# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The sum of value $\sum_{n=1}^{10} \frac{528}{n(n+1)(n+2)}$ is equal to

(1) 220 (2) 65

(3) 130 (4) 260

**Answer (3)**

**Sol.** $S = \sum_{n=1}^{10} \frac{528}{n(n+1)(n+2)}$

$= 264 \sum_{n=1}^{10} \left[ \frac{(n+2)-(n)}{n(n+1)(n+2)} \right]$

$= 264 \sum_{n=1}^{10} \left( \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right)$

$= 264 \left( \frac{1}{1\times 2} - \frac{1}{2\times 3} \right)$

$\left( \frac{1}{2\times 3} - \frac{1}{3\times 4} \right)$

.

.

.

$\left( \frac{1}{10\times 11} - \frac{1}{11\times 12} \right)$

$= 264 \left( \frac{1}{2} - \frac{1}{11\times 12} \right) = = 264 \times \frac{(66-1)}{11\times 12}$

= 130

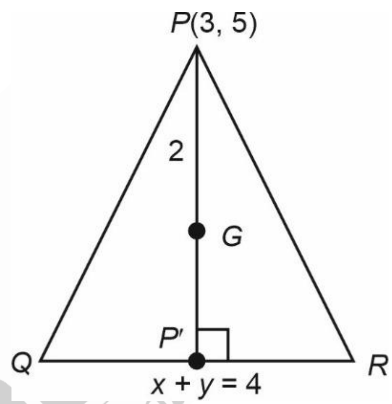
2. Consider an equilateral triangle *PQR*, where *P*(3, 5) and *QR* is *x* + *y* = 4. It the orthocentre of $\Delta PQR$ is ($\alpha$, $\beta$), then 9($\alpha$ + $\beta$) is equal to

(1) 56 (2) 48

(3) 64 (4) 36

**Answer (2)**

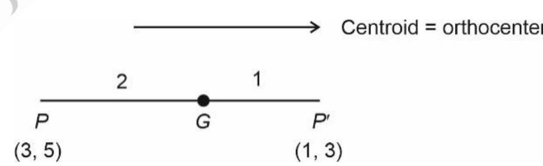
**Sol.** Foot of perpendicular P' $= \frac{x-3}{1} = \frac{y-5}{1} = -\frac{(8-4)}{2}$




$\Rightarrow x = 1$

$\Rightarrow y = 3$

$\therefore \quad P'(1,3)$




$G : \left( \frac{3+2}{3}, \frac{6+5}{3} \right)$

$= \left( \frac{5}{3}, \frac{11}{3} \right)$

$9(\alpha + \beta) = \frac{16}{3} \times 9 = 48$

3. Let $S_n$ is the sum of first $n$ terms of an A.P. If $S_n = 3n^2 + 5n$. Then, the sum of square of first ten terms of the given A.P is

(1) 15110 (2) 15220

(3) 14202 (4) 14308

**Answer (2)**

**Sol.** $S_n = 3n^2 + 5n$

$t_n = S_n - S_{n-1}$

$= 3n^2 + 5n - (3(n-1)^2 + 5(n-1))$

$= 3(2n-1) + 5 = 6n + 2$

$$\sum_{r=1}^{10} t_r^2 = \sum_{r=1}^{10} 4(3r+1)^2$$

$$= 4\left(\sum_{r=1}^{10} 9r^2 + 6\sum_{r=1}^{10} r + 10\right)$$

$$= 4\left[9\left(\frac{10\times 11\times 21}{6}\right) + 6\left(\frac{10\times 11}{2}\right) + 10\right]$$

$= 4[3465 + 330 + 10] = 15220$

4. Consider a set A = {1, 2, 3, 5, 6,}. The number of one-one functions $f : A \to A$ such that $f(1) \geq 3$, $f(3) \leq 4$ and $f(2) + f(3) = 5$ is equal to

(1) 144 (2) 72

(3) 36 (4) 24

**Answer (2)**

**Sol.** $f(1) \geq 3$

$f(3) \leq 4$

$f(2) + f(3) = 5$

$f(4)$, $f(5)$, $f(6)$ has no conditions.

| $(f(2), f(3))$ | $f(1)$ |
|---|---|
| (1, 4) | {3, 5, 6} $\to$ 3 choices |
| (4, 1) | {3, 5, 6} $\to$ 3 choices |
| (2, 3) | {4, 5, 6} $\to$ 3 choices |
| (3, 2) | {4, 5, 6} $\to$ 3 choices |

$\therefore$ Total function $\to 3 \times 4 \times 3!$

$= 72$

5. The value of $\displaystyle\int_0^{\infty} \frac{\ln x}{x^2+4}dx$ is equal to

(1) $\dfrac{\pi \ln 2}{4}$ (2) $\dfrac{\pi \ln 4}{2}$

(3) $\dfrac{\pi \ln 2}{2}$ (4) $\dfrac{\pi \ln 4}{3}$

**Answer (1)**

**Sol.** $I = \displaystyle\int_0^{\infty} \frac{\ln x}{x^2+4}dx$

put $x = \dfrac{4}{u}$

$dx = \dfrac{-4}{u^2}du$

$$I = \int_0^{\infty} \frac{\ln\left(\frac{4}{u}\right)}{\left(\frac{4}{u}\right)^2 + 4}\left(\frac{4}{u^2}\right)du$$

$$= \int_0^{\infty} \frac{\ln 4 - \ln u}{\frac{16}{u^2} + 4}\cdot\frac{4}{u^2}du$$

$$= \int_0^{\infty} \frac{\ln 4}{u^2+4}du - \int_0^{\infty} \frac{\ln u}{u^2+4}du$$

$$2I = \ln 4\int_0^{\infty} \frac{1}{u^2+4}du = \ln 4 \times \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\Big|_0^{\infty}$$

$$= \ln 4 \times \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi \ln 4}{4} = \frac{\pi \ln 2}{2}$$

$$I = \frac{\pi \ln 2}{4}$$

6. If tan $A$ and tan $B$ are the roots of the equation $x^2 - 2x - 5 = 0$. Then, the value of $20\sin^2\left(\dfrac{A+B}{2}\right)$ is

(1) $5 + 3\sqrt{6}$ (2) $10 + 3\sqrt{10}$

(3) $5 - 3\sqrt{2}$ (4) $10 - 3\sqrt{10}$

**Answer (4)**

**Sol.** $\tan A + \tan B = 2$

$\tan A \tan B = -5$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{2}{1-(-5)} = \frac{2}{6} = \frac{1}{3}$$

$$\cos(A+B) = \frac{3}{\sqrt{10}}$$

$$2\sin^2\left(\frac{A+B}{2}\right) = 1 - \cos(A+B)$$

$$2\sin^2\left(\frac{A+B}{2}\right) = 1 - \frac{3}{\sqrt{10}}$$

$$20\sin^2\left(\frac{A+B}{2}\right) = 10\left(1 - \frac{3}{\sqrt{10}}\right)$$

$$= 10 - 3\sqrt{10}$$

7. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\left(4 - \operatorname{cosec}^2 x\right)}{\cos^4 x} dx$ is equal to

(1) $\frac{16\sqrt{3}}{9}$ (2) $\frac{32\sqrt{3}}{9}$

(3) $\frac{16}{\sqrt{3}}$ (4) 32

**Answer (2)**

**Sol.** Consider, $\frac{4 - \operatorname{cosec}^2 x}{\cos^4 x} = \left(4 - (1 + \cot^2 x)\right)(\sec^2 x)^2$

$$= \left(3 - \frac{1}{\tan^2 x}\right)(1 + \tan^2 x)(\sec^2 x)$$

Put $u = \tan x$

$du = \sec^2 x dx$

$$\Rightarrow I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \left(3 - \frac{1}{u^2}\right)(1 + u^2) dx$$

$$= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} (3u^2 + 2 - u^{-2}) dx$$

$$= \left(\frac{u^3}{3} + 2u - \frac{u^{-1}}{-1}\right)\Bigg|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$= \frac{16\sqrt{3}}{3} - \frac{16\sqrt{3}}{9} = \frac{32\sqrt{3}}{9}$$

8. Let $\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{j} + 2\hat{k}$, $\vec{r} \times \vec{a} + \vec{b} \times \vec{a} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. The value of $|3\vec{r}|^2$ is equal to

(1) 56 (2) 44

(3) 42 (4) 48

**Answer (2)**

**Sol.** $\vec{r} \times \vec{a} + \vec{b} \times \vec{a} = 0$

$\Rightarrow (\vec{r} \times \vec{b}) \times \vec{a} = 0$ $(\vec{r} \cdot \vec{a} = 0 \therefore \vec{r} \neq -\vec{b})$

$\Rightarrow \vec{r} + \vec{b} = \lambda \vec{a}$

$\Rightarrow \vec{r} = \lambda \vec{a} - \vec{b}$

$\Rightarrow \vec{r} \cdot \vec{a} = \lambda a^2 - \vec{a} \cdot \vec{b}$

$$\Rightarrow \lambda = \frac{\vec{a} \cdot \vec{b}}{a^2} = \frac{1-2}{9} = \frac{-1}{9}$$

$$\therefore \vec{r} = \frac{-1}{9}\vec{a} - \vec{b}$$

$$\Rightarrow |\vec{r}|^2 = \frac{a^2}{81} + b^2 + 2.\left(\frac{\vec{a}}{9}\right)(\vec{b})$$

$$= \frac{9}{81} + 5 - \frac{2}{9}$$

$$= \frac{1}{9} + 5 - \frac{2}{9}$$

$$= 5 - \frac{1}{9} = \frac{44}{9}$$

$$|3\vec{r}|^2 = 9|\vec{r}|^2 = 44$$

9. Let $f(x+y)=\dfrac{f(x)+f(y)}{3}$ and $f'(0)$ = 3. If $g(x)=3+e^x f(x)$. Then, the minimum value of $g(x)$ is

(1) $2+\dfrac{2}{e}$

(2) $3-\dfrac{3}{e}$

(3) $1-\dfrac{1}{e}$

(4) $1+\dfrac{1}{e}$

**Answer (2)**

**Sol.** $f\left(\dfrac{x+y}{3}\right)=\dfrac{f(x)+f(4)}{3}$

$$f'(x)=\lim_{h\to 0}\frac{f\left(\dfrac{3x+3h}{3}\right)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{f(3x)+f(3h)}{3}-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{f(3x)+f(3h)}{3}-\left(f\left(\dfrac{3x+0}{3}\right)\right)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{f(3x)+f(3h)}{3}-\left(\dfrac{f(3x)+f(0)}{3}\right)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{f(0+3h)-f(0)}{3h}$$

$f'(x)=f'(0)=3$

$f'(x)=3$

$f(x)=3x+k$

also $f(0) = 0$

$\Rightarrow f(x) = 3x$

$g(x) = 3 + e^x f(x)$

$g(x) = 3 + e^x (3x)$

$g'(x) = 3e^x + 3xe^x = 0$

$\Rightarrow 3e^x(1 + x) = 0$

$\Rightarrow x = -1$

$g''(x) = 3e^x + 3e^x + 3xe^x$

$= 6e^x + 3xe^x$

$g''(-1) = \dfrac{6}{e}-\dfrac{3}{e}=\dfrac{3}{e}>0$

$\Rightarrow$ minima at $x = -1$

$g(-1) = 3 + e^{-1}(-3)$

$= 3-\dfrac{3}{e}$

10. Consider an Ellipse E: $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ with eccentricity $e=\dfrac{4}{5}$, focus at (4,0) and the point $P(3, \alpha)$ lie on $E$. Then the area of the triangle $POS$ (in sq. unit) is

(1) $\dfrac{12}{5}$

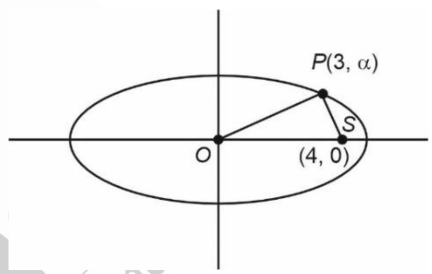
(2) $\dfrac{13}{5}$

(3) $\dfrac{24}{5}$

(4) $\dfrac{48}{5}$

**Answer (3)**

**Sol.**




$ae=4$

$\Rightarrow a=5$

$\left(\dfrac{4}{5}\right)^2=1-\dfrac{b^2}{a^2}$

$\dfrac{16}{25}=1-\dfrac{b^2}{25}$

$\Rightarrow b=3$

$\Rightarrow P$ lie on ellipse:

$\dfrac{9}{25}+\dfrac{\alpha^2}{9}=1$

$\Rightarrow 81+25\alpha^2=9\times 25$

$\Rightarrow \alpha^2=\dfrac{9\times 25-81}{25}$

$\alpha=\dfrac{12}{5}$

$\Rightarrow\ \Delta POS=\dfrac{1}{2}\times 4\times\dfrac{12}{5}=\dfrac{24}{5}$

11. The shortest distance of the point (2, 3, 4) from the line $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-2}{0}$ is equal to

(1) $\sqrt{2}$  (2) $\sqrt{5}$

(3) $\sqrt{\frac{21}{5}}$  (4) $\sqrt{7}$

**Answer (3)**

**Sol.** Any general point on Line $L:\ \frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-2}{0}$ is

$P(2\lambda+1,-\lambda+3,2)$

$A(2, 3, 4)$

Direction ratio of AP $(2\lambda-1,-\lambda,-2)$

$AP\perp L$

$2(2\lambda-1)+(-1)(-\lambda)+0(-2)=0$

$4\lambda-2+\lambda=0$

$5\lambda-2=0$

$\lambda=\frac{2}{5}$

$|AP|=\sqrt{\left(\frac{4}{5}-1\right)^2+\left(-\frac{2}{5}\right)^2+(-2)^2}$

$=\sqrt{\frac{1}{25}+\frac{4}{25}+4}$

$=\sqrt{\frac{1}{5}+4}$

$=\sqrt{\frac{21}{5}}$

12. Letter is posted from either KANPUR or ANANTPUR. When envelope was upon, only "AN" was visible. The probability that it came from ANANTPUR is equal to

(1) $\frac{9}{16}$  (2) $\frac{1}{11}$

(3) $\frac{10}{17}$  (4) $\frac{9}{13}$

**Answer (3)**

**Sol.** $E_1$ : The letter came from KANPUR

$E_2$ : The letter came from ANANTPUR

$E$ : Visible letter is 'AN"

$P(E_1)=P(E_2)=\frac{1}{2}$

$P\left(\frac{A}{E_1}\right)=\frac{1}{5}$

$P\left(\frac{A}{E_2}\right)=\frac{2}{7}$

$P\left(\frac{E_2}{A}\right)=\frac{\frac{1}{2}\times\frac{2}{7}}{\frac{1}{2}\times\frac{1}{5}+\frac{1}{2}\times\frac{2}{7}}=\frac{10}{17}$

13. If $\alpha=\frac{\pi}{4}+\sum_{P=1}^{11}\tan^{-1}\left(\frac{2^{P-1}}{1+2^{2P-1}}\right)$, then the value of $\tan\alpha$ is

(1) $2^9$  (2) $2^{10}$

(3) $2^{11}$  (4) $2^{12}$

**Answer (3)**

**Sol.** $\alpha=\frac{\pi}{4}+\sum_{p=1}^{11}\tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)$

$\tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right)=\tan^{-1}\left(\frac{2^p-2^{p-1}}{1+2^p\cdot 2^{p-1}}\right)$

$\tan^{-1}\left(2^p\right)-\tan^{-1}(2^{p-1})$

Now, $\sum_{p=1}^{11}\left[\tan^{-1}(2^p)-\tan^{-1}(2^{p-1})\right]$

$=\tan^{-1}(2^1)-\tan^{-1}(2^0)+\tan^{-1}(2^2)-\tan^{-1}(2^1)+......+\tan^{-1}(2^{11})-\tan^{-1}(2^{10})$

$=\tan^{-1}(2^{11})-\tan^{-1}(2^0)=\tan^{-1}(2^{11})-\tan^{-1}(1)$

$=\tan^{-1}(2^{11})-\frac{\pi}{4}$

$\therefore\ \alpha=\tan^{-1}(2^{11})$

$\tan\alpha=2^{11}$

14. *A* and *B* play a badminton series. First one to get 5 wins is the winner of the series. The number of ways in which the series can be played, if *A* is the winner of the tournament, is equal to

(1) 121 (2) 126

(3) 132 (4) 144

**Answer (2)**

**Sol.** Last match is won by *A*

Case I: *A* wins in 5 games : ${}^4C_4$

Case II: *A* wins in 6 games : ${}^5C_4$

Case III: *A* wins in 7 games : ${}^6C_4$

Case IV: *A* wins in 8 games : ${}^7C_4$

Case V: *A* wins in 9 games : ${}^8C_4$

$\therefore$ total ways ${}^4C_4 + {}^5C_4 + \ldots + {}^8C_4 = 126$

15. The sum of integral values of *P* for which the equation $3\sin^2 x - 12\cos x - 3 = P$ has at least one zero, is

(1) –60 (2) –75

(3) –90 (4) 110

**Answer (2)**

**Sol.** $3(1 - \cos^2 x) - 12\cos x - 3 = P$

$3\cos^2 x + 12\cos x + P = 0$ has at least one root.

$3(\cos^2 x + 4\cos x) = -P$

$= 3((\cos x + 2)^2 - 4) = -P$

$= 3(\cos x + 2)^2 - 12$

$-1 \le \cos x \le 1$

$1 \le \cos x + 2 \le 3$

$1 \le (\cos x + 2)^2 \le 9$

$3 \le 3(\cos x + 2)^2 \le 27$

$-9 \le 3(\cos x + 2)^2 - 12 \le 15$

$-9 \le -P \le 15$

$-15 \le P \le 9$

$\Rightarrow$ Sum of integral values

$-15 - 14 - 13 - 12 - 11 - 10 = -75$

16. The area bounded by the region in first quadrant $xy \le 27$ and $1 \le y \le x^2$ (in square units) is

(1) $52\ln 9 - \frac{52}{3}$ (2) $54\ln 3 - \frac{52}{3}$

(3) $54\ln 3 - \frac{53}{3}$ (4) $52\ln 9 - \frac{53}{3}$

**Answer (2)**

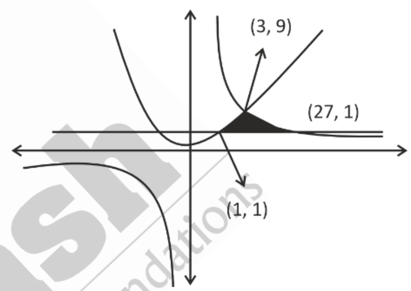
**Sol.** For point of intersection solve,

$xy = 27$ and $y = x^2$

$\Rightarrow x^2 = \frac{27}{x} \Rightarrow x^3 = 27 \Rightarrow x = 3$

Now, $y = \frac{27}{x}$ and $y = 1$

$\Rightarrow x = 27$




$$\text{Area} = \int_1^3 (x^2 - 1)dx + \int_3^{27}\left(\frac{27}{x} - 1\right)dx$$

$$= \left[\frac{x^3}{3} - x\right]_1^3 + \left[27\ln|x| - x\right]_3^{27}$$

$$= \frac{20}{3} + 27\ln 9 - 24$$

$$= 54\ln 3 - \frac{52}{3}$$

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In the expansion of $\left(\frac{1}{x^3}-x^4\right)^n$, the sum of coefficient of $x^7$ & $x^{14}$ is equal to 0. Then the value of $n$ is

**Answer (21)**

**Sol.** $\left(\frac{1}{x^3}-x^4\right)^n$

$$t_{r+1}={}^nC_r(x^{-3})^{n-r}\left(-x^4\right)^r$$

$$={}^nC_r(-1)^r x^{7r-3n}$$

for coeff. $X^7$

Now, $7r_1-3n=7$

$$\Rightarrow r_1=\frac{3n+7}{7}$$

Coeff. Is ${}^nC_{r_1}(-1)^{r_1}$

for coeff. $x^{14}$

$7r_2 - 3n = 14$

$$r_2=\frac{3n+14}{7}=\frac{3n+7}{7}+1$$

$r_2 = r_1 + 1$ & coeff. is $C_2={}^nC_{r_1+1}(-1)^{r_1+1}$

$C_1 + C_2 = 0$

$${}^nC_{r_1}(-1)^{r_1}+{}^nC_{r_1+1}(-1)^{r_1+1}=0$$

$${}^nC_{r_1}(-1)^{r_1}-{}^nC_{r_{1+1}}(-1)^{r_1}=0 \quad \left\{\because (-1)^{r_1+1}=-(-1)^{r_1}\right\}$$

$${}^nC_{r_1}={}^nC_{r_1+1}$$

$r_1 + r_1 + 1 = n$

$2r_1 + 1 = n$

$$2\left(\frac{3n+7}{7}\right)+1=n$$

$\Rightarrow n = 21$

22. If $y=y(x)$ is the solution of the differential equation

$$x\sin\left(\frac{y}{x}\right)dy=\left(y\sin\left(\frac{y}{x}\right)-x\right)dx$$

Given that $y(1)=\frac{\pi}{2}$ and $\alpha=\cos\left(\frac{y\left(e^{12}\right)}{e^{12}}\right)$.

If the radius of the circle $x^2+y^2-2px-2p-\alpha-2=0$ is $r$ where $r\le 6$. Then, the number of integral values of $p$ is

**Answer (9)**

**Sol.** $x\sin\left(\frac{y}{x}\right)dy=\left(y\sin\left(\frac{y}{x}\right)-x\right)dx$

$$\sin\left(\frac{y}{x}\right)dy=\left(\frac{y}{x}\sin\left(\frac{y}{x}\right)-1\right)dx$$

$$\frac{dy}{dx}=\frac{\frac{y}{x}\sin\left(\frac{y}{x}\right)-1}{\sin\left(\frac{y}{x}\right)}$$

Let $y=vx$

$$\frac{dy}{dx}=V+x\frac{dv}{dx}$$

$$V+x\frac{dv}{dx}=\frac{V\sin V-1}{\sin V}$$

$$x\frac{dv}{dx}=\frac{V\sin V-1}{\sin V}-V$$

$$x\frac{dv}{dx}=\frac{V\sin V-1-V\sin V}{\sin V}$$

$$x\frac{dv}{dx}=\frac{-1}{\sin V}$$

$$-\sin V dV=\frac{dx}{x}$$

Integrating both sides

$\cos V = \ln x + k$

$\cos\left(\frac{y}{x}\right) = \ln x + k$

at $y(1) = \frac{\pi}{2}$

$k = 0$

$\alpha = \cos\left(\frac{y(e^{12})}{e^{12}}\right)$

$\alpha = \ln(e^{12})$

$\alpha = 12$

C: $x^2 + y^2 - 2px - 2p - \alpha - 2 = 0$

Put $\alpha = 12$

$x^2 + y^2 - 2px - 2p - 14 = 0$

one $r \le 6$

$\sqrt{p^2 + 2p + 14} \le 6$

$p^2 + 2p + 14 \le 36$

$p^2 + 2p - 22 \le 0$

$p \in [-1-\sqrt{23}, -1+\sqrt{23}]$

$\Rightarrow$ Integral values of p are

$-5, -4, -3, -2, -1, 0, 1, 2, 3$

23.

24.

25.

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