MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\int_{0}^{20\pi}(\sin^4 x+\cos^4 x)dx$ is equal to

(1) 15π (2) 10π

(3) 20π (4) 40π

Answer (1)

Sol. $\sin^4 x+\cos^4 x = (\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$$=1-\frac{\sin^2(2x)}{2}=1-\frac{1-\cos(4x)}{4}=\left(\frac{3}{4}+\frac{\cos(4x)}{4}\right)$$

$$I=\int_0^{2\pi}(\sin^4 x+\cos^4 x)dx$$

$$=40\int_0^{\frac{\pi}{2}}\left(1-\frac{1}{2}\sin^2 x\right)dx$$

$$=40\int_0^{\frac{\pi}{2}}\left(1-\frac{1-\cos(4x)}{4}\right)dx$$

$$=40\int_0^{\frac{\pi}{2}}\left(\frac{3}{4}+\frac{\cos(4x)}{4}\right)dx$$

$$=40\left[\frac{3x}{4}+\frac{\sin(4x)}{16}\right]_0^{\frac{\pi}{2}}$$

$$=40\left(\frac{3\pi}{8}+0-0-0\right)=15\pi$$

2. If $A=\begin{pmatrix}2 & -2\\ 4 & -2\end{pmatrix}$, $B=\begin{pmatrix}3 & 3\\ 1 & 9\end{pmatrix}$ and $PA = B$ and $AQ = B$, then $2(P + Q)$ is

(1) $\begin{pmatrix}11 & 12\\ 24 & 13\end{pmatrix}$ (2) $\begin{pmatrix}-11 & 12\\ 24 & 13\end{pmatrix}$

(3) $\begin{pmatrix}-11 & 12\\ -24 & 13\end{pmatrix}$ (4) $\begin{pmatrix}11 & 12\\ -24 & 13\end{pmatrix}$

Answer (3)

Sol. $\because PA = B$

$P = BA^{-1}$

$$\begin{bmatrix}3 & 3\\ 1 & 9\end{bmatrix}\begin{bmatrix}-\frac{1}{2} & \frac{1}{2}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$P=\begin{pmatrix}-\frac{9}{2} & 3\\ -\frac{19}{2} & 5\end{pmatrix}$$

$\because AQ = B$

$Q = A^{-1}B$

$$=\begin{pmatrix}-\frac{1}{2} & \frac{1}{2}\\ -1 & \frac{1}{2}\end{pmatrix}\begin{pmatrix}3 & 3\\ 1 & 9\end{pmatrix}$$

$$=\begin{pmatrix}-1 & 3\\ -\frac{5}{2} & \frac{13}{2}\end{pmatrix}$$

Now, $2(P+Q)=2\begin{pmatrix}-\frac{11}{2} & 6\\ -12 & \frac{13}{2}\end{pmatrix}$

$$=\begin{pmatrix}-11 & 12\\ -24 & 13\end{pmatrix}$$

3. If $f(x)=\int\frac{16x+24}{x^2+2x-15}dx$, given that $f(4)$ = 14ln3, then the value of f(7) is

(1) 32ln2 + 7ln3 (2) 7ln2 + 32ln3

(3) 32ln2 – 7ln3 (4) 32ln3 – 7ln2

Answer (1)

Sol. $f(x)=\int\frac{16x+24}{x^2+2x-15}dx$

$\because f(x)=\frac{16x+24}{x^2+2x-15}=\frac{16x+24}{(x+5)(x-3)}$

$=\frac{7}{x+5}+\frac{9}{x+3}$

$\Rightarrow f(x)=\int\left(\frac{7}{x+5}+\frac{9}{x-3}\right)dx$

$\Rightarrow f(x)=7\ln|x+5|+9\ln|x-3|+c$

$\because f(4)=14\ln 3$

$\Rightarrow 14\ln 3=14\ln 3+c\Rightarrow c=0$

$\Rightarrow f(x)=7\ln|x+5|+9\ln|x-3|$

$\Rightarrow f(7)=7\ln 12+9\ln 4=32\ln 2+7\ln 3$

4. Let A = {2, 3, 4,5, 6}, consider R be relation of $A \times A$ such that (x, y) R (a, b) implies that x divides a and $y \le b$ then total number of elements in R is

(1) 24 (2) 120

(3) 720 (4) 144

Answer (2)

Sol. $A=\{2,3,7,5,6\}$ $(x,y)\in A\times A$

$(a,b)\in A\times A$ $(x,y)\,R\,(a,b)$

$\Rightarrow x\,|\,a$ and $y\le b$

$(A\times A)=\{(2,2)\,(2,3)\,(2,4)\,(2,5)\,(2,6)$

$(3,2)\,(3,3)\,(3,4)\,(3,5)\,(3,6)$

$(4,2),\,(4,3)\,(4,4)\,(4,5)\,(4,6)$

$(5,2)\,(5,3)\,(5,4)\,(5,5)\,(5,6)$

$(6,2)\,(6,3)\,(6,4)\,(6,5)\,(6,6)\}$

Consider $x=2\Rightarrow a$ can be $\{2,4,6\}$

$\Rightarrow$ (3 choices) $\times(15)=45$

$x=3, a$ can be $\{3,6\}$

$\Rightarrow 2\times 15=30$

$x=4, a$ can be $\{4\}\Rightarrow 1\times 15$

$x=5, a$ can be $\{5\}=1\times 15$

$x=6, a$ can be $\{6\}=1\times 15$

= 120

5. Let a_1, a_2, a_3..... be an arithmetic progression and g_1, g_2, g_3.... be an increasing geometric progression such that $g_1 = a_1 = a_2 + g_2 = 1$ and $g_3 + a_3 = 4$ then $a_{10} + g_5$ is equal to

(1) 34

(2) 35

(3) 55

(4) 54

Answer (3)

Sol. $a_1 = 1$, $a_2 + g_2 = a + d + br = 1$

$d = -br$

$a_3 + g_3 = (a + 2d) + br^2 = 4$

$\Rightarrow$ $1 + 2d + br^2 = 4$

$\Rightarrow$ $br^2 - 2br = 3$

$r^2-2r-3=0$, (b = 1)

$(r - 3)(r + 1) = 0$

$r = 3$

$d = -3$

$a_{10}+g_5=(a+9d)+(br^4)$

$= 1 + 9(-3) + (1)(3^4)$

= 1 – 27 + 81

= 82 – 27 = 55

6. If AB is a chord of hyperbola $xy = 12$, whose mid-point is $\left(\frac{1}{2}, \frac{-1}{2}\right)$, then the area of triangle ABO (where O is the origin) is

(1) $\frac{7}{2}$

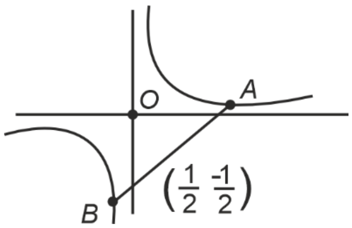
(2) $\frac{9}{2}$

(3) 7

(4) $\frac{3}{2}$

Answer (1)

Sol.



$xy = 12$, for chord with mid point

$T = S_1$

$$\Rightarrow \frac{xy_1 + yx_1}{2} - 12 = x_1y_1 - 12$$

$$\Rightarrow \frac{x\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)}{2} = \frac{-1}{4}$$

$\Rightarrow y - x + 1 = 0$

$$xy = 12 \Rightarrow y = x - 1 = \frac{12}{x}$$

$\Rightarrow x^2 - x + 12 = 0$

$(x - 7)(x + 3) = 0$

$x = -3, 4$

$\Rightarrow$ Point A and B

are $(-3, -4)$ and $(4, 3)$

$$\text{Area } = \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -3 & -4 & 1 \\ 4 & 3 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}(-9 + 16) = \frac{7}{2} = 3.5$$

7. Let $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{c} = \lambda\vec{a} + \mu\vec{b}$ and

$\vec{c} \cdot (3\hat{i} - 6\hat{j} + 3\hat{k}) = 10$

$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$

Then $|\vec{c}| =$

(1) $\frac{\sqrt{1920}}{18}$

(2) $\frac{\sqrt{1914}}{18}$

(3) $\frac{\sqrt{920}}{18}$

(4) $\frac{\sqrt{914}}{18}$

Answer (2)

Sol. $a = -\hat{i} + 2\hat{j} + \hat{k}$

$b = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow \vec{c} = \lambda(-\hat{i} + 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - 3\hat{k})$

$= (-\lambda + \mu)\hat{i} + (2\lambda + \mu)\hat{j} + (\lambda - 3\mu)\hat{k}$

Now

$\vec{c} \cdot (3\hat{i} - 6\hat{j} + 3\hat{k}) = 10$

$\Rightarrow 3(-\lambda + \mu) - 6(2\lambda + \mu) + 3(\lambda - 3\mu) = 10$ ...(1)

Also

$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$

$\Rightarrow (-\lambda + \mu) + (2\lambda + \mu) + (\lambda - 3\mu) = -2$ ...(2)

Solving (1) and (2)

$\lambda = \frac{-17}{18}$ $\quad \mu = \frac{1}{9}$

$$\vec{c} = \frac{19}{18}\hat{i} - \frac{16}{9}\hat{j} - \frac{23}{18}\hat{k}$$

$$|\vec{c}| = \sqrt{\frac{1}{18^2}(19^2 + 32^2 + 23^2)} = \frac{\sqrt{1914}}{18}$$

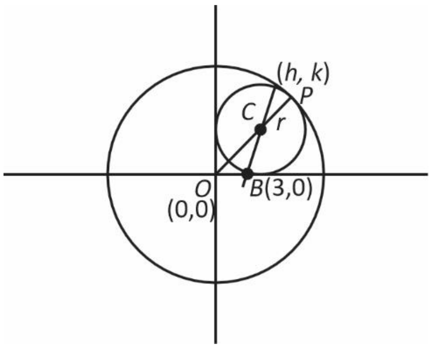
8. A variable circle with diameter AB where $A(3, 0)$ touching the circle $x^2 + y^2 = 36$ internally. The locus of centre of variable circle is a conic whose eccentricity is e, then $72e^2$ is

(1) 36 (2) 18

(3) 54 (4) 16

Answer (2)

Sol.



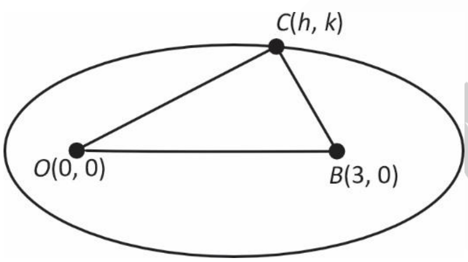
Let the centre be (h, k), then

$OP = 6, \quad OC = \sqrt{h^2 + k^2}, \quad CP = r = \sqrt{(h-3)^2 + y^2}$

$\Rightarrow OP = OC + CP$

$6 = \sqrt{h^2 + k^2} + \sqrt{(h-3)^2 + k^2}$

$\Rightarrow$ This is an ellipse



Such that: $OB = 2ae = 3$

$OC + CB = 6 = 2a$

$\Rightarrow a = 3, \quad e = \dfrac{1}{2}$

$\Rightarrow 72e^2 = 72 \times \dfrac{1}{4} = 18$

9. If $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1,$ $2\left(\cos^8\theta - \sin^8\theta\right)\sec 2\theta = a^2, a \in Z\}$ then the number of elements in set P is

(1) 4 (2) 3

(3) 2 (4) 0

Answer (4)

Sol. $a^2 = 2(\cos^8\theta - \sin^8\theta)\sec 2\theta$

$= 2(\cos^4\theta + \sin^4\theta)(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)\sec 2\theta$

$= 2(\cos^4\theta + \sin^4\theta)$

$= 2[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$= 2 - \sin^2 2\theta$

$\because a \in Z \Rightarrow a^2 \in \{0, 1, 4, 9, ...\}$

$\Rightarrow (2 - \sin^2 2\theta) \in \{0, 1, 4, 9...\}$

$\Rightarrow \sin^2 2\theta = 1$

$\Rightarrow 2\theta = n\pi \pm \dfrac{\pi}{2}, n \in Z$

$\Rightarrow \theta = \dfrac{n\pi}{2} \pm \dfrac{\pi}{4}, n \in Z$

When $\theta = \dfrac{n\pi}{2} \pm \dfrac{\pi}{4}, n \in Z,$ then $\tan^2\theta = 1$

$\because \tan^2\theta \neq 1$

$\Rightarrow P$ is null set

$\Rightarrow n(P) = 0$

10. If complex numbers z_1 and z_2 such that $|z_1| = 17$ and $|\bar{z}_2 - 3 - 4i| = 5$ then $\max(|z_1 + z_2|)$ is equal to

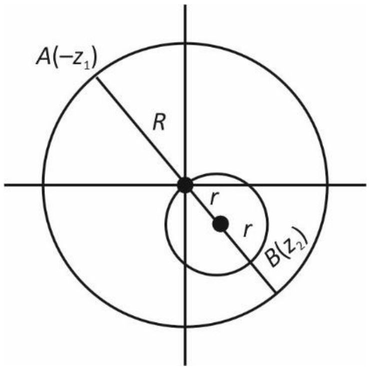
(1) 17 (2) 27

(3) 12 (4) 7

Answer (2)

Sol. $|\bar{z}_2 - (3 + 4i)| = 5$

$\Rightarrow \quad |z_2 - (3 - 4i)| = 5$



$\Rightarrow$ maximum distance $AB = |z_2 - (-z_1)|$

$= R + 2r$

$= 17 + 2 \times 5 = 27$

11. A fair coin is tossed four times such that for heads 10 points are allotted and tail gets 5 points then probability that total points are 30 points is equal to

(1) $\frac{5}{8}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{3}{8}$

Answer (4)

Sol. Let a heads and $(4 - a)$ tails

$\Rightarrow$ $10a + 5(4 - a) = 30$

$\Rightarrow$ $20 + 5a = 30 \Rightarrow a = 2$

$\Rightarrow$ Probability = ${}^4C_2 \times \left(\frac{1}{2}\right)^4$

$= \frac{6}{16} = \frac{3}{8}$

12. If roots of the equation $x^2 - 3x + n = 0$ are α and β and equation $x^2 + 3x + 4 = 0$ are $\frac{\alpha}{3}, 3\beta$ then $(2\alpha + \beta + 3n)$ is equal to

(1) $\frac{47}{4}$ (2) $\frac{-51}{4}$

(3) $\frac{-37}{4}$ (4) $\frac{-25}{4}$

Answer (2)

Sol. $\alpha + \beta = 3$

$$\frac{\alpha}{3} + 3\beta = -3$$

$$\Rightarrow \alpha + \frac{\alpha}{3} + \beta + 3\beta = 0$$

$$-\frac{4\alpha}{3} = 4\beta \Rightarrow \alpha = -3\beta$$

$$\alpha + \beta = 3 \Rightarrow -2\beta = 3 \Rightarrow \beta = \frac{-3}{2}$$

$$\alpha = \frac{9}{2}$$

$$\alpha\beta = n = \frac{-27}{4}$$

$$\Rightarrow 2\alpha + \beta + 3n$$

$$= 2\left(\frac{9}{2}\right) + \left(\frac{-3}{2}\right) + \left(\frac{-81}{4}\right)$$

$$= \frac{9 - (81 + 6)}{4} = \frac{36 - 87}{4} = \frac{-51}{4}$$

13. If parabola $y = x^2 + px + q$ passes through (1, –1) and whose vertex is at the minimum distance from x-axis then $p^2 + q^2$ is equal to

(1) 2 (2) 3

(3) 4 (4) 5

Answer (3)

Sol. $-1 = 1 + P + q \Rightarrow P + q = -2$

vertex is $\left(-\frac{P}{2}, -\left(\frac{P^2 - 4q}{4}\right)\right)$

$$= \left(-\frac{P}{2}, \frac{4q - P^2}{4}\right)$$

To minimise:

$$\frac{4q - P^2}{4} = \frac{4q - (-2 - q)^2}{4}$$

$= \dfrac{4q-(q^2+4q+4)}{4}$

$= \dfrac{-q^2-4}{4}$

distance $= \left|\dfrac{-q^2-4}{4}\right| = \dfrac{q^2+4}{4} \Rightarrow q = 0$

and $P = -2$

$\Rightarrow P^2 + q^2 = 4$

14. If $2y^2\dfrac{dx}{dy} + 2xy + x^2 = 0$, $y > 0$, where $x = x(y)$ and $x(e) = e$, then the value of $x(e^2)$ is

(1) $\dfrac{4e^2}{5e^2+1}$ (2) $\dfrac{4e^2}{5e^2-1}$

(3) $\dfrac{5e^2}{4e^2+1}$ (4) $\dfrac{5e^2}{4e^2-1}$

Answer (2)

Sol. $2y^2\dfrac{dx}{dy} + (2xy + x^2) = 0$

$\dfrac{2y^2}{x^2}\dfrac{dx}{dy} + \dfrac{2y}{x} = -1$

Put $\dfrac{1}{x} = t \Rightarrow \dfrac{dt}{dy} = -\dfrac{1}{x^2}\dfrac{dx}{dy}$

$\Rightarrow -\dfrac{dt}{dy}(2y^2) + 2yt = -1$

$\Rightarrow \dfrac{dt}{dy} + \left(-\dfrac{1}{y}\right)t = \dfrac{1}{2y^2}$

$\text{I.F.} = e^{\int -\frac{1}{y}dy} = \dfrac{1}{y}$

$\Rightarrow t \cdot \dfrac{1}{y} = \int \dfrac{1}{2y^3}dy + C$

$\Rightarrow \dfrac{t}{y} = -\dfrac{1}{4y^2} + C$

$\Rightarrow \dfrac{1}{xy} = -\dfrac{1}{4y^2} + C$

$\therefore x(e) = e$

$\Rightarrow \dfrac{1}{e^2} = -\dfrac{1}{4e^2} + C \Rightarrow C = \dfrac{5}{4e^2}$

$\Rightarrow \dfrac{1}{xy} = -\dfrac{1}{4y^2} + \dfrac{5}{4e^2}$

$\Rightarrow \dfrac{1}{x(e^2)e^2} = -\dfrac{1}{4e^4} + \dfrac{5}{4e^2} \Rightarrow \dfrac{1}{x(e^2)} = -\dfrac{1}{4e^2} + \dfrac{5}{4}$

$\Rightarrow \dfrac{1}{x(e^2)} = \dfrac{-1+5e^2}{4e^2} \Rightarrow x(e^2) = \dfrac{4e^2}{5e^2-1}$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The value of the sum $\sum_{n=1}^{8} \dfrac{1^3 + 2^3 + ... + \text{upto } n \text{ terms}}{1+3+5+... \text{upto } n \text{ terms}}$ is

Answer (71.00)

Sol. $\sum_{n=1}^{8} \dfrac{\left(\dfrac{n(n+1)}{2}\right)^2}{n^2} = \sum_{n=1}^{8} \dfrac{n^2(n+1)^2}{4n^2}$

$= \sum_{n=1}^{8} \dfrac{(n+1)^2}{4} = \sum_{n=0}^{8} \dfrac{(n+1)^2}{4} - \dfrac{1}{4}$

$= \dfrac{1^2 + 2^2 + \ldots + 8^2 + 9^2}{4} - \dfrac{1}{4}$

$\dfrac{9 \times 10 \times 19}{6 \times 4} - \dfrac{1}{4} = 71$

22. $f(x)$ is 5 degree polynomial has extremes at $x = \pm 1$ and $\lim_{x\to 0}\frac{f(x)}{x^3} = -5$, then $f(2) - f(-2)$ is

Answer (112)

Sol. $f'(x) = (x-1)(x+1)(ax^2 + bx + c)$

$$f'(x) = (x^2-1)(ax^2+bx+c)$$

$$f'(x) = ax^4 + bx^3 + cx^2 - ax^2 - bx - c$$

$$f'(x) = ax^4 + bx^3 + (c-a)x^2 - bx - c$$

$$f(x) = \frac{ax^5}{5} + \frac{bx^4}{4} + \frac{(c-a)x^3}{3} - \frac{bx^2}{2} - cx + d$$

$\lim_{x\to 0}\frac{f(x)}{x^3} = -5$ Then $b = c = d = 0$

and $\frac{c-a}{3} = -5$

$a = 15$

$$f(x) = 3x^5 - 5x^3$$

$$f(x) - f(-x) = f(x) + f(x)$$

$= 2f(x)$

$= 2[3(2)^5 - 5(2)^3]$

$= 112$

23. If ${}^{30}C_{30-r} + 3\left({}^{30}C_{31-r}\right) + 3\left({}^{30}C_{32-r}\right) + {}^{30}C_{33-r} = {}^{m}C_r \,\forall r \in \{0, 1, ...30\}$ then m is equal to

Answer (33.00)

Sol. $\left({}^{30}C_{30-r} + {}^{30}C_{31-r}\right) + 2\left({}^{30}C_{31-r} + {}^{30}C_{32-r}\right) + {}^{30}C_{32-r} + {}^{30}C_{33-r}$

using ${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r$

$${}^{31}C_{31-r} + \left({}^{31}C_{32-r}\right) + {}^{31}C_{33-r}$$

$$= {}^{31}C_{31-r} + {}^{31}C_{32-r} + {}^{31}C_{32-r} + {}^{31}C_{33-r}$$

$$= {}^{32}C_{32-r} + {}^{32}C_{33-r} = {}^{33}C_{33-r}$$

$$= {}^{32}C_r = {}^{m}C_r$$

$$\Rightarrow m = 33$$

24. If P_n denotes the number of triangles formed by the vertices of n-sides polygon and $P_{n+1} - P_n = 66$, then n is

Answer (12)

Sol. ${}^{n+1}C_3 - {}^{n}C_3 = 66$

$(n+1)\cdot n(n-1) - n(n-1)(n-2) = 66 \times 6$

$\Rightarrow \boxed{n = 12}$

25. Let $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, then the number of points of discontinuity in [2, 4] is/are (Where [·] denotes greatest integer function)

Answer (10)

Sol. Let $g(x) = x^2 - x - \frac{1}{2}$

$g(x)$ is ↑ing in [2, 4]

$\Rightarrow g(x) \in [f(2), f(4)]$

$\Rightarrow g(x) \in \left[\frac{3}{2}, \frac{23}{2}\right]$

$\therefore f(x)$ is discontinuity wherever $f(x)$ will become integer.

$\Rightarrow g(x)$ is discontinuous if $g(x) = 2, 3, 4, ..., 11$

So total 10 points of discontinuity

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