02/04/2026
Morning

Aakash
Medical | IIT-JEE | Foundations

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Memory Based
Answers & Solutions

Time : 3 hrs. M.M. : 300

for

JEE (Main)-2026 (Online) Phase-2

(Physics, Chemistry and Mathematics)

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The dimensions of $\frac{1}{2}\in_0 E^2$ is

(1) MLT^{-2} (2) $ML^{-1}T^{-2}$

(3) MLT^{-1} (4) $ML^{-1}T^{-1}$

Answer (2)

Sol. $\frac{1}{2}\in_0 E^2$ is energy density

So $\frac{ML^2T^{-2}}{L^3}=ML^{-1}T^{-2}$

2. A wooden cubic block of relative density 0.4 is floating in water. Side of cubic block is 10 cm. When a coin is placed on the block the block dips 0.3 cm in equilibrium. Weight of coin is

(1) 0.2 N (2) 30 N

(3) 0.3 N (4) 3 N

Answer (3)

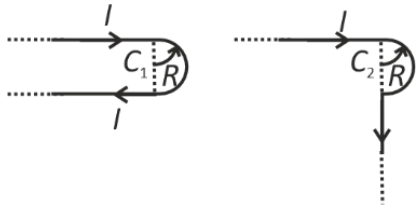
Sol. Weight of coin = additional buoyant force

$= \rho l a^2 \Delta x g$

$= 10^3 \times 10^{-2} \times 0.3 \times 10^{-2} \times 10$

$W = 0.3$ N

3. Consider two arrangements of wires, find ratio of magnetic field at centre of the semi-circular part.



(1) $\frac{\pi+4}{\pi-1}$ (2) $\frac{\pi+4}{\pi+2}$

(3) $\frac{\pi+2}{\pi+1}$ (4) $\frac{\pi-2}{\pi+1}$

Answer (3)

Sol. $B_1 = \frac{\mu_0 I}{4\pi R}+\frac{\mu_0 I}{2\times 2R}+\frac{\mu_0 I}{4\pi R}$

$B_1 = \frac{\mu_0 I}{4R}\left(\frac{2}{\pi}+1\right)$

$B_2 = \frac{\mu_0 I}{4\pi R}+\frac{\mu_0 I}{2\times 2R}$

$= \frac{\mu_0 I}{4R}\left(\frac{1}{\pi}+1\right)$

$\frac{B_1}{B_2}=\frac{2+\pi}{1+\pi}$

4. In isobaric reversible process on a diatomic gas, ratio of $\Delta Q : \Delta U : W$ shall be

(1) 7 : 5 : 2 (2) 5 : 3 : 2

(3) 3 : 2 : 1 (4) 6 : 5 : 1

Answer (1)

Sol. $\Delta Q = \mu C_P \Delta T$ $C_P : C_V : R$

$\Delta U = \mu C_V \Delta T$ $\frac{7}{5}:\frac{5}{2}:1$

$\Delta W = \mu R \Delta T$ 7 : 5 : 2

5. In circular motion, angular position θ and time t are related as

$\theta = \frac{5t^4}{4}-\frac{t^3}{3}$

Then angular acceleration at $t = 10$ sec is

(1) 1180 rad/s^2 (2) 130 rad/s^2

(3) 1480 rad/s^2 (4) 98 rad/s^2

Answer (3)

Sol. $\omega : \frac{5\times 4t^3}{4}-\frac{3t^2}{3}=5t^3-t^2$

$\alpha = 15t^2 - 2t = 1500 - 20 = 1480$

6. For an ideal gas having $C_p = 3R$ and $C_v = 2R$. Find work done by one mole of the gas in adiabatic expansion when pressure reduces from 8 bar to 1 bar. (Initial temperature = 140°C)

(1) $140R$ (2) $70R$

(3) $826R$ (4) $413R$

Answer (4)

Sol. Here $y=\frac{C_p}{C_v}=\frac{3}{2}$

And $\frac{(T_2)^y}{(P_2)^{y-1}}=\frac{(T_1)^y}{(P_1)^{y-1}}$

$\Rightarrow (T_2)^{3/2}=\frac{(1)\cdot(T_1)^{3/2}}{(8)^{1/2}}$

$\Rightarrow T_2=\frac{T_1}{(8)^{1/3}}=\frac{T_1}{2}=\frac{413}{2}K$

So $|\Delta w| = NC_v dT$

$=(1)2R\left(\frac{413}{2}\right)=413R$

7. In YDSE experiment wavelength of light used is 620 nm and separation between slits is 0.2 mm. Find angular fringe width.

(1) 3×10^{-4} (2) 3.1×10^{-3}

(3) 1.2×10^{-3} (4) 6.2×10^{-4}

Answer (2)

Sol. $\omega=\frac{\lambda}{d}$

$=\frac{620\times10^{-9}}{2\times10^{-4}}=310\times10^{-5}$

$= 3.1 \times 10^{-3}$

8. Photons of wavelength λ & 3λ incident on a metal surface. If stopping potential for the ejected photoelectrons are $4V_0$ & V_0 respectively, find threshold wavelength.

(1) 6λ (2) 9λ

(3) 2λ (4) 8λ

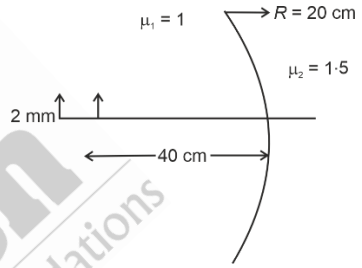
Answer (2)

Sol. $\frac{hc}{\lambda}=\phi+4V_0$

$\Rightarrow \frac{hc}{3\lambda}=\phi+V_0$

$\Rightarrow \frac{hc}{3\lambda}=3d, \Rightarrow \phi=\frac{hc}{9\lambda}$

9. For the given spherical surface of curvature radius R = 20 cm, object of height h_0 = 2 mm is placed at distance 40 cm from surface. Medium on both side of spherical surface have refractive index μ_1 = 1 and μ_2 = 1.5 respectively as shown. Find height of image.



(1) 2 mm (2) 1.5 mm

(3) 1 mm (4) 4 mm

Answer (C)

Sol. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$\Rightarrow \frac{3}{2v}+\frac{1}{40}=-\frac{-1}{2\times20}$

$\Rightarrow \frac{3}{2v}=-\frac{1}{40}-\frac{1}{40}=-\frac{1}{20}$

$\Rightarrow v=-30\,\text{cm}$

So, M = $\frac{\mu_1 v}{\mu_2 u}=\frac{3(-30)}{\left(\frac{3}{2}\right)(-40)}=0{\cdot}5$

So $h = mh_0 = \frac{1}{2}\times2(\text{mm})=1\,\text{mm}$

10. Find change in surface energy if 512 small drops of radius $r = 2$ mm merge into a single large drop. (surface tension = S).

(1) $\pi S \times 7.168 \times 10^{-3}$ Joule

(2) $\pi S \times 3.584 \times 10^{-3}$ Joule

(3) $\pi S \times 1.792 \times 10^{-3}$ Joule

(4) $\pi S \times 6.284 \times 10^{-3}$ Joule

Answer (1)

Sol. Total volume

$$512 \cdot \frac{4}{3}\pi r^2 = \frac{4}{3}\pi R^2$$

$\Rightarrow R = 8r$ Here $r = 2$ mm

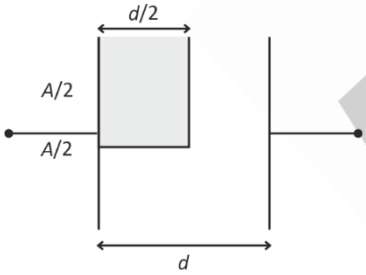
Now $14 = (512) \cdot 4\pi(r^2)S - 4\pi R^2 \cdot S$

$= 4\pi S(512r^2 - 64r^2)$

$= 4\pi S(448) \times 4 \times 10^{-6}$ Joule

$= \pi S \times 7.168 \times 10^{-3}$ Joule

11. One fourth volume of an empty capacitor of capacitance C_0 is filled with dielectric of constant $K = 5$. Surface area of dielectric is $A/2$ & width $d/2$. If $A >> d^2$, then, new capacitance is

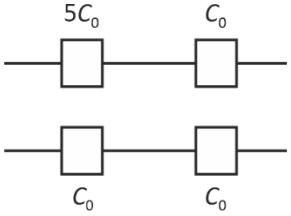


(1) $\frac{3}{2}C_0$ (2) $\frac{4}{3}C_0$

(3) $\frac{5}{4}C_0$ (4) $\frac{7}{6}C_0$

Answer (2)

Sol.



$$\frac{5C_0}{6} + \frac{C_0}{2} = \frac{4}{3}C_0$$

12. Two hollow conducting spheres are separated by large distance. They are connected by a conducting wire. If E_1 & E_2 are the magnitude of electric field near the surface of sphere (1) and sphere (2) respectively, then find the ratio of $\frac{E_1}{E_2}$.

(1) $\frac{9}{2}$ (2) $\frac{9}{4}$

(3) $\frac{2}{9}$ (4) $\frac{4}{9}$

Answer (2)

Sol. $\sigma_1 r_1 = \sigma_2 r_2 \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

And $E = \frac{\sigma}{\varepsilon_0}$

So $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{18}{8}$

$\Rightarrow \frac{E_1}{E_2} = \frac{9}{4}$

13. A charge particle of charge 1 μC moves along y-axis with constant speed 1.6×10^6 m/s. EM wave has electric field component $E = 600\sin(\omega t - kx)\hat{j}$ exist in region. Find maximum electric force on the charge.

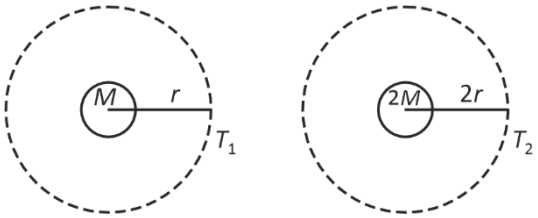
(1) 8×10^4 N (2) 6×10^4 N

(3) 2×10^{-4} N (4) 41×10^{-4} N

Answer (2)

Sol. $\vec{F} = 1.6 \times 10^{-6} \times 600 = 960\hat{j}$

14. Two satellites are revolving around two planets in circular orbits as shown. Find ratio of their time periods.



(1) $\frac{1}{2}$

(2) $\frac{1}{3}$

(3) $\frac{1}{4}$

(4) $\frac{1}{\sqrt{2}}$

Answer (1)

Sol. $T_1 = 2\pi\sqrt{\frac{r^3}{GM}}$

$T_2 = 2\pi\sqrt{\frac{8r^3}{2GM}} = 2T_1$

$\frac{T_1}{T_2} = \frac{1}{2}$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Power dissipated in an inductor coil is P. If coil of same material, but with number of turns, length and cross-section area being doubled, is used at same current then power dissipated is $\alpha\sqrt{2}P$ then α is _____

Answer (2)

Sol. $R \alpha$ length

Assuming tight, number of turns per unit length remain same.

$A \to 2A \Rightarrow r \to \sqrt{2}r$

Case 1 : Length = $(2\pi R)N = L$

Case 2 : Length = $(2\pi\sqrt{2}R)2N = 2\sqrt{2}L$

Power = L^2R

$R \to 2\sqrt{2}R$

$P \to 2\sqrt{2}P$

22. For a thin prism of angle A deviation of ray is 8. If refractive index of prism is 1.5 then $\frac{A}{\delta}$ is _____

Answer (2)

Sol. $(\mu - 1)A = 0.5A = \delta$

23.

24.

25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The correct increasing order of bond length among the following is

(1) $O_2^+, O_2, O_2^-, O_2^{2-}$

(2) $O_2^+, O_2, O_2^{2-}, O_2^-$

(3) $O_2^{2-}, O_2^-, O_2, O_2^+$

(4) $O_2^-, O_2^{2-}, O_2^+, O_2$

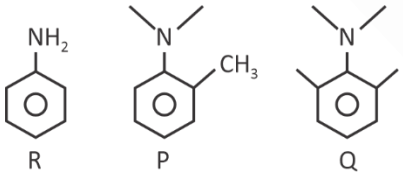
Answer (1)

Sol. More the bond order of molecule or ion less will be the bond length

$O_2^{2-} \Rightarrow BO = 1, O_2^- \Rightarrow BO = 1.5$

$O_2 \Rightarrow BO = 2, O_2^+ \Rightarrow BO = 2.5$

2. Write the correct order of rate of reaction of following with PhN_2Cl



(1) R > P > Q

(2) P > R > Q

(3) Q > P > R

(4) P > Q > R

Answer (1)

Sol. Due to S.I.R. Q and P are less reactive.

3. Match List-I with List-II

	List-I		List-II
(I)	Vitamin C	(A)	Thiamine
(II)	Vitamin B_1	(B)	Riboflavin
(III)	Vitamin B_6	(C)	Ascorbic Acid
(IV)	Vitamin B_2	(D)	Pyridoxine

(1) I-A, II-B, III-C, IV-D (2) I-C, II-A, III-D, IV-B

(3) I-A, II-C, III-B, IV-D (4) I-C, II-D, III-A, IV-B

Answer (2)

Sol. Vitamin C → Ascorbic Acid

Vitamin B_1 → Thiamine

Vitamin B_6 → Pyridoxine

Vitamin B_2 → Riboflavin

4. Arrange following functional group in the increasing order of priority of functional group:

$-CONH_2, -\overset{\overset{O}{\|}}{C}=O, -CHO, -CN, -C\equiv C-, -NH_2$

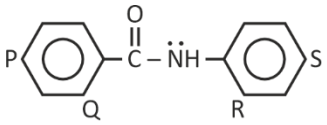
(1) 1 > 4 > 2 > 3 > 6 > 5 (2) 1 > 4 > 2 > 3 > 5 > 6

(3) 1 > 4 > 3 > 2 > 6 > 5 (4) 1 > 2 > 4 > 3 > 6 > 5

Answer (3)

Sol. Priority table.

5. In the given compound, the electrophile attack will be favoured at



(1) S (2) P

(3) Q (4) R

Answer (1)

Sol. Due to more electron density, E.A.S. will take place at position-S.

6. Which compound gives apple green colour in flame test?

(1) $BaSO_4$ (2) $Mn_3(PO_4)_2$

(3) $Ca_3(PO_4)_2$ (4) $Al_2(SO_4)_3$

Answer (1)

Sol. $Ba^{2+} \Rightarrow$ Apple green

$Ca^{2+} \Rightarrow$ Brick red

Mn^{2+} and Al^{3+} generally do not produce a distinct or characteristic colour to the flame in flame test.

7. One mole of alkane reacts with 8 mole of O_2. The molecular formula of alkane is

(1) C_4H_{10} (2) C_5H_{12}

(3) C_7H_{14} (4) C_3H_8

Answer (2)

Sol. $C_nH_{2n+2} + \frac{3n+1}{2}O_2 \rightarrow nCO_2 + (n+1)H_2O$

$\therefore \frac{3n+1}{2} = 8 \Rightarrow 3n = 15$

$\Rightarrow n = 5$

Alkane is C_5H_{12}

8. 18 g of H_2O reacts with x g of Fe to produce Fe_3O_4. 'x' is (Atomic mass of Fe = 56 u)

(1) 84 g (2) 63 g

(3) 42 g (4) 51 g

Answer (3)

Sol. $3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2$

3×56 g 4 mol

$1\,mol\,H_2O \rightarrow \frac{3\times 56}{4} = 42$ g of Fe

9. The K_{sp} values of Ag_2CrO_4 and AgBr are 32x and 4y respectively. The ratio of solubilities $\frac{S_1}{S_2}$ is

[Here, S_1 is solubility of Ag_2CrO_4 and S_2 is solubility of AgBr]

(1) $\frac{2x^{1/3}}{y^{1/2}}$ (2) $\frac{x^{1/3}}{y^{1/2}}$

(3) $\frac{x^{1/3}}{2y^{1/2}}$ (4) $\frac{(2x)^{1/3}}{(2y)^{1/2}}$

Answer (2)

Sol. $Ag_2CrO_4 \rightleftharpoons \underset{2S_1}{2Ag^+} + \underset{S_1}{CrO_4^{2-}}$

$AgBr \rightleftharpoons \underset{S_2}{Ag^+} + \underset{S_2}{Br^-}$

$K_{sp}[Ag_2CrO_4] = [Ag^+]^2[CrO_4^{2-}]$

$32x = (2S_1)^2 \times (S_1)$

$8x = S_1^3$

$2x^{1/3} = S_1$ ...(i)

$K_{sp}(AgBr) = [Ag^+][Br^-]$

$4y = S_2 \times S_2$

$4y = S_2^2$

$2y^{1/2} = S_2$ ...(ii)

$\frac{S_1}{S_2} = \frac{x^{1/3}}{y^{1/2}}$

10. The orbit angular momentum of e in n^{th} shell of H atom is $\frac{3h}{\pi}$. The total energy of e is

(1) –0.38 eV (2) –3.4 eV

(3) –0.544 eV (4) –0.85 eV

Answer (1)

Sol. Orbit angular momentum (mvr) $\Rightarrow \frac{h}{2\pi} \times n = \frac{3h}{\pi}$, So n = 6

Total energy of e = $-13.6 \times \frac{z^2}{n^2}$ eV

$= -13.6 \times \frac{1^2}{6^2} = -0.38$ eV

11. Arrange the following complexes in the increasing order of crystal field splitting energy (Δ_0)

(a) $[Cr(CN)_6]^{3-}$ (b) $[CrF_6]^{3-}$

(c) $[Cr(NH_3)_6]^{3+}$ (d) $[Cr(en)_3]^{3+}$

(1) (b) < (c) < (d) < (a) (2) (a) < (b) < (c) < (d)

(3) (a) < (d) < (c) < (b) (4) (b) < (a) < (c) < (d)

Answer (1)

Sol. More the strength of ligand, more will be splitting energy.

Ligand strength order $F^- < NH_3 < en < CN^-$

So order of $\Delta_0 \Rightarrow$ (a) > (d) > (c) > (b)

12. Calculate the energy required for the following process

$Li^{2+} \rightarrow Li^{3+} + e^-$

Given: Ground state energy of Hydrogen is –13.6 eV/atom

(1) 13.6 eV/atom (2) 122.4 eV/atom

(3) 54.4 eV/atom (4) 30.6 eV/atom

Answer (2)

Sol. The energy required for ionisation of

Li^{2+} ion = $+13.6\ z^2$

$= 13.6 \times 3^2$

$= 122.4$ eV/atom

13. 20 g fluoroacetic acid is dissolved in 500 gm water. If depression in freezing point is 1°C, then calculate K_a for fluoroacetic acid. [Assume molality is same as molarity]

(1) 1.18×10^{-3} (2) 1.5×10^{-5}

(3) 1.18×10^{-4} (4) 1.2×10^{-6}

Answer (1)

Sol. $\Delta T_f = i \times K_f \times m$

$$1 = (1+\alpha) \times 1.86 \times \frac{20 \times 1000}{78 \times 500}$$

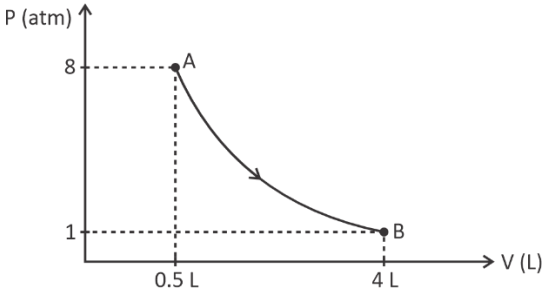
$$1 + \alpha = \frac{1}{0.954} = 1.048$$

$\alpha = 0.048$

4.8%

$$K_a = \alpha^2 c = (0.048)^2 \times \frac{20 \times 1000}{78 \times 500} = 1.18 \times 10^{-3}$$

14. Calculate work in following process during A to B, in following graph. (Given $C_V = 3R$)



(1) –6.2 L.atm (2) –8.3 L.atm

(3) –4.8 L.atm (4) –9.2 L.atm

Answer (2)

Sol. $W_{Rev} = -2.303 \times (P_f \cdot V_f) \times \log \frac{V_f}{V_i}$

$$= -2.303 \times (4 \times 1) \times \log \frac{4}{0.5}$$

$= -2.303 \times 4 \times 0.9$

$= -8.3$ L.atm

15. Match **List-I** with **List-II**

	List-I (Molecules)		**List-II (Test)**
(I)	Ethanol	(P)	Neutral $FeCl_3$
(II)	Phenol	(Q)	Cerric ammonium nitrate
(III)	Ethanoic Acid	(R)	Schiff reagent
(IV)	Acetaldehyde	(S)	$NaHCO_3$

(1) I-P, II-Q, III-R, IV-S (2) 1-Q, II-S, III-P, IV-R

(3) I-Q, II-P, III-S, IV-S (4) 1-P, II-Q, III-S, IV-R

Answer (3)

Sol. Ethanol → Cerric ammonium nitrate

Phenol → Neutral $FeCl_3$

Ethanoic Acid → $NaHCO_3$

Acetaldehyde → Schiff reagent

16. Which of the following relation about first order and zero order reaction is correct

(1) $\left(t_{1/2}\right)_{\text{first order}} = 2\left(t_{100\%}\right)_{\text{first order}}$

(2) $\left(t_{100\%}\right)_{\text{first order}} = \text{Infinite time of } t_{1/2} \text{ of first order}$

(3) $\left(t_{1/2}\right)_{\text{zero order}} = 2 \times \left(t_{100\%}\right)_{\text{zero order}}$

(4) $\left(t_{100\%}\right)_{\text{zero order}} = \text{inf inite time of } t_{1/2} \text{ of zero order}$

Answer (2)

Sol. For zero order

$[A_t]_0 = [A_0] - kt$

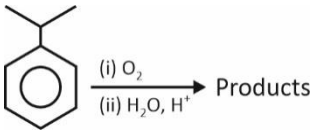
$t_{1/2} = \frac{[A_0]}{2k}$ $\quad t_{100\%} = \frac{[A_0]}{k}$

For first order

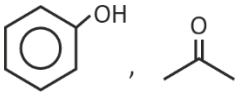
$t_{1/2} = \frac{0.693}{k}$

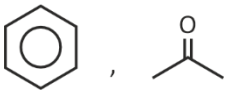
First order reaction never goes to completion.

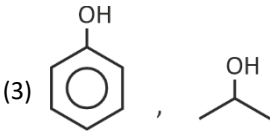
17. Consider the following reaction :



Products obtained are

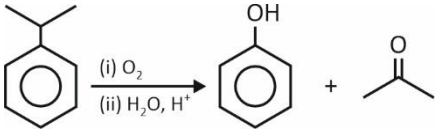
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

18. Given below are two statements.

Statement I : First I.E. order : Na > Mg > Al.

Statement II : 3rd I.E. order : Mg > Al > Na.

(1) Both Statement I and Statement II are incorrect

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are correct

Answer (1)

Sol. IE_1 (KJ/mol)

Na 496

Mg 737

Al 577

For third IE, electronic configuration drives the order.

$Mg^{2+} \xrightarrow{-1e^-} Mg^{3+}$

2, 8 2, 7

$Al^{2+} \longrightarrow Al^{3+}$

2, 8, 1 2, 8

$Na^{2+} \longrightarrow Na^{3+}$

2, 7 2, 6

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. For first order reaction rate constant at 27°C and t°C is 1.5×10^3 and 4.5×10^3 respectively. If the activation energy of reaction is 60 kJ, then find temperature t.

($R = 8.3\ Jmol^{-1}\ K^{-1}$)

Answer (41)

Sol. $\log\frac{k_1}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log\frac{4.5}{1.5} = \frac{E_a}{2.303R}\left(\frac{1}{300} - \frac{1}{T}\right)$

$\log 3 = \frac{60000}{2.303 \times 8.3}\left(\frac{(T-300)}{300T}\right)$

$\frac{0.48 \times 2.303 \times 8.3}{60000} = \frac{T-300}{300T}$

$0.0001529 \times 300T = T - 300$

On solving,

$T = \frac{300}{0.95413}$

$T = 314.4\ K$

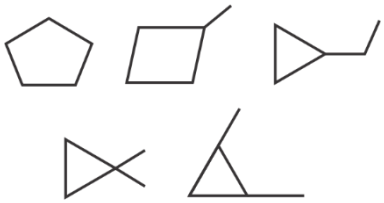
$= 41.4°C$

$\approx 41°C$

22. Calculate number of isomers of C_5H_{10}, which does not decolorise cold alkaline $KMnO_4$ solution.

Answer (7)

Sol. Cycloalkanes do not react with cold $\bar{O}H$ /$KMnO_4$, while alkenes do react.



(d + l + meso)

Total isomers = 1 + 1 + 1 + 1 + 3 = 7

23. 8 g of $[CrCl_x(H_2O)_y]Cl_{(3-x)} \cdot H_2O_{(6-y)}$ react with average $AgNO_3$ to form 8.61 g of AgCl. The value of (x × y) is _____.

Answer (5)

Sol. mol of $CrCl_3 \cdot 6H_2O = \frac{8}{266.5} = 0.03$

mol of AgCl formed = $\frac{8.61}{143.5} = 0.06$

0.03 mol of compound forms 0.06 mol AgCl

So, 1 mol of compound forms = $\frac{0.06}{0.03}$ = 2 mol AgCl

So formula is $[CrCl(H_2O)_5]Cl_2 \cdot H_2O$

3 – x = 2, x = 1

6–y = 1, y = 5

x × y = 1 × 5 = 5

24.

25.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If y = f(x) is the solution of the differential equation $(1+\sin x)\frac{dy}{dx}+\cos x=0$, such that $f(0)=0$ then $f\left(\frac{\pi}{2}\right)$ is equal to

(1) ln2 (2) –ln2

(3) ln3 (4) ln4

Answer (2)

Sol. $\frac{dy}{dx}=\frac{-\cos x}{1+\sin x}$

$\Rightarrow \int dy=\int\left(\frac{-\cos x}{1+\sin x}\right)dx$

$\Rightarrow y=-\ln|1+\sin x|+c$

Since, $f(0)=0$

$\Rightarrow 0=0+c \Rightarrow c=0$

$f(x)=\ln\left|\frac{1}{1+\sin x}\right|$

$f\left(\frac{\pi}{2}\right)=\ln\left|\frac{1}{2}\right|=-\ln 2$

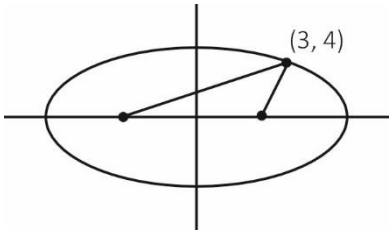
2. If eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, which passes through point (3, 4) is $\frac{\sqrt{5}}{3}$, then length of latus rectum of ellipse is

(1) $\frac{4\sqrt{5}}{3}$ (2) $\frac{8\sqrt{5}}{3}$

(3) $\frac{4\sqrt{7}}{3}$ (4) $\frac{8\sqrt{7}}{3}$

Answer (2)

Sol.



$e^2=\frac{5}{9}, \quad \frac{9}{a^2}+\frac{16}{b^2}=1$

$1-\frac{b^2}{a^2}=\frac{5}{9} \Rightarrow \boxed{\frac{b^2}{a^2}=\frac{4}{9}}$

$\frac{9}{a^2}+\frac{16}{\frac{4a^2}{9}}=1 \Rightarrow \frac{9}{a^2}+4\times\frac{9}{a^2}=1$

$\Rightarrow a^2=45$

$\Rightarrow b^2=\frac{4a^2}{9}=20$

$\Rightarrow$ length of latus rectum = $\frac{2b^2}{a}$

$=\frac{8\sqrt{5}}{3}$

3. Let $x\in[-\pi,\pi]$.

$S=\{x : \sin x(\sin x+\cos x)=a, a\in I\}$

Then number of elements in set S is equal to

(1) 5

(2) 10

(3) 9

(4) 4

Answer (3)

Sol. $x \in [-\pi, \pi]$

Let $E = \sin^2 x + \sin x \cdot \cos x$

$$\Rightarrow \frac{1}{2}(2\sin^2 x + 2\sin x \cdot \cos x) = E$$

$$\Rightarrow \frac{1}{2}(1 - \cos 2x + \sin 2x) = E$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2}[\sin 2x - \cos 2x] = E$$

$\sin 2x - \cos 2x \in [-\sqrt{2}, \sqrt{2}]$

$$\frac{1}{2} + \frac{1}{2}(\sin 2x - \cos 2x) \in \left[\frac{1}{2} - \frac{1}{\sqrt{2}}, \frac{1}{2} + \frac{1}{\sqrt{2}}\right]$$

a = 0 and 1

For a = 0

$\sin x(\sin x + \cos x) = 0 \Rightarrow \sin x = 0, \tan x = -1$

$$\Rightarrow x = -\pi, -\frac{\pi}{4}, 0, \frac{3\pi}{4}, \pi$$

For a = 1

$$\sin x \cdot \cos x = \cos^2 x$$

$$\Rightarrow \cos x(\sin x - \cos x) = 0$$

$$\Rightarrow \cos x = 0, \tan x = 1$$

$$x = -\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{2}$$

$\therefore$ Number of elements = 9

4. Number of seven-digit number formed by using the digits 1, 2, 3, 4, 5 at least once are

(1) 16800 (2) 13200

(3) 15200 (4) 15800

Answer (1)

Sol. **Case 1 :** One digit repeat 3 times and the other repeat once (x, x, x, a, b, c, d)

Total Number ${}^5C_1 \times \frac{7!}{3!}$ = 5 × 840 = 4200

Case 2 : Two digits repeat twice each and other repeated once (x, x, a, a, b, c, d)

Total Number ${}^5C_2 \times \frac{7!}{2!2!}$ = 10 × 1260 = 12600

Total numbers 4200 + 12600 = 16800

5. The value of $\int_0^3 \frac{e^x + e^{-x}}{([x])!} dx$ equals (Here [.] denotes the greatest integer function)

(1) $\frac{1}{2}\left(e^2 + e^3 - e^{-2} - e^{-3}\right)$

(2) $e^2 - e^3 + e^{-2} - e^{-3}$

(3) $\frac{1}{4}\left(e^2 + e^3 - e^{-2} - e^{-3}\right)$

(4) $\frac{1}{2}\left(e^2 + e - e^{-1} - e^{-2}\right)$

Answer (1)

Sol. $$\int_0^3 \frac{e^x + e^{-x}}{([x])!} dx$$

$$= \int_0^1 \frac{\left(e^x + e^{-x}\right)}{0!} dx + \int_1^2 \frac{e^x + e^{-x}}{1!} + \int_2^3 \frac{e^x + e^{-x}}{2!} dx$$

$$= \left(e^x - e^{-x}\right)\Big|_0^1 + \left(e^x - e^{-x}\right)\Big|_0^2 + \frac{\left(e^x - e^{-x}\right)}{2}\Bigg|_2^3$$

$$= \left(e - \frac{1}{e}\right) - (1 - 1) + \left(e^2 - \frac{1}{e^2}\right) - \left(e - \frac{1}{e}\right) + \frac{1}{2}\left(e^3 - \frac{1}{e^3}\right) - \frac{1}{2}\left(e^2 - \frac{1}{e^2}\right)$$

$$= \frac{1}{2}\left(e^2 - \frac{1}{e^2}\right) + \frac{1}{2}\left(e^3 - \frac{1}{e^3}\right)$$

$$= \frac{1}{2}\left(e^2 + e^3 - e^{-2} - e^{-3}\right)$$

6. The value of $\sin\frac{\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{7\pi}{18}$ is equal to

(1) $\frac{1}{8}$ (2) $\frac{1}{16}$

(3) $\frac{1}{4}$ (4) $\frac{1}{32}$

Answer (1)

Sol. We know that $\sin\theta\cdot\sin\left(\frac{\pi}{3}-\theta\right)\cdot\sin\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}\sin3\theta$

$\therefore \quad \sin\frac{\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{7\pi}{18}=\sin\frac{\pi}{18}\cdot\sin\left(\frac{\pi}{3}-\frac{\pi}{18}\right)\cdot\sin\left(\frac{\pi}{3}+\frac{\pi}{18}\right)$

$=\frac{1}{4}\sin\left(3\cdot\frac{\pi}{18}\right)$

$=\frac{1}{8}$

7. Let $f(x)=\begin{cases}\left|\frac{\sin x}{x}\right| & , \ x\neq 0\\ 1 & , \ x=0\end{cases}$. Then total number of critical point(s) in $x \in (-2\pi, 2\pi)$ is

(1) 3 (2) 5

(3) 9 (4) 1

Answer (2)

Sol. Critical points: Point at which derivative is zero or function is not differentiable

$$f(x)=\begin{cases}\left|\frac{\sin x}{x}\right| & , \ x-\{0\}\\ 1 & , \ x=0\end{cases}$$

$$f'(0^+)=\lim_{h\to0}\left|\frac{\frac{\sinh}{h}-1}{h}\right|=\lim_{h\to0}\frac{\sinh-h}{h^2}=\lim_{h\to0}\frac{\cosh-1}{2h}=0$$

Similarly, $f'(0^-)=0$

$f'(0)=0$ and differentiable but $f'(0)\Rightarrow$ critical point

$$f'(\pi^+)=\frac{\left|\frac{\sin(\pi+h)}{\pi+h}\right|-0}{h}=\lim_{h\to0}\frac{-\sin(\pi+h)}{(\pi+h)h}=\lim_{h\to0}\frac{-\cos(\pi+h)}{2h+\pi}=\frac{1}{\pi}$$

Similarly, $f'(\pi^-)=-\frac{1}{\pi}$

$\Rightarrow f(x)$ is not differentiable at $x=\pi$ and $-\pi$

And $f'(x)=\frac{x\cos x-\sin x}{x^2}\Rightarrow \tan x=x$

$\Rightarrow$ x $\simeq$ 4.49

$\Rightarrow \tan x=x$ has 2 solutions in $x\in(-2\pi,2\pi)$

$\Rightarrow 2$ critical points

$\Rightarrow$ Total 5 points

8. Consider two A.P.s such that A.P.$_1$ has 1st term 1, common difference 5 and total terms 101 and A.P.$_2$ has 1st term 9, common difference 7 and total terms 71 then the number of common terms which appear in both these A.P.s is

(1) 11 (2) 14

(3) 12 (4) 13

Answer (2)

Sol. (A.P.)$_1$: a_1 = 1, d_1 = 5, n_1 = 101

$\Rightarrow$ A_n = 5n – 4, last term = 501

(A.P.)$_2$: a_2 = 9, d_2 = 7, n_2 = 71

$\Rightarrow$ Last term $=499$

B_n = 7m + 2

$\Rightarrow$ 5n – 4 = 7m + 2

$\Rightarrow$ 5n = 7m + 6

$\Rightarrow 5 / 7m+6 \Rightarrow m_{min}=2$,

$\Rightarrow$ 1st common term is 7(2) + 2 = 16

Common difference = Lcm (5, 7) = 35

$\Rightarrow$ Common A.P. is

16, 51, 86, ... (≤499)

$\Rightarrow$ 16 + (N – 1) 35 ≤ 499

$\Rightarrow \quad N-1\le\frac{483}{35}\simeq13.8$

$\Rightarrow$ $N \le 14.8$

$\Rightarrow$ N = 14

9. If ${}^{36}C_{r+1} = \frac{6.{}^{35}C_r}{k^2-3}$ such that $S = \{(r,k)\}$ then number of elements in set S is

(1) 9 (2) 4

(3) 13 (4) 7

Answer (2)

Sol. ${}^{36}C_{r+1} = \frac{6.{}^{35}C_r}{k^2-3}$

$\Rightarrow \frac{36}{r+1}\left({}^{35}C_r\right) = \frac{6.{}^{35}C_r}{(k^2-3)}$

since ${}^{35}C_r > 0$

$\Rightarrow \frac{6}{r+1} = \frac{1}{k^2-3}$

$\Rightarrow 6(k^2-3) = r+1$

$\Rightarrow 6 | r+1$

$\Rightarrow r+1 \in \{0, 6, 12, 18, 24, 30, 36\}$

$\Rightarrow$

$r+1$	0	6	12	18	24	30	36
k^2-3	0	1	2	3	4	5	6
k^2	3	(4)	5	6	7	8	(9)

$k = \pm 2, \pm 3$

$r = 5 \Rightarrow k \pm 2$

$r = 35 \Rightarrow k \pm 3$

$S = \{(5, 2), (5, -2), (35, 3), (35, -3)\}$

$\Rightarrow n(S) = 4$

10. If the system of equations

$x + 2y + z = 5$

$2x + y + \alpha z = 5$

$8x + y + z = 18$ has no solution, then α is equal to

(1) 3 (2) $\frac{1}{3}$

(3) 4 (4) $\frac{9}{15}$

Answer (4)

Sol. $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & \alpha \\ 8 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow 15\alpha - 9 = 0$

$\Rightarrow \alpha = \frac{9}{15}$

$D_x = \begin{vmatrix} 1 & 5 & 5 \\ 2 & 1 & 5 \\ 8 & 1 & 18 \end{vmatrix} \neq 0$

11. The domain of $f(x) = \sqrt{\log_{0.6}\left(\frac{|2x-5|}{|x^2-4|}\right)}$ is

(1) $\left(-\infty, -1-\sqrt{10}\right) \cup \left(-1+\sqrt{10}, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

(2) $\left(-\infty, -1-\sqrt{10}\right) \cup \left(\sqrt{10}, \infty\right)$

(3) $\left(-1-\sqrt{10}, \frac{5}{2}\right)$

(4) $\left(-\infty, -1-\sqrt{10}\right) \cup \left(\sqrt{10}-1, \infty\right)$

Answer (1)

Sol. $\frac{|2x-5|}{|x^2-4|} \neq 0$

$x^2 - 4 \neq 0 \Rightarrow x \neq \pm 2$ and $x \neq \frac{5}{2}$

$\log_{0.6}\left(\frac{|2x-5|}{|x^2-4|}\right) \geq 0$

$\frac{|2x-5|}{|x^2-4|} \leq 1$ {$\because$ base < 1}

$|2x-5| \leq |x^2-4|$

$\Rightarrow (2x-5)^2 - (x^2-4)^2 \leq 0$

$\Rightarrow x^4 - 12x^2 + 20x - 9 \geq 0$

$\Rightarrow (x-1)^2 (x^2 + 2x - 9) \geq 0$

$(x-1)^2 \geq 0$ (always)

$x^2 + 2x - 9 \geq 0$

$\left(x+1-\sqrt{10}\right)\left(x+1+\sqrt{10}\right) \geq 0$

$\Rightarrow x \in (-\infty, -1-\sqrt{10}\,] \cup \left[-1+\sqrt{10}, \infty\right) - \left\{\frac{5}{2}\right\}$

12. If $L_1 : \frac{x-a}{2} = \frac{y-2}{3} = \frac{z-b}{6}$ and $L_2 : \frac{x-b}{3} = \frac{y-7}{6} = \frac{z-1}{3}$ intersect in xy plane. Then, the value of $|a + b|$ is

(1) 15 (2) 10

(3) 14 (4) 11

Answer (1)

Sol. General point on L_1 is $(2k+a, 3k+2, 6k+b)$

General point on L_3 is $(3\lambda+b, 6\lambda+7, 3\lambda+1)$

$2K + a = 3\lambda + b$ ...(1)

$3k + 2 = 6\lambda + 7$ ...(2)

Now Lines Intersect in xy plane

$\Rightarrow 6K + b = 0$ and $3\lambda + 1 = 0$

and $\boxed{b = -6K} \Rightarrow \lambda = \frac{-1}{3}$

Putting $\lambda = \frac{-1}{3}$ in eq^n (2)

$3K + 2 = -2 + 7$

$\Rightarrow 3K + 2 = 5$

$\Rightarrow \boxed{K = 1}$

$\Rightarrow \boxed{b = -6}$

Now putting value of b, k and λ in (1)

$2(1) + a = 3\left(\frac{-1}{3}\right) - 6$

$2 + a = -1 - 6$

$2 + a = -7$

$\boxed{a = -9}$

$|a + b| = |-9 - 6| = 15$

13. If $\frac{dy}{dx} = (2 + \ln x)$ has solution $y = y(x)$ such that $y(1) = e$ then $y(e)$ is equal to

(1) $2e - 1$ (2) $3e - 1$

(3) $2e + 1$ (4) $3e + 1$

Answer (2)

Sol. $\frac{dy}{dx} = (2 + \ln x)$

$\int dy = \int (2 + \ln x)\,dx$

$y = 2x + x \ln x - x + C$

$y = x + x \ln x + C$

$y(1) = e = 1 + C \Rightarrow C = e - 1$

$y(e) = e + e \ln e + e - 1$

$= 3e - 1$

14. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°, then $3\left|3\vec{a} + 4\vec{b}\right| + 4\left|3\vec{a} - 4\vec{b}\right|$ is equal to___.

(1) $9\sqrt{7} + 12\sqrt{3}$ (2) $18\sqrt{7} + 24\sqrt{3}$

(3) $18\sqrt{7} - 6\sqrt{3}$ (4) $12\sqrt{7} - 6\sqrt{3}$

Answer (2)

Sol. $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos 60° = 2 \times 3 \times \frac{1}{2} = 3$

$\left|13\vec{a} + 4\vec{b}\right|^2 = |3\vec{a}|^2 + \left|4\vec{b}\right|^2 + 2\left(3\vec{a}.4\vec{b}\right)$

$= 9|\vec{a}|^2 + 16|\vec{b}|^2 + 24 \times 3$

$= 9(4) + 16(9) + 72$

$= 252$

$\left|3\vec{a} + 4\vec{b}\right| = 6\sqrt{7}$

$\left|3\vec{a} - 4\vec{b}\right|^2 = |3\vec{a}|^2 + \left|3\vec{b}\right|^2 - 2\left(3\vec{a}.4\vec{b}\right) = 108$

$\left|3\vec{a} - 4\vec{b}\right| = 6\sqrt{3}$

$3\left|3\vec{a} + 4\vec{b}\right| + 4\left|3\vec{a} - 4\vec{b}\right| = 18\sqrt{7} + 24\sqrt{3}$

15. The value of $\int_2^4 \sqrt{2^x - 4}\, dx + \int_0^{2\sqrt{3}} \log_2(x^2+4)dx$ is equal to

(1) $4\sqrt{3}$ (2) $2\sqrt{3}$

(3) $8\sqrt{3}$ (4) $16\sqrt{3}$

Answer (3)

Sol. $y^2 = 2^x - 4$

$\Rightarrow y^2 + 4 = 2^x$

$\Rightarrow \log_2(y^2+4) = x$

$\sqrt{2^x - 4}$ and $\log_2(x^2+4)$ are inverse of each other

$\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx = b.f(b) - a.f(a)$

$\therefore\ a = 2,\ b = 4,\ f(a) = 0,\ f(b) = 2\sqrt{3}$

$\therefore\ 4\cdot 2\sqrt{3} - 0 = 8\sqrt{3}$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the roots of the equation $x(x+2) + (x+1)(x+3) + ... + (x + (n-1)) + (x + n + 1) = 4n$ are α and $\alpha + 2$. Then, the value of $|2\alpha + n|$ is

Answer (3)

Sol. Put $x = \alpha$

$\alpha(\alpha+2) + (\alpha+1)(\alpha+3) + (\alpha+2)(\alpha+4)+....+(\alpha + (n-1) + (\alpha +(n+1))) = 4n$ ....(i)

Put $x = \alpha + 2$

$(\alpha+2)(\alpha+4) + (\alpha+3)(\alpha+5)+....+(\alpha + (n-1)) + (\alpha +(n+1)) + (\alpha + n) + (\alpha + (n+2)) + (\alpha + n + 1)(\alpha + n + 3) = 4n$....(ii)

Subtracting equation (ii) from (i)

$\alpha(\alpha+2) + (\alpha+1)(\alpha+3) = (\alpha + n)(\alpha +(n+2)) + (\alpha + n + 1)(\alpha +(n+3))$

$\Rightarrow \alpha^2 + 2\alpha + \alpha^2 + 4\alpha + 3 = \alpha^2 + (2n+2)\alpha + n(\alpha+2) + \alpha^2 + (2n+4)\alpha + (n+1)(n+3)$

$\Rightarrow 6\alpha + 3 = 2n\alpha + 2\alpha + n^2 + 2n + 2n\alpha + 4\alpha + n^2 + 4n + 3$

$\Rightarrow 4n\alpha + 2n^2 + 6n = 0$

$\Rightarrow n(4\alpha + 2n + 6) = 0$

$n \neq 0$

$4\alpha + 2n = -6$

$2\alpha + n = -3$

$|2\alpha + n| = 3$

22.

23.

24.

25.

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