Aakash

+ BYJU'S

SOLUTIONS

FOR

MOCK TEST PAPER

Class - X

Mathematics

(Standard)

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005,
Ph.011-47623456

MM : 80 Mock Test Paper (Maths-Standard) - Class-X (2023-24) **Time : 180 Min.**

Hints and Solutions

MATHEMATICS

SECTION-A : Multiple Choice Type Questions (20×1 = 20 Marks)

A1. **Answer :** (3) **...[1 Mark]**

A2. **Answer :** (4) **...[1 Mark]**

A3. **Answer :** (4) **...[1 Mark]**

A4. **Answer :** (4) **...[1 Mark]**

A5. **Answer :** (3) **...[1 Mark]**

A6. **Answer :** (2) **...[1 Mark]**

A7. **Answer :** (1) **...[1 Mark]**

A8. **Answer :** (3) **...[1 Mark]**

A9. **Answer :** (3) **...[1 Mark]**

A10. **Answer :** (4) **...[1 Mark]**

A11. **Answer :** (2) **...[1 Mark]**

Solution:

$$\frac{\cos^2 60^\circ - 7\tan^2(45^\circ) + 8\,\text{cosec}^2\, 60^\circ}{\sin^2 45^\circ - 5\cos^2 90^\circ + 3\sin(30^\circ)}$$

$$= \frac{\frac{1}{4} - 7 + 8 \times \frac{4}{3}}{\frac{1}{2} - 0 + 3 \times \frac{1}{2}}$$

$$= \frac{\frac{1}{4} - 7 + 8 \times \frac{4}{3}}{\frac{1}{2} + \frac{3}{2}}$$

$$= \frac{\frac{1}{4} - 7 + \frac{32}{3}}{\frac{4}{2}} = \frac{3 - 84 + 128}{12} \times \frac{1}{2}$$

$$= \frac{47}{12} \times \frac{1}{2} = \frac{47}{24}$$

A12. **Answer :** (2) **...[1 Mark]**

A13. **Answer :** (4) **...[1 Mark]**

A14. **Answer :** (2) **...[1 Mark]**

A15. **Answer :** (2) **...[1 Mark]**

A16. **Answer :** (1) **...[1 Mark]**

A17. **Answer :** (3) **...[1 Mark]**

A18. **Answer :** (4) **...[1 Mark]**

A19. **Answer :** (4) **...[1 Mark]**

A20. **Answer :** (1) **...[1 Mark]**

SECTION-B : Very Short Answer Type Questions (5×2 = 10 Marks)

A21. Solution:

Discriminant, $D = (1)^2 - 4(1)(1) = -3 < 0$

...[1 Mark]

No real roots

...[1 Mark]

OR

Solution:

$64 = 2^6$ and $200 = 2^3 \times 5^2$

...[1 Mark]

$\therefore$ HCF (64, 200) $= 2^3 = 8$

...[1 Mark]

A22. Solution:

We have,

$2x^2 - x - 15 = 2x^2 - 6x + 5x - 15$

$= (2x + 5)(x - 3)$

...[0.5 Mark]

So, the value of $2x^2 - x - 15$ is zero when $2x + 5 = 0$ or $x - 3 = 0$.

...[0.5 Mark]

Therefore, the zeroes of $2x^2 - x - 15$ are $\frac{-5}{2}$ and 3.

Now,

Sum of zeroes = $\frac{-5}{2} + 3 = \frac{1}{2} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

...[0.5 Mark]

Product of zeroes = $\frac{-5}{2} \times 3 = \frac{-15}{2} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

...[0.5 Mark]

A23. Solution:

Here, $(1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta)$

$= (1 + \cot^2\theta)(1 - \cos^2\theta)$

...[1 Mark]

$= (1 + \cot^2\theta)\sin^2\theta$

$= \text{cosec}^2\theta \sin^2\theta$

$= 1$

...[1 Mark]

OR

Solution:

Given $\tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60°$

...[1 Mark]

$\therefore 2\sin^2 60° - 3\cos^2 60° = 2 \times \frac{3}{4} - 3 \times \frac{1}{4}$

$= \frac{6}{4} - \frac{3}{4}$

$= \frac{3}{4}$

...[1 Mark]

A24. Solution:

The integers divisible by 5 between 1 and 80 are 5, 10, 15, 20, 25, 30, …… 75

Let n be the number of integers which is a multiple of 5.

$T_n = a + (n - 1)d$

$75 = 5 + (n - 1)5$

$n = 15$

...[1 Mark]

$P(\text{Not divisible by 5}) = \frac{63}{78} = \frac{21}{26}$

...[1 Mark]

A25. Solution:

$\frac{3}{6} = \frac{4}{8} = \frac{12}{24} = \frac{1}{2}$, pair of linear equation represents coincident lines.

...[2 Mark]

SECTION-C : Short Answer Type Questions (6×3 = 18 Marks)

A26. Solution:

Let us assume that $\sqrt{7}$ is rational. Then, there exists co-prime integers *a* and *b* such that

$\sqrt{7} = \frac{a}{b}$

$\Rightarrow \quad b\sqrt{7} = a$

...[1 Mark]

Squaring both sides, we get

$a^2 = 7b^2$

Therefore, a^2 is divisible by 7 and hence, *a* is also divisible by 7.

...[1 Mark]

So, we can write $a = 7r$, for some integer *r*.

Substituting for *a*, we get $49r^2 = 7b^2 \Rightarrow 7r^2 = b^2$.

This means, b^2 is also divisible by 7 and so, *b* is also divisible by 7.

So, our assumption is wrong because *a* and *b* are not co-prime.

...[1 Mark]

Hence, $\sqrt{7}$ is irrational.

A27. Solution:

Let the two digit number be $10x + y$ ($y > x$).

According to the question,

$10x + y + 10y + x = 110$

...[0.5 Mark]

$\Rightarrow 11x + 11y = 110$

$\Rightarrow x + y = 10$ ...(i)

...[0.5 Mark]

Also, $y - x = 4$ ...(ii)

...[0.5 Mark]

On adding equations (i) and (ii), we get

$2y = 14$

$y = 7$

...[0.5 Mark]

On putting value of '*y*' in equation (i), we get

$x = 3$

...[0.5 Mark]

Thus, the number is 37.

...[0.5 Mark]

A28. Solution:

$\because$ Angle swept by minute hand in an hour = 360°

$\therefore$ Angle swept by minute hand in a minute $= \frac{360^\circ}{60} = 6^\circ$

...[1 Mark]

$\therefore$ Angle swept by minute hand in 35 minutes = 6° × 35 = 210°

According to the question,

Area swept by the hour in the time period 4:15 am and 4:50 am $= \frac{\pi r^2}{360^\circ} \times 210^\circ$

...[1 Mark]

$= \frac{22}{7} \times \frac{(14)^2 \times 210^\circ}{360^\circ}$

$= 359\frac{1}{3}$ cm^2

...[1 Mark]

OR

Solution:

$PQ = \sqrt{[3-(-3)]^2 + [-4-(-4)]^2} = 6$ units

...[0.5 Mark]

$QR = \sqrt{(3-3)^2 + [2-(-4)]^2}$ = 6 units

...[0.5 Mark]

$RS = \sqrt{(3-(3))^2 + (2-2)^2} = 6$ units

...[0.5 Mark]

$SP = \sqrt{[-3-(-3)]^2 + [(2-(-4))]^2} = 6$ units

...[0.5 Mark]

$PR = \sqrt{[3-(-3)]^2 + [2-(-4)]^2} = 6\sqrt{2}$ units

...[0.5 Mark]

As, $PQ = QR = RS = SP$, and $PR^2 = PQ^2 + QR^2$, So, *PQRS* is a square

...[0.5 Mark]

A29. Solution:

Radius of the cylindrical part = $\frac{8}{2} = 4\ m$

...[0.5 Mark]

Height of the cylindrical part = 4.2 m

∴ Curved surface area of cylindrical part = $2\pi rh$

= $2 \times \frac{22}{7} \times 4 \times 4.2$

= 105.6 m^2

...[1 Mark]

∴ Curved surface area of the conical top = πrl

= $\frac{22}{7} \times 4 \times 5.6\ m^2$

= 70.4 m^2

...[0.5 Mark]

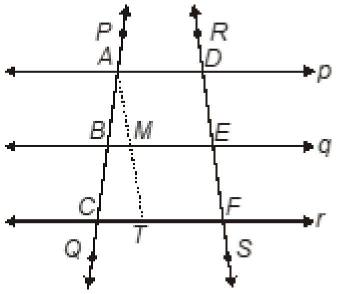
⇒ Total area of canvas = 105.6 m^2 + 70.4 m^2

= 176 m^2

...[1 Mark]

A30. **Solution:**

Given : Three parallel lines *p*, *q* and *r* which are cut by the transversals and at *A, B, C* and *D, E, F* respectively.



To Prove : $\frac{AB}{BC} = \frac{DE}{EF}$

Construction : Draw *AT* || *RS* meeting the lines *q* and *r* in *M* and *T* respectively.

...[0.5 Mark]

Proof : Since *AD* || *ME* and *AM* || *DE*,

∴ *AMED* is a parallelogram.

⇒ *AM* = *DE* ...(i)

...[0.5 Mark]

Also, *ME* || *TF* and *MT* || *EF*.

∴ *MTFE* is a parallelogram.

...[0.5 Mark]

⇒ *MT* = *EF* ...(ii)

...[0.5 Mark]

Now, in Δ*ACT*, *BM* || *CT*

∴ $\frac{AB}{BC} = \frac{AM}{MT}$ [By BPT]

...[0.5 Mark]

⇒ $\frac{AB}{BC} = \frac{DE}{EF}$

[From (i) and (ii)]

...[0.5 Mark]

Hence proved.

OR

Solution:

L.H.S. = (tanθ + 3)(3 tanθ + 1)

$= \left(\frac{\sin\theta}{\cos\theta} + 3\right)\left(\frac{3\sin\theta}{\cos\theta} + 1\right)$

...[0.5 Mark]

$= \left(\frac{\sin\theta + 3\cos\theta}{\cos\theta}\right)\left(\frac{3\sin\theta + \cos\theta}{\cos\theta}\right)$

...[0.5 Mark]

$= \left(\frac{3\sin^2\theta + 9\sin\theta\cos\theta + \sin\theta\cos\theta + 3\cos^2\theta}{\cos^2\theta}\right)$

...[1 Mark]

$= \frac{3 + 10\sin\theta\cos\theta}{\cos^2\theta}$

...[0.5 Mark]

= $3\sec^2\theta + 10\tan\theta$ = R.H.S.

...[0.5 Mark]

A31. **Solution:**

Total numbers of outcomes = 50

...[1 Mark]

Prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

...[1 Mark]

Number of favourable outcomes = 15

∴ Probability = $\frac{15}{50} = \frac{3}{10}$.

...[1 Mark]

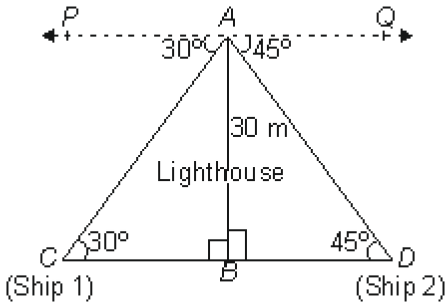
SECTION-D : Long Answer Type Questions (4×5 = 20 Marks)

A32. Solution:

In right Δ*ABC*,

$\frac{AB}{BC} = \tan 30^\circ$

...[1 Mark]



$\frac{30}{BC} = \frac{1}{\sqrt{3}}$

...[0.5 Mark]

BC = $30\sqrt{3}$ m

...[1 Mark]

BC = 30 × 1.73

BC = 51.90 metres

In right Δ*ABD*

$\frac{AB}{BD} = \tan 45^\circ$

...[1 Mark]

$\frac{30}{BD} = 1$

BD = 30 metres

...[1 Mark]

As *CB* > *DB*, so ship 2 reach the lighthouse first.

...[0.5 Mark]

OR

Solution:

Number of Social Science books = 153

Number of Science books = 527 – 153 = 374

…[1 Mark]

Let us find the HCF (374, 153)

374 = 17 × 22

153 = 17 × 9

…[1 Mark]

Thus, we have HCF (374, 153) = 17.

…[1 Mark]

Now, each bundle has to contain 17 books so that the number of bundles should be least.

The number of bundles of Social Science books $= \frac{153}{17} = 9$

…[1 Mark]

The number of bundles of Science books $= \frac{374}{17} = 22$

…[0.5 Mark]

Total number of bundles made = 9 + 22 = 31.

…[0.5 Mark]

A33. **Solution:**

We have,

$\frac{XY}{YP} = \frac{4}{1}$

$\Rightarrow \frac{XY}{YP} - 1 = \frac{4}{1} - 1$

...[0.5 Mark]

$\Rightarrow \frac{XY - YP}{YP} = \frac{4-1}{1}$

...[1 Mark]

$\Rightarrow \frac{XP}{YP} = \frac{3}{1}$

So *P* divides *XY* in the ratio 3 : 1

...[0.5 Mark]

Coordinates of *P* are $\left(\frac{3\times1+6\times3}{3+1}, \frac{1\times1+3\times(-8)}{3+1}\right)$

...[1 Mark]

$= \left(\frac{21}{4}, \frac{-23}{4}\right)$

...[0.5 Mark]

Since $\left(\frac{21}{4}, \frac{-23}{4}\right)$ lies on the line $2x + y + k = 0$

$\therefore 2 \times \frac{21}{4} - \frac{23}{4} = -k$

...[0.5 Mark]

$\Rightarrow \frac{42-23}{4} = -k$

...[0.5 Mark]

$\Rightarrow k = \frac{-19}{4}$

...[0.5 Mark]

OR

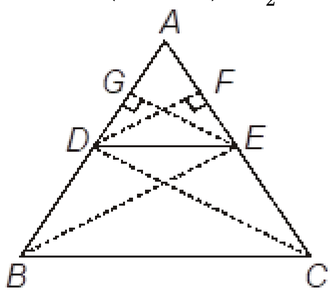
Solution:
Statement : In a triangle, a line drawn parallel to one side of a triangle intersecting the other two sides in distinct points, divides the other two sides in the same ratio.

...[0.5 Mark]

Proof of the Theorem
Given : ΔABC, in which DE is drawn parallel to BC.
To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$
Construction : Join CD and BE. Draw $DF \perp AE$ and $EG \perp AD$.

...[0.5 Mark]

Proof : ar$(\Delta ADE) = \frac{1}{2} \times AD \times EG$ ...(i)



ar$(\Delta BDE) = \frac{1}{2} \times BD \times EG$ ...(ii)

...[1 Mark]

On dividing (i) by (ii), we get

$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EG}{\frac{1}{2} \times BD \times EG} = \frac{AD}{BD}$ ...(iii)

...[1 Mark]

Similarly,
ar$(\Delta ADE) = \frac{1}{2} \times DF \times AE$
and ar$(\Delta CDE) = \frac{1}{2} \times CE \times DF$

$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)} = \frac{\frac{1}{2} \times DF \times AE}{\frac{1}{2} \times DF \times CE} = \frac{AC}{CE}$ ...(iv)

...[1 Mark]

Now, ar(ΔBDE) = ar(ΔCDE) [$\because$ Triangles on the same base and between the same parallel lines are equal in area]

...[0.5 Mark]

$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta CDE)}$

...[0.5 Mark]

$\therefore$ From (iii) and (iv), we get
$\frac{AD}{DB} = \frac{AE}{EC}$
Hence proved.

A34. Solution:

Classes	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	10	15
20 – 30	11	26
30 – 40	x	26 + x
40 – 50	27	53 + x
50 – 60	38	91 + x
60 – 70	40	131 + x
70 – 80	29	160 + x
80 – 90	14	174 + x
90 – 100	6	180 + x
Total	180 + x	

...[2 Marks]

$\frac{N}{2} = \frac{180+x}{2}$

Median is 57.5, so median class is 50 – 60, l = 50, h = 10, cf = 53 + x, f = 38

...[1 Mark]

$$\therefore \text{ Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

...[1 Mark]

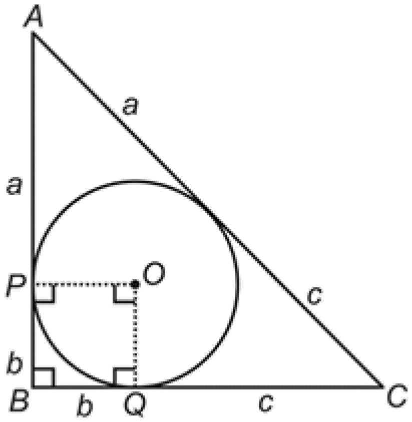
$$57.5 = 50 + \left(\frac{\frac{180+x}{2} - \left(53+x\right)}{38}\right) \times 10$$

...[0.5 Mark]

⇒ x = 17

...[0.5 Mark]

A35. Solution:



The triangle ABC is right-angled at B.

$\therefore AC = 50$ m,

Length of tangents drawn from an external point are equal.

$\therefore a + b = 40$ ...(i)

$b + c = 30$ ...(ii)

and $c + a = 50$ ...(iii)

...[1 Mark]

Now adding (i), (ii) and (iii), we get

$2(a + b + c) = 120$

$\Rightarrow a + b + c = 60$ ...(iv)

...[1 Mark]

Now from (iv) and (i), we get

$c = 20$ m

...[1 Mark]

From (iv) and (ii), we get

$a = 30$ m

and $b = 10$ m

...[1 Mark]

Now as $\angle B = 90°$, $\angle OPB = 90°$, $\angle OQB = 90°$, $\angle POQ = 90°$.

$\therefore OPBQ$ is a square.

Hence, radius of the circle = b *i.e.*, 10 m.

...[1 Mark]

SECTION-E : Source-based / Case-based Units of Assessment (3×4 = 12 Marks)

A36.

(a) Solution:

$a_{80} = a + 79d = 4+79(5)$

$= 399$

...[1 Mark]

(b) Solution:

$a_1 + a_{80} = 4 + 399 = 403$

...[1 Mark]

(c) Solution:

Middle most terms are a_{40} and a_{41}

$a_{40} = a + 39d = 4 + 39(5) = 199$

...[1 Mark]

$a_{41} = a + 40 = 4+40(5) = 204$

...[1 Mark]

or

(c) Solution:

$S_{80} = \frac{80}{2}\left[2\left(4\right) + \left(80-1\right)5\right]$

...[1 Mark]

$= 16120$

...[1 Mark]

A37.

(a) Solution:

Class	Class marks (x_i)	Frequency(f_i)	Cumulative frequency (cf)	f_ix_i
3 – 6	4.5	2	2	9
6 – 9	7.5	5	7	37.5
9 – 12	10.5	10	17	105
12 – 15	13.5	23	40 = N	310.5
		$\sum f_i = 40$		$\sum f_i x_i = 462$

Modal class is the class having highest frequency, i.e. 12 – 15

...[1 Mark]

(b) Solution:

Cumulative frequency = (N) = 40

then, $\frac{N}{2} = 20$

Median class is 12 – 15

Lower limit of median class is 12

...[1 Mark]

(c) Solution:

Median $= l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$

$= 12 + \left(\frac{20 - 17}{23}\right) \times 3$ = 12.39

...[2 Marks]

or

(c) Solution:

Mean = $\frac{\sum f_i x_i}{\sum f_i}$

= $\frac{462}{40}$ = 11.55

...[2 Marks]

A38.

(a) Solution:

Let height of pole 'B' be h meters

...[1 Mark]

In ΔRSP

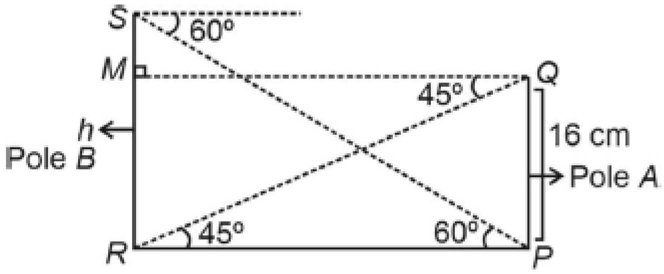
= $\tan 60^\circ = \frac{h}{16}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{16}$

$\Rightarrow \quad h = 16\sqrt{3}$ m

(b) **Solution:**

As shown in the figure,



In ΔPQR

$\tan 45° = \frac{PQ}{PR}$

$\Rightarrow \quad 1 = \frac{16}{PR}$

$\Rightarrow$ $PR = 16$

...[1 Mark]

(c) **Solution:**

PS is the distance between top of pole *B* and foot of pole *A*.

In ΔPRS

$\sin 60° = \frac{RS}{PS} = \frac{16\sqrt{3}}{PS}$

...[1 Mark]

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{16\sqrt{3}}{PS}$

$\Rightarrow$ $PS = 32$ m

...[1 Mark]

or

(c) **Solution:**

QR is the distance between top of pole *A* and foot of pole *B*

In ΔPRQ

$\sin 45° = \frac{16}{QR}$

...[1 Mark]

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{16}{QR}$

$\Rightarrow \quad QR = 16\sqrt{2}$ m

...[1 Mark]