}$$

5. If L_1 and L_2 are two parallel lines and ΔABC is an equilateral triangle then area of triangle ABC is



(1) $7\sqrt{3}$

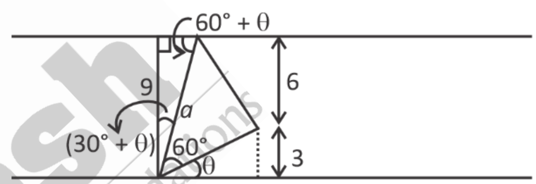
(2) $4\sqrt{3}$

(3) $21\sqrt{3}$

(4) 84

Answer (3)

Sol. Let a be the side of ΔABC



$$\sin\theta = \frac{3}{a}$$

$$\sin(60° + \theta) = \frac{9}{a}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} \times \sqrt{1 - \frac{9}{a^2}} + \frac{3}{a} \times \frac{1}{2} = \frac{9}{a}$$

$$\Rightarrow \quad \sqrt{3}\left(\sqrt{a^2 - 9}\right) + 3 = 18$$

$\Rightarrow \quad 3(a^2 - 9) = 15^2 \qquad \Rightarrow a^2 - 9 = 15 \times 5$

$\Rightarrow a^2 = 84$

Area = $\frac{\sqrt{3}}{4} a^2 = 21\sqrt{3}$

6. Consider a set $S = \{a, b, c, d\}$. Then number of reflexive as well as symmetric relations from $S \to S$ are

(1) 1024 (2) 256

(3) 16 (4) 64

Answer (4)

Sol. Number of such relations:

From $A \to A$ such that

$n(A) = N$ is

$\Rightarrow 2^{\left(\frac{N^2-N}{2}\right)}$

for $N = 4 \Rightarrow 2^6 = 64$

Alter :

$$\begin{bmatrix} O & - & - & - \\ \Theta & O & - & - \\ \Theta & \Theta & O & - \\ \Theta & \Theta & \Theta & O \end{bmatrix} \Rightarrow \text{each } \Theta \text{ has two choices}$$

$\Rightarrow 2^6 = 64$

7. The locus of point of intersection of tangent drawn to the circle $(x - 2)^2 + (y - 3)^2 = 16$, which sub-stends an angle of 120° is

(1) $3x^2 + 3y^2 - 12x - 18y - 25 = 0$

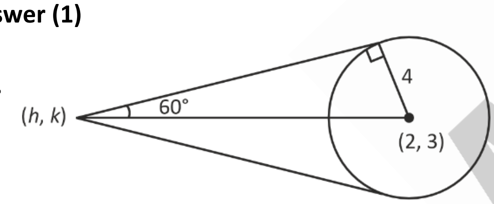
(2) $x^2 + y^2 - 12x - 18y - 25 = 0$

(3) $3x^2 + 3y^2 + 12x + 18y - 25 = 0$

(4) $x^2 + y^2 + 12x + 18y - 25 = 0$

Answer (1)

Sol.



$x^2 + y^2 - 14x - 6y - 3 = 0$

$$\tan 60° = \frac{4}{\sqrt{h^2 + k^2 - 4h - 6k - 3}}$$

squaring both side

$3(h^2 + k^2 - 4h - 6k - 3) = 16$

To get locus replace h, k by x and y;

$3x^2 + 3y^2 - 12x - 18y - 9 - 16 = 0$

$3x^2 + 3y^2 - 12x - 18y - 25 = 0$

8. If $a_1, a_2, a_3, \ldots$ are the terms of an increasing geometric progression such that

$a_1 + a_3 + a_5 = 21,$

$a_1a_3a_5 = 64$

then $a_1 + a_2 + a_3$ is

(1) 5 (2) 7

(3) 10 (4) 15

Answer (2)

Sol. Let $a_3 = P$, $a_1 = \frac{P}{r^2}$, $a_5 = Pr^2$

$$\Rightarrow P + \frac{P}{r^2} + Pr^2 = 21$$

and $P \times \frac{P}{r^2} \times Pr^2 = 64 \Rightarrow P^3 = 64 = 4^3$

$\Rightarrow P = 4$

$$\Rightarrow 1 + \frac{1}{r^2} + r^2 = \frac{21}{4} \Rightarrow r^4 - \frac{17}{4}r^2 + 1 = 0$$

$$r^2 = \frac{\frac{17}{4} \pm \sqrt{\frac{289}{16} - 4}}{2} = \frac{\frac{17}{4} \pm \frac{15}{4}}{2} = 4, \frac{1}{4}$$

$$\Rightarrow r = \pm 2, \pm \frac{1}{2}$$

$$\Rightarrow a_1 + a_2 + a_3 = \frac{P}{r^2} + \frac{P}{r} + P$$

$$= 4\left(\frac{1}{4} + \frac{1}{2} + 1\right)$$

$$= 1 + 2 + 4 = 7$$

9. Ellipse $E : \frac{x^2}{36} + \frac{y^2}{16} = 1$, A hyperbola confocal with ellipse E and eccentricity of hyperbola is equal to 5. The length of latus rectum of hyperbola is, if principle axis of hyperbola is x-axis?

(1) $\frac{96}{\sqrt{5}}$ (2) $24\sqrt{5}$

(3) $18\sqrt{5}$ (4) $12\sqrt{5}$

Answer (1)

Sol. $E: \dfrac{x^2}{A^2} + \dfrac{y^2}{B^2} = 1$

$H: \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

For con-focal E and H

$\because \quad A^2 - B^2 = a^2 + b^2$ (Given that $A^2 = 36$ and $B^2 = 16$)

$36 - 16 = a^2 + b^2$

$a^2 + b^2 = 20 \qquad \ldots(1)$

$e_H = 5$

$\Rightarrow \quad 25 = 1 + \dfrac{b^2}{a^2}$

$\Rightarrow \quad 25a^2 = 20 \Rightarrow a^2 = \dfrac{4}{5} \Rightarrow a = \dfrac{2}{\sqrt{5}}$

Also $a^2 + b^2 = 20$

$b^2 = 20 - \dfrac{4}{5} \Rightarrow \dfrac{96}{5}$

L(LR) = $= 2 \times \dfrac{b^2}{a} = \dfrac{2 \times \frac{96}{5}}{\frac{2}{\sqrt{5}}} = \dfrac{96}{\sqrt{5}}$

10. If the mean and variance of observations x, y, 12, 14, 4, 10, 2 is 8 and 16 respectively where $x > y$. Then, the value of $3x - y$ is

(1) 18 (2) 20

(3) 22 (4) 24

Answer (1)

Sol. $\dfrac{x + y + 12 + 14 + 4 + 10 + 2}{7} = 8$

$\Rightarrow x + y = 14$

also var = 16

$\dfrac{x^2 + y^2 + 144 + 196 + 16 + 100 + 4}{7} - (8)^2 = 16$

$\Rightarrow x^2 + y^2 = 100$

By solving we get

$x = 8$, $y = 6$

$\Rightarrow 3x - y = 18$

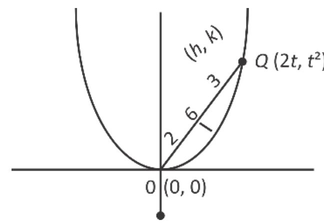
11. If O is the vertex of the of the parabola $x^2 = 4y$, Q is the point on parabola. If C is the locus of point which divides OQ in ratio 2:3. The equation of chord of C which bisected at point (1, 2).

(1) $5x + 4y + 3 = 0$ (2) $5x - 4y - 3 = 0$

(3) $5x - 4y + 3 = 0$ (4) $5x + 4y - 3 = 0$

Answer (C)

Sol.



$h = \dfrac{4t}{5}$, $k = \dfrac{2t^2}{5}$

$\Rightarrow \dfrac{2t^2}{5} = t$, $\dfrac{2t^2}{5} = t^2$

$\dfrac{5k}{2} = \left(\dfrac{5h}{4}\right)^2$

Replace (h, k) with (x, y)

$\dfrac{5y}{2} = \dfrac{25x^2}{16} =$

$C: \boxed{8y = 5x^2}$

chord with given middle point

$T = S_1$

$5xx_1 - 4(y + y_1) = 5x_1^2 - 8y_1$

$5x - 4(y + 2) = 5 - 16$

$5x - 4y - 8 = -11$

$5x - 4y + 3 = 0$

12. The value of $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\pi + 4x^{11}}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx$

(1) 3π (2) 4π

(3) 6π (4) 12π

Answer (2)

Sol. $\int_{0}^{\frac{\pi}{6}} \frac{\pi+4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}+\frac{\pi-4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}dx$

$=2\pi\int_{0}^{\frac{\pi}{6}} \frac{1}{1-\sin\left(x+\frac{\pi}{6}\right)}dx$

$=2\pi\int_{0}^{\frac{\pi}{6}} \frac{1+\sin\left(x+\frac{\pi}{6}\right)}{\cos^2\left(x+\frac{\pi}{6}\right)}dx$

$=2\pi\int_{0}^{\frac{\pi}{6}} \sec^2\left(x+\frac{\pi}{6}\right)+\tan\left(x+\frac{\pi}{6}\right)\sec\left(x+\frac{\pi}{6}\right)dx$

$\Rightarrow\ 2\pi\left[\tan\left(x+\frac{\pi}{6}\right)+\sec\left(x+\frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$

$\Rightarrow 4\pi$

13. If $f(3) = 18$, $f'(3) = 0$ and $f''(3) = 4$. Then, the value of $\lim_{x\to1}\ln\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}$ is equal to

(1) 2
(2) 4
(3) 6
(4) 8

Answer (1)

Sol. $\ln\left(\lim_{x\to1}\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)(1^{\infty})$

$\ln\left(e^{\lim_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)}{f(3)}-1\right)}\right)$

$\ln\left(e^{\lim_{x\to1}\frac{f'(x+2)}{2(x-1)}}\right)$

$\ln\left(e^{\lim_{x\to1}\frac{f''(x+2)}{2}}\right)$

$\lim_{x\to1}\frac{f''(3)}{2}=\frac{4}{2}=2$

14. If the domain of the function $\cos^{-1}\left(\frac{2x-5}{11x-7}\right)+\sin^{-1}(2x^2-3x+1)$ is $[0,a]\cup\left[\frac{12}{13},b\right]$ then $\frac{1}{ab}$ is equal to

(1) –3
(2) 3
(3) 2
(4) 4

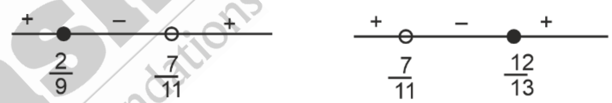
Answer (2)

Sol. $\cos^{-1}\left(\frac{2x-5}{11x-7}\right)$ is defined as

$\frac{2x-5}{11x-7}\le1,\ \frac{2x-5}{11x-7}\ge-1$

$\frac{-9x+2}{11x-7}\le0,\ \frac{13x-12}{11x-7}\ge0$

$\Rightarrow\frac{\left(x-\frac{2}{9}\right)}{\left(x-\frac{7}{11}\right)}\ge0,\ \frac{\left(x-\frac{12}{13}\right)}{\left(x-\frac{7}{11}\right)}\ge0$



$\Rightarrow\left[\left(-\infty,\frac{2}{9}\right]\cup\left(\frac{7}{11},\infty\right)\right]\cap\left[\left(-\infty,\frac{2}{11}\right)\cup\left[\frac{12}{13},\infty\right)\right]$

$\Rightarrow x\in\left(-\infty,\frac{2}{9}\right]\cup\left[\frac{12}{13},\infty\right)$

Similarly, $\sin^{-1}(2x^2-3x+1)$ is defined

When $-1\le2x^2-3x+1\le1$

$\Rightarrow2x^2-3x\le0\ \Rightarrow x\left(x-\frac{3}{2}\right)\le0$

$\Rightarrow x\in\left[0,\frac{3}{2}\right]$

and $2x^2-3x+2\ge0\Rightarrow x\in R\ \Rightarrow x\in\left[0,\frac{3}{2}\right]$

$$\Rightarrow \cos^{-1}\left(\frac{2x-5}{11x-7}\right)+\sin^{-1}(2x^2-3x+1) \text{ is defined}$$

for $\left[0,\frac{2}{9}\right]\cup\left[\frac{12}{13},\frac{3}{2}\right]$

$$\Rightarrow ab=\frac{2}{9}\times\frac{3}{2}=\frac{1}{3}$$

$$\Rightarrow \frac{1}{ab}=3$$

15. In binomial expansion of $(ax^2 + bx + c)(1 - 2x)^{26}$, the coefficients of x, x^2 and x^3 are –56, 0 and 0 respectively, then $(a + b + c)$ is equal to

(1) 1500 (2) 1403

(3) 1300 (4) 1483

Answer (2)

Sol. $(ax^2 + bx + c)\ (1 - 2x)^{26}$

$ax^2(1-2x)^{26} + bx(1-2x)^{26} + c(1-2x)^{26}$

Coefficient of

$x \Rightarrow (0 + b\cdot{}^{26}C_0 + c\cdot{}^{26}C_1(-2)x$

$= b - 52c$

Coefficient of

$x^2 = (a({}^{26}C_0) + b\cdot{}^{26}C_1\,(-2) + c\ {}^{26}C_2\,(-2)^2)x^2$

$= a - 52b + {}^{26}C_2 \times 4 = a - 52b + 1300c$

Coefficient of

$x^3 = [a\cdot{}^{26}C_1(-2) + b\cdot{}^{26}C_2(26)^2 + c\ {}^{26}C_3(-2)^3]$

$= -52a + 1300b - 20800c$

$\Rightarrow\ b - 52c = -56$

$\Rightarrow\ b - 52c = -56$

$a - 52b + 1300c = 0 \qquad \Rightarrow \quad a = 1300$

$-a + 25b - 400c = 0 \qquad b = 100$

$c = 3$

$\Rightarrow \quad a + b + c = 1403$

16. If $a_1 = 1$ and for $\forall\, n \geq 1$ $a_{n+1} = \frac{1}{2}a_n + \frac{n^2-2n-1}{n^2(n+1)^2}$ then

$\left|\sum_{n=1}^{\infty}\left(a_n-\frac{2}{n^2}\right)\right|$ is equal to

(1) 3 (2) 4

(3) 5 (4) 2

Answer (4)

Sol. $a_{n+1} = \frac{1}{2}a_n+\frac{1}{(n+1)^2}-\frac{\left((n+1)^2-n^2\right)}{n^2(n+1)^2}$

$a_{n+1} = \frac{a_n}{2}+\frac{2}{(n+1)^2}-\frac{1}{n^2}$

$a_{n+1} - \frac{2}{(n+1)^2}=\frac{1}{2}\left(a_n-\frac{2}{n^2}\right)$

Let $b_n = a_n\frac{-2}{n^2}$

then $<b>$ is geometric progression with ratio = $\frac{1}{2}$

$b_1=a_1-\frac{2}{1}=1-2=(-1)$

$$\sum_{n=1}^{\infty}\left(a_n-\frac{2}{n^2}\right)=\sum_{n=1}^{\infty}b_n=\frac{(-1)}{1-\left(\frac{1}{2}\right)}=\frac{-1}{1/2}=-2$$

$$\Rightarrow \left|\sum_{n=1}^{\infty}\left(a_n-\frac{2}{n^2}\right)\right|=2$$

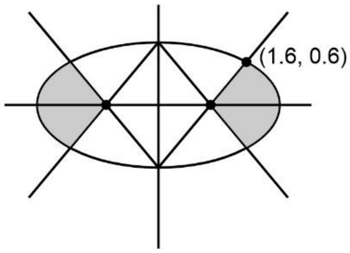
17. Area enclosed by $x^2 + 4y^2 \leq 4$, $y \leq |x| - 1$, $y \geq 1 - |x|$ is

(1) $4\sin^{-1}\left(\frac{3}{5}\right)+\frac{6}{5}$ (2) $\sin^{-1}\left(\frac{3}{5}\right)-\frac{6}{5}$

(3) $4\sin^{-1}\left(\frac{3}{5}\right)+\frac{12}{5}$ (4) $4\sin^{-1}\left(\frac{3}{5}\right)-\frac{6}{5}$

Answer (4)

Sol.



$$4\int_0^{0.6}\left[\sqrt{4-4y^2}-(1+y)\right]dy$$

$I_1 = 4\int_0^{\frac{3}{5}}\sqrt{4-4y^2dy}$, put $y = \sin\theta$, $dy = \cos\theta d\theta$

$$\Rightarrow I_1 = \int_0^{\theta}2\cos^2\theta d\theta = \int_0^{\theta}(1+\cos^2\theta)d\theta = \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\theta}$$

$$\therefore \sin = \frac{3}{5}, \cos\theta = \frac{4}{5}, \sin 2\theta = \frac{24}{25}$$

$$\Rightarrow I_1 = \left(\sin^{-1}\frac{3}{5} + \frac{12}{25}\right), I_2 = \int_0^{\frac{3}{5}}(1+y)dy = \frac{39}{50}$$

$$\Rightarrow \text{Area} = 4\sin^{-1}\left(\frac{3}{5}\right) - \frac{6}{5}$$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $x^2 + x + 1 = 0$, then

$$\left(x+\frac{1}{x}\right)^4 + \left(x^2+\frac{1}{x^2}\right)^4 + \left(x^3+\frac{1}{x^3}\right)^4 + \ldots + \left(x^{25}+\frac{1}{x^{25}}\right)^4 \text{ is}$$

Answer (145)

Sol. $x^2 + x + 1 = 0 \begin{smallmatrix} w \\ w^2 \end{smallmatrix}$

$$\left(w+\frac{1}{w}\right)^4 + \left(w^2+\frac{1}{w^2}\right)^4 + \left(w^3+\frac{1}{w^3}\right)^4 + \ldots. \left(w^{25}+\frac{1}{w^{25}}\right)^4$$

$$\sum\left(w^k+\frac{1}{w^k}\right)^4$$

$$K = 3x \Rightarrow w^{3x} + \frac{1}{w^{3x}} = 2$$

$$k \neq 3x \Rightarrow w^k + \frac{1}{w^k} = -1$$

$$\sum_{k=1}^{25}\left(w^k+\frac{1}{w^k}\right)^4 \Rightarrow 8\left(1+1+2^4\right)+1$$

$= 145$

22. The sum of roots of the equation $|x-1|^2 - 5|x-1| + 6 = 0$ is

Answer (4)

Sol. $|x-1|2 - 5|x-1| + 6 = 0$

Let $|x-1| = t$

$t^2 - 5 + 6 = 0$

$(t-3)(t-2) = 0$

$t = 2, 3$

$|x-1| = 2$ or $|x-1| = 3$

$x - 1 = \pm 2$ or $x - 1 = \pm 3$

$\Rightarrow x = 3, -1, 4, -2$

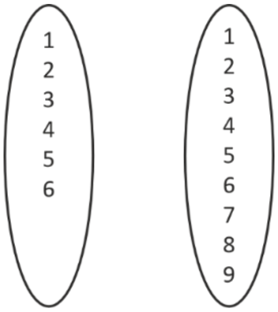
$\Rightarrow \text{Sum} = 3 - 1 + 4 - 2$

$= 4$

23. If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 2, 3, \ldots 8, 9\}$. Then the number of strictly increasing functions from $A \to B$ such that $f(i) \neq i \ \forall \ i = 1, 2, 3, 4, 5, 6$ are

Answer (28)

Sol.



Case (i) If $f(1) = 2$ then ${}^{7}C_5$ functions

Case (ii) $f(1) = 3$ then ${}^{6}C_5$

Case (iii) $f(1) = 4$ then ${}^{5}C_5$

$\Rightarrow 21 + 6 + 1 = 28$

24.

25.

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