

Date: 09/03/2026



Question Paper Code

65/1/1

Aakash

Medical | IIT-JEE | Foundations

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Time: 3 hrs.

Class-XII

Max. Marks: 80

MATHEMATICS (Theory)

(CBSE 2026)

GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them:

- (i) This question paper contains **38** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) **In Section – A** : Question Numbers **1** to **18** are Multiple Choice Questions (MCQ) type and question number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) **In Section – B** : Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) **In Section – C** : Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) **In Section – D** : Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) **In Section – E** : Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculators is **NOT** allowed.

SECTION-A

This Section comprises of **20** Multiple Choice Questions (MCQs) of **1** mark each.

20 × 1 = 20

1. If $2\cos^{-1}x = y$, then [1]
- (A) $0 \leq y \leq \pi$ (B) $-\pi \leq y \leq \pi$
 (C) $0 \leq y \leq 2\pi$ (D) $-\pi \leq y \leq 0$

Answer (C)

Sol. $\cos^{-1}x \in [0, \pi]$

$$2\cos^{-1}x \in [0, 2\pi]$$

$$\text{and } y = 2\cos^{-1}x$$

$$\Rightarrow y \in [0, 2\pi]$$

2. Which of the following cannot be the order of a row-matrix? [1]
- (A) 2×1 (B) 1×2
 (C) 1×1 (D) $1 \times n$

Answer (A)

Sol. A row matrix is a matrix with only one horizontal row and n columns denoted by the order $1 \times n$
 $\Rightarrow 2 \times 1$ cannot be a row matrix.

3. Which of the following properties is/are true for two matrices of suitable orders? [1]
- (i) $(A+B)' = A' + B'$
 (ii) $(A-B)' = B' - A'$
 (iii) $(AB)' = A'B'$
 (iv) $(kAB)' = kB'A'$ (k is a scalar)
- (A) (i) only (B) (i), (ii) and (iii)
 (C) (i) and (ii) (D) (i) and (iv)

Answer (D)

Sol. (i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

(iii) $(AB)' = B'A'$

(iv) $(KAB)' = K(AB)'$
 $= KB'A'$

\Rightarrow (i) and (iv) are true.

4. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then [1]
- (A) $\Delta_1 = 2\Delta_2$ (B) $\Delta_2 = -2\Delta_1$
 (C) $\Delta_1 = \Delta_2$ (D) $\Delta_2 = -\Delta_1$

Answer (B)

Sol. $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 1(6) = 6$

$$\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix} = -2(6) = -12$$

$$\Delta_2 = -2\Delta_1$$

5. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is **[1]**

- (A) 0 (B) $\frac{\pi}{4}$
 (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Answer (B)

Sol. $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$

$$\Rightarrow \cos x \sin x + \cos x \sin x = 1$$

$$\Rightarrow 2\cos x \sin x = 1$$

$$\Rightarrow \sin 2x = 1$$

$$\Rightarrow 2x = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{4}$$

6. If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric? **[1]**

- (A) AB (B) $AB + BA$
 (C) $(A+B)^2$ (D) $A - B$

Answer (D)

Sol. A and B are skew symmetric matrix

$$A^T = -A$$

$$B^T = -B$$

(A) $(AB)^T = B^T A^T = BA$

(B) $(AB + BA)^T = (AB)^T + (BA)^T$

$$= B^T A^T + A^T B^T$$

$$= BA + AB$$

$$= AB + BA$$

$$\begin{aligned} \text{(C)} \quad & ((A+B)^T)^T = (A^T + B^T)^T \\ & = (-A - B)^T \\ & = (A+B)^T \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad & (A-B)^T = A^T - B^T \\ & = -A + B \\ & = -(A-B) \\ & \Rightarrow (A-B) \text{ is a skew symmetric matrix} \end{aligned}$$

7. The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is

[1]

- (A) -16 (B) -9
(C) 0 (D) 16

Answer (A)

Sol. $f(x) = x^3 - 12x$, $x \in [0, 3]$

$$f'(x) = 3x^2 - 12 = 0$$

$$\Rightarrow x^2 = \frac{12}{3} \Rightarrow x = \pm 2$$

$$f(0) = 0, f(3) = -9, f(2) = -16$$

Least value of $f(x)$ is -16 at $x = 2$

8. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log|b^2 + c^2x^2| + K$, then the value of A is

[1]

- (A) $3a$ (B) $\frac{3a}{2b^2}$
(C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$

Answer (D)

Sol. $\int \frac{3ax}{b^2 + c^2x^2} dx$

$$u = b^2 + c^2x^2$$

$$du = 2c^2x dx$$

$$\int \frac{3ax}{b^2 + c^2x^2} = \frac{3a}{2c^2} \int \frac{1}{u} du$$

$$= \frac{3a}{2c^2} \ln|u| + k$$

$$= \frac{3a}{2c^2} \ln|b^2 + c^2x^2| + k$$

$$\Rightarrow A = \frac{3a}{2c^2}$$

9. The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx$ is [1]

- (A) 0 (B) $\log 2$
(C) $2\log 2$ (D) $\frac{1}{2}\log 2$

Answer (A)

Sol. $I = \int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx \quad \dots(i)$

Then, by using the property

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$I = \int_{-1}^1 \frac{-x^3}{x^2 + 2|x| + 1} dx \quad \dots(ii)$$

adding (i) & (ii)

$$2I = 0$$

$$\Rightarrow I = 0$$

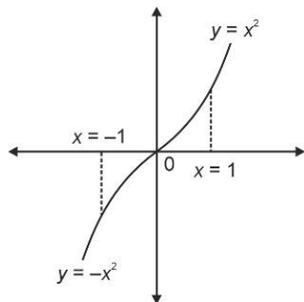
10. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by [1]

- (A) 0 (B) $\frac{1}{3}$
(C) $\frac{2}{3}$ (D) $\frac{4}{3}$

Answer (C)

Sol. $y = x|x|$

$$y = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



$$\text{Area} = 2 \int_0^1 x^2 dx$$

$$= 2 \times \left[\frac{x^3}{3} \right]_0^1$$

$$= \frac{2}{3}$$

11. The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is [1]

- (A) $e^{\int \frac{P}{Q} dy}$ (B) $e^{\int P dy}$
 (C) $e^{\int \frac{P}{R} dy}$ (D) $e^{\int \frac{P}{R} dx}$

Answer (C)

Sol. $R \frac{dx}{dy} + Px = Q$

$$\frac{dx}{dy} + \frac{P}{R}x = \frac{Q}{R}$$

$$\text{I.F} = e^{\int \frac{P}{R} dy}$$

12. The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are [1]

- (A) 0, 1 (B) 1, 1
 (C) 2, 1 (D) 1, not defined

Answer (B)

Sol. $\frac{de^y}{dx} = 0$

$$\Rightarrow e^y \frac{dy}{dx} = 0$$

Order = 1

Degree = 1

13. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is [1]

- (A) 0 (B) 1
 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$

Answer (C)

Sol. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\vec{b} = 2\hat{i} - p\hat{j} + \hat{k}$$

$$\vec{a} \perp \vec{b}$$

$$\therefore \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} - p\hat{j} + \hat{k}) = 0$$

$$\Rightarrow (1)(2) + (2)(-p) + (3)(1) = 0$$

$$\Rightarrow 2 - 2p + 3 = 0$$

$$\Rightarrow 2p = 5$$

$$\Rightarrow p = \frac{5}{2}$$

14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is [1]

- (A) 8 (B) -8
(C) 2 (D) $\frac{5}{2}$

Answer (A)

Sol. Let the three points be A , B & C

Their position vectors are

$$\vec{OA} = 2\hat{i} + m\hat{j} + 5\hat{k}$$

$$\vec{OB} = 3\hat{i} + 11\hat{j} + 6\hat{k}$$

$$\vec{OC} = -\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = (3-2)\hat{i} + (11-m)\hat{j} + (6-5)\hat{k}$$

$$= \hat{i} + (11-m)\hat{j} + \hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = -4\hat{i} - 12\hat{j} - 4\hat{k}$$

$$\text{Now, } \frac{1}{-4} = \frac{11-m}{-12} = \frac{1}{-4}$$

$$\Rightarrow \frac{11-m}{-12} = \frac{1}{-4}$$

$$\Rightarrow 11-m=3$$

$$\Rightarrow m=11-3$$

$$\Rightarrow m=8$$

15. If $|\vec{a}|=8$, $|\vec{b}|=3$ and $|\vec{a} \times \vec{b}|=12$, then the value of $|\vec{a} \cdot \vec{b}|$ [1]

- (A) $6\sqrt{3}$ (B) $8\sqrt{3}$
(C) $12\sqrt{3}$ (D) $3\sqrt{12}$

Answer (C)

Sol. $|\vec{a}|=8$, $|\vec{b}|=3$, $|\vec{a} \times \vec{b}|=12$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$12 = 8 \times 3 \times \sin \theta$$

$$\sin \theta = \frac{12}{24} = \frac{1}{2}$$

$$\text{Now, } \cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\therefore \cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$= 8 \times 3 \times \frac{\sqrt{3}}{2}$$

$$= 12\sqrt{3}$$

16. The length of perpendicular drawn from point (2, 5, 7) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is

[1]

(A) 2

(B) 5

(C) $\sqrt{74}$

(D) $\sqrt{78}$

Answer (C)

Sol. Line is $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ and let point P(2, 5, 7)

Direction ratios of line are $\langle 1, 0, 0 \rangle$

This line is x-axis.

Any point on the x-axis has the general co-ordinate (x, 0, 0)

For the x-axis, the foot of perpendicular is simply the x-coordinate of the point P with other co-ordinate being zero.

\therefore Foot of perpendicular is Q(2, 0, 0)

\therefore Length of the perpendicular is the distance between P(2, 5, 7) & Q(2, 0, 0)

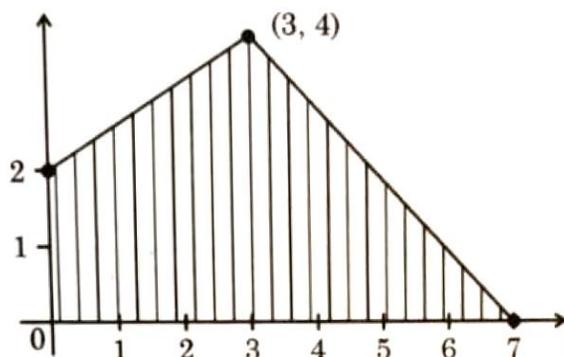
$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(2-2)^2 + (5-0)^2 + (7-0)^2}$$

$$= \sqrt{0+25+49}$$

$$= \sqrt{74}$$

17. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below :



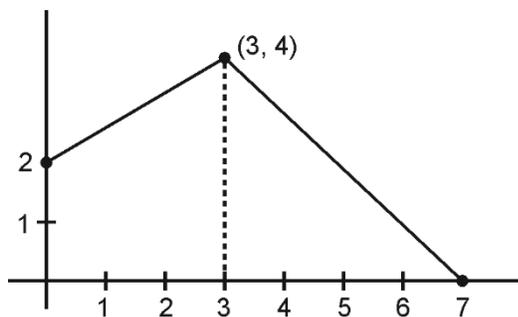
The maximum value of Z – minimum value of Z is

[1]

- (A) 8 (B) 29
(C) 35 (D) 43

Answer (D)

Sol. The corner points of the bounded feasible region are $(0, 0)$, $(7, 0)$, $(3, 4)$, $(0, 2)$



Now, $Z = 5x + 7y$

At $(0, 0)$; $Z = 5(0) + 7(0) = 0$

At $(7, 0)$; $Z = 5(7) + 7(0) = 35$

At $(3, 4)$; $Z = 5(3) + 7(4) = 43$

At $(0, 2)$; $Z = 5(0) + 7(2) = 14$

$\therefore Z_{\max} = 43$ (at point $(3, 4)$)

$Z_{\min} = 0$ (at point $(0, 0)$)

$\therefore Z_{\max} - Z_{\min} = 43 - 0 = 43$

18. The degree of an objective function of a linear programming problem is

[1]

- (A) 0 (B) 1
(C) 2 (D) Any natural number

Answer (B)

Sol. $Z = 5x + 7y$

Since highest power of variables x and y is 1

\therefore Degree of the function is 1

Assertion – Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. [1]

Reason (R) : For any two events A and B , $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Answer (C)

Sol. Here, we define event

A = Getting a prime number on die.

B = Getting an odd number on die.

Given that number appearing on the die is odd, so we need to calculate probability that number is prime too.

Prime number on die = 2, 3, 5

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

Odd number on die = 1, 3, 5

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

$$A \cap B = \{3, 5\}$$

$$P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{1/2} = \frac{2}{3}$$

So assertion is true.

In reason, given that $P(A|B) = \frac{P(A \cap B)}{P(B)}$, which is wrong statement, as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Option (C) is correct.

20. **Assertion (A) :** Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. [1]

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

Answer (D)

Sol. Line 1:

$$x = py + q, z = ry + s$$

here, we put $y = t$

$$x = pt + q, z = rt + s$$

$$(x, y, z) = (pt + q, t, rt + s)$$

$$\text{Direction vector: } \vec{d}_1 = (p, 1, r)$$

Line 2:

$$x = p'y + q', z = r'y + s'$$

here, we put $y = t'$

$$x = p't' + q', z = r't' + s'$$

$$(x, y, z) = (p't' + q', t', r't' + s')$$

$$\text{Direction vector: } \vec{d}_2 = (p', 1, r')$$

Two lines are perpendicular when dot product of their direction vectors is zero.

$$\vec{d}_1 \cdot \vec{d}_2 = 0$$

$$pp' + 1 + rr' = 0$$

$pp' + rr' = -1$, but in assertion it is given that $pp' + rr' = 1$. So assertion is false.

Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular when the dot product of their direction vector is zero.

here, direction vectors are \vec{b}_1 and \vec{b}_2 respectively.

$$\text{So } \vec{b}_1 \cdot \vec{b}_2 = 0$$

Reason is true.

Option (D) is correct

SECTION-B

This section comprises of **5** Very Short Answer (VSA) type questions of **2** marks each.

5 × 2 = 10

21. (a) Check whether function $f(x)$ defined as

[2]

$$f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases} \text{ is continuous at } x = 3 \text{ or not?}$$

OR

(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

[2]

Sol. (a) $f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$

A function is continuous at $x = a$ if :

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

left hand limit $x \rightarrow 3^-$

for $x < 3$

$$f(x) = \frac{|x-3|}{2(x-3)}$$

as $x < 3$

$$f(x) = \frac{-(x-3)}{2(x-3)} = \frac{-1}{2} \Rightarrow \lim_{x \rightarrow 3^-} f(x) = \frac{-1}{2}$$

Right hand limit $x \rightarrow 3^+$

for $x > 3$

$$f(x) = \frac{x-6}{6}$$

$$\lim_{x \rightarrow 3^+} f(x) = \frac{3-6}{6} = \frac{-1}{2}$$

$$\text{and } f(3) = \frac{3-6}{6} = \frac{-1}{2}$$

$$\text{as, } \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

OR

$$(b) \quad \sqrt{3}(x^2 + y^2) = 4xy$$

$$\sqrt{3}x^2 + \sqrt{3}y^2 = 4xy$$

differentiating both sides with respect to x .

$$\sqrt{3}(2x) + \sqrt{3}(2y) \frac{dy}{dx} = 4y + 4x \frac{dy}{dx}$$

$$(2\sqrt{3}y - 4x) \frac{dy}{dx} = 4y - 2\sqrt{3}x$$

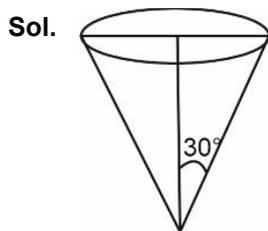
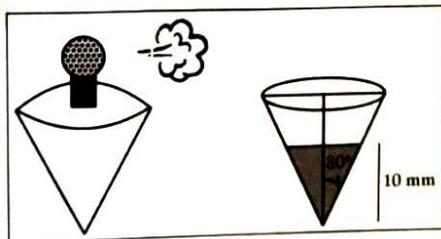
$$\text{Putting } x = \frac{1}{2} \text{ and } y = \frac{\sqrt{3}}{2}$$

$$\left[2\sqrt{3} \left(\frac{\sqrt{3}}{2} \right) - 4 \left(\frac{1}{2} \right) \right] \frac{dy}{dx} = \frac{4 \cdot \sqrt{3}}{2} - \frac{2\sqrt{3}}{2}$$

$$\frac{dy}{dx} = \frac{\sqrt{3}}{3-2} = \sqrt{3}$$

$$\boxed{\frac{dy}{dx} = \sqrt{3}}$$

22. A room freshener bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1 \text{ mm}^3 / \text{min}$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$. [2]



Let the radius of cone be r and height be h

$$\tan \alpha = \frac{r}{h}$$

$$\Rightarrow r = h \tan \alpha \text{ here } \alpha = \frac{\pi}{6}$$

$$\Rightarrow r = \frac{h}{\sqrt{3}}$$

Now, volume of perfume in the cone $V = \frac{1}{3} \pi r^2 h$

$$\text{Put } r = \frac{h}{\sqrt{3}}$$

$$v = \frac{1}{3} \pi \left(\frac{h}{\sqrt{3}} \right)^2 h$$

$$= \frac{\pi h^3}{9}$$

$$\frac{dV}{dt} = -1 \text{ mm}^3 / \text{min}$$

$$\text{and } \frac{dv}{dt} = \frac{3\pi h^2}{9} \frac{dh}{dt}$$

$$-1 = \frac{\pi h^2}{3} \frac{dh}{dt}$$

At $h = 10 \text{ mm}$

$$-1 = \frac{\pi(10)^2}{3} \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-3}{100\pi} \text{ mm/min}$$

⇒ The level of perfume is dropping at the rate of $\frac{3}{100\pi}$ mm/min.

23. Find the vector of magnitude 14 in the direction of \overrightarrow{QP} , where P and Q are the points $(1, 3, 2)$ and $(-1, 0, 8)$ respectively. [2]

Sol. The vector $\overrightarrow{QP} = \vec{P} - \vec{Q}$

$$= (\hat{i} + 3\hat{j} + 2\hat{k}) - (-\hat{i} + 0\hat{j} + 8\hat{k})$$

$$= 2\hat{i} + 3\hat{j} - 6\hat{k}$$

The unit vector in the direction of \overrightarrow{QP} is $\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{2^2 + 3^2 + (-6)^2}} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

The vector with magnitude 14 in

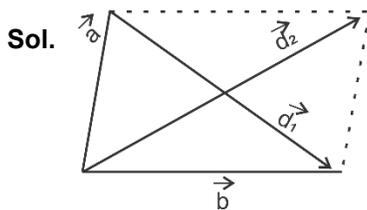
the direction of \overrightarrow{QP} is given by $(14) \overrightarrow{QP}$

$$= (14) \left[\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k} \right]$$

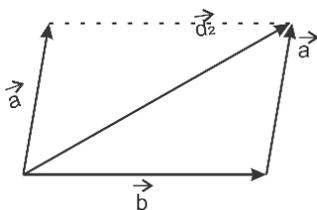
$$= 4\hat{i} + 6\hat{j} - 12\hat{k}$$

Answer is $4\hat{i} + 6\hat{j} - 12\hat{k}$

24. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths. [2]



Consider,



using triangular law of addition

$$\vec{b} + \vec{a} - \vec{d}_2 = \vec{0}$$

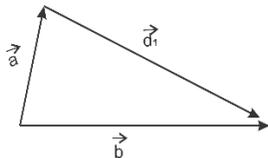
$$\Rightarrow \vec{d}_2 = \vec{a} + \vec{b}$$

$$= 4\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\Rightarrow |\vec{d}_2| = \sqrt{16+4+16} = 6 \text{ units}$$

$\Rightarrow d$ the diagonal length $|\vec{d}_2|$ is 6

again, considering other diagonal,



again using triangular law of addition of vectors,

$$\vec{a} + \vec{d}_1 - \vec{b} = 0$$

$$\Rightarrow \vec{d}_1 = \vec{b} - \vec{a} = -2\hat{i} + 2\hat{j}$$

$$\Rightarrow |\vec{d}_1| = \sqrt{(-2)^2 + (2)^2} = 2\sqrt{2} \text{ units}$$

\Rightarrow diagonals are $4\hat{i} - 2\hat{j} + 4\hat{k}$ and $-2\hat{i} + 2\hat{j}$

with lengths 6 and $2\sqrt{2}$ units respectively.

25. (a) Simplify : $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right), 0 < x < \frac{\pi}{4}$. [2]

OR

(b) Evaluate : $\tan\left(\sin^{-1}1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$ [2]

Sol. (a) Since, $x \in \left(0, \frac{\pi}{4}\right) \Rightarrow 2x \in \left(0, \frac{\pi}{2}\right)$

$$\sin 2x, \cos x \in (0, 1)$$

$$\Rightarrow \text{Let } E = \tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$$

divide numerator and denominator by $\cos 2x$

$$E = \tan^{-1}\left(\frac{1 - \tan 2x}{1 + \tan 2x}\right) = \tan^{-1}\left(\frac{\tan\left(\frac{\pi}{4}\right) - \tan(2x)}{1 + \tan\frac{\pi}{4}\tan 2x}\right)$$

$$\text{since } \tan\left(\frac{\pi}{4}\right) = 1,$$

$$\Rightarrow E = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - 2x\right)\right), \text{ using } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

since $\frac{\pi}{4} - 2x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$, which lie in principal value branch,

$$\Rightarrow E = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - 2x\right)\right) = \frac{\pi}{4} - 2x$$

as $\tan^{-1}(\tan x) = x \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\Rightarrow E = \boxed{\frac{\pi}{4} - 2x}$$

OR

(b) Let $E = \tan\left(\sin^{-1}(1) - \cos^{-1}\left(\frac{-1}{2}\right)\right)$

$$\begin{aligned} \sin^{-1}1 &= \frac{\pi}{2} \text{ and } \cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) \\ &= \pi - \frac{\pi}{3} = \frac{2\pi}{3} \end{aligned}$$

$$\Rightarrow E = \tan\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) = \tan\left(\frac{3\pi - 4\pi}{6}\right)$$

$$= \tan\left(\frac{-\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = \frac{-1}{\sqrt{3}}$$

Answer is $\frac{-1}{\sqrt{3}}$

SECTION-C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

6 × 3 = 18

26. Evaluate : $\int_0^1 x \tan^{-1} x \, dx$.

[3]

Sol. Let $I = \int_0^1 x \tan^{-1} x \, dx$

using integration by parts,

$$I = \left[(\tan^{-1} x) \left(\frac{x^2}{2} \right) \right]_0^1 - \int_0^1 \left(\frac{1}{1+x^2} \right) \times \frac{x^2}{2} \, dx$$

$$= \left[(\tan^{-1} x) \left(\frac{x^2}{2} \right) \right]_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{1+x^2} \, dx$$

$$= \left(\frac{\pi}{8} - 0 \right) - \frac{1}{2} \int_0^1 \frac{(x^2+1)-1}{(1+x^2)} \, dx$$

$$= \frac{\pi}{8} - \frac{1}{2} \int_0^1 \left(1 - \frac{1}{1+x^2} \right) \, dx$$

distributing integration,

$$\begin{aligned} &= \frac{\pi}{8} - \frac{1}{2} \left(\int_0^1 (1) dx - \int_0^1 \frac{1}{1+x^2} dx \right) \\ &= \frac{\pi}{8} - \frac{1}{2} \left(|x|_0^1 - |\tan^{-1}x|_0^1 \right) \\ &= \frac{\pi}{8} - \frac{1}{2} \left((1-0) - \left(\frac{\pi}{4} - 0 \right) \right) = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} \\ &= \frac{\pi}{4} - \frac{1}{2} \end{aligned}$$

Alternatively, $\left(\frac{\pi - 2}{4} \right)$.

27. (a) Find $\int \sqrt{\frac{x+2}{x-2}} dx$ [3]

OR

(b) Find: $\int \frac{x^2}{(x^2+9)(x^2+16)} dx$ [3]

Sol. (a) $I = \int \sqrt{\frac{x+2}{x-2}} dx$

Multiply the numerator and denominator by $(x+2)$

$$\begin{aligned} \Rightarrow I &= \int \frac{\sqrt{(x+2)(x+2)}}{\sqrt{(x-2)(x+2)}} dx \\ &= \int \frac{(x+2)}{\sqrt{x^2-4}} dx \\ &= \int \frac{x}{\sqrt{x^2-4}} dx + 2 \int \frac{dx}{\sqrt{x^2-4}} \quad \dots(i) \end{aligned}$$

Let $\int \frac{x}{\sqrt{x^2-4}} dx = I_1$

Let $u = x^2 - 4$

$du = 2x dx$

Substituting the values

$$I_1 = \frac{1}{2} \int \frac{du}{\sqrt{u}}$$

$$I_1 = \frac{1}{2} \int u^{-1/2} du$$

Using $\int u^n du = \frac{u^{n+1}}{n+1}$

$$I_1 = \frac{1}{2} \left(\frac{u^{1/2}}{1/2} \right) = \sqrt{u} + c_1$$

$$I_1 = \sqrt{x^2 - 4} + c_1 \quad (\because u = x^2 - 4)$$

Let $I_2 = 2 \int \frac{dx}{\sqrt{x^2 - 2^2}}$

Applying the formula:

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

here $a = 2$

$$\therefore I_2 = 2 \log \left| x + \sqrt{x^2 - 4} \right| + c_2$$

$$I = I_1 + I_2 \dots \text{(from eq. i)}$$

$$I = \sqrt{x^2 - 4} + 2 \log \left| x + \sqrt{x^2 - 4} \right| + (c_1 + c_2)$$

$$= \sqrt{x^2 - 4} + 2 \log \left| x + \sqrt{x^2 - 4} \right| + c$$

OR

(b) $I = \int \frac{x^2}{(x^2 + 9)(x^2 + 16)} dx$

Let $x^2 = t$ and consider the rational function

$$\frac{t}{(t + 9)(t + 16)} = \frac{A}{t + 9} + \frac{B}{t + 16} \quad \dots(i)$$

Multiply both sides by $(t + 9)(t + 16)$

$$t = A(t + 16) + B(t + 9) \quad \dots(ii)$$

Put $t = -9$ in eq (ii)

$$-9 = A(16 - 9) + 0$$

$$\Rightarrow A = -\frac{9}{7}$$

Put $t = -16$ in eq (ii)

$$-16 = A(16 - 16) + B(-16 + 9)$$

$$\Rightarrow -16 = -7B$$

$$\Rightarrow B = \frac{16}{7}$$

Substitute the value of A and B in eq(i)

$$\frac{x^2}{(x^2 + 9)(x^2 + 16)} = \frac{-9}{7(x^2 + 9)} + \frac{16}{7(x^2 + 16)}$$

$$I = \int \left[\frac{-9}{7(x^2 + 9)} + \frac{16}{7(x^2 + 16)} \right] dx$$

Use the formula : $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$

$$= \frac{-9}{3 \times 7} \tan^{-1} \left(\frac{2}{3} \right) + \frac{16}{4 \times 7} \tan^{-1} \left(\frac{x}{4} \right) + C$$

$$= -\frac{3}{7} \tan^{-1} \left(\frac{x}{3} \right) + \frac{4}{7} \tan^{-1} \left(\frac{x}{4} \right) + C$$

28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x| dx$, then show that $I_1 - 4I_2 = 0$. [3]

Sol. $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$

By using the formula: $1 + \cos 2x = 2\cos^2 x$

$$I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x}$$

$$= \frac{1}{2} \int_{-\pi/4}^{\pi/4} \sec^2 x dx$$

$$= \frac{1}{2} (\tan x) \Big|_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2} \left(\tan \left(\frac{\pi}{4} \right) - \left(\tan \left(-\frac{\pi}{4} \right) \right) \right)$$

$$= \frac{1}{2} (1 - (-1))$$

$$= \frac{1}{2} (2)$$

$$= 1$$

$$I_2 = \int_{-1/2}^{1/2} |x| dx$$

Using $|x| = \begin{cases} -x & x \leq 0 \\ x & x > 0 \end{cases}$

$$= \int_{-1/2}^0 |x| dx + \int_0^{1/2} |x| dx$$

$$= \int_{-1/2}^0 (-x) dx + \int_0^{1/2} x dx$$

$$= -\frac{x^2}{2} \Big|_{-1/2}^0 + \frac{x^2}{2} \Big|_0^{1/2}$$

$$= -\left(\frac{0^2}{2} - \frac{\left(-\frac{1}{2}\right)^2}{2} \right) + \left(\frac{\left(\frac{1}{2}\right)^2}{2} - 0 \right)$$

$$= \frac{1}{8} + \frac{1}{8}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

$$\therefore I_1 = 1 \text{ and } I_2 = \frac{1}{4}$$

To prove:

$$I_1 - 4I_2 = 0$$

$$\text{LHS: } 1 - 4\left(\frac{1}{4}\right) = 1 - 1 = 0$$

$$\text{RHS} = 0$$

Since LHS = RHS, hence proved.

29. (a) Find the general solution of the following differential equation : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ [3]

OR

- (b) Find the particular solution of the differential equation $xy \frac{dy}{dx} = (x+2)(y+2)$, given that $y(1) = -1$. [3]

Sol. (a) $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2 \quad \dots(i)$$

This is a homogenous differential equation

For homogenous differential equation, we substitute $y = vx$ differentiating both sides w.r.t x

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substitute $\frac{dy}{dx}$ and y in equation (i)

$$v + x \frac{dv}{dx} = 1 + v + v^2$$

$$\Rightarrow \frac{xdv}{dx} = 1+v^2$$

$$\Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$$

Integrate both sides

Using the formula: $\int \frac{dx}{1+x^2} = \tan^{-1}x$

$$\int \frac{dx}{x} = \ln|x|$$

$$\tan^{-1}v = \ln|x| + c$$

substitute $v = y/x$ back in the equation

$$\tan^{-1}(y/x) = \log|x| + c$$

OR

(b) Given: $xy \frac{dy}{dx} = (x+2)(y+2)$, $y(1) = -1$

$$xy \frac{dy}{dx} = (x+2)(y+2)$$

$$\Rightarrow \frac{ydy}{y+2} = \frac{(x+2)dx}{x}$$

Integrating both sides

$$\int \frac{y}{y+2} dy = \int \left(\frac{x+2}{x} \right) dx$$

$$\Rightarrow \int \frac{y+2-2}{y+2} dy = \int \left(1 + \frac{2}{x} \right) dx$$

$$\Rightarrow \int \left(1 - \frac{2}{y+2} \right) dy = \int \left(1 + \frac{2}{x} \right) dx$$

$$y - 2\log|y+2| = x + 2\log|x| + c \quad \dots(i)$$

putting $x = 1$, $y = -1$ in equation (i)

$$-1 - 2\log|-1+2| = 1 + 2\log|1| + c$$

$$\Rightarrow -1 - 2\log(1) = 1 + 2\log 1 + c$$

$$\Rightarrow -1 = 1 + c$$

$$\Rightarrow c = -2$$

put $c = -2$ in equation (i)

$$y = 2\log|y+2| + x + 2\log|x| - 2$$

$$y - x + 2 = \log(y + 2)^2 + \log(x^2) \quad [\text{using } a \log b = \log b^a]$$

$$\Rightarrow y - x + 2 = \log((y + 2)^2 \cdot (x^2)) \quad [\text{using } \log a + \log b = \log(ab)]$$

30. Solve the following linear programming problem graphically:

[3]

Minimize $Z = 13x - 15y$

Subject to constraints

$$x + y \leq 7,$$

$$2x - 3y + 6 \geq 0,$$

$$x \geq 0, y \geq 0$$

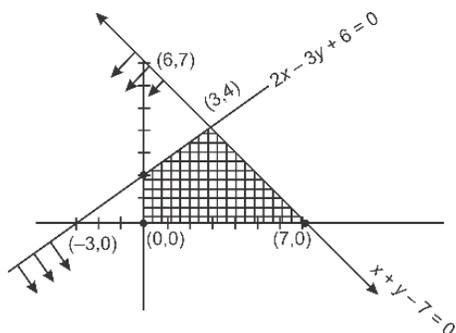
Sol. we have to minimize, $Z = 13x - 15y$

subject to constraints,

$$x + y \leq 7$$

$$2x - 3y + 6 \geq 0$$

$$x \geq 0, y \geq 0$$



$$x + y - 7 = 0$$

| | | |
|---|---|---|
| x | 0 | 7 |
| y | 7 | 0 |

, $x + y - 7 \leq 0$ includes (0, 0)

$$2x - 3y + 6 = 0$$

| | | |
|---|---|----|
| x | 0 | -3 |
| y | 2 | 0 |

, $2x - 3y + 6 \geq 0$ include (0, 0)

$2x - 3y + 6 = 0$ and $x + y = 7$ intersect at (3, 4)

\Rightarrow critical points are

$$(x, y) \equiv (0, 0) (0, 2), (3, 4), (7, 0)$$

\Rightarrow minimum with occur at these points

| Corner point (x, y) | Value of $z = 13x - 15y$ |
|---------------------|-----------------------------------|
| 0(0,0) | $13(0) - 15(0) = 0$ |
| A(7,0) | $13 \times 7 - 15 \times 0 = 91$ |
| B(0,2) | $13 \times 0 - 15 \times 2 = -30$ |
| C(3,4) | $13 \times 3 - 15 \times 4 = -21$ |

\Rightarrow minimum value of z is -30 which occurs at $x = 0, y = 2$

31. (a) Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball. **[3]**

OR

- (b) The probability of simultaneous occurrence of atleast one of the two events X and Y is a . If the probability that exactly one of the events X, Y occurs is b , prove that $P(X') + P(Y') = 2 - 2a + b$. **[3]**

Sol. (a) If the number is less than 3 (i. e. 1 or 2) \rightarrow choose Bag I

otherwise (3, 4, 5, 6) \rightarrow choose Bag II

$$P(\text{Bag I}) = P(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{Bag II}) = P(3, 4, 5, 6) = \frac{4}{6} = \frac{2}{3}$$

$$P(\text{Red} | \text{Bag I}) = \frac{3}{7}, P(\text{Red} | \text{Bag II}) = \frac{8}{14} = \frac{4}{7}$$

using total probability theorem,

$$P(\text{Red}) = P(\text{Bag I}) P(\text{Red} | \text{Bag I}) + P(\text{Bag II}) P(\text{Red} | \text{Bag II})$$

$$= \frac{1}{3} \times \frac{3}{7} + \frac{2}{3} \times \frac{4}{7}$$

$$= \frac{3}{21} + \frac{8}{21} = \frac{11}{21}$$

\Rightarrow probability that the ball drawn from one of the bags is a red ball is $\frac{11}{21}$

OR

- (b) Probability that atleast one of X or Y occurs

$$P(X \cup Y) = a$$

Probability that exactly one of X or Y occurs,

$$P(X \cap Y') + P(X' \cap Y) = b$$

also

$$P(X \cup Y) = P(X \cap Y') + P(X' \cap Y) + P(X \cap Y)$$

$$\Rightarrow a = b + P(X \cap Y)$$

$$\Rightarrow P(X \cap Y) = a - b \quad \dots(1)$$

No, we know

$$P(X') = 1 - P(X)$$

$$P(Y') = 1 - P(Y)$$

$$P(X') + P(Y') = 2 - [P(X) + P(Y)] \quad \dots(2)$$

and $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$a = P(X) + P(Y) - (a - b) \text{ [from (1)]}$$

$$P(X) + P(Y) = 2a - b \quad \dots(3)$$

from (2) and (3)

$$P(x') + P(y') = 2 - (2a - b)$$

$$= 2 - 2a + b$$

Hence, proved

SECTION-D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

4 × 5 = 20

32. (a) A relation R is defined on Z , the set of integers, as $R = \{(x, y) : |x - y| \text{ is divisible by a prime number 'p', } x, y \in Z\}$ check whether R is an equivalence relation or not. **[5]**

OR

- (b) A function $f : R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto. **[5]**

- Sol.** (a) $R = \{(x, y) : |x - y| \text{ is divisible by a prime number 'p', } x, y \in Z\}$

Z is the set of integers

- (1) **Symmetric Relation :**

$$\text{Let } (x, y) \in R$$

$$\Rightarrow |x - y| = p\lambda$$

$$\Rightarrow |y - x| = p\lambda \Rightarrow |(y - x)| \text{ is divisible by prime } p$$

$$\Rightarrow (y, x) \in R$$

$\Rightarrow R$ is symmetric relation

- (2) **Reflexive Relation :**

$$\text{Let } (x, y) \in R \Rightarrow |x - y| = p\lambda$$

$$\because |x - x| = 0 \Rightarrow |x - x| \text{ is divisible by prime } p$$

$$\Rightarrow (x, x) \in R$$

$\Rightarrow R$ is reflexive relation

- (3) **Transitive Relation:**

$$\text{Let } \Rightarrow (x, y) \in R, (y, z) \in R$$

$$|x - y| = p\lambda_1 \text{ and } |y - z| = p\lambda_2$$

$$\Rightarrow |x - z| = |(x - y) + (y - z)| = p|\pm\lambda_1 \pm \lambda_2| \Rightarrow |x - z| \text{ is divisible by prime } p$$

$$\Rightarrow (x, z) \in R$$

R is reflexive relation

$\because R$ is symmetric, reflexive and transitive relation

$\Rightarrow R$ is equivalence relation.

OR

(b) $f : R - \left\{ \frac{3}{5} \right\} \rightarrow R - \left\{ \frac{3}{5} \right\}, f(x) = \frac{3x+2}{5x-3}$

(1) One-one function

$$\text{Let } f(x_1) = f(x_2), x_1, x_2 \in R - \left\{ \frac{3}{5} \right\}$$

$$\Rightarrow \frac{3x_1+2}{5x_1-3} = \frac{3x_2+2}{5x_2-3}$$

$$\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6$$

$$\Rightarrow 19x_1 = 19x_2 \Rightarrow x_1 = x_2$$

$\Rightarrow f(x)$ is one-one function

(2) Onto function

$$f(x) = y = \frac{3x+2}{5x-3}$$

$$\Rightarrow 5xy - 3y = 3x + 2$$

$$5xy - 3x = 3y + 2$$

$$\Rightarrow x = \frac{3y+2}{5y-3}$$

$$\therefore x \in R - \left\{ \frac{3}{5} \right\} \Rightarrow y \in R - \left\{ \frac{3}{5} \right\}$$

$$\Rightarrow \text{Range of } f(x) \text{ is } R - \left\{ \frac{3}{5} \right\}$$

\Rightarrow Range of $f(x)$ is codomain of $f(x)$

$\Rightarrow f(x)$ is onto function

$\Rightarrow y = f(x)$ is one-one onto function

33. (a) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$$

[5]

OR

(b) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2} \right]$.

[5]

Sol. (a) $A^{-1} = \frac{\text{adj}(A)}{|A|}$

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$$

$$|A| = 0(0-2) - 2(0+2) + 1(2+1) = -1$$

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} -1 & -2 \\ -1 & 0 \end{vmatrix} = -2$$

$$A_{12} = (-1)^{1+2} M_{12} = - \begin{vmatrix} -2 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} -2 & -1 \\ 1 & -1 \end{vmatrix} = 3$$

$$A_{21} = (-1)^{2+1} M_{21} = - \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = -1$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$A_{23} = (-1)^{2+3} M_{23} = - \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = 2$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 2 & 1 \\ -1 & -2 \end{vmatrix} = -3$$

$$A_{32} = (-1)^{3+2} M_{32} = - \begin{vmatrix} 0 & 1 \\ -2 & -2 \end{vmatrix} = -2$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 0 & 2 \\ -2 & -1 \end{vmatrix} = 4$$

$$\Rightarrow \text{adj}(A) = \begin{bmatrix} -2 & -2 & 3 \\ -1 & -1 & 2 \\ -3 & -2 & 4 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj}(A)}{|A|} = - \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$$

$$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$$

$$A^T = \begin{bmatrix} 0 & -2 & 1 \\ 2 & -1 & -1 \\ 1 & -2 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$$

$$\therefore A^T X = B$$

Pre-multiplying $(A^T)^{-1}$ on both sides

$$\Rightarrow A^T (A^T)^{-1} X = (A^T)^{-1} B$$

$$\therefore A^{-1} A = I$$

$$\Rightarrow IX = (A^T)^{-1} B$$

$$\Rightarrow X = (A^T)^{-1} B$$

$$\because (A^T)^{-1} = (A^{-1})^T$$

$$\Rightarrow X(A^{-1})^T B = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 7 & +2 \times 8 & -3 \times 10 \\ 1 \times 7 & +1 \times 8 & -2 \times 10 \\ 3 \times 7 & +2 \times 8 & -4 \times 10 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow x = 0, y = -5, z = -3$$

OR

(b) $\because \begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix

if matrix M is a singular matrix, then $|M| = 0$

$$\Rightarrow \begin{vmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 3(8 - 7\cos 2x) + 1(-14 + 11\cos 2x) + \sin 3x(-49 + 44) = 0$$

$$\Rightarrow 24 - 21\cos 2x - 14 + 11\cos 2x - 5\sin 3x = 0$$

$$\Rightarrow 10 - 10\cos 2x - 5\sin 3x = 0$$

$$\Rightarrow 2 - 2\cos 2x - \sin 3x = 0$$

$$\because \sin 3x = 3\sin x - 4\sin^3 x \text{ and } \cos 2x = 1 - 2\sin^2 x$$

$$\Rightarrow 2 - 2 + 4\sin^2 x - 3\sin x + 4\sin^3 x = 0$$

$$\Rightarrow \sin x(4\sin^2 x + 4\sin x - 3) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } 4\sin^2 x + 4\sin x - 3 = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \sin x = \frac{1}{2} \text{ or } \sin x = \frac{-3}{2}$$

$$\because -1 \leq \sin x \leq 1$$

$$\Rightarrow \sin x = 0 \text{ or } \sin x = \frac{1}{2}$$

$$\because x \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow x = 0, \frac{\pi}{6}$$

34. If $x = \cos t$, $y = \cos mt$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.

[5]

Sol. If $x = \cos t$, $y = \cos mt$

prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

$$\therefore x = \cos t \quad \dots(a)$$

d.w.r. to t

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} \cos t = -\sin t \quad \dots(i)$$

$$\therefore y = \cos mt \quad \dots(b)$$

d.w.r. to t

$$\frac{dy}{dt} = \frac{d}{dt} (\cos mt) = -\sin(mt) \frac{d}{dt} (mt) = -m \sin(mt) \quad \dots(ii)$$

dividing (ii) by (i), we get

$$\Rightarrow \frac{dy}{dx} = \frac{m \sin(mt)}{\sin t} \quad \dots(iii)$$

d. w. r. to x

$$\Rightarrow \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left(\frac{m \sin mt}{\sin t} \right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dt}{dx} \frac{d}{dt} \left(\frac{m \sin(mt)}{\sin t} \right) = \left(\frac{dt}{dx} \right) \left(\frac{\sin t \cdot \frac{d}{dt} m \sin mt - m \sin(mt) \frac{d}{dt} \sin t}{\sin^2 t} \right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{dt}{dx} \right) \left(\frac{\sin t \cdot m^2 \cos mt - m \sin(mt) \cos t}{\sin^2 t} \right)$$

$$\therefore \frac{dt}{dx} = -\frac{1}{\sin t} \text{ (from (i)), } \sin^2 t = 1 - \cos^2 t = 1 - x^2 \text{ (from (a))}$$

$$y = \cos mt \text{ (from (b)), } \frac{dy}{dx} = \frac{m \sin(mt)}{\sin t} \text{ (from iii)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{\sin t} \left(\frac{m^2 \sin t \cos mt - m \sin(mt) \cos t}{1-x^2} \right)$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - \cos t \frac{(m \sin mt)}{\sin t} + m^2 \cos mt = 0$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0 \text{ (Hence proved)}$$

35. Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting. [5]

Sol. Line $L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t$

Line $L_2: \frac{x-4}{5} = \frac{y-1}{2} = z = \lambda$

direction ratios of line L_1 are $\langle 2, 3, 4 \rangle$

direction ratios of line L_2 are $\langle 5, 2, 1 \rangle$

$$\therefore \frac{2}{5} \neq \frac{3}{2} \neq \frac{4}{1}$$

\Rightarrow lines L_1 and L_2 are not parallel.

Let any point on lines L_1 is $A(2t + 1, 3t + 2, 4t + 3)$

Let any point on lines L_2 is $B(5\lambda + 4, 2\lambda + 1, \lambda)$

for intersection point of lines L_1 and L_2

$$2t + 1 = 5\lambda + 4 \quad \dots(i)$$

$$3t + 2 = 2\lambda + 1 \quad \dots(ii)$$

$$4t + 3 = \lambda \quad \dots(iii)$$

From (i) and (iii)

$$2t + 1 = 5(4t + 3) + 4 \Rightarrow 18t + 18 = 0 \Rightarrow t = -1$$

put $t = -1$ in (iii) $\Rightarrow \lambda = -1$

$\lambda = -1, t = -1$ satisfy equation (ii)

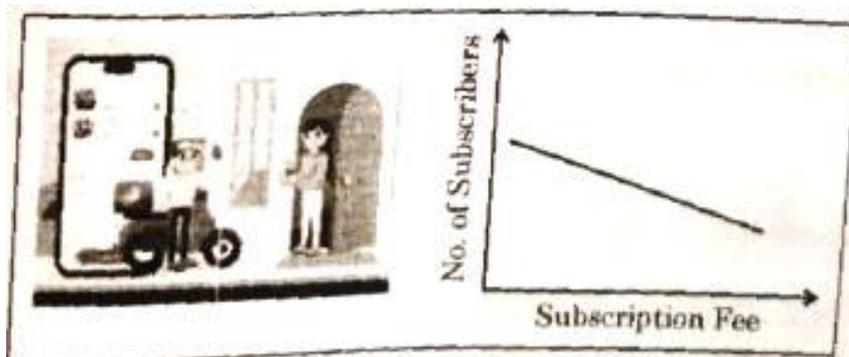
\Rightarrow Intersection point of lines L_1 and L_2 is $(-1, -1, -1)$

SECTION-E

This section comprises of 3 case study/passage based questions of 4 marks each.

3 × 4 = 12

36. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.



The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹x.

Based on the given information, answer the following questions:

- (i) How many subscribers will discontinue after an increase of ₹x in annual fee?

[1]

- (ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x . [1]
- (iii) (a) Find the value of x for which $R(x)$ is maximum. [2]

OR

- (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing. [2]

Sol. Total number of subscriber: 5000

Annual subscription fees per subscribe : ₹300

- (i) On increasing ₹ x annually no. of subscribes discontinue is $(5000 - 10x)$

- (ii) Total Revenue = Total number of subscribes \times Subscription fees

$$R(x) = (5000 - 10x)(300 + x)$$

$$R'(x) = (5000 - 10x)(1) + (300 + x)(-10)$$

$$R'(x) = 2000 - 20x$$

$$\text{Put } R'(x) = 0 \Rightarrow x = 100$$

$$\text{Where } R''(x) = -20 < 0$$

$$\text{as } R''(x) < 0$$

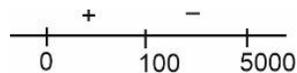
- (iii) (a) $R(x)$ is maximum at $x = 100$

OR

(b) $R'(50) = 1000 > 0$

$$R'(150) = -1000 < 0$$

Increasing $(0, 100)$



Decreasing $(100, 5000)$

37. In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.



A total of 1,00,000 jackpot tickets each costing ₹100 were sold there by raising a fund of ₹1,00,00,000. Rohan bought one ticket.

Based on given information, answer the following questions :

- (i) What are the possible amounts, the person can win?
- (ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ?

OR

- (b) What is the probability that the person does not win any amount?
- (iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.

Sol. Total tickets sold : 1,00,000

Ticket cost : 100/ ticket

Prizes : 1st Prize : 3,00,000 (1 winner)

2nd Prize : 2,00,000 (2 winner)

3rd Prize : 50,000 (3 winner)

Total Winning Tickets : 1 + 2 + 3 = 6 tickets

(i) Possible amount, the person can win are

Rs. 3,00,000 if win 1st prize

Rs. 2,00,000 if win 2nd prize

Rs. 50,000 if win 3rd prize

Rs. 0 in case of no winning.

(ii) (a) $P(\text{atleast } 200000) = P(2,00,000) + P(3,00,000)$

$$= \frac{2}{100,000} + \frac{1}{100,000} = 0.00003$$

OR

(b) $P(\text{Doesn't win any amount}) = 1 - P(\text{winning prizes})$

$$= 1 - \frac{6}{1,00,000} = 0.99994$$

(iii) P_1 : probability of winning jackpot 1 is $\frac{6}{100000}$

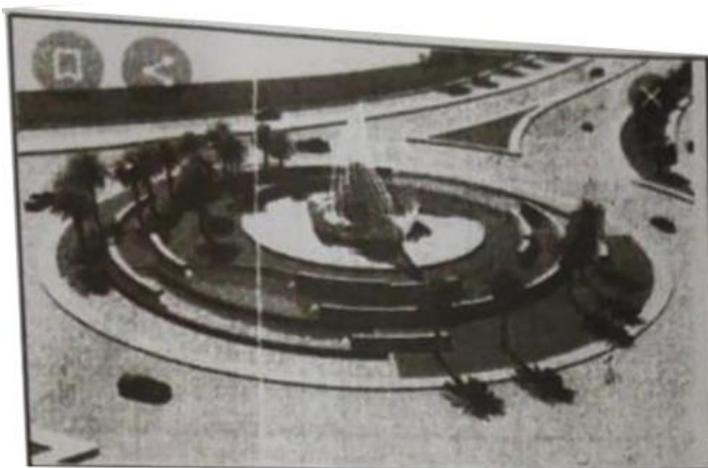
P_2 : Probability of winning jackpot 2 is $\frac{1}{100000}$

$$P(\text{Exactly one Jackpot}) = P_1\bar{P}_2 + P_2\bar{P}_1$$

$$= \frac{6}{100000} \times \frac{99999}{100000} + \frac{1}{100000} \times \frac{99994}{100000}$$

$$= \frac{699988}{10,000,000,000}$$

38. Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1; x^2 + y^2 = 64$.

There is a circular pond with a fountain in the middle of the roundabout whose equation is given by

$$C_2 : x^2 + y^2 = 4.$$

Based on the given information, answer the following questions:

- (i) Represent the given equations C_1 and C_2 with the help of a diagram.
- (ii) Express y as a function of $x, (y = f(x))$, for both C_1 and C_2 .
- (iii) (a) Using integration find the area of region covered by the roundabout.

OR

- (b) Using integration, find the area of region covered by circular pond.

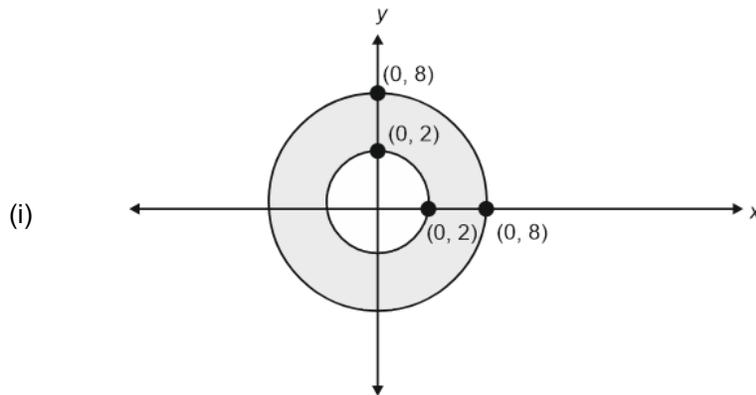
Sol. $C_1 : x^2 + y^2 = 64 \Rightarrow (x-0)^2 + (y-0)^2 = (8)^2$

$$C_2 : x^2 + y^2 = 4 \Rightarrow (x-0)^2 + (y-0)^2 = (2)^2$$

Center of both curve C_1 & C_2 is origin $(0,0)$

radius of $C_1 (R_1) = 8$ unit

radius of $C_2 (R_2) = 2$ unit



(ii) $C_1 : x^2 + y^2 = 64 \Rightarrow y^2 = 64 - x^2 \Rightarrow y = \pm\sqrt{64 - x^2}$

$$C_2 : x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2 \Rightarrow y = \pm\sqrt{4 - x^2}$$

- (iii) (a) Area of region covered by round about is

$$= 4 \left[\int_0^8 C_1 dx - \int_0^2 C_2 dx \right]$$

$$= 4 \left[\int_0^8 \sqrt{64 - x^2} dx - \int_0^2 \sqrt{4 - x^2} dx \right]$$

$$\text{By using } \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$\begin{aligned}\text{Area} &= 4 \left[\left(\frac{x}{2} \sqrt{64 - x^2} + \frac{64}{2} \sin^{-1} \frac{x}{8} \right) \Big|_0^8 + \left(\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right) \Big|_0^2 \right] \\ &= 4 [16\pi + \pi] \\ &= 68\pi \text{ square units.}\end{aligned}$$

OR

$$\begin{aligned}\text{(b) Area of region covered by circular pond } (C_2) &= 4 \int_0^2 \sqrt{4 - x^2} dx \\ &= 4 \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= 4[\pi] = 4\pi \text{ square units}\end{aligned}$$

□ □ □