MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The coefficient of x^2 in the binomial expansion of $\left(2x^2+\frac{1}{x}\right)^{10}$ is

(1) 3260 (2) 3360

(3) 1760 (4) 1890

Answer (2)

Sol. $\left(2x^2+\frac{1}{x}\right)^{10}$

$$T_{r+1}={}^{10}C_r(2x^2)^{10-r}\left(\frac{1}{x}\right)^r$$

$20 - 2r - r = 2$

$r = 6$

coff of $x^2 = {}^{10}C_6(2)^4 = 3360$

2. If it is given that probability of A becoming captain of the team is 0.8 and B becomes captain is 0.4. If A becomes captain then chances of winning the match by the team is 0.6 and when B becomes captain then chances of team winning the match is 0.7. Then the probability that team wins the match is

(1) 0.76 (2) 0.78

(3) 0.74 (4) 0.84

Answer (1)

Sol. $P(w)=P(A)P\left(\frac{w}{A}\right)+P(B)P\left(\frac{w}{B}\right)$

$= 0.8 \times 0.6 + 0.4 \times 0.7$

$=\frac{48+28}{100}=0.76$

3. Let $A = \{2, 3\}$ and $B = \{5, 6\}$. Then, the number of relations from $A \times B$ to $A \times B$ are

(1) 2^{12} (2) 2^{16}

(3) 2^{10} (4) 2^{15}

Answer (2)

Sol. $n(A \times B) = 2 \times 2$

$= 4$

Number of relations from $A \times B$ to $A \times B$

$= 2^{4\times 4}$

$= 2^{16}$

4. Let $A_1, A_2, A_3, \ldots, A_{49}$ be 49 AM's between 49 and 149. Then the mean of A_1, A_{25}, A_{47} and A_{49} is

(1) 110 (2) 120

(3) 130 (4) 140

Answer (1)

Sol. $d=\frac{149-49}{49+1}$

$\boxed{d=2}$

$A_1 = 49 + 2$

$A_{25} = 49 + 25.2$

$A_{47} = 49 + 47.2$

$A_{49} = 49 + 49.2$

$$\text{Mean}=\frac{49.4+2[1+25+47+49]}{4}=110$$

5. The value of $\sum_{r=1}^{10}\frac{r}{4+r^4}$ is equal to

(1) $\frac{4832}{123565}$ (2) $\frac{4565}{12322}$

(3) $\frac{4565}{12321}$ (4) $\frac{4564}{12321}$

Answer (2)

Sol. $\displaystyle\sum_{r=1}^{10}\frac{r}{4+r^4}=\sum_{r=1}^{10}\frac{r}{r^4+4-4r^2+4r^2}$

$\displaystyle=\sum_{r=1}^{10}\frac{r}{(r^2+2)^2-(2r)^2}$

$\displaystyle=\sum_{r=1}^{10}\frac{r}{(r^2+2r+2)-(r^2-2r+2)}$

$\displaystyle=\frac{1}{4}\sum_{r=1}^{10}\left(\frac{(r^2+2r+2)-(r^2-2r+2}{(r^2+2r+2)(r^2-2r+2)}\right)$

$\displaystyle=\frac{1}{4}\sum_{r=1}^{10}\left(\frac{1}{r^2-2r+2}-\frac{1}{r^2+2r+2}\right)$

$\displaystyle=\frac{1}{4}\left[1-\frac{1}{5}+\frac{1}{2}-\frac{1}{10}+\frac{1}{5}-\frac{1}{17}+\frac{1}{10}-\frac{1}{26}+\ldots+\frac{1}{65}-\frac{1}{101}+\frac{1}{82}-\frac{1}{122}\right]$

$\displaystyle=\frac{1}{4}\left[1+\frac{1}{2}-\frac{1}{101}-\frac{1}{122}\right]$

$\displaystyle=\frac{4565}{12322}$

6. If α, β are the roots of the equation $x^2 - 4x + p = 0$ and γ and δ are the roots of the equation $x^2 - x + q = 0$. If α, β, γ and δ form a GP with positive common ratio, then the value of $p + q$ is

(1) $\frac{22}{9}$ (2) $\frac{33}{9}$

(3) $\frac{21}{9}$ (4) $\frac{34}{9}$

Answer (4)

Sol. $\because\ \alpha,\beta,\gamma,\delta\rightarrow GP$

$\Rightarrow\alpha=a\quad\beta=ar,\gamma=ar^2,\delta=ar^3$

$\because\ x^2-4x+P=0<^{\alpha}_{\beta}$

$\Rightarrow a+ar=4$ ...(1)

$a^2r=P$

Also $x^2-x+q=0<^{\gamma}_{\delta}$

$ar^2+ar^3=1$ ...(2)

$a^2r^5=q$

from (1) and (2)

$\dfrac{a(1+r)}{ar^2(1+r)}=4$

$\boxed{r=\pm\frac{1}{2}}$

If $r=\dfrac{1}{2}$ $a\left(\dfrac{3}{2}\right)=4$

$\Rightarrow a=\dfrac{8}{3}$

Now $P=\left(\dfrac{8}{3}\right)^2\times\dfrac{1}{2}$

$q=\left(\dfrac{8}{3}\right)^2\left(\dfrac{1}{2}\right)^5$

$p+q=\boxed{\frac{34}{9}}$

7. If Z_1 and Z_2 are the solutions of the equation $6Z^2 - 11Z + 59 - 3i = 0$ then $|Z_1|^2 + |Z_2|^2$ is equal to

(1) $\frac{709}{36}$ (2) $\frac{703}{18}$

(3) $\frac{705}{36}$ (4) $\frac{701}{36}$

Answer (1)

Sol. $6z^2 - 11z + 59 - 3i = 0$

$D = (-11)^2 - 4(6)(59 - 3i)$

$= -1295 + 72i$

$\sqrt{-1295+72i}=x+iy$

$\Rightarrow$ $x^2 - y^2 = -1295$

$xy = 36$

$\Rightarrow$ $x = \pm1$ then $y = \pm 36$

$$z = \frac{11 \pm (1 \pm 36i)}{12}$$

$$= \quad 1+3i, \frac{5}{6} - 3i$$

$$\Rightarrow \quad |z_1|^2 = 10$$

$$|z_2|^2 = \left(\frac{25}{36} + 9\right)$$

$$|z_1|^2 + |z_2|^2 = \frac{709}{36}$$

8. If $f(x) = \lim\limits_{y \to 0} \frac{(1-\cos xy)\tan(xy)}{y^3}$ then number of roots of the equation $f(x) = \sin x$ is

(1) 0

(2) 2

(3) 3

(4) 1

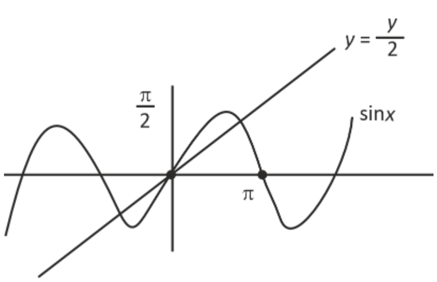
Answer (3)

Sol. $f(x) = \lim\limits_{y \to 0} \frac{(1-\cos xy)\tan(xy)}{y^3}$

$$= \lim_{y \to 0} \frac{2\sin^2\left(\frac{xy}{2}\right)}{\left(\frac{xy}{2}\right)^2 \times \left(\frac{y}{x^2}\right)} \times \frac{\tan(xy)}{(xy) \times \frac{1}{x}}$$

$$= \lim_{y \to 0} \frac{x^2}{2} \times \frac{1}{x} = \frac{x}{2}$$

$$f(x) = \frac{x}{2}$$



9. Let $\overrightarrow{OP} = \vec{a}, \overrightarrow{OQ} = \vec{b}$. Let a point R be such that $\overrightarrow{OP} = 5\overrightarrow{OR}$. Let a point M be such that $\overrightarrow{OQ} = 5\overrightarrow{RM}$ then $\overrightarrow{PM}$ is equal to

(1) $\frac{1}{5}\left(\vec{b} - 4\vec{a}\right)$ (2) $\frac{1}{5}\left(4\vec{b} - \vec{a}\right)$

(3) $\frac{1}{5}\left(\vec{b} + 4\vec{a}\right)$ (4) $\frac{1}{5}\left(4\vec{b} + \vec{a}\right)$

Answer (1)

Sol. $\overrightarrow{OP} = \vec{a}, \overrightarrow{OQ} = \vec{b}$

$$\overrightarrow{OP} = 5\overrightarrow{OR} \Rightarrow \overrightarrow{OR} = \frac{1}{5}\vec{a}$$

$$\overrightarrow{OQ} = 5\overrightarrow{RM} \Rightarrow \overrightarrow{RM} = \frac{1}{5}\vec{b}$$

$$OM = \overrightarrow{OR} + \overrightarrow{RM} = \frac{1}{5}\left(\vec{a} + \vec{b}\right)$$

$$\overrightarrow{PM} = \overrightarrow{OM} - \overrightarrow{OP}$$

$$= \frac{1}{5}\vec{a} + \frac{1}{5}\vec{b} - \vec{a}$$

$$= \frac{1}{5}\vec{b} - \frac{4}{5}\vec{a}$$

$$= \frac{1}{5}\left(\vec{b} - 4\vec{a}\right)$$

10. If $f(x) = \int_1^x f(t)dt + (1-x)(\ln x - 1) + e$. Then, the value of $f(f(1))$ is

(1) $e^e + 1$ (2) $e^e - 1$

(3) $e^e + 2$ (4) $e^e - 2$

Answer (1)

Sol. $f(n) = \left(\int_1^x f(t)dt\right) + (1-x)(\ln x - 1) + e$

$$f'(x) = f(x) + \frac{(1-x)}{x} + (\ln x - 1)(-1)$$

$$\frac{dy}{dx}=y+\frac{1}{x}-1-\ln x+1$$

$$\frac{dy}{dx}+(-1)y=\frac{1}{x}-\ln x$$

$$\text{I.F}=e^{-x}$$

$$y(e^{-x})=\int\left(\frac{e^{-x}}{x}-e^{-x}\ln x\right)dx$$

$$=\int\frac{e^{-x}}{x}dx-\left(-\ln xe^{-x}+\int\frac{e^{-x}}{x}dx\right)+c$$

$$y(e^{-x})=e^{-x}\ln x+c$$

$$y=\ln x+ce^{x}$$

$$y(1)=e$$

$$\Rightarrow \boxed{c=1}$$

$$y=e^{x}+\ln x$$

$$f(x)=e^{x}+\ln x$$

$$f(1)=e$$

$$f(f(1))=e^{e}+\ln(e^{e})$$

$$=e^{e}+1$$

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A bag contains 4 red balls, 6 yellow balls and 5 blue balls. In how many ways we can select 8 balls such that we get at least two balls of each colour, is

Answer (4100)

Sol.

R	Y	B	ways
4	2	2	${}^4C_4\cdot{}^6C_2\cdot{}^5C_2=150$
2	4	2	${}^4C_2\cdot{}^6C_4\cdot{}^5C_2=900$
2	2	4	${}^4C_2\cdot{}^6C_2\cdot{}^5C_4=450$
3	3	2	${}^4C_3\cdot{}^6C_3\cdot{}^5C_2=800$
3	2	3	${}^4C_3\cdot{}^6C_2\cdot{}^5C_3=600$
2	3	3	${}^4C_2\cdot{}^6C_3\cdot{}^5C_3=1200$

Total ways = 4100

22. The number of solution of equation $\cos\theta\cos\frac{5\theta}{2}=\cos 7\theta\cos\frac{7\theta}{2}$, $\theta\in[-\pi,\pi]$ is equal to

Answer (19)

Sol. $\cos\theta\cos\frac{5\theta}{2}=\cos 7\theta\cos\frac{7\theta}{2}$

$$\cos\left(\theta+\frac{5\theta}{2}\right)+\cos\left(\theta-\frac{5\theta}{2}\right)=\cos\left(7\theta+\frac{7\theta}{2}\right)+\cos\left(7\theta-\frac{7\theta}{2}\right)$$

$$\cos\frac{7\theta}{2}+\cos\frac{3\theta}{2}=\cos\cos\frac{21\theta}{2}+\cos\frac{7\theta}{2}$$

$$\cos\frac{3\theta}{2}=\cos\frac{21\theta}{2}$$

$$\frac{21\theta}{2}=2k\pi\pm\frac{3\theta}{2}$$

$$\frac{21\theta}{2}=2k\pi+\frac{3\theta}{2}\Rightarrow\theta=\frac{2k\pi}{9},\ k\in I$$

$\frac{21\theta}{2} = 2k\pi - \frac{3\theta}{2} \Rightarrow \theta = \frac{k\pi}{6},\ k \in I$

when $\theta = \frac{2k\pi}{9}$

$-\pi \le \frac{2k\pi}{9} \le \pi \Rightarrow -\frac{9}{2} \le k \le \frac{9}{2}$

$k \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} \to 9$ sol^n

for $\theta = \frac{k\pi}{6}$

$-\pi \le \frac{k\pi}{6} \le \pi \Rightarrow -6 \le k \le 6$

$k \in \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\} \to 13$ sol^n

$\theta = \left\{-\frac{2\pi}{3}, 0, \frac{2\pi}{3}\right\}$ is common

$\therefore$ Total $\text{Sol}^n = 19$

23. If $y = y(x)$ be the solution of the differential equation $\left(\sqrt{\tan x}\right)dy = \left(\sec^3 x - (\tan x)^{\frac{3}{2}} y\right)dx$ and $y\left(\frac{\pi}{4}\right) = \frac{6\sqrt{2}}{5}$ and $y\left(\frac{\pi}{3}\right) = \frac{4\alpha}{5}$ then α^4 is equal to

Answer (48.00)

Sol. $\frac{dy}{dx} = \frac{\sec^3 x - (\sqrt{\tan x})^3}{\sqrt{\tan x}}$

$\frac{dy}{dx} + (\tan x)y = (\sec^2 x)\frac{\sec x}{\sqrt{\tan x}}$

$\text{I.F.} = e^{\int \frac{\sin x}{\cos x}dx} = e^{-\ln|\cos x|} = \sec x$

$\Rightarrow y\sec x = \int \frac{(\sec x)^2 (\sec x)^2}{\sqrt{\tan x}} dx$

Let $\tan x = t^2$

$(\sec^2 x)dx = 2t\,dt$

$y\sec x = \int \frac{(1+t^4)(2t\,dt)}{t}$

$= 2t + \frac{2t^5}{5} + C$

$y\sec x = 2\sqrt{\tan x} + \frac{2}{5}(\sqrt{\tan x})^5 + C$

$y\left(\frac{\pi}{4}\right) \times \sqrt{2} = 2 + \frac{2}{5} + C \Rightarrow \frac{6\sqrt{2} \times \sqrt{2}}{5} = \frac{12}{5} + C$

$\Rightarrow\quad C = 0$

$y\left(\frac{\pi}{3}\right)(2) = 2 \times \sqrt{(\sqrt{3})}\left[1 + \frac{1}{5}(\sqrt{3})^4\right]$

$= 2\sqrt{\sqrt{3}} \times \left(1 + \frac{3}{5}\right)$

$\Rightarrow y\left(\frac{\pi}{3}\right) = 3^{\frac{1}{4}} \times \frac{8}{5} = \frac{4\alpha}{5} \quad \Rightarrow \quad \alpha = 2 \cdot 3^{\frac{1}{4}}$

$\Rightarrow\quad \alpha^4 = 16 \times 3 = 48$

24. If the mean of the frequency distribution, table

x_i	0	2	6	12	. .	.	n(n+1)
f_i	${}^nC_0$	${}^nC_2$	${}^nC_3$	.	.	.	${}^nC_n$

is 45. Then, the mean will be

Answer (42)

Sol. Mean = 45

$\frac{\sum f_i x_i = 45}{\sum f_i}$

$\sum_{r=0}^{n} \frac{r(r+1)}{2^n}\,{}^nC_r = 45$

$\sum_{r=0}^{n} r^2 + {}^nC_r + \sum_{r=0}^{n} r\,{}^nC_r \;= 45 \times 2^n$

$\Rightarrow n(n+1)2^{n-2} + n(2^{n-1}) = 45 \times 2^n$

$\Rightarrow n^{n-2}(n^2 + n + 2n) = 45 \times 2^n$

$\Rightarrow n^2 + 3n = 180$

$\Rightarrow n^2 + 3n - 180 = 0$

$\Rightarrow \boxed{n = 12}$

Total frequency = 2^{12}

$$\sum_{i=0}^{5} {}^{12}C_i = \sum_{i=7}^{12} {}^{12}C_i$$

$\Rightarrow$ Median will be at $f_i = {}^{12}C_6$

$\Rightarrow$ Median = 6 × 7 = 42

25.

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