CHAPTER-WISE PREVIOUS YEARS' QUESTIONS

MATHEMATICS

HINTS & SOLUTIONS

Class X (CBSE)

MATHEMATICS

1 : Real Numbers

1. Answer (b) [1]

$144 = 2^4 \times 3^2$

$198 = 2 \times 3^2 \times 11$

_____

$HCF = 2 \times 3^2$

$= 18$

Hence, option (b) is correct.

2. Answer (c) [1]

Prime factorisation of 225 is given below,

3	225
3	75
5	25
5	5
	1

$\therefore \quad 225 = 3^2 \times 5^2$

Option (c) is correct.

3. Answer (c) [1]

Total number of factors of a prime number is 2

Hence, option (c) is correct.

4. Answer (c) [1]

$12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 5 \times 3$

$\therefore \quad HCF = 3$

$L.C.M = 2 \times 2 \times 3 \times 5 \times 7$

$= 420$

Hence, option (c) is correct.

5. Answer (a) [1]

$92 = 2 \times 2 \times 23$

$152 = 2 \times 2 \times 2 \times 19$

$H.C.F (92, 152) = 2 \times 2 = 4$

6. Answer (c) [1]

Let numbers be $2x$ and $2x + 2$.

$2x = 2 \times x$

$2x + 2 = 2(x + 1)$

$H.C.F = 2$

7. Answer (a) [1]

$HCF \times LCM = $ Product of numbers

$= 50 \times 20$

$= 1000$

8. Answer (d) [1]

For 6^n, where n belongs to natural number, the given number never ends with zero. [1]

9. Answer (b)

$3750 = 2 \times 3 \times 5 \times 5 \times 5 \times 5$

$= 2^1 \times 3^1 \times 5^4$

10. Answer (b) [1]

$95 = 5 \times 19$ and $171 = 9 \times 19$

$\Rightarrow \quad HCF (95, 171) = 19$

11. Answer (c) [1]

$LCM (20, 25, 30) = 300$ minutes

$= 5$ hours

12. Answer (d) [1]

Greatest number =

$H.C.F.\left[(1251-1), (9377-2) \text{ and } (15628-3)\right]$

$= H.C.F.[1250, 9375, 15625]$

$= 625$

13. Answer (a) [1]

a^3 and b^3 will be co-prime, if a, b are co-prime.

14. Answer (d) [1]

Unit digit of 5^n and 6^n are 5 and 6 respectively. [$\because$ n is a natural number]

$\therefore \quad$ Unit's digit of $2(5^n + 6^n) = 2 \times (5 + 6)$

$= 2 \times 11$

$= 22$ (i.e. 2)

15. Answer (c)   [1]

2400 is not divisible by 500.

16. Answer (a)   [1]

HCF × LCM = 30 × 70

      = 2100

17. Answer (c)   [1]

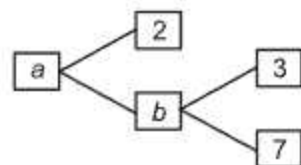   = $5 - 2\sqrt{5}$ (An irrational number)

18. Answer (a)   [1]

Least composite number is 4

and least prime number is 2

$\Rightarrow \dfrac{\text{HCF }(2, 4)}{\text{LCM }(2, 4)} = \dfrac{2}{4} = \dfrac{1}{2}$

19.

Let assume the missing entries be a, b.

$b = 3 \times 7 = 21$ [½]

$a = 2 \times b = 2 \times 21 = 42$ [½]

20. Given two numbers 100 and 190.

$\therefore$ HCF × LCM = 100 × 190   [½]

             = 19000   [½]

21. Smallest prime number is 2.

Smallest composite number is 4.

Therefore, HCF is 2.   [1]

22. Let us assume that $\left(5 + 3\sqrt{2}\right)$ is rational. Then there exist co-prime positive integers a and b such that

$5 + 3\sqrt{2} = \dfrac{a}{b}$ [½]

$3\sqrt{2} = \dfrac{a}{b} - 5$

$\sqrt{2} = \dfrac{a - 5b}{3b}$ [½]

$\Rightarrow \sqrt{2}$ is irrational.

$\left[\because a, b \text{ are integers}, \therefore \dfrac{a - 5b}{3b} \text{ is rational}\right].$

   [½]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.   [½]

Hence, $\left(5 + 3\sqrt{2}\right)$ is an irrational number.

23. Let the numbers be $2x$ and $3x$ [½]

Given,

LCM = 180

Clearly, HCF = x [½]

LCM (a, b) × HCF $(a, b) = a \times b$

$\Rightarrow$ $180 \times x = 2x \times 3x$ [½]

$\Rightarrow$ $x^2 - 30x = 0$

$\Rightarrow$ $x(x - 30) = 0$

       $x = 0$ or $x = 30$

$\because$ $x = 0$ is not possible as HCF can't be 0

      HCF = $x = 30$ [½]

24. Let assume $3 + \sqrt{2}$ is a rational number.

$\therefore$ $3 + \sqrt{2} = \dfrac{p}{q}$

$\{p, q$ are co-prime integers and $q \neq 0\}$ [1]

$\Rightarrow \sqrt{2} = \dfrac{p}{q} - 3$

$\Rightarrow \sqrt{2} = \dfrac{p - 3q}{q}$ [1]

Since, $\dfrac{p - 3q}{q}$ is a rational number but we know

$\sqrt{2}$ is an irrational.

$\therefore$ Irrational $\neq$ rational

$\therefore$ $3 + \sqrt{2}$ is not a rational number.   [1]

25. Let assume $2 - 3\sqrt{5}$ is a rational number.

$\Rightarrow 2 - 3\sqrt{5} = \dfrac{p}{q},$

(where p, q are co-prime integers and $q \neq 0$)

$\Rightarrow 2 - \dfrac{p}{q} = 3\sqrt{5}$ [1]

$\Rightarrow \dfrac{2q - p}{3q} = \sqrt{5}$

Since, $\dfrac{2q-p}{3q}$ is a rational number but we also

know $\sqrt{5}$ is an irrational   [1]

$\therefore$ Rational $\neq$ irrational.

$\Rightarrow$ Our assumption is wrong.

$\therefore$ $2-3\sqrt{5}$ is an irrational number.   [1]

26. Using the factor tree for the prime factorization of 404 and 96, we have

$404 = 2^2 \times 101$ and $96 = 2^5 \times 3$

To find the HCF, we list common prime factors and their smallest exponent in 404 and 96 as under :

Common prime factor = 2, Least exponent = 2

$\therefore$ HCF $= 2^2 = 4$ [1]

To find the LCM, we list all prime factors of 404 and 96 and their greatest exponent as follows :

Prime factors of 404 and 96	Greatest Exponent
2	5
3	1
101	1

$\therefore$ LCM $= 2^5 \times 3^1 \times 101^1$

$= 2^5 \times 3^1 \times 101^1$

$= 9696$ [1]

Now,

HCF $\times$ LCM $= 9696 \times 4 = 38784$

Product of two numbers $= 404 \times 96 = 38784$

Therefore, HCF $\times$ LCM = Product of two numbers.   [1]

27. Let $\sqrt{2}$ be rational. Then, there exist positive

integers a and b such that $\sqrt{2} = \dfrac{a}{b}$. [where a

and b are co-prime, $b \neq 0$].   [½]

$\Rightarrow (\sqrt{2})^2 = \left(\dfrac{a}{b}\right)^2$ [½]

$\Rightarrow 2 = \dfrac{a^2}{b^2}$

$\Rightarrow 2b^2 = a^2$

$\therefore$ 2 divides a^2

$\Rightarrow$ 2 divides a ...(i)

Let $a = 2c$ for some integer c.   [½]

$a^2 = 4c^2$

$\Rightarrow 2b^2 = 4c^2$

$\Rightarrow b^2 = 2c^2$

$\therefore$ 2 divides b^2

$\Rightarrow$ 2 divides b ...(ii)   [½]

From (i) and (ii), we get

2 is common factor of both a and b.

But this contradicts the fact that a and b have no common factor other than 1.   [½]

$\therefore$ Our supposition is wrong.

Hence, $\sqrt{2}$ is an irrational number.   [½]

28. Let $5+2\sqrt{3}$ be a rational number.

$5+2\sqrt{3} = \dfrac{p}{q}$, where p and q are co-prime

integers.   [½]

$\Rightarrow 2\sqrt{3} = \dfrac{p}{q} - 5$

$= \dfrac{p-5q}{q}$ [½]

$\Rightarrow \sqrt{3} = \dfrac{p-5q}{2q}$ [½]

Here, $\dfrac{p-5q}{2q}$ is rational as p and q are integers.

[½]

But it is given that $\sqrt{3}$ is irrational.

$\Rightarrow$ LHS is irrational and RHS is rational.   [½]

which contradicts our assumption that $5+2\sqrt{3}$ is a rational number.

$\therefore$ $5+2\sqrt{3}$ is an irrational number.   [½]

OR

For maximum number of columns, we need to find highest common factor i.e., HCF of 612 and 48.   [½]

Now,

$612 = 48 \times 12 + 36$ [½]

$48 = 36 \times 1 + 12$ [½]

$36 = 12 \times 3 + 0$ [½]

$\therefore$ HCF of 612 and 48 is 12.   [½]

$\therefore$ Maximum number of columns in which they can march is 12.   [½]

29. Let us assume $\sqrt{3}$ is rational number

So there exists co-prime integers p and q, $q \neq 0$

such that $\sqrt{3} = \dfrac{p}{q}$ **[½]**

Squaring both sides,

$3 = \dfrac{p^2}{q^2}$

$p^2 = 3q^2$ ...(i) **[½]**

$\Rightarrow$ 3 is a factor of p^2

$\Rightarrow$ 3 is a factor of p **[½]**

$\Rightarrow$ $p = 3m$, where m is an integer

$\Rightarrow$ $p^2 = 9m^2$ ...(ii) [Squaring both sides]

From equation (i) and (ii), we get

$\Rightarrow$ $3q^2 = 9m^2$

$\Rightarrow$ $q^2 = 3m^2$ **[½]**

$\Rightarrow$ 3 is a factors of q^2

$\Rightarrow$ 3 is a factor of q also **[½]**

So both p and q have 3 as their common factor, which contradicts the fact that p and q are co-prime

So are assumption is wrong **[½]**

Hence $\sqrt{3}$ is a irrational

30. Number of apples = 36

Number of bananas = 60

Number of mangoes = 42

$\therefore$ $36 = 2^2 \times 3^2$

$60 = 2^2 \times 3 \times 5$

$42 = 2 \times 3 \times 7$

(i) HCF (36, 60) $= 2^2 \times 3$

$= 12$

$\therefore$ Khushi can invite atmost 12 guests. **[1]**

(ii) Each guest will get 3 apples and 5 bananas. **[1]**

(iii) (a) HCF (36, 60, 42) $= 2 \times 3$ **[1]**

$= 6$ **[1]**

$\therefore$ Khushi can invite atmost 6 guests.

OR

(b) Cost of 1 dozen of bananas = ₹60

Cost of 1 apple = ₹15

Cost of 1 mango = ₹20 **[1]**

$\therefore$ Total amount spent on 60 bananas, 36 apples and 42 mangoes

$= 5 \times 60 + 15 \times 36 + 20 \times 42$ **[1]**

$= ₹1680$

2 : Polynomials

1. $(x + a)$ is factor of the polynomial $p(x) = 2x^2 + 2ax + 5x + 10$.

$\therefore$ $p(-a) = 0$ {By factor theorem}

$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$ **[½]**

$2a^2 - 2a^2 - 5a + 10 = 0$ **[½]**

$a = 2$

2. If $x = 1$ is the zero of the polynomial

$\therefore$ $p(x) = ax^2 - 3(a - 1)x - 1$

Then $p(1) = 0$ **[½]**

$\therefore$ $a(1)^2 - 3(a - 1) - 1 = 0$

$-2a + 2 = 0$

$a = 1$ **[½]**

3. Given α and β are the zeroes of quadratic polynomial with $\alpha + \beta = 6$ and $\alpha\beta = 4$.

Quadratic polynomial $= k[x^2 - 6x + 4]$, where k is real. **[1]**

4. Answer (d) **[1]**

2 is a zero of polynomial $p(x) = kx^2 + 3x + k$.

$\Rightarrow$ $p(2) = 0$

$\Rightarrow$ $k(2^2) + 3(2) + k = 0$

$\Rightarrow$ $4k + 6 + k = 0$

$\Rightarrow$ $5k = -6$

$\therefore$ $k = \dfrac{-6}{5}$

Option (d) is correct.

5.　Answer (a)　　　　　　　　　　　[1]

Graph of given polynomial cuts the x-axis at 3 distinct points.

∴ Number of zeroes is 3.

6.　Answer (b)　　　　　　　　　　　[1]

Let $f(x) = x^2 + 3x + k$

$f(2) = (2)^2 + 3(2) + k = 0$

$\Rightarrow 4 + 6 + k = 0$

$\Rightarrow k = -10$

Hence, option (b) is correct.

7.　Answer (a)　　　　　　　　　　　[1]

Quadratic polynomial

$= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$= x^2 - (-5)x + 6$

$= x^2 + 5x + 6$

Hence, option (a) is correct.

8.　Answer (b)　　　　　　　　　　　[1]

$K[x^2 - (\text{sum of zeroes}) \, x + (\text{product of zeroes}],$ where K is a non-zero constant.

∴　$p(x) = K[x^2 - 5x]$

9.　Answer (c)　　　　　　　　　　　[1]

$p(x) = x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2, x = 3$

10.　Answer (a)　　　　　　　　　　　[1]

Polynomial, $p(x) = x^2 + 99x + 127$

Sum of zeroes $= -\dfrac{b}{a} = -99 = $ negative

Product of zeroes $= \dfrac{c}{a} = 127 = $ positive

So, both zeroes must be negative.

11.　Answer (c)　　　　　　　　　　　[1]

As, we can see from the graph maximum height is achieved at $t = 1$ s.

Height attained at $t = 1$ s

$h = -(1)^2 + 2(1) + 8 = 9$ m

12.　Answer (b)　　　　　　　　　　　[1]

Quadratic polynomial

13.　Answer (c)　　　　　　　　　　　[1]

As, we can see from the graph ball reach maximum height at $t = 1$ s.

14.　Answer (b)　　　　　　　　　　　[1]

Since, it is a quadratic polynomial so, it will have 2 zeroes.

15.　Answer (b)　　　　　　　　　　　[1]

Zeroes of the polynomial, $h = -t^2 + 2t + 8 = 0$

$-(t^2 - 2t - 8) = 0$

$\Rightarrow (t - 4)(t + 2) = 0$

$t = 4$ and $t = -2$

16.　Answer (d)　　　　　　　　　　　[1]

Zeroes of a polynomial $f(x)$ would be those points where the graph $f(x)$ will touch or cut the x-axis.

∴　Number of zeroes = 5

17.　Answer (c)　　　　　　　　　　　[1]

Graph intersects x-axis at 3 points.

18.　Answer (a)　　　　　　　　　　　[1]

Required polynomial $= k[x^2 - 8x + 5]$

19.　Answer (b)　　　　　　　　　　　[1]

$p(1) = 1 + a + 2b = 0$

$\Rightarrow a + 2b = -1$

and $a + b = 4$

$\Rightarrow b = -5$ and $a = 9$

20.　Answer (b)　　　　　　　　　　　[1]

$\alpha + \beta = k + 6$ and $\alpha\beta = 4k - 2$

$\alpha + \beta = \dfrac{\alpha\beta}{2}$

$\Rightarrow k + 6 = 2k - 1$

∴　$k = 7$

21.　Answer (b)　　　　　　　　　　　[1]

$p(x) = x^2 + 5x + 6$

$p(-2) = (-2)^2 + 5(-2) + 6$

$\qquad = 0$

22.　Answer (b)　　　　　　　　　　　[1]

$x^2 - 2x - 1$ is the required polynomial.

23.　Answer (d)　　　　　　　　　　　[1]

$\alpha + \beta = 0$　　　$\left[\because \alpha + \beta = \dfrac{-b}{a} \right]$

24. Answer (d)   [1]

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$$

$$\because \quad \alpha+\beta=\frac{3}{4}, \quad \alpha\beta=\frac{-7}{4}$$

$$\Rightarrow \quad \frac{\alpha+\beta}{\alpha\beta}=\frac{3\times 4}{4(-7)}=\frac{-3}{7}$$

25. $p(x)=6x^2+37x-(k-2)$

Let α, β be the zeroes of $p(x)$

$$\because \quad \beta=\frac{1}{\alpha} \quad \text{[Given condition]}$$

$$\alpha\beta=1 \qquad \text{...(i)} \qquad [\tfrac{1}{2}]$$

Also, $\alpha\beta=\dfrac{-(k-2)}{6}$ ...(ii) $[\tfrac{1}{2}]$

From (i) and (ii),

$$\frac{-(k-2)}{6}=1 \qquad\qquad [\tfrac{1}{2}]$$

$$\Rightarrow \quad 2-k=6$$

$$\Rightarrow \quad k=-4 \qquad\qquad [\tfrac{1}{2}]$$

26. For given polynomial

$x^2-(k+6)x+2(2k-1),$ $[\tfrac{1}{2}]$

Let the zeroes be α and β.

So, $\alpha+\beta=-\dfrac{b}{a}=k+6, \quad \alpha\beta=\dfrac{c}{a}=\dfrac{4k-2}{1}$ [1]

$$\because \quad \text{Sum of zeroes} = \frac{1}{2}\ (\text{product of zeroes})$$

$$\Rightarrow \quad \alpha+\beta=\frac{1}{2}\alpha\beta \qquad\qquad [\tfrac{1}{2}]$$

$$\Rightarrow \quad k+6=\frac{1}{2}(4k-2)$$

$$\Rightarrow \quad k+6=2k-1$$

$$\therefore \quad k=7$$

So, the value of k is 7.   [1]

27. α and β are zeroes of the polynomial $f(x)=x^2-4x-5$

$$\Rightarrow \quad \alpha+\beta=-\frac{b}{a}=4 \text{ and } \alpha\beta=\frac{c}{a}=-5, \text{ where } a =$$

$1, b=-4, c=-5$ [1]

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $[\tfrac{1}{2}]$

$$= (4)^2-2(-5) \qquad\qquad [\tfrac{1}{2}]$$

$$= 16+10 \qquad\qquad [\tfrac{1}{2}]$$

$$= 26 \qquad\qquad [\tfrac{1}{2}]$$

28. Let α and β are the zeroes of the polynomial $f(x) = ax^2 + bx + c$.

$$\therefore \quad (\alpha+\beta)=\frac{-b}{a} \qquad \text{...(i)} \qquad [\tfrac{1}{2}]$$

and $\alpha\beta=\dfrac{c}{a}$ ...(ii) $[\tfrac{1}{2}]$

According to the question, $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ are the zeroes of the required quadratic polynomial

$\therefore$ Sum of zeroes of required polynomial

$$S'=\frac{1}{\alpha}+\frac{1}{\beta}$$

$$=\frac{\alpha+\beta}{\alpha\beta}$$

$$=\frac{-b}{c} \qquad\qquad \text{...(iii)} \qquad [\tfrac{1}{2}]$$

[From equation (i) and (ii)]

and product of zeroes of required polynomial

$$=\frac{1}{\alpha}\times\frac{1}{\beta}.$$

$$P'=\frac{1}{\alpha\beta}$$

$$=\frac{a}{c} \qquad\qquad \text{...(iv)} \qquad [\tfrac{1}{2}]$$

[From equation (ii)]

$\therefore$ Equation of the required quadratic polynomial

$= k\left(x^2-S'x+P'\right)$, where k is any non-zero constant $[\tfrac{1}{2}]$

$$= k\left(x^2-\left(\frac{-b}{c}\right)x+\frac{a}{c}\right)$$

[From equation (iii) and (iv)]

$$= k\left(x^2+\frac{b}{c}x+\frac{a}{c}\right) \qquad\qquad [\tfrac{1}{2}]$$

29. α, β are zeroes of the polynomial x^2-5x+6.

$\alpha+\beta=5$ and $\alpha\beta=6$

Let S and P be the sum and product of zeroes $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$

$$S=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{5}{6}$$

$$P=\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{1}{6}$$

Required quadratic polynomial is

$k\left(x^2-\dfrac{5}{6}x+\dfrac{1}{6}\right)$, where k is any non-zero constant. [3]

3 : Pair of Linear Equations in Two Variables

1. $x + 2y - 8 = 0$

    $2x + 4y - 16 = 0$

    For any pair of linear equations

    $a_1x + b_2y + c_1 = 0$

    $a_2x + b_2y + c_2 = 0$

    If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$, then   [½]

    There exists infinite solutions

    Here $\dfrac{a_1}{a_2} = \dfrac{1}{2}, \dfrac{b_1}{b_2} = \dfrac{2}{4}, \dfrac{c_1}{c_2} = \dfrac{-8}{-16}$

    $\therefore \quad \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \dfrac{1}{2}$

    $\therefore$ Lines are coincident and will have infinite solutions.   [½]

2. For any real number except $k = -6$ [1]

    $kx - 2y = 3$ and $3x + y = 5$ represent lines intersecting at a unique point.

    $\Rightarrow \quad \dfrac{k}{3} \neq \dfrac{-2}{1}$

    $\Rightarrow k \neq -6$

    For any real number except $k = -6$

    The given equation represent two intersecting lines at unique point.

3. Answer (d)   [1]

    For no solution; $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

    $\therefore \quad \dfrac{1}{2} = \dfrac{1}{k} \neq \dfrac{-4}{-3}$

    $\Rightarrow \boxed{k = 2}$

    Hence, option (d) is correct.

4. Answer (b)   [1]

    For no solution,

    $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

    $\Rightarrow \quad \dfrac{1}{2} = \dfrac{1}{k} \neq \dfrac{-4}{-3}$

    $\Rightarrow k = 2$

5. Answer (d)   [1]

    Perimeter, $2(l + b) = 14$ ...(i)

    $l = 2b + 4$ ...(ii)

6. Answer (a)   [1]

    $(-5, 6)$ is the solution of $x = -5$ and $y = 6$.

7. Answer (b)   [1]

    For, infinitely many solutions

    $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

    $\Rightarrow \quad \dfrac{3}{k} = \dfrac{5}{15} = \dfrac{8}{24}$

    $k = 9$

8. Answer (a)   [1]

    $32x + 33y = 34$ ...(i)

    $33x + 32y = 31$ ...(ii)

    Adding equation (i) and (ii) and subtracting equation (ii) from (i), we get

    $65x + 65y = 65$ or $x + y = 1$ ...(iii)

    and $-x + y = 3$ ...(iv)

    Adding equation (iii) and (iv), we get

    $y = 2$

    Substituting the value of y in equation (iii),

    $x = -1$

9. Answer (c)   [1]

    If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

    $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

    It can only possible between $3x - 2y = 5$ and $-12x + 8y = 7$.

Solution for 10 to 14 :

    For Amruta, $x + (6 - 2)y = 22$

    i.e., $x + 4y = 22$ ...(i)

    For Radhika, $x + (4 - 2)y = 16$

    i.e., $x + 2y = 16$ ...(ii)

    Solving equation (i) and (ii), we get

 $x = 10$ and $y = 3$

    i.e., Fixed charges $(x) = ₹10$ ...(iii)

    and additional charges per subsequent day $(y) = ₹3$ ...(iv)

10. Answer (d)   [1]

$x + 2y = 16$ [From equation (ii)]

11. Answer (c)   [1]

$x + 4y = 22$ [From equation (i)]

12. Answer (b)   [1]

$x = ₹10$ [From equation (iii)]

13. Answer (d)   [1]

$y = ₹3$ [From equation (iv)]

14. Answer (c)   [1]

Total amount paid for 2 more days by both

$= (x + 4y) + 2y + (x + 2y) + 2y$

$= 2x + 10y$

$= 2 \times 10 + 10 \times 3$

$= ₹50$

15. Answer (c)   [1]

$x + ky = 5$

At $x = 2$, $y = 1$

$2 + k.1 = 5$

$\therefore \quad k = 3$

16. Answer (d)   [1]

$\dfrac{1}{-3} = \dfrac{2}{-6} \neq \dfrac{5}{1}$

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

17. Answer (c)   [1]

$2x - 5y - 6 = 0$

$-6x + 15y + 18 = 0$

$\dfrac{1}{3} = \dfrac{-1}{3} = \dfrac{-1}{3}$

18. $2x + 3y = 7$

$(k - 1)x + (k + 2)y = 3k$

For this pair of linear equations to have infinitely many solutions, they need to be coincident [½]

$\Rightarrow \dfrac{2}{k-1} = \dfrac{3}{k+2} = \dfrac{7}{3k}$ [½]

Upon solving we get

$\boxed{k = 7}$ [1]

19. Since it is a rectangle

$\ell(AB) = \ell(CD)$

$x + y = 30$ ...(i)   [½]

$\ell(AD) = \ell(BC)$

$x - y = 14$ ...(ii)   [½]

Adding (i) and (ii), we get

$2x = 44$

$x = 22$ [½]

Putting $x = 22$ in equation (ii)

$22 - y = 14 \Rightarrow 22 - 14 = y$

$\therefore \quad y = 8$

$\therefore \quad x = 22$ and $y = 8$ [½]

20. For infinitely many solutions

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ [½]

$$\underset{\text{I}}{\dfrac{c}{12}} = \underset{\text{II}}{\dfrac{3}{c}} = \underset{\text{III}}{\dfrac{3-c}{-c}}$$

(i) $c^2 = 12 \times 3$ [From I and II]

$c = \pm 6$ [½]

(ii) $\dfrac{3}{c} = \dfrac{3-c}{-c}$ [From II and III]

$-3c = 3c - c^2$

$c^2 - 6c = 0$

$c = 0, 6$

(iii) $c^2 = 12(c - 3)$ [From I and III]   [½]

$c^2 - 12c + 36 = 0$

$(c - 6)^2 = 0$

$c = 6$

Hence the value of c is 6.   [½]

21. $x + 3y = 6$

$2x - 3y = 12$

Graph of $x + 3y = 6$:

When $x = 0$, we have $y = 2$ and when $y = 0$, we have $x = 6$.   [½]

Therefore, two points on the line are $(0, 2)$ and $(6, 0)$.   [½]

The line $x + 3y = 6$ is represented in the given graph.

Graph of $2x - 3y = 12$:

When $x = 0$, we have $y = -4$ and when $y = 0$, we have $x = 6$.   [½]

Hence, the two points on the line are $(0, -4)$ and $(6, 0)$.   [½]

The line $2x - 3y = 12$ is shown in the graph.
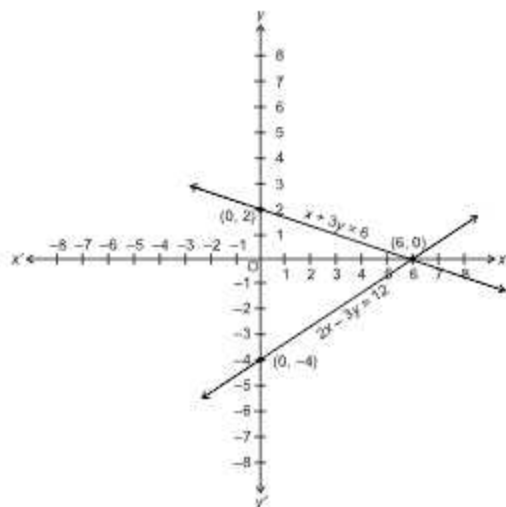
The line $x + 3y = 6$ intersects y-axis at $(0, 2)$ and the line $2x - 3y = 12$ intersects y-axis at $(0, -4)$. **[½]**

22. $\dfrac{ax}{b} - \dfrac{by}{a} = a + b$ ...(i)

     $ax - by = 2ab$ ...(ii)   **[½]**

Multiply (ii) with $\dfrac{1}{b}$ and subtract (i) from (ii)

$\dfrac{a}{b}x - y = 2a$

$-\dfrac{ax}{b} - \dfrac{by}{a} = -a + b$ **[1]**

_____

$y\left(\dfrac{b-a}{a}\right) = a - b$ **[½]**

$y = -a$

Substituting $y = -a$ in (i)

$\dfrac{a}{b}x - \dfrac{b}{a}(-a) = a + b$ **[½]**

$\dfrac{a}{b}x = a$

$x = b$

$\therefore$ $x = b$ and $y = -a$ **[½]**

23. Lets say numerator = x

Denominator = y

Given $x + y = 2y - 3$

$\Rightarrow$ $\boxed{x - y + 3 = 0}$ ...(i)   **[1]**

From the next condition

$\dfrac{x-1}{y-1} = \dfrac{1}{2}$

$\boxed{2x - y - 1 = 0}$ ...(ii)   **[1]**

Solving (i) and (ii)

$x = 4$

$y = 7$

$\therefore$ Fraction $= \dfrac{4}{7}$ **[1]**

24. $\dfrac{4}{x} + 3y = 8$ ...(i)   **[½]**

     $\dfrac{6}{x} - 4y = -5$ ...(ii)   **[½]**

Multiplying 4 to (i) and 3 to (ii)

$\dfrac{16}{x} + 12y = 32$

$\dfrac{18}{x} - 12y = -15$ **[½]**

$\dfrac{34}{x} = 17$

$\boxed{x = 2}$ **[½]**

Substitute

$x = 2$ in (i)

$2 + 3y = 8$

$3y = 6$

$y = 2$ **[½]**

$\therefore$ $x = 2$

     $y = 2$ **[½]**

25. Let the present age of father be x years and the sum of present ages of his two children be y years.   **[½]**

According to question

$x = 3y$ **[½]**

$\Rightarrow$ $x - 3y = 0$ ...(i)

After 5 years,

     $x + 5 = 2(y + 10)$

$\Rightarrow$ $x - 2y = 15$ ...(ii)   **[½]**

On subtracting equation (i) from (ii), we get :

$\begin{array}{ccccc} x & - & 2y & = & 15 \\ x & - & 3y & = & 0 \\ - & & + & & - \\ \hline & & y & = & 15 \end{array}$ **[1]**

On substituting the value of $y = 15$ in (i), we get :

$x - 3 \times 15 = 0$

$\therefore$ $x = 45$ **[½]**

Hence, the present age of father is 45 years.

26. Let the numerator of required fraction be x and the denominator of required fraction be y $(y \neq 0)$

According to question;   [½]

$$\frac{x-2}{y} = \frac{1}{3}$$

$\Rightarrow 3x - 6 = y$ [½]

$\Rightarrow 3x - y = 6$ ...(i)

and

$$\frac{x}{y-1} = \frac{1}{2}$$

$\Rightarrow 2x = y - 1$ [½]

$\Rightarrow 2x - y = -1$ ...(ii)

On subtracting (ii) from (i), we get :

$$
\begin{array}{rcrcr}
3x & - & y & = & 6 \\
2x & - & y & = & -1 \\
- & + & & & + \\
\hline
 & & x & = & 7
\end{array}
$$
    [1]

On substituting $x = 7$ in (i), we get :

$3(7) - y = 6$

$\Rightarrow -y = 6 - 21$

$\therefore y = 15$ [½]

Hence, the required fraction is $\dfrac{x}{y} = \dfrac{7}{15}$.

27. Given lines are $2x + 3y = 2$ and $x - 2y = 8$

$2x + 3y = 2$

$\Rightarrow y = \dfrac{2-2x}{3}$

x	1	-2	4
y	0	2	-2

    [½]

and $x - 2y = 8$

$\Rightarrow y = \dfrac{x-8}{2}$

x	0	8	2
y	-4	0	-3

    [½]

$\therefore$ We will plot the points $(1, 0)$, $(-2, 2)$ and $(4, -2)$ and join them to get the graph of $2x + 3y = 2$ and we will plot the points $(0, -4)$, $(8, 0)$ and $(2, -3)$ and join them to get the graph of $x - 2y = 8$
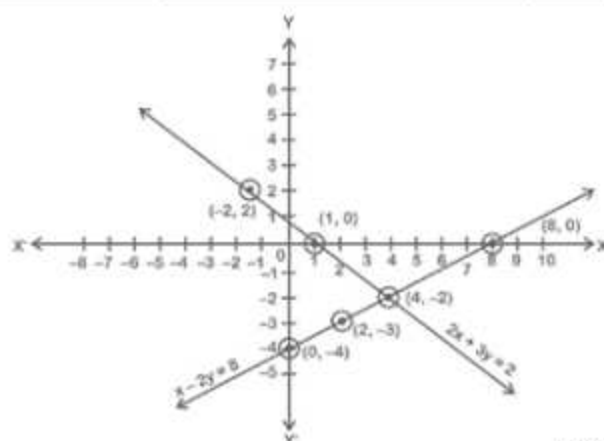
    [1½]

The graph of two given equations intersect at $(4, -2)$

$\therefore$ Solution of $2x + 3y = 2$ and $x - 2y = 8$ is $x = 4$ and $y = -2$ [½]

28. $2y - x = 8$ [½]

x	0	-8
y	4	0

    $5y - x = 14$

x	-4	6
y	2	4

    [½]

    $y - 2x = 1$

x	0	1
y	1	3

    [½]
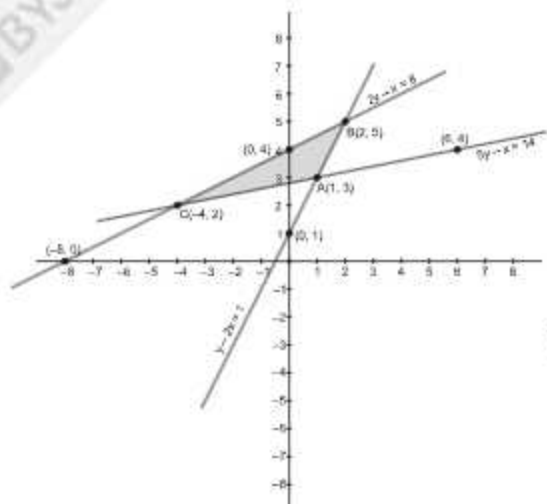
    [1½]

29. (A) Let required fraction be $\dfrac{x}{y}$

According to question,

$$\frac{x+1}{y-1} = 1$$
    [1]

$\Rightarrow x + 1 = y - 1$

$\Rightarrow x = y - 2$ ...(i)

Also, $\dfrac{x}{y+1} = \dfrac{1}{2}$

$\Rightarrow 2x = y + 1$　　　...(ii)　　[1]

From equations (i) and (ii), we get

$2y - 4 = y + 1$

$y = 5$

$\therefore x = 3$

Required fraction $\dfrac{x}{y}$ is $\dfrac{3}{5}$　　　[1]

OR

(B)　$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

For no solution; $\dfrac{3}{2k-1} = \dfrac{1}{k-1} \neq \dfrac{1}{2k+1}$　[½]

$\Rightarrow \dfrac{3}{2k-1} = \dfrac{1}{k-1}$　　　[½]

$2k - 1 = 3k - 3$

$\Rightarrow k = 2$　　　[½]

Also, $\dfrac{1}{k-1} \neq \dfrac{1}{2k+1}$　　　[½]

$2k + 1 \neq k - 1$　　　[½]

$\Rightarrow k \neq -2$　　　[½]

30.　(i)　$5x + 4y = 9500$　　...(i)

$4x + 3y = 7370$　　...(ii)　　[1]

(ii)　(a)　Multiplying (i) by 3 and (ii) by 4; we get

$15x + 12y = 28{,}500$　...(iii)

$16x + 12y = 29{,}480$　...(iv)　　[1]

Subtracting (iii) from (iv)

$16x + 12y = 29{,}480$

$15x + 12y = 28{,}500$

(−)　　(−)　　(−)　　　[1]

$x = 980$

Prize amount for Hockey = ₹980 per student

OR

(b)　Multiplying (i) by 3 and (ii) by 4; we get

$15x + 12y = 28{,}500$　...(iii)

$16x + 12y = 29{,}480$　...(iv)　　[½]

Subtracting (iii) from (iv)

$16x + 12y = 29{,}480$

$15x + 12y = 28{,}500$

(−)　　(−)　　(−)　　　[½]

$x = ₹980$

Putting $x = 980$ in (i) :

$5(980) + 4y = 9500$

$\Rightarrow y = ₹1150$　　　[½]

$\because$　Prize amount of Hockey = ₹980 per student Prize amount of Cricket = ₹1150 per student

$\Rightarrow$　Prize amount per student of Cricket is greater by ₹170　　　[½]

(iii)　Total prize amount if there are 2 students each from 2 games　= $2(x + y)$

$= 2(980 + 1150)$　[½]

$= ₹4{,}260$　[½]

31.　Let AB be the pillar of height 9 meter. The peacock is sitting at point A on the pillar and B is the foot of the pillar. ($AB = 9$)

Let C be the position of the snake which is at 27 meters from B. ($BC = 27$ and $\angle ABC = 90°$)

As the speed of the snake and of the peacock is same they will travel the same distance in the same time

Now take a point D on BC that is equidistant from A and C (Please note that snake is moving towards the pillar)　　　[½]

[½]
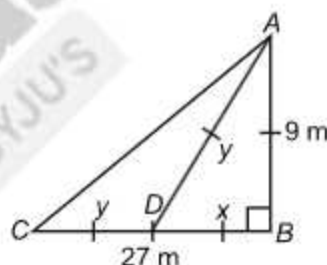
Hence by condition $AD = DC = y$(say)

Take $BD = x$

Now consider triangle ABD which is a right angled triangle

Using Pythagoras theorem ($AB^2 + BD^2 = AD^2$)

$9^2 + x^2 = y^2$　　　[½]

$81 = y^2 - x^2 = (y - x)(y + x)$　　　[½]

$81/(y + x) = (y - x)$　　　[½]

$y + x = BC = 27$

Hence, $81/27 = (y - x) = 3$　　　[½]

$y - x = 3$　　　...(i)

$y + x = 27$　　　...(ii)　　[½]

Adding (i) and (ii), gives $2y = 30$ or $y = 15$　[1]

$x = 12, y = 5$　　　[1]

Thus the snake is caught at a distance of x meters or 12 meters from the hole.　[½]

4 : Quadratic Equations

1. Answer (b)

    Given a quadratic equation

    $x^2 - 3x - m(m + 3) = 0$

    $\Rightarrow x^2 - (m + 3)x + mx - m(m + 3) = 0$ [½]

         $x(x - (m + 3)) + m(x - (m + 3)) = 0$

         $(x - (m + 3))(x + m) = 0$

    $\therefore \quad x = -m, \, m + 3$ [½]

2. Answer (a)

    It is given that 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$.

    Therefore, $y = 1$ will satisfy both the equations.

    $\therefore \quad a(1)^2 + a(1) + 3 = 0$

    $\Rightarrow a + a + 3 = 0$

    $\Rightarrow 2a + 3 = 0$ [½]

    $\Rightarrow a = \dfrac{-3}{2}$

    Also, $(1)^2 + (1) + b = 0$

    $\Rightarrow 1 + 1 + b = 0$

    $\Rightarrow b = -2$

    $\therefore \quad ab = \dfrac{-3}{2} \times -2 = 3$ [½]

3. Answer (d)   [1]

    Let the roots be 2 and β

    $2 + \beta = 0$

    $\Rightarrow \beta = -2$

    $\Rightarrow$ Product of roots = $(2)(-2) = -4$

    $\Rightarrow$ Quadratic equation = $x^2 - 4 = 0$

4. Answer (c)   [1]

    $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$

    $\Rightarrow 2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$

    $\Rightarrow 2\sqrt{6}x + 5x + 3 = 0$

    $\therefore$ Not a quadratic equation

5. Answer (a)   [1]

    $x^2 + 3x - 10 = 0$

    $(x + 5)(x - 2) = 0$

    $\Rightarrow \quad x = -5, \, 2$

6. $x^2 + 6x + 9 = 0$

    $x^2 + 2.3x + (3)^2 = 0$ [½]

    $(x + 3)^2 = 0$

    $\Rightarrow \quad x = -3$ is the solution of $x^2 + 6x + 9 = 0$.   [½]

7. $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.

    Discriminant for $ax^2 + bx + c = 0$ will be $b^2 - 4ac$.   [½]

    $\therefore$ For the given quadratic equation

         $= (10)^2 - 4\left(3\sqrt{3}\right)\left(\sqrt{3}\right)$

         $= 100 - 36$

         $= 64$ [½]

8. Given quadratic equation is,

    $px^2 - 2\sqrt{5}px + 15 = 0$

    Here, $a = p, \, b = -2\sqrt{5}p, \, c = 15$

    For real equal roots, discriminant = 0

    $\therefore \quad b^2 - 4ac = 0$ [½]

    $\therefore \quad \left(-2\sqrt{5}p\right)^2 - 4p(15) = 0$

    $\therefore \quad 20p^2 - 60p = 0$

    $\therefore \quad 20p(p - 3) = 0$

    $\therefore \quad p = 3$ or $p = 0$

    But, $p = 0$ is not possible.

    $\therefore \quad p = 3$ [½]

9. $\because \quad x = 3$ is one of the root of $x^2 - 2kx - 6 = 0$

    $(3)^2 - 2k(3) - 6 = 0$

    $9 - 6k - 6 = 0$

    $3 - 6k = 0$ [½]

    $3 = 6k$

    $k = \dfrac{3}{6} = \dfrac{1}{2}$ [½]

10. $x^2 + 4x + k = 0$

    $\because$ Roots of given equation are real,

    $D \geq 0$ [½]

    $\Rightarrow (4)^2 - 4 \times k \geq 0$

    $\Rightarrow -4k \geq -16$

    $\Rightarrow k \leq 4$

    $\therefore \quad k$ has all real values ≤ 4 [½]

11. $3x^2 - 10x + k = 0$

∵ Roots of given equation are reciprocal of each other.

Let the roots be α and $\dfrac{1}{\alpha}$.   **[½]**

Product of roots $= \dfrac{c}{a}$

$\Rightarrow \alpha \cdot \dfrac{1}{\alpha} = \dfrac{k}{3}$

$\therefore k = 3$ **[½]**

12. Quadratic equation $3x^2 - 4x + k = 0$ has equal roots

$\Rightarrow D = b^2 - 4ac = 0$, where $a = 3$, $b = -4$ and $c = k$

$\Rightarrow (-4)^2 - 4 \times 3 \times k = 0$

$\Rightarrow 16 - 12k = 0$

$\Rightarrow k = \dfrac{16}{12} = \dfrac{4}{3}$ **[1]**

13. Answer (c)   **[1]**

(A) is true but (R) is false.

14. Given; $mx(x - 7) + 49 = 0$

$\Rightarrow mx^2 - 7mx + 49 = 0$

$D = (7m)^2 - 4m \times 49$ **[1]**

$49m^2 - 4m \times 49 = 0$

$49 m^2 = 4m \times 49$

$m = 4$ $[\because m \neq 0]$ **[1]**

15. Given quadratic equation is $3x^2 - 2kx + 12 = 0$

Here $a = 3$, $b = -2k$ and $c = 12$.

The quadratic equation will have equal roots if $\Delta = 0$

$\therefore b^2 - 4ac = 0$

Putting the values of a, b and c we get

$(2k)^2 - 4(3)(12) = 0$ **[1]**

$\Rightarrow 4k^2 - 144 = 0$

$\Rightarrow 4k^2 = 144$

$\Rightarrow k^2 = \dfrac{144}{4} = 36$

Considering square root on both sides,

$k = \sqrt{36} = \pm 6$

Therefore, the required values of k are 6 and –6. **[1]**

16. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$ **[1]**

$\Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$

$\therefore x = \dfrac{\sqrt{3}}{4}$ or $x = -\dfrac{2}{\sqrt{3}}$ **[1]**

17. Comparing the given equation with the standard quadratic equation $(ax^2 + bx + c = 0)$, we get $a = 2$, $b = a$ and $c = -a^2$

Using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get :

$x = \dfrac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a)^2}}{2 \times 2}$ **[1]**

$= \dfrac{-a \pm \sqrt{9a^2}}{4}$

$= \dfrac{-a \pm 3a}{4}$

$\Rightarrow x = \dfrac{-a + 3a}{4} = \dfrac{a}{2}$ or $\dfrac{-a - 3a}{4} = -a$

So, the solutions of the given quadratic equation are $x = \dfrac{a}{2}$ or $x = -a$.   **[1]**

18. $4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow x^2 + bx - \left(\dfrac{a^2 - b^2}{4}\right) = 0$

$\Rightarrow x^2 + 2\left(\dfrac{b}{2}\right)x = \dfrac{a^2 - b^2}{4}$

$\Rightarrow x^2 + 2\left(\dfrac{b}{2}\right)x + \left(\dfrac{b}{2}\right)^2 = \dfrac{a^2 - b^2}{4} + \left(\dfrac{b}{2}\right)^2$ **[1]**

$\Rightarrow \left(x + \dfrac{b}{2}\right)^2 = \dfrac{a^2}{4}$

$\Rightarrow x + \dfrac{b}{2} = \pm\dfrac{a}{2}$

$\Rightarrow x = \dfrac{-b}{2} \pm \dfrac{a}{2}$

$\Rightarrow x = \dfrac{-b - a}{2}, \dfrac{-b + a}{2}$

Hence, the roots are $-\left(\dfrac{a + b}{2}\right)$ and $\left(\dfrac{a - b}{2}\right)$. **[1]**

19. Given -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$.

$\therefore$ -5 satisfies the given equation.

$\therefore$ $2(-5)^2 + p(-5) - 15 = 0$

$\therefore$ $50 - 5p - 15 = 0$

$\therefore$ $35 - 5p = 0$

$\therefore$ $5p = 35$

$\Rightarrow$ $p = 7$ [1]

Substituting $p = 7$ in $p(x^2 + x) + k = 0$, we get

$7(x^2 + x) + k = 0$

$\therefore$ $7x^2 + 7x + k = 0$

The roots of the equation are equal.

$\therefore$ Discriminant $= b^2 - 4ac = 0$

Here, $a = 7$, $b = 7$, $c = k$

$b^2 - 4ac = 0$

$\therefore$ $(7)^2 - 4(7)(k) = 0$

$\therefore$ $49 - 28k = 0$

$\therefore$ $28k = 49$

$\therefore$ $k = \dfrac{49}{28} = \dfrac{7}{4}$ [1]

20. Quadratic equation $px^2 - 14x + 8 = 0$

Also, one root is 6 times the other

Let say one root $= x$

Second root $= 6x$

From the equation : Sum of the roots $= +\dfrac{14}{p}$

Product of roots $= \dfrac{8}{p}$

$\therefore$ $x + 6x = \dfrac{14}{p}$.

$x = \dfrac{2}{p}$ [1]

$\Rightarrow$ $6x^2 = \dfrac{8}{p}$

$\Rightarrow$ $6\left(\dfrac{2}{p}\right)^2 = \dfrac{8}{p}$

$\dfrac{6 \times 4}{p^2} = \dfrac{8}{p}$

$p = 3$ [1]

21. $4x^2 - 5x - 1 = 0$

$D = b^2 - 4ac$, where $a = 4$, $b = -5$ and $c = -1$ [½]

$\Rightarrow$ $D = 25 + 16 = 41$ [½]

$\Rightarrow$ $D > 0$ [½]

$\therefore$ The given equation has real and distinct roots [½]

22. $x^2 + 2\sqrt{2}x - 6 = 0$

$\therefore$ $x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$ [1]

$x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0$

$(x + 3\sqrt{2})(x - \sqrt{2}) = 0$

$\Rightarrow$ $x = -3\sqrt{2}, \sqrt{2}$ [1]

23. (A) Discriminant (D) of a quadratic equation

$ax^2 + bx + c = 0$ is $b^2 - 4ac$

Discriminant for $3x^2 - 2x + \dfrac{1}{3} = 0$ is

$D = (-2)^2 - 4(3)\left(\dfrac{1}{3}\right) = 0$ [1]

Hence, roots are real and equal. [1]

OR

(B) $x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x - 2) + 1(x - 2) = 0$ [1]

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x - 2 = 0$ and $x + 1 = 0$

$\Rightarrow x = 2$ and $x = -1$ [1]

Hence, roots are 2 and -1

24. (A) $\because$ For $ax^2 + bx + c = 0$,

Sum of roots $= \dfrac{-b}{a}$ [½]

Product of roots $= \dfrac{c}{a}$ [½]

$\Rightarrow$ For $2x^2 - 9x + 4 = 0$

$\therefore$ Sum of roots $= \dfrac{-(-9)}{2} = \dfrac{9}{2}$ [½]

Product of roots $= \dfrac{4}{2} = 2$ [½]

OR

(B) For $ax^2 + bx + c = 0$, $D = b^2 - 4ac$

∵ For $4x^2 - 5 = 0$

∴ $D = (0)^2 - 4(4)(-5)$ [½]

$= 80$ [½]

∵ $D > 0$ [½]

Roots are real and distinct. [½]

25. Let assume two numbers be x, y.

Given, $x + y = 8 \Rightarrow x = 8 - y$ ...(i)

$\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{8}{15}$ [1]

$\dfrac{x+y}{xy} = \dfrac{8}{15} \Rightarrow \dfrac{8}{xy} = \dfrac{8}{15}$

$\Rightarrow xy = 15$ [1]

From (i) $xy = y(8 - y) = 15$

∴ $y^2 - 8y + 15 = 0$

$y = 3, 5 \Rightarrow x = 5, 3$

∴ The numbers are 3 and 5. [1]

26. $x^2 - 3\sqrt{5}x + 10 = 0$

For any quadratic equation

$ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ [1]

∴ For the given equation

$x = \dfrac{3\sqrt{5} \pm \sqrt{45 - 40}}{2}$ [1]

$x = \dfrac{3\sqrt{5} \pm \sqrt{5}}{2}$

$\Rightarrow \boxed{x = \sqrt{5}, 2\sqrt{5}}$ [1]

27. $4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$ [1]

$\Rightarrow [(2x^2) - 2.2x.a + a^2] - b^2 = 0$

$\Rightarrow [(2x - a)^2] - b^2 = 0$ [1]

$\Rightarrow [(2x - a) - b][(2x - a) + b] = 0$

$\Rightarrow [(2x - a) - b] = 0$ or $[(2x - a) + b] = 0$

$\Rightarrow x = \dfrac{a+b}{2}; x = \dfrac{a-b}{2}$ [1]

28. $3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3} \times [\sqrt{3}x - \sqrt{2}] - \sqrt{2}[\sqrt{3}x - \sqrt{2}] = 0$ [1]

$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})^2 = 0$

∴ $\sqrt{3}x - \sqrt{2} = 0$ [1]

$\Rightarrow \sqrt{3}x = \sqrt{2}$

$\Rightarrow x = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{2} \times \sqrt{3}}{(\sqrt{3})^2} = \dfrac{\sqrt{6}}{3}$ [1]

29. $(k + 4)x^2 + (k + 1)x + 1 = 0$

$a = k + 4$, $b + k + 1$, $c = 1$

For equal roots, discriminant, $D = 0$ [1]

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$ [1]

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k = 5$ or $k = -3$

Thus, for $k = 5$ or $k = -3$, the given quadratic equation has equal roots. [1]

30. For the given equation, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{3}$, $b = -2\sqrt{2}$, $c = -2\sqrt{3}$

Now, $\sqrt{D} = \sqrt{b^2 - 4ac}$

$= \sqrt{(-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})}$

$= \sqrt{8 + 24} = \sqrt{32} = 4\sqrt{2}$ [1]

Using quadratic formula, we obtain

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-2\sqrt{2}) \pm 4\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x = \dfrac{2\sqrt{2} + 4\sqrt{2}}{2\sqrt{3}}$ or $\dfrac{2\sqrt{2} - 4\sqrt{2}}{2\sqrt{3}}$ [1]

$\Rightarrow x = \dfrac{\sqrt{2} + 2\sqrt{2}}{\sqrt{3}}$ or $\dfrac{\sqrt{2} - 2\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \quad x = \dfrac{3\sqrt{2}}{\sqrt{3}}$ or $x = \dfrac{-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \quad x = \sqrt{3}\sqrt{2}$ or $x = \dfrac{-\sqrt{2}}{\sqrt{3}}$

$\therefore \quad x = \sqrt{6}$ or $x = \dfrac{-\sqrt{2}}{\sqrt{3}}$ **[1]**

31. Given $ad \ne bc$ for the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$.

    For this equation not to have real roots its discriminant < 0.   **[1]**

    $D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

    $D = 4a^2c^2 + 4b^2d^2 + 8acbd - 4a^2c^2 - 4b^2d^2 - 4b^2c^2 - 4a^2d^2$ **[1]**

    $D = -4(a^2d^2 + b^2c^2 - 2acbd)$

    $D = -4(ad - bc)^2$

    Given $ad \ne bc$

    $\therefore \quad D < 0$

    Quadratic equation has no real roots.   **[1]**

32. Let the usual speed of the plane be x km/hr.

    Time taken to cover 1500 km with usual speed $= \dfrac{1500}{x}$ hrs

    Time taken to cover 1500 km with speed of $(x + 100)$ km/hr $= \dfrac{1500}{x + 100}$ hrs.   **[1]**

    $\therefore \quad \dfrac{1500}{x} = \dfrac{1500}{x + 100} + \dfrac{1}{2}$

    $\dfrac{1500}{x} - \dfrac{1500}{x + 100} = \dfrac{1}{2}$

    $1500\left(\dfrac{x + 100 - x}{x(x + 100)}\right) = \dfrac{1}{2}$ **[1]**

    $150000 \times 2 = x(x + 100)$

    $x^2 + 100x - 300000 = 0$

    $x^2 + 100x - 300000 = 0$

    $x = -600$ or $x = 500$

    But speed can't be negative.

    Hence, usual speed 500 km/hr.   **[1]**

33. Let the duration of the flight be x hours

    Speed $= \dfrac{\text{Distance}}{\text{time}} = \dfrac{600}{x}$ km/h   **[½]**

    Duration of the flight due to slow down

    $= x + \dfrac{30}{60} = x + \dfrac{1}{2}$ According to question   **[½]**

$\dfrac{600}{x} - \dfrac{600}{x + \dfrac{1}{2}} = 200$ **[½]**

$\Rightarrow \quad \dfrac{3}{x} - \dfrac{3}{x + \dfrac{1}{2}} = 1$

$\Rightarrow \quad \dfrac{3(2x + 1) - 6x}{x(2x + 1)} = 1$ **[½]**

$\Rightarrow \quad \dfrac{6x + 3 - 6x}{x(2x + 1)} = 1$

$\Rightarrow \quad \dfrac{3}{x(2x + 1)} = 1$

$\Rightarrow \quad 2x^2 + x - 3 = 0$ **[½]**

$\Rightarrow \quad 2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow \quad x(2x + 3) - 1(2x + 3) = 0$

$\Rightarrow \quad (2x + 3)(x - 1) = 0$

$x = 1$ **[½]**

Original duration of the flight is 1 hour.

34. For equal real roots,   **[½]**

    Discriminant, $D = 0$

    Here, equation is

    $px(x - 2) + 6 = 0$

    $\Rightarrow \quad px^2 - 2px + 6 = 0$ **[½]**

    $\because \quad D = b^2 - 4ac$ for $ax^2 + bx + c = 0$

    Here,

    $D = (-2p)^2 - 4(p)(6)$ **[½]**

    $\Rightarrow \quad 0 = 4p^2 - 24p$ **[½]**

    $\Rightarrow \quad 0 = 4p(p - 6)$

    $p = 0$ or $p = 6$ **[½]**

    But $p \ne 0$ as coefficient of x^2 should be non-zero.

    $p = 6$ **[½]**

35. Let the sides of the two squares be x cm and y cm where $x > y$.

    Then, their areas are x^2 and y^2 and their perimeters are $4x$ and $4y$.

    By the given condition :

    $x^2 + y^2 = 400$ ...(i)

    and $4x - 4y = 16$

    $\Rightarrow \quad 4(x - y) = 16 \Rightarrow x - y = 4$

    $\Rightarrow \quad x = y + 4$ ...(ii)   **[1]**

    Substituting the value of x from (ii) in (i), we get :

    $(y + 4)^2 + y^2 = 400$

$\Rightarrow y^2 + 16 + 8y + y^2 = 400$

$\Rightarrow 2y^2 + 16 + 8y = 400$

$\Rightarrow y^2 + 4y - 192 = 0$

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y + 16) - 12(y + 16) = 0$ [1]

$\Rightarrow (y + 16)(y - 12) = 0$

$\Rightarrow y = -16 \quad \text{or} \quad y = 12$ [1]

Since, y cannot be negative, $y = 12$.

So, $x = y + 4 = 12 + 4 = 16$

Thus, the sides of the two squares are 16 cm and 12 cm.   [1]

36. Let the two natural numbers be x and y such that $x > y$.

Given :

Difference between the natural numbers = 5

$\therefore \quad x - y = 5$ ...(i)

Difference of their reciprocals $\dfrac{1}{10}$ (given)

$\dfrac{1}{y} - \dfrac{1}{x} = \dfrac{1}{10}$ [1]

$\Rightarrow \dfrac{x - y}{xy} = \dfrac{1}{10}$

$\Rightarrow \dfrac{5}{xy} = \dfrac{1}{10}$

$\Rightarrow xy = 50$ ...(ii)   [1]

Putting the value of x from equation (i) in equation (ii), we get

$(y + 5) y = 50$

$\Rightarrow y^2 + 5y - 50 = 0$

$\Rightarrow y^2 + 10y - 5y - 50 = 0$

$\Rightarrow y(y + 10) - 5(y + 10) = 0$

$\Rightarrow (y - 5)(y + 10) = 0$

$\Rightarrow y = 5 \text{ or } -10$ [1]

As y is a natural number, therefore $y = 5$

Other natural number = $y + 5 = 5 + 5 = 10$

Thus, the two natural numbers are 5 and 10.   [1]

37. Given quadratic equation :

$(k + 4)x^2 + (k + 1)x + 1 = 0$

Since the given quadratic equation has equal roots, its discriminant should be zero.

$\therefore \quad D = 0$ [1]

$\Rightarrow (k + 1)^2 - 4 \times (k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k - 5 = 0 \quad \text{or} \quad k + 3 = 0$

$\Rightarrow k = 5 \text{ or } -3$ [1]

Thus, the values of k are 5 and –3.

For $k = 5$, $(k + 4)x^2 + (k + 1)x + 1 = 0$

$\Rightarrow 9x^2 + 6x + 1 = 0$

$\Rightarrow (3x)^2 + 2(3x) + 1 = 0$

$\Rightarrow (3x + 1)^2 = 0$

$\Rightarrow x = -\dfrac{1}{3}, -\dfrac{1}{3}$

$\Rightarrow x^2 - 2x + 1 = 0$ [For $k = -3$]

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1, 1$ [1]

Thus, the equal roots of the given quadratic equation is either 1 or $-\dfrac{1}{3}$.   [1]

38. Let l be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 metres more than the shorter side.

Thus, diagonal = $16 + b$

Since longer side is 14 metres more than shorter side, we have,

$l = 14 + b$

Diagonal is the hypotenuse of the triangle.   [1]

Consider the following figure of the rectangular field.
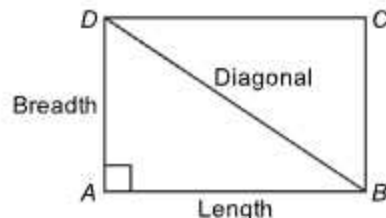
By applying Pythagoras Theorem in $\triangle ABD$, we have,

Diagonal^2 = Length^2 + Breadth^2 [1]

$\Rightarrow (16 + b)^2 = (14 + b)^2 + b^2$

$\Rightarrow 256 + b^2 + 32b = 196 + b^2 + 28b + b^2$

$\Rightarrow \quad 256 + 32b = 196 + 28b + b^2$

$\Rightarrow \quad 60 + 32b = 28b + b^2$

$\Rightarrow \quad b^2 - 4b - 60 = 0$ **[1]**

$\Rightarrow \quad b^2 - 10b + 6b - 60 = 0$

$\Rightarrow \quad b(b - 10) + 6(b - 10) = 0$

$\Rightarrow \quad (b + 6)(b - 10) = 0$

$\Rightarrow \quad (b + 6) = 0$ or $(b - 10) = 0$

$\Rightarrow \quad b = -6$ or $b = 10$

As breadth cannot be negative, breadth = 10 m

Thus, length of the rectangular field = 14 + 10 = 24 m.   **[1]**

39. Let x be the first speed of the train.

We know that, $\dfrac{\text{Distance}}{\text{Speed}} = \text{time}$

Thus, we have,

$\dfrac{54}{x} + \dfrac{63}{x+6} = 3$ **[1]**

$\Rightarrow \quad \dfrac{54(x+6)+63x}{x(x+6)} = 3$

$\Rightarrow \quad 54(x + 6) + 63x = 3x(x + 6)$

$\Rightarrow \quad 54x + 324 + 63x = 3x^2 + 18x$

$\Rightarrow \quad 117x + 324 = 3x^2 + 18x$ **[1]**

$\Rightarrow \quad 3x^2 - 117x - 324 + 18x = 0$

$\Rightarrow \quad 3x^2 - 99x - 324 = 0$

$\Rightarrow \quad x^2 - 33x - 108 = 0$

$\Rightarrow \quad x^2 - 36x + 3x - 108 = 0$

$\Rightarrow \quad x(x - 36) + 3(x - 36) = 0$

$\Rightarrow \quad (x + 3)(x - 36) = 0$ **[1]**

$\Rightarrow \quad (x + 3) = 0$ or $(x - 36) = 0$

$\Rightarrow \quad x = -3$ or $x = 36$

Speed cannot be negative. Hence, initial speed of the train is 36 km/hour.   **[1]**

40. Let the speed of the stream be s km/h.

Speed of the motor boat 24 km/h

Speed of the motor boat (upstream) = 24 − s

Speed of the motor boat (downstream) = 24 + s **[1]**

According to the given condition,

$\dfrac{32}{24-s} - \dfrac{32}{24+s} = 1$

$\therefore \quad 32\left(\dfrac{1}{24-s} - \dfrac{1}{24+s}\right) = 1$ **[1]**

$\therefore \quad 32\left(\dfrac{24+s-24+s}{576-s^2}\right) = 1$

$\therefore \quad 32 \times 2s = 576 - s^2$

$\therefore \quad s^2 + 64s - 576 = 0$

$\therefore \quad (s + 72)(s - 8) = 0$ **[1]**

$\therefore \quad s = -72$ or $s = 8$

Since, speed of the stream cannot be negative, the speed of the stream is 8 km/h.   **[1]**

41. Two taps when run together fill the tank in $3\dfrac{1}{13}$ hrs

Say taps are A, B and

A fills the tank by itself in x hrs

B fills tank in $(x + 3)$ hrs   **[1]**

Portion of tank filled by A (in 1 hr) $= \dfrac{1}{x}$

Portion of tank filled by B (in 1hr) $= \dfrac{1}{x+3}$

Portion of tank filled by A and B (both in 1hr) $= \dfrac{13}{40}$

$\therefore \quad \dfrac{1}{x} + \dfrac{1}{x+3} = \dfrac{13}{40}$ **[1]**

$(x + 3 + x)40 = 13(x)(x + 3)$

$80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$

$\Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow \quad x = 5$ or $\dfrac{-24}{13}$

[But negative value not be taken]   **[1]**

$\therefore \quad A$ fills tank in 5 hrs

B fills tank in 8 hrs   **[1]**

42. Let the speed of stream be x km/ hr.

Now, for upstream: speed = $(18 - x)$ km/hr

$\therefore \quad$ Time taken $= \left(\dfrac{24}{18-x}\right)$ hr   **[½]**

Now, for downstream: speed = $(18 + x)$ km/hr

$\therefore \quad$ Time taken $= \left(\dfrac{24}{18+x}\right)$ hr   **[½]**

Given that,

$\dfrac{24}{18-x} = \dfrac{24}{18+x} + 1$ **[½]**

$-1 = \dfrac{24}{18+x} - \dfrac{24}{18-x}$

$-1 = \dfrac{24[(18-x)-(18+x)]}{(18)^2 - x^2}$ [½]

$-1 = \dfrac{24[-2x]}{324 - x^2}$ [½]

$-324 + x^2 = -48x$

$x^2 + 48x - 324 = 0$ [½]

$x^2 + 54x - 6x - 324 = 0$

$(x + 54)(x - 6) = 0$

$x = -54$ or $x = 6$ [½]

$x = -54$ km/hr (not possible)   [½]

Therefore, speed of the stream = 6 km/hr.

43. Let x be the original average speed of the train for 63 km.

Then, $(x + 6)$ will be the new average speed for remaining 72 km.   [½]

Total time taken to complete the journey is 3 hrs.

$\therefore \quad \dfrac{63}{x} + \dfrac{72}{(x+6)} = 3$ [½]

$\left(\therefore \text{ Time} = \dfrac{\text{Distance}}{\text{Speed}} \right)$

$\therefore \quad \dfrac{63x + 378 + 72x}{x(x+6)} = 3$ [½]

$\Rightarrow 135x + 378 = 3x^2 + 18x$ [½]

$\Rightarrow x^2 - 39x - 126 = 0$ [½]

$\Rightarrow (x - 42)(x + 3) = 0$ [½]

$\Rightarrow \boxed{x = 42} \quad \text{OR} \quad \boxed{x = -3}$ [½]

Since, speed cannot be negative.

Therefore $x = 42$ km/hr.   [½]

44. Let the time in which tap with longer and smaller diameter can fill the tank separately be x hours and y hours respectively.   [½]

According to the question

$\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{8}{15}$ ...(i)   [½]

and $x = y - 2$ ...(ii)   [½]

On substituting $x = y - 2$ from (ii) in (i), we get

$\dfrac{1}{y-2} + \dfrac{1}{y} = \dfrac{8}{15}$ [½]

$\Rightarrow \dfrac{y + y - 2}{y^2 - 2y} = \dfrac{8}{15}$

$\Rightarrow 15(2y - 2) = 8(y^2 - 2y)$

$\Rightarrow 30y - 30 = 8y^2 - 16y$

$\Rightarrow 8y^2 - 46y + 30 = 0$ [½]

$\Rightarrow 4y^2 - 20y - 3y + 15 = 0$

$\Rightarrow (4y - 3)(y - 5) = 0$

$\Rightarrow y = \dfrac{3}{4}, y = 5$ [½]

Substituting values of y in (ii), we get

$\begin{array}{c|c} x = \dfrac{3}{4} - 2 & x = 5 - 2 \\[2mm] x = \dfrac{-5}{4} & x = 3 \\[2mm] \therefore \ x \neq \dfrac{-5}{4} & \end{array}$ [½]

$\left(\begin{array}{l} \text{time cannot} \\ \text{be negative} \end{array} \right)$

Hence, the time taken by tap with longer diameter is 3 hours and the time taken by tap with smaller diameter is 5 hours, in order to fill the tank separately.   [½]

45. Let the units digit of the two digit number be x.

$\therefore$ Ten's digit will be $\dfrac{14}{x}$ [½]

According to question,

$10 \times \dfrac{14}{x} + x + 45 = 10x + \dfrac{14}{x}$ [1]

$\Rightarrow \dfrac{140}{x} + x + 45 = \dfrac{10x^2 + 14}{x}$

$\Rightarrow \dfrac{140 + x^2 + 45x}{x} = \dfrac{10x^2 + 14}{x}$ [½]

$\Rightarrow 9x^2 - 45x - 126 = 0$

$\Rightarrow 9x^2 - 63x + 18x - 126 = 0$

$\Rightarrow 9x(x - 7) + 18(x - 7) = 0$ [½]

$\Rightarrow (x - 7)(9x + 18) = 0$

$\Rightarrow$ Either $x = 7$ or $x = -2$ [½]

$\therefore \quad x = 7$ $[\because x \neq -2]$

$\therefore$ Ten's digit $= \dfrac{14}{7} = 2$ [½]

So, the number is 27.   [½]

46. Let age of boy be x years, then age of his sister will be $(25 - x)$ years   [½]

Product of their ages, $(x)(25 - x) = 150$ [½]

$\Rightarrow 25x - x^2 = 150$ [½]

$\Rightarrow x^2 - 25x + 150 = 0$ [½]

$\Rightarrow (x - 15)(x - 10) = 0$ [1]

$\Rightarrow x = 10$ and 15 [½]

Hence, their present age's are 10 years and 15 years. [½]

47. (a) Let x be the digit at 10th place of given two digit number and y be the unit's place of given two digit number.

According to the question,

$xy = 24$

$\Rightarrow \quad y = \dfrac{24}{x}$ ...(i) [1]

and

$10x + y - 18 = 10y + x$

$\Rightarrow \quad 9x - 9y = 18$

$\Rightarrow \quad x - y = 2$ ...(ii) [1]

From equation (i) and (ii), we get

$x - \dfrac{24}{x} = 2$

or $x^2 - 2x - 24 = 0$

or $x^2 - 6x + 4x - 24 = 0$

or $(x - 6)(x + 4) = 0$

$x = 6$ or $x = -4$ [1]

$\therefore \quad x = 6$ [Because x can't be negative]

From (i),

$y = 4$

$\therefore$ Original number is 64. [1]

OR

(b) Let x and y be the two numbers such that $x > y$

According to question,

$x^2 - y^2 = 180$ ...(i) [½]

and $y^2 = 8x$ ...(ii) [½]

From (i) and (ii), we get

$x^2 - 8x - 180 = 0$ [½]

or $(x - 18)(x + 10) = 0$ [½]

$x = 18, -10$

$x = 18$ [Because x cannot be negative] [½]

From (ii)

Put $x = 18$ in equation (ii), we get

$y^2 = 144$ [½]

or $y = \pm 12$ [½]

$\therefore$ Required numbers are (18, 12) and (18, −12) [½]

48. Let assume the two numbers to be x, y ($y > x$)

Given that $y - x = 4 \Rightarrow y = 4 + x$ ...(i) [1]

$\dfrac{1}{x} - \dfrac{1}{y} = \dfrac{4}{21}$ [1]

$\Rightarrow \dfrac{y - x}{xy} = \dfrac{4}{21}$

$\Rightarrow \dfrac{4}{xy} = \dfrac{4}{21}$ [1]

$\Rightarrow xy = 21$

$x(4 + x) = 21$ [1]

$x^2 + 4x - 21 = 0$

$(x + 7)(x - 3) = 0$

$x = -7, 3$ [1]

$y = -3, 7$

$\therefore$ Numbers are −7, −3 or 3, 7 [1]

49. $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Discriminant

$D = 81(a + b)^2 - 36(2a^2 + 5ab + 2b^2)$ [1]

$D = 9[9a^2 + 9b^2 + 18ab - 8a^2 - 8b^2 - 20ab]$

$D = 9[a^2 + b^2 - 2ab]$ [1]

$\therefore \quad \boxed{D = 9(a - b)^2}$ [1]

$\therefore \quad x = \dfrac{+9(a + b) \pm \sqrt{9(a - b)^2}}{2 \times 9}$ [1]

$x = \dfrac{9(a + b) \pm 3(a - b)}{18}$

$x = \dfrac{3a + 3b + a - b}{6}, \dfrac{3a + 3b - a + b}{6}$ [1]

$\therefore \quad x = \dfrac{2a + b}{3}, \dfrac{a + 2b}{3}$ [1]

50. −5 is root of $2x^2 + px - 15 = 0$

$\therefore \quad 2(-5)^2 + p(-5) - 15 = 0$ [1]

$10 - p - 3 = 0$

$\therefore \quad p = 7$ [1]

$p(x^2 + x) + k = 0$ has equal roots. [1]

$\therefore \quad 7x^2 + 7x + k = 0$ [As we know $p = 7$] [1]

$\therefore$ Discriminant $= 0$

$D = 49 - 28k$ [1]

$28k = 49$

$k = \dfrac{7}{4}$ [1]

51. Let the required three integers be $(x-1)$, x and $(x+1)$.   [1]

    Now, $(x-1)^2 + [x.(x+1)] = 46$

    $(x^2 - 2x + 1) + [x^2 + x] = 46$ [1]

    $2x^2 - x - 45 = 0$

    $2x^2 - 10x + 9x - 45 = 0$ [1]

    $2x(x-5) + 9(x-5) = 0$

    $(x-5)(2x+9) = 0$ [1]

    $x = 5$ or $x = -9/2$

    So, $x = 5$ [Because it is given that x is a positive integer]   [1]

    Thus, the required integers are $(5-1)$, i.e. 4, 5 and 6.   [1]

52. Let the smaller number be x and larger number be y.

    $y^2 - x^2 = 88$ ...(i)

    $y = 2x - 5$ ...(ii)   [1]

    In equation (i)

    $(2x-5)^2 - x^2 = 88$ [1]

    $4x^2 - 20x + 25 - x^2 = 88$

    $3x^2 - 20x - 63 = 0$ [1]

    By splitting the middle term,

    $3x^2 - 27x + 7x - 63 = 0$

    $3x(x-9) + 7(x-9) = 0$ [1]

    $(x-9)(3x+7) = 0$

    $\Rightarrow x = 9$ and $x = -7/3$ [1]

    We cannot take negative value because x must be greater than 5.

    So, smaller number = 9

    And larger number = $2x - 5 = 18 - 5 = 13$ [1]

53.
    A ——————————— B
         180 km

    Distance travelled by train = 180 km, let say speed = s km/hr

    $\boxed{\text{Time taken } (t) = \dfrac{180}{s}}$ [1]

    It is given if speed had been $(s+9)$ km/hr

    Train would have travelled AB in $(t-1)$ hrs.   [1]

    $\therefore \quad t - 1 = \dfrac{180}{s+9}$

    $\Rightarrow \boxed{t = \dfrac{180}{s+9} + 1}$ [1]

    $\therefore \quad \dfrac{180}{s+9} + 1 = \dfrac{180}{s}$

    $(189 + s)s = 180s + 1620$ [1]

    $189s + s^2 = 180s + 1620$

    $s^2 + 9s - 1620 = 0$ [1]

    $\Rightarrow s^2 + 45s - 36s - 1620 = 0$

    $\Rightarrow s = -45, 36$ [$\because s$ cannot negative]   [1]

    $\therefore \quad \boxed{s = 36 \text{ km/hr}}$

54. Total cost of books = ₹80

    Let the number of books be x.

    So, the cost of each book = ₹$\dfrac{80}{x}$ [1]

    Cost of each book if he buy 4 more book

    = ₹$\dfrac{80}{x+4}$ [1]

    As per given in question :

    $\dfrac{80}{x} - \dfrac{80}{x+4} = 1$ [1]

    $\Rightarrow \dfrac{80x + 320 - 80x}{x(x+4)} = 1$

    $\Rightarrow \dfrac{320}{x^2 + 4x} = 1$

    $\Rightarrow x^2 + 4x - 320 = 0$ [1]

    $\Rightarrow (x+20)(x-16) = 0$

    $\Rightarrow x = -20, 16$ [1]

    Since, number of books cannot be negative.

    So, the number of books he bought is 16.   [1]

55. Let the first number be x then the second number be $(9-x)$ as the sum of both numbers is 9.   [1]

    Now, the sum of their reciprocals is $\dfrac{1}{2}$, therefore

    $\dfrac{1}{x} + \dfrac{1}{9-x} = \dfrac{1}{2}$ [1]

    $\Rightarrow \dfrac{9-x+x}{x(9-x)} = \dfrac{1}{2}$ [1]

    $\Rightarrow \dfrac{9}{9x-x^2} = \dfrac{1}{2}$

    $\Rightarrow 18 = 9x - x^2$ [1]

    $\Rightarrow x^2 - 9x + 18 = 0$

    $\Rightarrow (x-6)(x-3) = 0$

    $\Rightarrow x = 6, 3$ [1]

    If $x = 6$ then other number is 3.

    and if $x = 3$ then other number is 6.

    Hence, numbers are 3 and 6.   [1]

5 : Arithmetic Progressions

1. Answer (c)

   Given common difference of the

   $AP = d = 3$

   Lets say the first term $= a$

   $a_{20} = a + 19d = a + 19 \times 3$

   $\qquad = a + 57$

   $a_{15} = a + 14d = a + 14 \times 3$ [½]

   $\qquad = a + 42$

   $a_{20} - a_{15} = a + 57 - a - 42$

   $\qquad = 15$ [½]

2. Answer (c)

   The first 20 odd numbers are 1, 3, 5, ... 39

   This is an AP with first term 1 and the common difference 2. [½]

   Sum of 20 terms $= S_{20}$

   $S_{20} = \dfrac{20}{2}[2(1) + (20-1)(2)] = 10[2+38] = 400$ [½]

   Thus, the sum of first 20 odd natural numbers is 400.

3. Answer (c)

   Common difference =

   $\dfrac{1-6q}{3q} - \dfrac{1}{3q} = \dfrac{1-6q-1}{3q} = \dfrac{-6q}{3q} = -2$ [1]

4. Answer (c)

   The first three terms of an AP are $3y - 1$, $3y + 5$ and $5y + 1$, respectively.

   We need to find the value of y.

   We know that if a, b and c are in AP, then :

   $b - a = c - b$

   $\Rightarrow 2b = a + c$

   $\therefore\ 2(3y + 5) = 3y - 1 + 5y + 1$ [½]

   $\Rightarrow 6y + 10 = 8y$

   $\Rightarrow 10 = 8y - 6y$

   $\Rightarrow 2y = 10$

   $\Rightarrow y = 5$

   Hence the correct option is c. [½]

5. Answer (a) [1]

   $2x$, $(x + 10)$, $(3x + 2)$ are in A.P.

   $\therefore\ x + 10 - 2x = 3x + 2 - x - 10$

   $\Rightarrow x = 6$

   Hence, option (a) is correct.

6. Answer (c) [1]

   $\therefore\ 10^{\text{th}}$ term $= p + (10 - 1)q$

   $\boxed{a_{10} = p + 9q}$

   Hence, option (c) is correct.

7. Answer (a) [1]

   $(4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

   $\Rightarrow 3k - 8 = -k + 4$

   $\Rightarrow 4k = 12$

   $\Rightarrow k = 3$

8. Answer (b) [1]

   A : $b - a = c - b$

   $\qquad \Rightarrow 2b = a + c$

   R : Sum of first n odd natural numbers $= n^2$

9. First term of an AP $= p$

   Common difference $= q$

   $T_{10} = p + (10 - 1)q$ [½]

   $T_{10} = p + 9q$ [½]

10. Given $\dfrac{4}{5}$, a, 2 are in AP

    $\therefore\ a - \dfrac{4}{5} = 2 - a$ [½]

    $\Rightarrow 2a = \dfrac{4}{5} + 2$

    $2a = \dfrac{14}{5}$

    $\therefore\ a = \dfrac{7}{5}$ [½]

11. Given an AP which has sum of first p terms $= ap^2 + bp$

    Lets say first term $= k$ & common difference $= d$

    $\therefore\ ap^2 + bp = \dfrac{p}{2}[2k + (p-1)d]$

    $2ap + 2b = 2k + (p-1)d$

    $2b + 2ap = (2k - d) + pd$ [½]

Comparing terms on both sides,

$\Rightarrow$ $\boxed{2a = d}$

$2k - d = 2b$

$2k = 2b + 2a$

$\boxed{k = a + b}$

Common difference = $2a$

First term = $a + b$ [½]

12. If $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of AP, then the common difference will be the same.

$\therefore$ $(2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$ [½]

$\therefore$ $k - 10 = 8$

$\therefore$ $k = 18$ [½]

13. Given

$a_{21} - a_7 = 84$ ...(i)

In an AP $a_1, a_2, a_3, a_4 .....$

$a_n = a_1 + (n - 1)d$ d = common difference

$a_{21} = a_1 + 20d$ ...(ii)

$a_7 = a_1 + 6d$ ...(iii) [½]

Substituting (ii) and (iii) in (i)

$a_1 + 20d - a_1 - 6d = 84$

$14d = 84$

$d = 6$

$\therefore$ Common difference = 6 [½]

14. $a_7 = 4$

$a + 6d = 4$ (as $a_n = a + (n - 1)d$)

but $d = -4$

$a + 6(-4) = 4$ [½]

$a + (-24) = 4$

$a = 4 + 24 = 28$

Therefore first term $a = 28$ [½]

15. Two digit numbers divisible by 3 are

12, 15, 18, ..., 99.

$a = 12$, $d = 15 - 12 = 3$ [½]

$\Rightarrow$ $T_n = 99$

$\Rightarrow$ $a + (n - 1)d = 99$

$\Rightarrow$ $12 + (n - 1)3 = 99$

$\Rightarrow$ $n = 30$

$\therefore$ Number of two digit numbers divisible by 3 are 30. [½]

16. $T_n = 7 - 4n$

$T_1 = 7 - 4(1) = 3$

$T_2 = 7 - 4(2) = 7 - 8 = -1$ [½]

$\therefore$ Common difference = $T_2 - T_1$

$= -1 - 3 = -4$ [½]

17. Given an AP 3, 15, 27, 39, ...

Lets say n^{th} term is 120 more than 21st term

$\therefore$ $T_n = 120 + T_{21}$

$a + (n - 1)d = 120 + (a + 20d)$ [1]

$(n - 1)12 = 120 + 20 \times 12$

$n - 1 = 30$

$\therefore$ 31st term is 120 more than 12th term. [1]

18. Given an AP with first term (a) = 2

Last term (ℓ) = 29

Sum of the terms = 155

Common difference (d) = ?

Sum of the n terms = $\dfrac{n}{2}(a + \ell)$ [½]

$\Rightarrow$ $155 = \dfrac{n}{2}(2 + 29)$

$\Rightarrow$ $\boxed{n = 10}$ [½]

Last term which is T_n

$= a + (n - 1)d$ [½]

$= a + (9)d$

$\therefore$ $29 = 2 + 9d$

$\boxed{d = 3}$

Common difference = 3 [½]

19. Two digit numbers divisible by 6 are,

12, 18..... 96 [1]

$\Rightarrow$ $96 = 12 + (n - 1) \times 6$

$[\because a_n = a + (n - 1)d]$

$\Rightarrow$ $n = \dfrac{96 - 12}{6} + 1 = 15$ [½]

$\therefore$ Two digit numbers divisible by 6 are 15. [½]

20. First three– digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore the series is 105, 112, 119,...

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly, 999 – 5 = 994 is the maximum possible three – digit number divisible by 7.

The series is as follows :

105, 112, 119, ..., 994   [½]

Here $a = 105$, $d = 7$

Let 994 be the nth term of this AP.

$a_n = a + (n - 1)d$

$\Rightarrow$ 994 = 105 + (n – 1)7

$\Rightarrow$ (n – 1)7 = 889

$\Rightarrow$ (n – 1) = 127

$\Rightarrow$ n = 128   [½]

So, there are 128 terms in the AP.

$\therefore$ Sum $= \dfrac{n}{2}$ {first term + last term}

$= \dfrac{128}{2}\{a_1 + a_{128}\}$

$64\{105 + 994\} = (64)(1099) = 70336$ [1]

21. Let a be the first term and d be the common difference.

Given : $a = 5$

$T_n = 45$

$S_n = 400$

We know :

$T_n = a + (n - 1)d$

$\Rightarrow$ 45 = 5 + (n – 1)d

$\Rightarrow$ 40 = (n – 1)d   ...(i)   [1]

And $S_n = \dfrac{n}{2}(a + T_n)$

$\Rightarrow$ 400 $= \dfrac{n}{2}(5 + 45)$

$\Rightarrow \dfrac{n}{2} = \dfrac{400}{50}$

$\Rightarrow$ n = 2 × 8 = 16   [½]

On substituting n = 16 in (i), we get :

40 = (16 – 1)d

$\Rightarrow$ 40 = (15)d

$\Rightarrow d = \dfrac{40}{15} = \dfrac{8}{3}$

Thus, the common difference is $\dfrac{8}{3}$.   [½]

22. $S_5 + S_7 = 167$ and $S_{10} = 235$

Now, $S_n = \dfrac{n}{2}\{2a + (n - 1)d\}$

$\therefore$ $S_5 + S_7 = 167$

$\Rightarrow \dfrac{5}{2}\{2a + 4d\} + \dfrac{7}{2}\{2a + 6d\} = 167$

$\Rightarrow$ 5a + 10d + 7a + 21d = 167

$\Rightarrow$ 12a + 31d = 167   ...(i)   [½]

Also, $S_{10} = 235$

$\therefore \dfrac{10}{2}\{2a + 9d\} = 235$

$\Rightarrow$ 10a + 45d = 235

$\Rightarrow$ 2a + 9d = 47   ...(ii)   [½]

Multiplying equation (ii) by 6, we get

12a + 54d = 282   ...(iii)

Subtracting (i) from (iii), we get

$\quad$ 12a $\quad$ + $\quad$ 54d $\quad$ = $\quad$ 282
$(-)$12a $\quad$ + $\quad$ 31d $\quad$ = $\quad$ –167
$\overline{\qquad\qquad\qquad\qquad 23d \quad = \quad 115}$

$\therefore$ $d = 5$ [½]

Substituting value of d in (ii), we have

2a + 9(5) = 47

$\Rightarrow$ 2a + 45 = 47

$\Rightarrow$ 2a = 2

$\Rightarrow$ a = 1

Thus, the given AP is 1, 6, 11, 16,.....   [½]

23. 4th term of an AP = $a_4 = 0$

$\therefore$ a + (4 – 1)d = 0

$\therefore$ a + 3d = 0

$\therefore$ a = –3d   ...(i)   [½]

25th term of an AP = a_{25}

= a + (25 – 1)d

= –3d + 24d   ...[From (i)]   [½]

= 21d

3 times 11th term of an AP = $3a_{11}$

= 3[a + (11 – 1)d]

= 3[a + 10d]

= 3[–3d + 10d]

= 3 × 7d

= 21d   [½]

$\therefore$ $a_{25} = 3a_{11}$

i.e., the 25th term of the AP is three times its 11th term.   [½]

24. Given progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$

This is an Arithmetic progression because
Common difference

$(d) = 19\frac{1}{4} - 20 = 18\frac{1}{2} - 19\frac{1}{4} = ...$

$d = \frac{-3}{4}$ 　　　　　[1]

Any n^{th} term $a_n = 20 + (n-1)\left(\frac{-3}{4}\right) = \frac{83 - 3n}{4}$

Any term $a_n < 0$ when $83 < 3n$

$\Rightarrow n > \frac{83}{3}$

$\Rightarrow n = 28$

∴ 28th term will be the first negative term. [1]

25. First 8 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24

The above sequence is an AP 　　　　　[1]

$a = 3$, $d = 3$ and last term $l = 24$

$S_n = \frac{n}{2}(a+l) = \frac{8}{2}[3 + 24] = 4(27)$

$S_n = 108$ 　　　　　[1]

26. $S_n = 3n^2 - 4n$

Let S_{n-1} be sum of $(n-1)$ terms

$t_n = S_n - S_{n-1}$ 　　　　　[½]

　$= (3n^2 - 4n) - [3(n-1)^2 - 4(n-1)]$ 　[½]

　$= (3n^2 - 4n) - [3n^2 - 6n + 3 - 4n + 4]$ 　[½]

　$= 3n^2 - 4n - 3n^2 + 10n - 7$

∴ $t_n = 6n - 7$

So, required n^{th} term $= 6n - 7$ 　　　[½]

27. Common difference must be equal

∴ $(a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$ 　　　　　[½]

$\Rightarrow (a^2 + b^2) - (a^2 + b^2 - 2ab) = (a^2 + b^2 + 2ab) - a^2 - b^2$ 　　　[½]

$\Rightarrow a^2 + b^2 - a^2 - b^2 + 2ab = a^2 + b^2 + 2ab - a^2 - b^2$ 　　[½]

$\Rightarrow 2ab = 2ab$ 　　　　　[½]

Hence, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.

28. (a) Given A.P. is 3, 8, 13, 18, ...

Here, $a = 3$ and $d = 8 - 3 = 5$ 　[½]

$a_n = a + (n-1)d$ 　　[n^{th} term] 　[½]

$\Rightarrow 78 = 3 + (n-1)5$

$\Rightarrow \frac{75}{5} = n - 1$ 　　　　　[½]

$\Rightarrow n = 16$

∴ 78 is 16th term of the given A.P. 　[½]

OR

(b) n^{th} term of A.P. is

$a_n = 6n - 5$

if $n = 1$,

$\Rightarrow a_1 = 6 - 5 = 1$ 　　　　　[½]

if $n_2 = 1$,

$a_2 = 6 \times 2 - 5 = 7$ 　　　　　[½]

∴ Common difference $(d) = 7 - 1$ 　[½]

　　　　　　　　　$= 6$ 　　　　[½]

29. First fifteen multiples of 8 are

8, 16, 24..... 　　　　　　　[½]

Here, $a = 8$ and $d = 8$

$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1)8]$ 　　[½]

　　$= \frac{15}{2}[16 + 112]$ 　　　　[½]

　　$= \frac{15 \times 128}{2}$

　　$= 960$

∴ Sum of first fifteen multiples of 8 is 960. [½]

30. (a) Given A.P.

$-\frac{11}{2}, -3, -\frac{1}{2}, ...$

Here,

$a = -\frac{11}{2}$, $d = -3 + \frac{11}{2} = \frac{11 - 6}{2} = \frac{5}{2}$ 　[½]

$t_n = \frac{49}{2}$

$a + (n-1)d = \frac{49}{2}$ 　　　　　[½]

or $-\frac{11}{2} + (n-1)\left(\frac{5}{2}\right) = \frac{49}{2}$

or $-11 + 5n - 5 = 49$ 　　　　　[½]

$\Rightarrow \quad 5n = 49 + 16$

$\Rightarrow \quad 5n = 65$

$\Rightarrow \quad n = \dfrac{65}{5} = 13$

$\Rightarrow \quad n = 13$ [½]

OR

(b) Given,

a, 7, b, 23 are in A.P.

$\therefore \quad 7 - a = b - 7 = 23 - b$ [½]

$\Rightarrow \quad 7 - a = b - 7$

$\Rightarrow \quad a + b = 14$ ...(i)   [½]

and $b - 7 = 23 - b$

$\Rightarrow \quad 2b = 30$

$\Rightarrow \quad b = 15$ [½]

From (i)

$a = 14 - 15$

$a = -1$ [½]

31. Sum of n terms of A.P. if n^{th} term of A.P. is given by,

$$S_n = \dfrac{n}{2}[a + a_n]$$

If $n = 1$

$a_1 = 5 - 2 = 3$ [½]

and if $n = 20$

$a_{20} = 5 - 40 = -35$ [½]

$\therefore \quad S_{20} = \dfrac{20}{2}[a_1 + a_{20}]$

$\qquad = \dfrac{20}{2}[3 + (-35)]$

$\qquad = 10[-32]$ [½]

$\qquad S_{20} = -320$ [½]

32. n^{th} term of 63, 65, 67, ...

$= 63 + (n - 1)(2)$

$= 63 + 2n - 2$

$= 61 + 2n$ ...(i)   [1]

n^{th} term of 3, 10, 17, ...

$= 3 + (n - 1)7$

$= 3 + 7n - 7$

$= 7n - 4$ ...(ii)   [1]

Given that n^{th} terms of two AP's are equal.

$61 + 2n = 7n - 4$ [Using (i) and (ii)]

$65 = 5n$

$\boxed{n = 13}$ [1]

33. Lets assume first term = a

Common difference = d

$T_m = a + (m - 1)d$

$T_n = a + (n - 1)d$

Given $m.T_m = n.T_n$ [1]

$m(a + (m - 1)d) = n(a + (n - 1)d)$

$ma + m(m - 1)d = na + n(n - 1)d$

$(m - n)a + d(m^2 - m - n^2 + n) = 0$ [1]

$a(m - n) + d(m - n)(m + n - 1) = 0$

$(m - n)[a + (m + n - 1)d] = 0$

$m \neq n$

$\therefore \quad a + (m + n - 1)d = 0$

$\boxed{T_{m+n} = 0}$ [1]

34. First term (a) = 5

$T_n = 33$

Sum of first n terms = 123

$\therefore \quad \dfrac{n}{2}[a + T_n] = 123$ [1]

$\dfrac{n}{2}[8 + 33] = 123$

$\boxed{n = 6}$ [1]

$T_n = a + (n - 1)d$

$33 = 8 + (5)d$

$\boxed{d = 5}$ [1]

35. Lets say first term of given AP = a

Common difference = d

Sum of first six terms = 42

$\therefore \quad \dfrac{6}{2}(2a + 5d) = 42$

$2a + 5d = 14$ ...(i)   [1]

Also given $T_{10} : T_{30} = 1 : 3$

$\Rightarrow \quad \dfrac{a + 9d}{a + 29d} = \dfrac{1}{3}$

$3a + 27d = a + 29d$

$\Rightarrow \quad 2a = 2d$

$\Rightarrow \quad \boxed{a = d}$ ...(ii)   [1]

Substituting (ii) in (i)

$\Rightarrow \ 2a + 5a = 14$

$a = 2$ and $d = 2$

$T_{13} = a + 12d$

$\qquad = 2 + 24$

$T_{13} = 26$ **[1]**

36. Sum of first ten terms = -150

Sum of next ten terms = 550

Lets say first term of AP = a

Common difference = d

Sum of first ten terms $= \dfrac{10}{2}[2a + 9d]$

$-150 = 5[2a + 9d]$

$\boxed{2a + 9d = -30}$ ...(i) **[1]**

For sum of next ten terms the first term would be $T_{11} = a + 10d$

$\Rightarrow \ -550 = \dfrac{10}{2}[2(a + 10d) + 9d]$

$\Rightarrow \ \boxed{-110 = 2a + 29d}$ ...(ii) **[1]**

Solving (i) and (ii)

$d = -4$

$a = 3$

$\therefore$ AP will be 3, -1, -5, -9, -13, ... **[1]**

37. Given an AP

Say first term = a

Common difference = d

Given $T_4 = 9$

$a + 3d = 9$ ...(i) **[1]**

Also $T_6 + T_{13} = 40$

$a + 5d + a + 12d = 40$

$2a + 17d = 40$ ...(ii) **[1]**

Solving (i) and (ii)

$a = 3 \qquad d = 2$

$\therefore$ AP will be 3, 5, 7, 9, ... **[1]**

38. Let a and d respectively be the first term and the common difference of the AP.

We know that the nth term of an AP is given by
$a_n = a + (n - 1)d$

According to the given information,

$A_{16} = 1 + 2a_8$

$\Rightarrow \ a + (16 - 1)d = 1 + 2[a + (8 - 1)d]$

$\Rightarrow \ a + 15d = 1 + 2a + 14d$

$\Rightarrow \ -a + d = 1$ ...(i) **[1]**

Also, it is given that, $a_{12} = 47$

$\Rightarrow \ a + (12 - 1)d = 47$

$\Rightarrow \ a + 11d = 47$ ...(ii) **[1]**

Adding (i) and (ii), we have :

$12d = 48$

$\Rightarrow \ d = 4$

From (i),

$-a + 4 = 1$

$\Rightarrow \ a = 3$ **[½]**

Hence, $a_n = a + (n - 1)d = 3 + (n - 1)(4)$
$\qquad = 3 + 4n - 4 = 4n - 1$

Hence, the nth term of the AP is $4n - 1$. **[½]**

39. $S_n = 3n^2 + 4n$

First term $(a_1) = S_1 = 3(1)^2 + 4(1) = 7$

$S_2 = a_1 + a_2 = 3(2)^2 + 4(2) = 20$ **[1]**

$a_2 = 20 - a_1 = 20 - 7 = 13$

So, common difference $(d) = a_2 - a_1 = 13 - 7 = 6$ **[1]**

Now, $a_n = a + (n - 1)d$

$\therefore \ a_{25} = 7 + (25 - 1) \times 6 = 7 + 24 \times 6 = 7 + 144 = 151$ **[1]**

40. Let a be the first term and d be the common difference of the given AP

Given :

$a_7 = \dfrac{1}{9}$

$a_9 = \dfrac{1}{7}$

$a_7 = a + (7 - 1)d = \dfrac{1}{9}$

$\Rightarrow \ a + 6d = \dfrac{1}{9}$ ...(i) **[1]**

$a_9 = a + (9 - 1)d = \dfrac{1}{7}$

$\Rightarrow \ a + 8d = \dfrac{1}{7}$ ...(ii) **[1]**

Subtracting equation (i) from (ii), we get :

$2d = \dfrac{2}{63}$

$\Rightarrow \ d = \dfrac{1}{63}$ **[½]**

Putting $d = \dfrac{1}{63}$ in equation (i), we get :

$$a + \left(6 \times \dfrac{1}{63}\right) = \dfrac{1}{9}$$

$$\Rightarrow \quad a = \dfrac{1}{63}$$

$$\therefore \quad a_{63} = a + (63 - 1)d = \dfrac{1}{63} + 62\left(\dfrac{1}{63}\right) = \dfrac{63}{63} = 1$$

Thus, the 63rd term of the given AP is 1. **[½]**

41. Here it is given that,

$$T_{14} = 2(T_8)$$

$$\Rightarrow \quad a + (14 - 1)d = 2[a + (8 - 1)d]$$

$$\Rightarrow \quad a + 13d = 2[a + 7d]$$

$$\Rightarrow \quad a + 13d = 2a + 14d$$

$$\Rightarrow \quad 13d - 14d = 2a - a$$

$$\Rightarrow \quad -d = a \qquad \qquad \text{...(i)} \qquad \textbf{[1]}$$

Now, it is given that its 6th term is –8.

$$T_6 = -8$$

$$\Rightarrow \quad a + (6 - 1)d = -8$$

$$\Rightarrow \quad a + 5d = -8$$

$$\Rightarrow \quad -d + 5d = -8 \qquad \text{[∵ Using (i)]}$$

$$\Rightarrow \quad 4d = -8$$

$$\Rightarrow \quad d = -2$$

Substituting this in eq. (i), we get $a = 2$ **[1]**

Now, the sum of 20 terms,

$$S_n = \dfrac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \dfrac{20}{2}[2a + (20 - 1)d]$$

$$= 10[2(2) + 19(-2)]$$

$$= 10[4 - 38]$$

$$= -340 \qquad \qquad \textbf{[1]}$$

42. Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given AP's.

Thus, we have $S_n = \dfrac{n}{2}[2a_1 + (n - 1)d_1]$ and

$$S_n' = \dfrac{n}{2}[2a_2 + (n - 1)d_2]$$

$$\therefore \quad \dfrac{S_n}{S_n'} = \dfrac{\dfrac{n}{2}[2a_1 + (n-1)d_1]}{\dfrac{n}{2}[2a_2 + (n-1)d_2]} = \dfrac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} \quad \textbf{[½]}$$

It is given that $\dfrac{S_n}{S_n'} = \dfrac{7n + 1}{4n + 27}$

$$\therefore \quad \dfrac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \dfrac{7n + 1}{4n + 27} \quad \text{...(i)} \qquad \textbf{[½]}$$

To find the ratio of the mth term of the two given AP's, replace n by $(2m - 1)$ in equation (i).

$$\therefore \quad \dfrac{2a_1 + (2m - 1 - 1)d_1}{2a_2 + (2m - 1 - 1)d_2} = \dfrac{7(2m - 1) + 1}{4(2m - 1) + 27}$$

$$\therefore \quad \dfrac{2a_1 + (2m - 2)d_1}{2a_2 + (2m - 2)d_2} = \dfrac{14m - 7 + 1}{8m - 4 + 27} \qquad \textbf{[1]}$$

$$\therefore \quad \dfrac{a_1 + (m - 1)d_1}{a_2 + (m - 1)d_2} = \dfrac{14m - 6}{8m + 23}$$

Hence, the ratio of the mth term of the two AP's is $14m - 6 : 8m + 23$. **[1]**

43. (A) Given : $a = 15$, $S_{15} = 750$

To find : a_{20}

Let d and n be common difference and number of terms respectively

$$\because \quad S_n = \dfrac{n}{2}(2a + (n - 1)d) \qquad \textbf{[½]}$$

$$\Rightarrow \quad S_{15} = \dfrac{15}{2}(2(15) + (15 - 1)d) \qquad \textbf{[½]}$$

$$\Rightarrow \quad 750 = \dfrac{15}{2}(30 + 14d)$$

$$\Rightarrow \quad \dfrac{750 \times 2}{15} = 30 + 14d$$

$$\Rightarrow \quad 14d + 30 = 50 \times 2$$

$$\Rightarrow \quad 14d = 100 - 30$$

$$\Rightarrow \quad 14d = 70 \qquad \qquad \textbf{[½]}$$

$$d = 5 \qquad \qquad \textbf{[½]}$$

Also, $a_n = a + (n - 1)d$ **[½]**

$$\Rightarrow \quad a_{20} = 15 + (20 - 1)(5)$$

$$= 15 + 19(5)$$

$$= 15 + 95$$

$$= 110 \qquad \qquad \textbf{[½]}$$

OR

(B) Instalments to be paid by Rohan

1000, 1100, 1200, ...

This sequence is an A.P

Here, a = 1000 and d = 100 **[½]**

To find : a30 and S30

$\because$ $a_n = a + (n-1)d$ **[½]**

$\Rightarrow$ $a_{30} = 1000 + (30-1)(100)$

$= 1000 + 29(100)$

$= 1000 + 2900$

$= 3900$ **[½]**

Also, $S_n = \dfrac{n}{2}(a + a_n)$ **[½]**

$\Rightarrow$ $S_{30} = \dfrac{30}{2}(1000 + 3900)$

$\Rightarrow$ $S_{30} = 15 \times 4900$

$= 73500$ **[½]**

$\therefore$ In the 30th instalment, he will pay ₹3900 and he has paid ₹73500 after 30 instalments. **[½]**

44. Given an A.P with first (a) = 8

Last term (ℓ) = 350

Common difference (d) = 9

$T_n = a + (n-1)d$

$= a + (n-1)d = 350$

$\Rightarrow$ $8 + (n-1)9 = 350$ **[1]**

$\boxed{n = 39}$

$\therefore$ Number of terms = 39 **[1]**

Sum of the terms

$= \dfrac{n}{2}[a + \ell]$

$= \dfrac{39}{2}[8 + 350]$ **[1]**

$= 6981$ **[1]**

45. Multiples of 4 between 10 and 250 are 12, 16, ... 248. **[1]**

We now have an A.P with first term = 12 and last term = 248 **[1]**

Common difference = 4

$\therefore$ $248 = 12 + (n-1)4$

$[\because a_n = a + (n-1)d]$ **[1]**

$\Rightarrow$ $\boxed{n = 60}$

$\therefore$ Multiples of 4 between 10 and 250 are 60. **[1]**

46. Given : $S_{20} = -240$ and a = 7

Consider, $S_{20} = -240$

$\Rightarrow$ $\dfrac{20}{2}(2 \times 7 + 19d) = -240$ **[1]**

$\left[\because S_n = \dfrac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ $10(14 + 19d) = -240$

$\Rightarrow$ $14 + 19d = -24$ **[1]**

$\Rightarrow$ $19d = -38$

$\Rightarrow$ $d = -2$ **[1]**

Now, $a_{24} = a + 23d = 7 + 23 \times -2 = -39$

$[\because a_n = a + (n-1)d]$

Hence, $a_{24} = -39$ **[1]**

47. Given AP is $-12, -9, -6, ..., 21$

First term, $a = -12$

Common difference, $d = 3$ **[1]**

Let 12 be the nth term of the AP.

$12 = a + (n-1)d$

$\Rightarrow$ $12 = -12 + (n-1) \times 3$ **[1]**

$\Rightarrow$ $24 = (n-1) \times 3$

$\Rightarrow$ $n = 9$

Sum of the terms of the AP = S_9

$= \dfrac{n}{2}(2a + (n-1)d) = \dfrac{9}{2}(-24 + 8 \times 3) = 0$ **[1]**

If 1 is added to each term of the AP, the sum of all the terms of the new AP will increase by n, i.e., 9.

$\therefore$ Sum of all the terms of the new AP = 0 + 9

$= 9$ **[1]**

48. Let a and d be the first term and the common difference of an AP respectively.

n^{th} term of an AP, $a_n = a + (n-1)d$

Sum of n terms of an AP, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

We have :

Sum of the first 10 terms $= \dfrac{10}{2}[2a + 9d]$

$\Rightarrow$ $210 = 5[2a + 9d]$

$\Rightarrow$ $42 = 2a + 9d$ ...(i) **[1]**

15th term from the last = $(50 - 15 + 1)^{th}$ = 36th term from the beginning

Now, $a_{36} = a + 35d$

∴   Sum of the last 15 terms

$$= \frac{15}{2}(2a_{36} + (15 - 1)d)$$ [1]

$$= \frac{15}{2}[2(a + 35d) + 14d]$$

$= 15[a + 35d + 7d]$

⇒ $2565 = 15[a + 42d]$

⇒ $171 = a + 42d$ ...(ii)   [1]

From (i) and (ii), we get,

$d = 4$

$a = 3$

So, the AP formed is 3, 7, 11, 15... and 199.   [1]

49. Consider the given AP 8, 10, 12, ...

Here the first term is 8 and the common difference is $10 - 8 = 2$

General term of an AP is t_n is given by,

$t_n = a + (n - 1)d$

⇒ $t_{60} = 8 + (60 - 1) \times 2$

⇒ $t_{60} = 8 + 59 \times 2$

⇒ $t_{60} = 8 + 118$

⇒ $t_{60} = 126$ [1]

We need to find the sum of the last 10 terms.

Thus,

Sum of last 10 terms = Sum of first 60 terms – Sum of first 50 terms

[½]

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

⇒ $S_{60} = \frac{60}{2}[2 \times 8 + (60 - 1) \times 2]$

⇒ $S_{60} = 30[16 + 59 \times 2]$

⇒ $S_{60} = 30[134]$

⇒ $S_{60} = 4020$ [1]

Similarly,

⇒ $S_{50} = \frac{50}{2}[2 \times 8 + (50 - 1) \times 2]$

⇒ $S_{50} = 25[16 + 49 \times 2]$

⇒ $S_{50} = 25[114]$

⇒ $S_{50} = 2850$ [1]

Thus the sum of last 10 terms = $S_{60} - S_{50}$ = 4020 – 2850 = 1170   [½]

50. Let there be a value of X such that the sum of the numbers of the houses preceding the house numbered X is equal to the sum of the numbers of the houses following it.

That is, $1 + 2 + 3 + \ldots + (X - 1) = (X + 1) + (X + 2) \ldots + 49$

∴ $[1 + 2 + 3 + \ldots + (X - 1)$

$= [1 + 2 + \ldots + X + (X - 1) + \ldots + 49]$
$- (1 + 2 + 3 + \ldots + X)$ [1]

∴ $\frac{X - 1}{2}[1 + X - 1] = \frac{49}{2}[1 + 49] - \frac{X}{2}[1 + X]$

∴ $X(X - 1) = 49 \times 50 - X(1 + X)$

∴ $X(X - 1) + X(1 + X) = 49 \times 50$ [1]

∴ $X^2 - X + X + X^2 = 49 \times 50$

∴ $2X^2 = 49 \times 50$ [1]

∴ $X^2 = 49 \times 25$

∴ $X = 7 \times 5 = 35$

Since X is not a fraction, the value of x satisfying the given condition exists and is equal to 35.   [1]

51. Let the numbers be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$

∴ $(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

⇒ $4a = 32$

$a = 8$ [1]

Also, $\frac{(a - 3d)(a + 3d)}{(a - d)(a + d)} = \frac{7}{15}$

⇒ $15a^2 - 135d^2 = 7a^2 - 7d^2$

⇒ $8a^2 = 128d^2$ [1]

$$d^2 = \frac{8a^2}{128} = \frac{8 \times 8 \times 8}{128}$$

$d^2 = 4$

$d = \pm 2$ [1]

If $d = 2$ numbers are : 2, 6, 10, 14

If $d = -2$ numbers are 14, 10, 6, 2   [1]

52. Let the first four terms be a, $a + d$, $a + 2d$, $a + 3d$

$a + a + d + a + 2d + a + 3d = 40$ [½]

$\Rightarrow 2a + 3d = 20$ ...(i) [½]

Sum of first 14 terms = 280

$\dfrac{n}{2}[2a+(n-1)d] = 280$ [½]

$\Rightarrow \dfrac{14}{2}[2a+13d] = 280$

$\Rightarrow 2a + 13d = 40$ ...(ii) [1]

On subtracting (i) from (ii), we get $d = 2$

Substituting the value of d in (i) [½]

$a = 7$

$\therefore$ Sum of n terms $= \dfrac{n}{2}[2a+(n-1)d]$ [½]

$= \dfrac{n}{2}[14+(n-1)2]$

$= n^2 + 6n$ [½]

53. Let the first term and common difference be a and d.

According to the question,

$4(a + 3d) = 18 \times (a + 17d)$ [1]

$\Rightarrow 4a + 12d = 18a + 306d$

$\Rightarrow 14a + 294d = 0$ [1]

$\Rightarrow a + 21d = 0$ [1]

$\therefore a_{22} = a + 21d$

$= 0$ [1]

OR

Given A.P.

24, 21, 18,.....

$\therefore$ First term = 24 = a

and common difference = $-3 = d$ ...(i) [1]

Let number of terms is n.

$\therefore$ Sum of n terms $= \dfrac{n}{2}[2a+(n-1)d]$ [1]

According to question

$\Rightarrow 78 = \dfrac{n}{2}[2 \times 24 - 3 \times (n-1)]$ [from (i) and given]

$\Rightarrow 78 = \dfrac{n}{2}[51-3n]$

$\Rightarrow n^2 - 17n + 52 = 0$ [1]

$\Rightarrow n^2 - 13n - 4n + 52 = 0$

$\Rightarrow n(n - 13) - 4(n - 13) = 0$

$(n - 13)(n - 4) = 0$

$n = 13, 4$

For first 4 terms and first 13 terms in both case we get sum 78. [1]

54. Let the four consecutive numbers in A.P. are $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$. [½]

$\therefore$ According to the condition given,

$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow 4a = 32$

$\Rightarrow a = 8$ ...(i) [1]

and, according to the 2nd condition given,

$\dfrac{(a-3d)\times(a+3d)}{(a-d)\times(a+d)} = \dfrac{7}{15}$ [½]

$\Rightarrow \dfrac{(8-3d)\times(8+3d)}{(8-d)\times(8+d)} = \dfrac{7}{15}$

$\Rightarrow \dfrac{64-9d^2}{64-d^2} = \dfrac{7}{15}$ [½]

$\Rightarrow 15(64 - 9d^2) = 7(64 - d^2)$

$\Rightarrow 128d^2 = 512$

$\Rightarrow d^2 = 4$

$\Rightarrow d = \pm 2$ [½]

$\therefore$ Numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2. [1]

OR

Here 1, 4, 7, 10, ... x is an A.P.

With first term $a = 1$ and common difference $d = 3$. [½]

Let there be n terms in the A.P. Then,

$x = n^{th}$ term

$\Rightarrow x = 1 + (n - 1) \times 3$ [½]

$= 3n - 2$ ...(i)

Now, $1 + 4 + 7 + 10 + ... + x = 287$

$\Rightarrow \dfrac{n}{2}[1+x] = 287 \quad \left[S_n = \dfrac{n}{2}(a+l)\right]$ [½]

$\Rightarrow \dfrac{n}{2}[1+3n-2] = 287$

$\Rightarrow 3n^2 - n - 574$

$\Rightarrow 3n^2 - n - 574 = 0$ [1]

$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow 3n(n - 14) + 41(n - 14) = 0$

$\Rightarrow (n - 14)(3n + 41) = 0$

$\Rightarrow n - 14 = 0$ $[\because 3n + 41 \neq 0]$

$\Rightarrow n = 14$ [½]

Putting $n = 14$ in eqn (i), we get

$x = 3 \times 14 - 2$

$x = 40$ [1]

55. (A) Let first term and common difference of A.P. be a and d respectively   [1]

$a_2 = 14$ and $a_3 = 18$

$a + d = 14$ ...(i)

$a + 2d = 18$ ...(ii)   [1]

Subtracting (i) from (ii), we get

$d = 4$

Putting $d = 4$ in equation (i), we get

$a = 10$ [1]

$S_{51} = \dfrac{51}{2}[2(10) + (50)(4)]$ [1]

$= \dfrac{51}{2}[220]$

$S_{51} = 5610$ [1]

OR

(B) Let d and n be the common difference and number of terms of A.P. respectively

$a_n = a + (n - 1)d$

$45 = 5 + (n - 1)d$

$\Rightarrow 40 = (n - 1)d$ ...(i)   [1]

and $S_n = \dfrac{n}{2}[a + a_n]$

$400 = \dfrac{n}{2}[5 + 45]$ [1]

$400 = 25n$

$\Rightarrow n = 16$ ...(ii)   [1]

From (i) and (ii), we get

$(16 - 1)d = 40$ [1]

$15d = 40$

$d = \dfrac{8}{3}$ [1]

Hence, number of terms and common difference is 16 and $\dfrac{8}{3}$ respectively.

6 : Triangles

1. Answer (c)

Applying B.P.T. in $\triangle ABC$, $DE \parallel BC$

$\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \dfrac{4}{6} = \dfrac{5}{EC}$

$\Rightarrow EC = 7.5$ cm   [1]

2. Answer (a)   [1]

Two congruent figures are always similar.

3. Answer (b)

$DE \parallel BC$

$\therefore \dfrac{DE}{BC} = \dfrac{AD}{AB}$

$\Rightarrow DE : BC = 2 : 5$ [1]

4. Answer (d)

Because, according to criteria of similarity RHS similarity is not possible.   [1]

5. Answer (a)

$BC = \sqrt{AC^2 + AB^2} = x\sqrt{2}$ units   [1]

6. Answer (c)

$\dfrac{BC}{PR} = \dfrac{x\sqrt{2}}{2x\sqrt{2}} = \dfrac{1}{2}$

$\therefore BC : PR = 1 : 2$ [1]

7. Answer (d)

$\because \angle A = 90°$ and $\angle P = 45°$

$\therefore \triangle PQR$ is not similar to $\triangle ABC$.   [1]

8. Answer (b)

Here, $\angle F = \angle C$, $\angle B = \angle E$ and $AB = \dfrac{1}{2}DE$

Since, AB and DE are not equal.

So, $\triangle ABC \sim \triangle DEF$.   **[1]**

9. Answer (d)
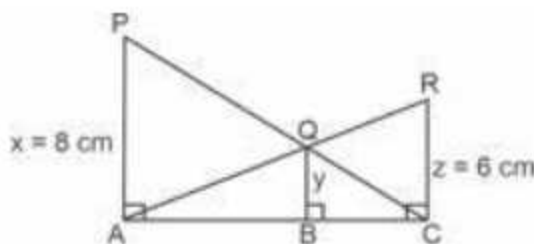
$$\frac{y}{x} = \frac{BC}{AC} \qquad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{AB + BC}{BC} = \frac{AB}{BC} + 1 \qquad \text{...(i)}$$

and $\dfrac{z}{y} = \dfrac{AC}{AB} = 1 + \dfrac{BC}{AB} = 1 + \dfrac{y}{x - y}$ [By BPT]

$$\text{[From (i)]}$$

$$\Rightarrow \quad \frac{6}{y} = 1 + \frac{y}{8 - y}$$

$$\Rightarrow \quad 8y = 48 - 6y$$

$$\Rightarrow \quad y = \frac{24}{7} \text{ cm} \qquad \text{[1]}$$

10. Answer (a)

$$y^\circ - (3x - 2)^\circ = 9^\circ$$

$$\Rightarrow \quad 3x^\circ - y^\circ = -7 \qquad \text{...(i)}$$

and $x^\circ + (3x - 2)^\circ + y^\circ = 180^\circ$

$$\Rightarrow \quad 4x^\circ + y^\circ = 182^\circ \qquad \text{...(ii)}$$

$$\Rightarrow \quad x^\circ = \frac{182^\circ - 7^\circ}{7} = 25^\circ \text{ and } y^\circ = 82^\circ$$

$$\Rightarrow \quad \angle A = 25^\circ, \angle B = 73^\circ \text{ and } \angle C = 82^\circ$$

$\therefore$ Sum of greatest and smallest angles

$$= 82^\circ + 25^\circ = 107^\circ \qquad \text{[1]}$$

11. Answer (a)   **[1]**

$\triangle AOB \sim \triangle COD$ [By AA similarity]

12. Answer (d)

$$\frac{\text{Perimeter of } \triangle AOB}{\text{Perimeter of } \triangle COD} = \frac{AB}{CD}$$

or $\dfrac{AB}{CD} = \dfrac{1}{4}$ **[1]**

13. Answer (b)

If $\dfrac{AO}{BC} = \dfrac{AD}{BO} = \dfrac{OD}{OC}$

or $\triangle AOD \sim \triangle BCO$ **[1]**

14. Answer (b)   **[1]**

$\angle E = \angle B = 83^\circ$

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 50^\circ$

15. Answer (b)   **[1]**

$$\frac{AP}{AB} = \frac{AQ}{AC} \qquad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{4}{10} = \frac{8}{AC}$$

$$AC = 20 \text{ cm}$$

16. Answer (b)   **[1]**

$\triangle ABC \sim \triangle QPR$

$$\Rightarrow \quad \frac{AC}{BC} = \frac{QR}{PR}$$

$$\Rightarrow \quad \frac{6}{5} = \frac{3}{x}$$

$$\Rightarrow \quad x = 2.5 \text{ cm}$$

17. Length of the diagonals of a rhombus are 30 cm and 40 cm.
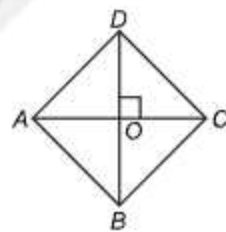
i.e., $BD = 30$ cm

$AC = 40$ cm

$\therefore \quad OD = OB = 15$ cm

$\quad OA = OC = 20$ cm   **[½]**

In $\triangle AOD$,

$OA^2 + OD^2 = AD^2$

$(20)^2 + (15)^2 = AD^2$

$AD = 25$ cm

Side of rhombus = 25 cm   **[½]**

18.

Given $\triangle LMN \sim \triangle PQR$

In similar triangles, corresponding angles are equal.

$\therefore \quad \angle L = \angle P$

$\angle M = \angle Q$

$\angle N = \angle R$

In $\triangle LMN$,

$\angle L + \angle M + \angle N = 180°$

$\angle M = 180° - 50° - 60°$

$\angle M = 70°$

$\therefore \quad \angle Q = 70°$ **[1]**

19.
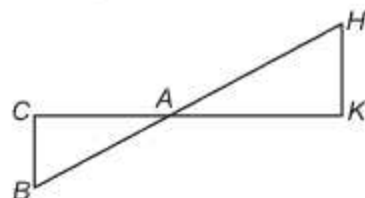
Given $\triangle AHK \sim \triangle ABC$

$\Rightarrow \dfrac{AH}{AB} = \dfrac{HK}{BC} = \dfrac{AK}{AC}$ **[½]**

Also, we know $AK = 10$ cm, $BC = 3.5$ cm and $HK = 7$ cm.

$\Rightarrow \dfrac{AK}{AC} = \dfrac{HK}{BC}$

$\Rightarrow \dfrac{10}{AC} = \dfrac{7}{3.5}$

$\boxed{AC = 5 \text{ cm}}$ **[½]**

20. Let perimeters of two similar triangles be P_1 and P_2 and their corresponding sides be a_1 and a_2

$\therefore \quad \dfrac{P_1}{P_2} = \dfrac{a_1}{a_2}$

$\Rightarrow \dfrac{25}{15} = \dfrac{9}{a_2}$

$\Rightarrow a_2 = 5.4$ cm **[1]**

21. $\because \ DE \parallel BC$

$\therefore \quad \dfrac{AD}{AB} = \dfrac{AE}{AC}$ **[½]**

$\Rightarrow \dfrac{2.4}{3.2} = \dfrac{AE}{8}$

$\therefore \quad AE = \dfrac{24}{32} \times 8 = 6$ cm **[½]**

22.
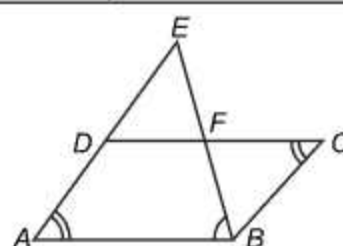
In $\triangle ABE$ and $\triangle CFB$,

$\angle A = \angle C$ (Opposite angles of a parallelogram)

[1]

$\angle AEB = \angle CBF$

(Alternate interior angles as $AE \parallel BC$)

$\therefore \quad \triangle ABE \sim \triangle CFB$ (By AA similarly criterion)

[1]

23. In $\triangle BAC$; $DE \parallel AC$

$\dfrac{BE}{EC} = \dfrac{BD}{DA}$ ...(i) {By B.P.T} **[½]**

In $\triangle BAP$; $DC \parallel AP$

$\dfrac{BC}{CP} = \dfrac{BD}{DA}$ ...(ii) {By B.P.T} **[½]**

From (i) and (ii), we have **[½]**

$\dfrac{BE}{EC} = \dfrac{BC}{CP}$ Hence proved. **[½]**

24. In POR,

$AC \parallel PR$

$\dfrac{OA}{AP} = \dfrac{OC}{CR}$ [BPT] ...(i) **[½]**

In $\triangle OPQ$,

$AB \parallel PQ$

$\dfrac{OA}{AP} = \dfrac{OB}{BQ}$ [BPT]...(ii) **[½]**

From (i) and (ii) we get

$\dfrac{OC}{CR} = \dfrac{OB}{BQ}$ **[½]**

So, by converse of BPT

$BC \parallel QR$ **[½]**

25.
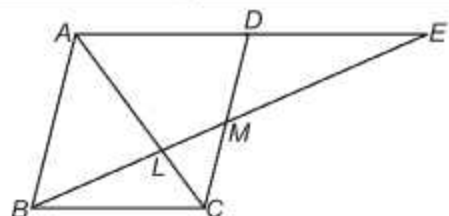
In $\triangle DME$ and $\triangle CMB$

$\angle EDM = \angle MCB$ [Alternate angles]

$DM = CM$ [M is mid-point of CD]

$\angle DME = \angle BMC$ [Vertically opposite angles]

By ASA congruency $\triangle DME \cong \triangle CMB$ **[1]**

By CPCT

$BM = ME$

$DE = BC$

Now in

$\triangle ALE$ and $\triangle BLC$

$\angle ALE = \angle BLC$ [VOA]

$\angle LAE = \angle LCB$ [Alternate angles]

By AA similarly

$\triangle ALE \sim \triangle CLB$ **[1]**

$\Rightarrow \dfrac{AE}{BC} = \dfrac{AL}{CL} = \dfrac{LE}{LB}$

$\Rightarrow \dfrac{EL}{BL} = \dfrac{AE}{BC}$

$\Rightarrow \dfrac{EL}{BL} = \dfrac{AD + DE}{BC}$

$\Rightarrow \dfrac{EL}{BL} = \dfrac{BC + BC}{BC}$

$\Rightarrow \boxed{EL = 2BL}$ **[1]**

26. Given : $\angle D = \angle E$
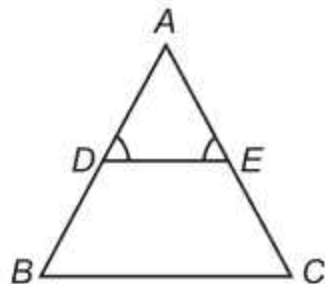
$\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **[½]**

To Prove : $\triangle BAC$ is an isosceles triangle.

Proof : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (Given)   **[½]**

$\therefore$ $DE \parallel BC$ [By converse of B.P.T]

$\Rightarrow \angle D = \angle B$ ...(i)   **[½]**

[Corresponding angles]

$\angle E = \angle C$ ...(ii)

[Corresponding angles]

But $\angle D = \angle E$ (Given)   **[½]**

From (i) and (ii)

$\therefore$ $\angle B = \angle C \Rightarrow AB = AC$ **[½]**

Hence, $\triangle BAC$ is an isosceles triangle.   **[½]**

27. **Given :** $DEFG$ is a square and $\triangle ABC$ is a right triangle right angled at A.
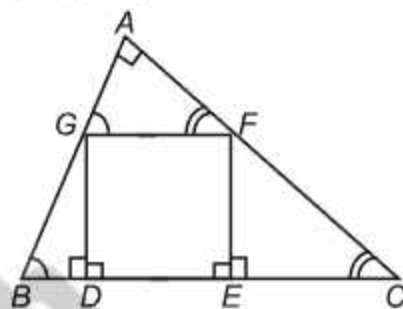
To prove : (i) $\triangle AGF \sim \triangle DBG$

(ii) $\triangle AGF \sim \triangle EFC$

Proof :

(i) In $\triangle AGF$ and $\triangle DBG$

$\angle A = \angle D = 90°$

and $\angle AGF = \angle GBD = 90°$

($\because GF \parallel BC \Rightarrow$ Corresponding angles)   **[1]**

By AA similarity

$\triangle AGF \sim \triangle DBG$ **[1]**

(ii) In $\triangle AGF$ and $\triangle EFC$

$\angle A = \angle E = 90°$

$\angle AFG = \angle ECF = 90°$

($\because GF \parallel BC \Rightarrow$ Corresponding angles)   **[1]**

By AA similarity

$\triangle AGF \sim \triangle EFC$ **[1]**

Hence proved.

28. (i) Figure A and figure C **[1]**

(ii) Figure C **[1]**

(iii) (a) Let $\triangle ABC \cong \triangle PQR$

By CPCT,

$\angle ABC = \angle PQR$ [½]

$\angle BAC = \angle QPR$

Using AA similarity criterion,

$\triangle ABC \sim \triangle PQR$ [½]

Converse may not be true, for example :

Let ABC and PQR be two triangles such that

$AB = 3$ cm, $BC = 4$ cm, $AC = 5$ cm

$PQ = 6$ cm, $QR = 8$ cm, $PR = 10$ cm

Here, $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = \dfrac{1}{2}$

$\Rightarrow \triangle ABC \sim \triangle PQR$

    [By SSS similarity criterion] [½]

    But,

$\triangle ABC \ncong \triangle PQR$ as $AB \ne PQ$. [½]

OR

(iii) (b) For two similar triangles to be congruent, ratio of length of corresponding sides should be 1 : 1

    i.e. if $\triangle ABC \sim \triangle PQR$, then

$\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = 1$ [1]

$\Rightarrow \triangle ABC \cong \triangle PQR$ [1]

29. (A)
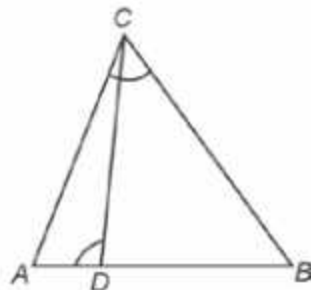
In $\triangle ACB$ and $\triangle ADC$

$\angle CAB = \angle CAD$ [Common] [½]

$\angle ADC = \angle BCA$ [Given] [½]

$\therefore \triangle ACB \sim \triangle ADC$ [By AA similarity criterion] [1]

$\dfrac{AC}{AD} = \dfrac{BC}{CD} = \dfrac{AB}{AC}$ [½]

$\Rightarrow AC \times AC = AD \times AB$ [½]

$\Rightarrow 8 \times 8 = 3(AB)$ [½]

$\Rightarrow AB = \dfrac{64}{3}$ [½]

$\therefore BD = AB - AD$ [½]

$= \dfrac{64}{3} - 3$

$= \dfrac{55}{3}$ [½]

$\therefore BD = \dfrac{55}{3}$ cm

OR

(B)
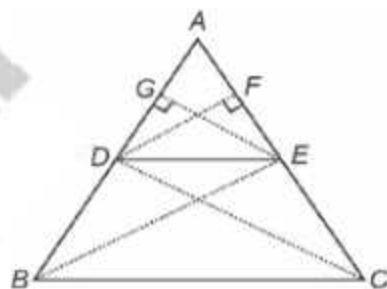
Given : $\triangle ABC$, in which DE is drawn parallel to BC [½]

To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction : Join CD and BE. Draw $DF \perp AE$ and $EG \perp AD$ [½]

Proof : $\text{ar}(\triangle ADE) = \dfrac{1}{2} \times AD \times EG$ ...(i) [½]

$\text{ar}(\triangle BDE) = \dfrac{1}{2} \times BD \times EG$ ...(ii)

Dividing (i) by (ii), we get

$\dfrac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \dfrac{\dfrac{1}{2} \times AD \times EG}{\dfrac{1}{2} \times BD \times EG} = \dfrac{AD}{BD}$ ...(iii) [½]

Similarly,

$$ar(\triangle ADE) = \frac{1}{2} \times DF \times AE$$ [½]

$$ar(\triangle CDE) = \frac{1}{2} \times CE \times DF$$ [½]

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle CDE)} = \frac{\frac{1}{2} \times DF \times AE}{\frac{1}{2} \times DF \times CE} = \frac{AE}{CE} \dots(iv)$$ [½]

$$ar(\triangle BDE) = ar(\triangle CDE)$$

[∵ Triangles on the same base and between

the same parallel lines are equal in area]   [½]

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{ar(\triangle ADE)}{ar(\triangle CDE)}$$ [½]

∴   From (iii) & (iv), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ [½]

Hence proved.

30.
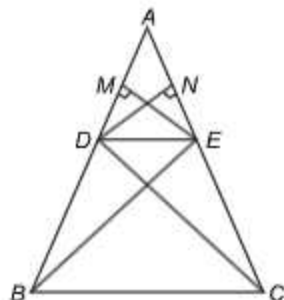
Construction: Join BE and CD and draw perpendicular DN and EM to AC and AB respectively.

$$\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{\frac{1}{2} \times EM \times AD}{\frac{1}{2} \times BD \times EM}$$

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{AD}{BD} \dots(i)$$ [1]

Similarly,

$$\frac{ar(\triangle ADE)}{ar(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$$

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle CDE)} = \frac{AE}{EC} \dots(ii)$$ [1]

But ar(△BDE) = ar(△CDE) (∵ Triangles on same base DE and between the same parallels DE and BC)

Thus, equation (ii) becomes,

$$\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{AE}{EC} \dots(iii)$$ [1]

From equations (i) and (iii), we have,

$$\frac{AD}{BD} = \frac{AE}{EC}$$ [1]

In the given figure, AB || DE and BC || EF.
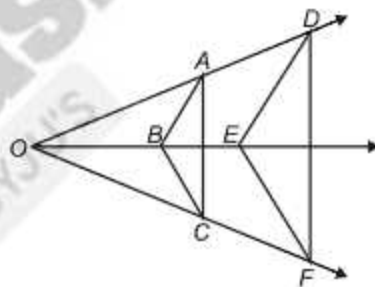
In △ODE, AB || DE (Given)

∴   By basic proportionality theorem,

$$\frac{OA}{AD} = \frac{OB}{BE} \dots(i)$$ [1]

Similarly, in △OEF, BC || EF (Given)

∴ $$\frac{OB}{BE} = \frac{OC}{CF} \dots(ii)$$

Comparing (i) and (ii), we get

$$\frac{OA}{AD} = \frac{OC}{CF}$$

Hence, AC || DF   [1]

[By the converse of BPT]

7 : Coordinate Geometry

1. Answer (a)

Given a line segment joining

$A(-6, 5)$ and $B(-2, 3)$ [½]
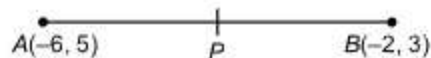
Midpoint of A & B is $P\left(\dfrac{a}{2}, 4\right)$

$\left(\dfrac{a}{2}, 4\right) = \left(\dfrac{-6-2}{2}, \dfrac{5+3}{2}\right)$

$\dfrac{a}{2} = -\dfrac{8}{2}$ [On comparing]

$\boxed{a = -8}$ [½]

2. Answer (b)

Given 2 points are $A(-6, 7)$ and $B(-1, -5)$

Distance between the points = AB

$= \sqrt{(-6+1)^2 + (7+5)^2}$ [½]

$= \sqrt{25 + 144}$

$\Rightarrow AB = 13$

$\Rightarrow 2AB = 26$ [½]

3. Answer (b)

It is given that the point P divides AB in the ratio $2 : 1$.

Using section formula, the coordinates of the point P are

$\left(\dfrac{1 \times 1 + 2 \times 4}{2+1}, \dfrac{1 \times 3 + 2 \times 6}{2+1}\right) = \left(\dfrac{1+8}{3}, \dfrac{3+12}{3}\right) = (3, 5)$

    [½]

Hence the coordinates of the point P are $(3, 5)$.

    [½]

4. Answer (a)

Let the coordinates of the other end of the diameter be (x, y).

We know that the centre is the mid-point of the diameter. So, $O(-2, 5)$ is the mid-point of the diameter AB.

The coordinates of the point A and B are $(2, 3)$ and (x, y) respectively.

Using mid-point formula, we have,

$-2 = \dfrac{2+x}{2} \Rightarrow -4 = 2 + x \Rightarrow x = -6$

$5 = \dfrac{3+y}{2} \Rightarrow 10 = 3 + y \Rightarrow y = 7$ [½]

Hence, the coordinates of the other end of the diameter are $(-6, 7)$.   [½]

5. Using distance formula

$\ell(OP) = \sqrt{(x-0)^2 + (y-0)^2}$ [½]

$\ell(OP) = \sqrt{x^2 + y^2}$ [½]

6. Let the centre be O and coordinates of point A be (x, y)

$\dfrac{x+1}{2} = 2$ [By Mid-point formula]

$\Rightarrow x = 3$ [½]

$\dfrac{y+4}{2} = -3$

$\Rightarrow y = -10$ [½]

$\therefore$ Coordinates of $A = (3, -10)$

7. Answer (b)   [1]

Distance of point $(3, 4)$ from x-axis is its y-coordinate.

8. Answer (c)   [1]

$A(4, p)$

$B(1, 0)$

$AB = 5$

$\therefore \sqrt{(4-1)^2 + (p-0)^2} = 5$

$\Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 16$

$\Rightarrow p = \pm 4$

9.

$C(k, 4)$ divides AB in the ratio $2 : 3$

$\Rightarrow C(k, 4) = \left(\dfrac{2 \times 3 + 5 \times 2}{2+3}, \dfrac{6 \times 3 + 1 \times 2}{2+3}\right)$

$\Rightarrow (k, 4) = \left(\dfrac{16}{5}, \dfrac{20}{5}\right)$

$\Rightarrow k = \dfrac{16}{5}$ [1]

10. Answer (c)   [1]

Distance between $A(a\cos\theta + b\sin\theta, 0)$ and $B(0, a\sin\theta - b\cos\theta)$ is

$$AB = \sqrt{\left((a\cos\theta + b\sin\theta) - 0\right)^2 + \left(0 - (a\sin\theta - b\cos\theta)\right)^2}$$

$$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (b\cos\theta - a\sin\theta)^2}$$

$$= \sqrt{a^2 + b^2}$$

Option (c) is correct.

11. Answer (d)   [1]
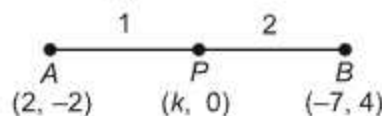
$$\therefore \quad k = \frac{(1 \times -7) + (2 \times 2)}{1+2} \quad \text{[Using section formula]}$$

$$\boxed{k = -1}$$

Hence, option (d) is correct.

12. Answer (a)

$$(x - 0)^2 + (1 - 0)^2 = (x - 2)^2 + (1 - 0)^2$$

$$x^2 + 1 = x^2 + 4 - 4x + 1$$

$$x = 1 \quad [1]$$

13. Answer (b)   [1]

Let $P(4, 0)$ divides $A(4, 6)$ and $B(4, -8)$ in $k : 1$.

Applying section formula

$$\therefore \quad 4 = \frac{k(4) + 1(4)}{k+1}$$
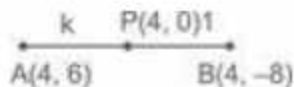
$$0 = \frac{-8k + 1(6)}{k+1} \Rightarrow k = \frac{3}{4} \text{ or } 3 : 4$$

14. Answer (b)   [1]

$$OD = \frac{OB}{2} = 3 \text{ units}$$

$$OA = 4 \text{ units} \quad \text{[Given]}$$

$$\therefore \quad AD = \sqrt{OD^2 + OA^2} \quad [\because \angle AOD = 90°]$$

$$AD = 5 \text{ units}$$

15. Answer (b)   [1]

Let $(0, 0)$ divides the line segment AB in $k : 1$.

$$\therefore \quad \frac{1 - 3k}{k+1} = 0 \text{ and } \frac{-3 + 9k}{k+1} = 0$$

$$\Rightarrow \quad k = \frac{1}{3}$$

Required ratio = 1 : 3

16. Answer (d)   [1]

Let $O(0, 0)$ be centre of circle.

and $A(-1, -1)$, $B(0, 3)$, $C(1, 2)$, $D(3, 1)$

$$OA = \sqrt{(0+1)^2 + (0+1)^2} = \sqrt{2} \text{ units}$$

$$OB = \sqrt{(0-0)^2 + (3-0)^2} = 3 \text{ units}$$

$$OC = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5} \text{ units}$$

$$OD = \sqrt{(3-0)^2 + (1-0)^2} = \sqrt{10} \text{ units} > 3 \text{ units}$$

So, $(3, 1)$ lies outside the circle.

17. Answer (c)   [1]

Let $C(x, y)$ be the mid-point.

Applying mid-point formula
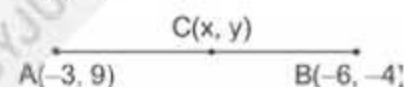
$$x = \frac{-3 - 6}{2} = \frac{-9}{2}$$

$$y = \frac{9 - 4}{2} = \frac{5}{2}$$

So, mid-point is $\left(\frac{-9}{2}, \frac{5}{2}\right)$.

18. Answer (c)   [1]

A, B and C are the vertices of an equilateral triangle, then $AB = BC$

$$\sqrt{(3-0)^2 + (\sqrt{3} - 0)^2} = \sqrt{(3-0)^2 + (k - 0)^2}$$

$$\Rightarrow \quad \sqrt{9 + 3} = \sqrt{9 + k^2}$$

Squaring both sides,

$$12 = 9 + k^2$$

$$k^2 = 3 \text{ or } k = \pm\sqrt{3}$$

19. Answer (b)   **[1]**

ABCD is a parallelogram.

Hence, O is the mid-point of both AC and BD.
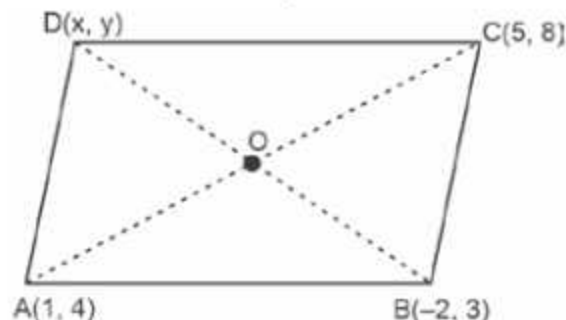
∴ For ordinate of point D,

$$\frac{y+3}{2} = \frac{4+8}{2}$$

$$\Rightarrow \quad y = 9$$

20. Answer (b)   **[1]**

$$\sqrt{(2+1)^2 + (y+3y)^2} = \sqrt{(5-2)^2 + (7+3y)^2}$$

$$\Rightarrow \quad 9 + 16y^2 = 9 + 49 + 9y^2 + 42y$$

$$\Rightarrow \quad 7y^2 - 42y - 49 = 0$$

$$\Rightarrow \quad y^2 - 6y - 7 = 0$$

$$\Rightarrow \quad (y - 7)(y + 1) = 0$$

$$\therefore \quad y = 7, -1$$

21. Answer (c)   **[1]**

Let $3x + y - 9 = 0$ divides the line segment formed by joining the point $A(1, 3)$ and $B(2, 7)$ in $k : 1$

(i.e., at point C).
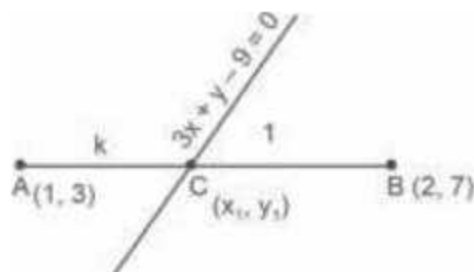
Now, $x_1 = \dfrac{2k+1}{k+1}$ and $y_1 = \dfrac{7k+3}{k+1}$

Point C lies on $3x + y - 9 = 0$, then

$$3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) - 9 = 0$$

$$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0$$

$$\Rightarrow \quad 4k - 3 = 0$$

$$\Rightarrow \quad k = \frac{3}{4}$$

22. Answer (c)   **[1]**
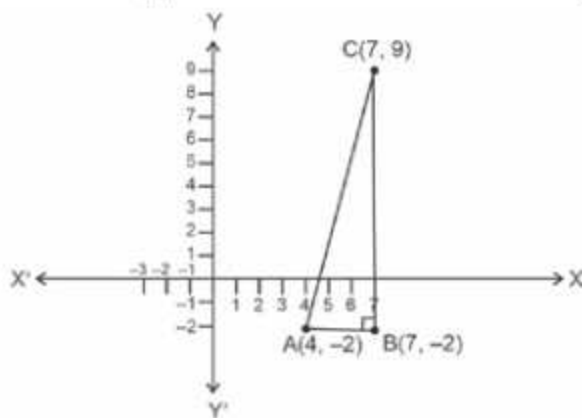
∴ ΔABC is a right angled triangle

23. Answer (d)   **[1]**

Any point on y-axis is of the form $(0, y)$

Let R divides PQ in the ratio $k : 1$

$$\therefore \quad 0 = \frac{5k + 1(-3)}{k+1}$$
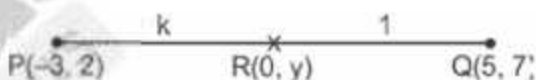
$$\Rightarrow \quad 5k - 3 = 0$$

or $\quad k = \dfrac{3}{5}$

24. Answer (d)   **[1]**

Required distance $= \sqrt{(-6)^2 + (8)^2}$

$$= \sqrt{100}$$

$$= 10 \text{ units}$$

25. Answer (b)   **[1]**

Distance of point $(-1, 7)$ from x-axis is 7.

26.
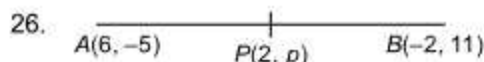
Given P is midpoint of AB

$$\therefore \quad (2, p) = \left(\frac{6-2}{2}, \frac{-5+11}{2}\right)$$ **[½]**

$$(2, p) = (2, 3)$$

$$\therefore \quad \boxed{p = 3}$$ **[½]**

27.
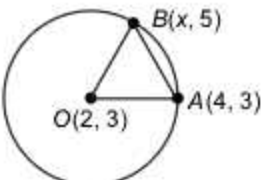
Let O be the mid-point of diagonals AC and BD of the parallelogram $ABCD$ and coordinates of D is (x, y) then

$$\left(\frac{6+1}{2}, \frac{6+2}{2}\right) = \left(\frac{x+4}{2}, \frac{y+3}{2}\right)$$ [½]

On comparing

$\frac{x+4}{2} = \frac{7}{2},$ $\frac{8}{2} = \frac{y+3}{2}$

$x = 7 - 4$ $8 = y + 3$

$x = 3$ $y = 8 - 3 = 5$

Hence coordinates of $D = (3, 5)$ [½]

28.

$OA = \sqrt{(2-4)^2 + (3-3)^2} = 2$ [½]

$OB = \sqrt{(2-x)^2 + (3-5)^2} = \sqrt{(2-x)^2 + 4}$ [½]

$\Rightarrow 2 = \sqrt{(2-x)^2 + 4}$ [∵ $OA = OB$ (radii)]

$4 = (2-x)^2 + 4$ [½]

$\Rightarrow \boxed{x = 2}$ [½]

29. Distance between the points $A(3, -1)$ and $B(11, y)$ is 10 units

$AB = 10$

$\sqrt{(3-11)^2 + (-1-y)^2} = 10$ [½]

$64 + (y+1)^2 = 100$ [½]

$(y+1)^2 = 36$

$y + 1 = 6$ or $y + 1 = -6$ [½]

$\therefore \boxed{y = -7, 5}$ [½]

30. It is given that the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$.

So, $AB = AC \Rightarrow AB^2 = AC^2$ [½]

Using distance formula, we have :

$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$ [½]

$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$

$\Rightarrow 4 - 4p = 0$ [½]

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$ [½]

31. $\triangle ABC$ is right angled at B.

$\therefore AC^2 = AB^2 + BC^2$ ...(i) [Pythagoras]

Now, $AC^2 = (7-4)^2 + (3-7)^2 = (3)^2 + (-4)^2 = 9 + 16 = 25$

$AB^2 = (p-4)^2 + (3-7)^2 = p^2 - 8p + 16 + (-4)^2$

$= p^2 - 8p + 16 + 16$

$= p^2 - 8p + 32$

$BC^2 = (7-p)^2 + (3-3)^2 = 49 - 14p + p^2 + 0$

$= p^2 - 14p + 49$ [1]

From (i), we have

$25 = (p^2 - 8p + 32) + (p^2 - 14p + 49)$

$\Rightarrow 25 = 2p^2 - 22p + 81$

$\Rightarrow 2p^2 - 22p + 56 = 0$

$\Rightarrow p^2 - 11p + 28 = 0$

$\Rightarrow p^2 - 7p - 4p + 28 = 0$

$\Rightarrow p(p-7) - 4(p-7) = 0$

$\Rightarrow (p-7)(p-4) = 0$

$\Rightarrow p = 7$ and $p = 4$ [1]

32. Let $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ be the given points of the vertices of triangle.

Now,

$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25}$ ...(i)   [½]

$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2}$

$= \sqrt{49+1} = \sqrt{50}$ ...(ii)   [½]

$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16+9} = \sqrt{25}$ ...(iii)   [½]

$\therefore BC^2 = AB^2 + AC^2$ and $AB = AC$

Hence triangle is isosceles right triangle.   [½]

Thus, $\triangle ABC$ is a right-angled isosceles triangle.

33. Let the coordinates of points P and Q be $P(0, a)$ and $Q(b, 0)$ respectively.

    [$\because$ P on y-axis Q on x-axis]   [½]

    Coordinates of mid-point of PQ

    $$= \left(\frac{0+b}{2}, \frac{0+a}{2} \right)$$

    $$= \left(\frac{b}{2}, \frac{a}{2} \right)$$ [½]

    On comparing with $(2, -5)$

    $\frac{b}{2} = 2$ and $\frac{a}{2} = -5$

    $b = 4$, $a = -10$ [½]

    Hence coordinates of $P = (0, -10)$

    Hence coordinates of $Q = (4, 0)$ [½]

34. Given that

    $PA = PB$

    By using distance formula

    $\sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$ [½]

    Squaring on both sides

    $\Rightarrow$ $x^2 + 25 - 10x + y^2 - 2y + 1$

    $= x^2 + 2x + 1 + y^2 - 10y + 25$ [½]

    $\Rightarrow$ $-10x - 2y = 2x - 10y$ [½]

    $\Rightarrow$ $8y = 12x$

    $\therefore$ $3x = 2y$ [½]

35. Suppose the point $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$ in the ratio $K : 1$.

    ```
    A       K       P       1       B
    •───────────────•───────────────•
    (2, 3)          (4, m)          (6, -3)
    ```
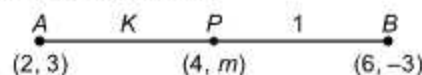
    Co-ordinates of point $P = \left(\frac{6K+2}{K+1}, \frac{-3K+3}{K+1} \right)$ [½]

    But the co-ordinates of point P are given as $(4, m)$

    $\frac{6K+2}{K+1} = 4$ ...(i)

    $\frac{-3K+3}{K+1} = m$ ...(ii)   [½]

    $\Rightarrow$ $6K + 2 = 4K + 4$ [From (i)]

    $\Rightarrow$ $2K = 2$

    $\Rightarrow$ $K = 1$

    Putting $K = 1$ in equation (ii)

    $\frac{-3(1)+3}{1+1} = m$

    $\therefore$ $m = 0$ [½]

Ratio is $1 : 1$ and $m = 0$

i.e. P is the mid-point of AB [½]

36. Let $P(x, y)$ divides the line segment joining the points $A(1, -3)$ and $B(4, 5)$ internally in the ratio $k : 1$.

    Using section formula, we get

    $x = \frac{4k+1}{k+1}$ ...(i)

    $y = \frac{5k-3}{k+1}$ ...(ii)   [½]

    Since, P lies on x-axis. So its ordinate will be zero.

    ```
            P
    •───────┼───────•
    A(1,-3)  k:1   B(4, 5)
    ```
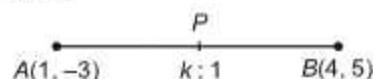
    $\Rightarrow$ $\frac{5k-3}{k+1} = 0$

    $\Rightarrow$ $k = \frac{3}{5}$

    Hence, the required ratio is $3 : 5$.   [½]

    Now putting the value of k in (i) and (ii), we get

    $x = \frac{17}{8}$ and $y = 0$

    So, coordinates of point P are $\left(\frac{17}{8}, 0 \right)$ [1]

37. (A) Let the point be $P(h, k)$

    ```
    A(-1,7)      P(h, k)      B(4, -3)
    ←── 2 ──→P←── 3 ──→
    ```
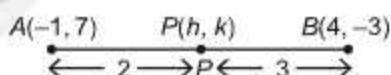
    using section formula,

    $h = \frac{2(4) + 3(-1)}{2+3}$ [½]

    $= \frac{5}{5}$

    $= 1$ [½]

    $k = \frac{2(-3) + 3(7)}{2+3} = 3$

    $P(h, k) = P(1, 3)$ [1]

    OR

    (B)

    ```
    D(p, 4)            C(6, 7)

    A(2, 3)            B(-5, 6)
    ```
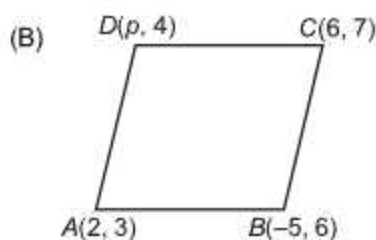
ABCD is a parallelogram

$\Rightarrow AB = CD$ [½]

$\Rightarrow \sqrt{(6-3)^2+(-5-2)^2} = \sqrt{(7-4)^2+(6-p)^2}$ [½]

$\Rightarrow 9 + 49 = 9 + (6-p)^2$

$\Rightarrow 6 - p = \pm 7$ [½]

$\Rightarrow p = 6 - 7$ and $p = 6 + 7$

$\Rightarrow p = -1, 13$

$\Rightarrow p = -1$ (Not possible as in this case $AD \neq BC$)

$\Rightarrow p = 13$ [½]

38.

$\dfrac{AP}{AB} = \dfrac{3}{7}$

As, $AB = 7a$, $AP = 3a$

$\Rightarrow AB = AP + PB$

$\Rightarrow 7a = 3a + PB$

$\Rightarrow PB = 7a - 3a = 4a$ [1]

Let the point $P(x, y)$ divide the line segment joining the points $A(-2, -2)$ and $B(2, -4)$ in the ratio $AP : PB = 3 : 4$ [½]

$\Rightarrow x = \dfrac{2(3)+(-2)(4)}{3+4}$ and $y = \dfrac{(-4)(3)+(4)(-2)}{3+4}$

[1]

$\Rightarrow x = \dfrac{6-8}{7}$ and $y = \dfrac{-12-8}{7}$

$\Rightarrow x = \dfrac{-2}{7}$ and $y = \dfrac{-20}{7}$

$\Rightarrow$ The coordinate of $P(x, y) = \left(\dfrac{-2}{7}, \dfrac{-20}{7}\right)$ [½]

39.
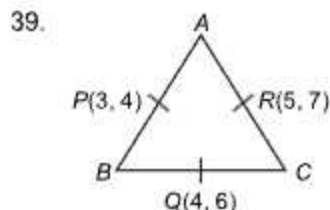
Consider a $\triangle ABC$ with $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, $P(3, 4)$, $Q(4, 6)$ and $R(5, 7)$ are the mid-points of *AB*, *BC* and *CA*. Then,

$3 = \dfrac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 6$ ...(i)

$4 = \dfrac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 8$ ...(ii)

$4 = \dfrac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 8$ ...(iii)

$5 = \dfrac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 12$ ...(iv)

$6 = \dfrac{x_3 + x_1}{2} \Rightarrow x_3 + x_1 = 10$ ...(v)

$7 = \dfrac{y_3 + y_1}{2} \Rightarrow y_2 + y_1 = 14$ ...(vi) [½]

On adding (i), (iii) and (v) we get

$2(x_1 + x_2 + x_3) = 6 + 8 + 10 = 24$

$\Rightarrow x_1 + x_2 + x_3 = 12$ ...(vii) [½]

From (i) and (vii), we get $x_3 = 12 - 6 = 6$

From (iii) and (vii) we get $x_1 = 12 - 8 = 4$

From (v) and (vii), we get $x_2 = 12 - 10 = 2$ [½]

Now, adding (ii), (iv) and (vi), we get

$20(y_1 + y_2 + y_3) = 8 + 12 + 14 = 34$

$\Rightarrow y_1 + y_2 + y_3 = 17$ ...(viii) [½]

From (ii) and (viii), we get $y_3 = 17 - 8 = 9$

From (iv) and (viii), we get $y_1 = 17 - 12 = 5$

From (vi) and (viii), we get $y_2 = 17 - 14 = 3$ [½]

Hence, the vertices of $\triangle ABC$ are $A(4, 5)$, $B(2, 3)$, $C(6, 9)$. [½]

40.
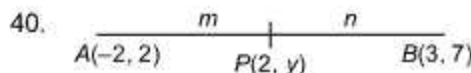
Lets say ratio = $m : n$

$\therefore (2, y) = \left(\dfrac{3m - 2n}{m+n}, \dfrac{2n + 7m}{m+n}\right)$ [1]

$2 = \dfrac{3m - 2n}{m+n}$

$2m + 2n = 3m - 2n$

$m : n = 4 : 1$ [1]

$y = \dfrac{2 + 7 \times 4}{5}$

$y = \dfrac{30}{5}$

$y = 6$ [1]

41.

A(2, 1) —— 1 —— P —— 2 —— B(5, −8)
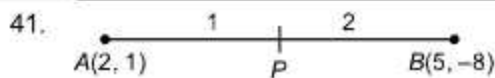
Given :

$$\frac{AP}{AB} = \frac{1}{3}$$

$$\Rightarrow \frac{AP}{AP+PB} = \frac{1}{3}$$

$$PB = 2AP$$

$$\Rightarrow AP : PB = 1 : 2 \qquad [1]$$

By section formula

$$\therefore \quad P = \left(\frac{2\times2+5}{3}, \frac{2-8}{3}\right)$$

$$P = (3, -2) \qquad [1]$$

Also it is given that P lies on $2x − y + k = 0$

$$\therefore \quad 2(3) − (−2) + k = 0$$

$$\boxed{k = -8} \qquad [1]$$

42.

A(3, −5) —— K —— P(x, y) —— l —— B(−4, 8)
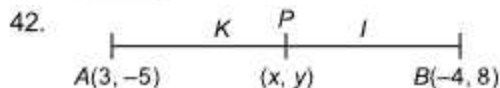
Let the co-ordinates of point P be (x, y)

By using the section formula co-ordinates of P are.

$$x = \frac{-4K+3}{K+1} \qquad y = \frac{8K-5}{K+1} \qquad [1]$$

Since P lies on $x + y = 0$

$$\therefore \quad \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$$

[On putting the values of x and y] [½]

$$\Rightarrow 4K − 2 = 0$$

$$\Rightarrow K = \frac{2}{4} \qquad [½]$$

$$\Rightarrow K = \frac{1}{2}$$

Hence the value of $K = \frac{1}{2}$ \qquad [1]

43. Let the y-axis divide the line segment joining the points (−4, −6) and (10, 12) in the ratio $k : 1$ and the point of the intersection be (0, y). Using section formula, we have:

$$\left(\frac{10k+(-4)}{k+1}, \frac{12k+(-6)}{k+1}\right) = 0, y$$

$$\therefore \quad \frac{10k-4}{k+1} = 0 \Rightarrow 10k - 4 = 0$$

$$\Rightarrow k = \frac{4}{10} = \frac{2}{5} \qquad [1]$$

Thus, the y-axis divides the line segment joining the given points in the ratio 2 : 5

$$\therefore \quad y = \frac{12k+(-6)}{k+1} = \frac{12\times\frac{2}{5}-6}{\frac{2}{5}+1} = \frac{\left(\frac{24-30}{5}\right)}{\left(\frac{2+5}{5}\right)} = \frac{-6}{7}$$

[1]

Thus, the coordinates of the point of division are $\left(0, -\frac{6}{7}\right)$ \qquad [1]

44. The given points are A(−2, 3) B(8, 3) and C(6, 7). Using distance formula, we have :

$$AB^2 = (8 + 2)^2 + (3 − 3)^2$$

$$\Rightarrow AB^2 = 10^2 + 0$$

$$\Rightarrow AB^2 = 100 \qquad [½]$$

$$BC^2 = (6 − 8)^2 + (7 − 3)^2$$

$$\Rightarrow BC^2 = (−2)^2 + 4^2$$

$$\Rightarrow BC^2 = 4 + 16$$

$$\Rightarrow BC^2 = 20 \qquad [½]$$

$$CA^2 = (2 − 6)^2 + (3 − 7)^2$$

$$\Rightarrow CA^2 = (−8)^2 + (−4)^2$$

$$\Rightarrow CA^2 = 64 + 16$$

$$\Rightarrow CA^2 = 80 \qquad [½]$$

It can be observed that :

$$BC^2 + CA^2 = 20 + 80 = 100 = AB^2 \qquad [1]$$

So, by the converse of Pythagoras Theorem,

ΔABC is a right triangle right angled at C. [½]

45. The given points are A(0, 2), B(3, p) and C(p, 5).

It is given that A is equidistant from B and C.

$$\therefore \quad AB = AC$$

$$\Rightarrow AB^2 = AC^2$$

$$\Rightarrow (3 − 0)^2 + (p − 2)^2 = (p − 0)^2 + (5 − 2)^2 \quad [1]$$

$$\Rightarrow 9 + p^2 + 4 − 4p = p^2 + 9$$

$$\Rightarrow 4 − 4p = 0$$

$$\Rightarrow 4p = 4$$

$$\Rightarrow p = 1 \qquad [1]$$

Thus, the value of p is 1

Length of $AB = \sqrt{(3-0)^2 + (1-2)^2} = \sqrt{3^2 + (-1)^2}$

$$= \sqrt{9+1} = \sqrt{10} \text{ units.} \qquad [1]$$

46. Here, $P(x, y)$ divides line segment AB, such that

$$AP = \frac{3}{7}AB$$

$\Rightarrow \quad \dfrac{AP}{AB} = \dfrac{3}{7}$

$\Rightarrow \quad \dfrac{AB}{AP} = \dfrac{7}{3}$

$\Rightarrow \quad \dfrac{AB}{AP} - 1 = \dfrac{7}{3} - 1$ [½]

$\Rightarrow \quad \dfrac{AB - AP}{AP} = \dfrac{7-3}{3}$

$\Rightarrow \quad \dfrac{BP}{AP} = \dfrac{4}{3}$

$\Rightarrow \quad \dfrac{AP}{BP} = \dfrac{3}{4}$ [1]

∴ P divides AB in the ratio $3 : 4$

$x = \dfrac{3\times 2 + 4(-2)}{3+4}; \ y = \dfrac{3\times(-4)+4(-2)}{3+4}$ [½]

$x = \dfrac{6-8}{7}; \ y = \dfrac{-12-8}{7}$

$x = \dfrac{-2}{7}; \ y = \dfrac{-20}{7}$

∴ The coordinates of P are $\left(\dfrac{-2}{7}, \dfrac{-20}{7}\right)$ [1]

47. $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$.

∴ $AP = BP$

$\Rightarrow \sqrt{[x-(a+b)]^2 + [y-(b-a)]^2}$

$\qquad = \sqrt{[x-(a-b)]^2 + [y-(a+b)]^2}$ [1]
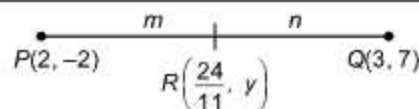
$\Rightarrow [x - (a + b)]^2 + [y - (b - a)]^2$

$\qquad = [x - (a - b)]^2 + [y - (a + b)]^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2$

$\qquad + y^2 - 2y(b - a) + (b - a)^2$

$\qquad = x^2 - 2x(a - b) + (a - b)^2$

$\qquad + y^2 - 2y(a + b) + (a + b)^2$ [1]

$\Rightarrow -2x(a + b) - 2y(b - a)$

$\qquad = -2x(a - b) - 2y(a + b)$

$\Rightarrow ax + bx + by - ay = ax - bx + ay + by$

$\Rightarrow 2bx = 2ay$

∴ $bx = ay$ ....(proved)   [1]

48.

$P(2, -2) \qquad R\left(\dfrac{24}{11}, y\right) \qquad Q(3, 7)$

Lets say ratio is $m + n$

Then

$\left(\dfrac{24}{11}, y\right) = \left(\dfrac{3m+2n}{m+n}, \dfrac{7m-2n}{m+n}\right)$ [1]

$\dfrac{24}{11} = \dfrac{3m+2n}{m+n}, \ y = \dfrac{7m-2n}{m+n}$

∴ $24(m + n) = 11(3m + 2n)$

$24m + 24n = 33m + 22n$

$2n = 9n$

∴ $\dfrac{m}{n} = \dfrac{2}{9} \Rightarrow$ Ratio $= 2 : 9$ [1]

$m = 2, n = 9$

$y = \dfrac{7\times 2 - 2\times 9}{11}$

$y = \dfrac{-4}{11}$ [1]

49. Let the point on y-axis be $P(0, y)$ which is equidistant from the points $A(5, -2)$ and $B(-3, 2)$.   [½]

We are given that $AP = BP$

So, $AP^2 = BP^2$ [½]

i.e., $(5 - 0)^2 + (-2 - y)^2 = (-3 - 0)^2 + (2 - y)^2$ [1]

$\Rightarrow 25 + y^2 + 4 + 4y = 9 + 4 + y^2 - 4y$

$\Rightarrow 8y = -16$

$\Rightarrow y = -2$

Hence, the required point is $(0, -2)$ [1]

50. $AD = 100 \times 1 \text{ m}$

$\qquad = 100 \text{ m}$

Niharika runs $\dfrac{1}{4}$th of $AD = \dfrac{100}{4} = 25 \text{ m}$ on 2nd line.

∴ Coordinates of green flag posted by Niharika are $(2, 25)$

Preet runs $\dfrac{1}{5}$th of $AD = \dfrac{100}{5} = 20 \text{ m}$ on 8th line.

∴ Coordinates of red flag posted by Preet are $(8, 20)$ [1]

(i) Distance between two flags

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{6^2 + (-5)^2}$$

$$= \sqrt{36 + 25}$$

$$= \sqrt{61}\,\text{m} \qquad \text{[1]}$$

(ii) Mid-point of line segment joining the two

$$\text{flags} = \left(\frac{8+2}{2}, \frac{25+20}{2} \right)$$

$$= \left(5, \frac{45}{2} \right) = (5, 22.5)$$

∴ Rashmi will post a blue flag on fifth line at the distance of 22.5 m.   **[1]**

51. Now,

Using section formula

$$\overset{3}{\underset{A(2,5)}{\bullet}} \quad \overset{4}{\underset{C(-1,2)}{\bullet}} \quad \underset{B(x,y)}{\bullet}$$

$$\Rightarrow \quad -1 = \frac{(3 \times x) + (4 \times 2)}{3 + 4} \qquad \text{[½]}$$

$$\Rightarrow \quad -1 = \frac{3x + 8}{7}$$

$$\Rightarrow \quad 3x + 8 = -7 \qquad \text{[½]}$$

$$\Rightarrow \quad 3x = -15$$

$$\Rightarrow \quad x = -5$$

Also,

$$2 = \frac{(3 \times y) + (4 \times 5)}{3 + 4} \qquad \text{[½]}$$

$$\Rightarrow \quad 2 = \frac{3y + 20}{7}$$

$$\Rightarrow \quad 3y + 20 = 14$$

$$\Rightarrow \quad 3y = -6$$

$$\Rightarrow \quad y = -2 \qquad \text{[½]}$$

∴ Coordinates of B are (−5, −2)   **[1]**

52. Let the Point $P(x, 2)$ divide the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $k : 1$

Then, the coordinates of P are

$$\left(\frac{4k + 12}{k + 1}, \frac{-3k + 5}{k + 1} \right) \qquad \text{[½]}$$

Now, the coordinates of P are $(x, 2)$

∴ $\dfrac{4k + 12}{k + 1} = x$ and $\dfrac{-3k + 5}{k + 1} = 2$ **[1]**

$$\frac{-3k + 5}{k + 1} = 2$$

$$\Rightarrow \quad -3k + 5 = 2k + 2$$

$$\Rightarrow \quad 5k = 3$$

$$\Rightarrow \quad k = \frac{3}{5} \qquad \text{[1]}$$

Substituting $k = \dfrac{3}{5}$ in $\dfrac{4k + 12}{k + 1} = x$, we get

$$x = \frac{4 \times \dfrac{3}{5} + 12}{\dfrac{3}{5} + 1} \qquad \text{[½]}$$

$$\Rightarrow \quad x = \frac{12 + 60}{3 + 5}$$

$$\Rightarrow \quad x = \frac{72}{8}$$

$$\Rightarrow \quad x = 9$$

Thus, the value of x is 9   **[½]**

Also, the point P divides the line segment joining the points $A(12, 5)$ and $(4, -3)$ in the ratio $\dfrac{3}{5} : 1$, i.e. $3 : 5$.   **[½]**

53. (i) Coordinates of R are (200, 400)   **[½]**

[∵ Abscissa of R = Abscissa of Q and side of square PQRS = 400 units]

Similarly, coordinates of S are (−200, 400)   **[½]**
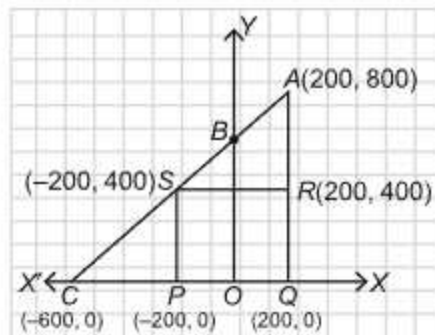
(ii) (a) Area of square $PQRS = PQ^2$ sq. units

$$= \left(\sqrt{(200 + 200)^2 + (0 - 0)^2} \right)^2 \text{ sq. units} \qquad \text{[1]}$$

$$= 160000 \text{ sq. units} \qquad \text{[1]}$$

OR

(b) Length of diagonal $PR = \sqrt{PQ^2 + QR^2}$

[1]

$$= \sqrt{400^2 + 400^2}$$

$$= 400\sqrt{2} \text{ units} \quad [1]$$

(iii) Here,

⇒ Coordinates of C are $(-600, 0)$,

Coordinates of A are $(200, 800)$.

And Coordinates of S are $(-200, 400)$

⇒ $(-200, 400) = \left(\dfrac{200K + (-600)}{K+1}, \dfrac{800K + 0}{K+1} \right)$

[Using section formula] [½]

⇒ $400K + 400 = 800K + 0$

[Comparing y –coordinates]

⇒ $400K = 400$ [½]

⇒ $K = 1$

54. $ABCD$ is a rectangle where $A(1, 1)$, $B(7, 1)$, $C(7, 5)$ and $D(1, 5)$

(i) Coordinates of the point of intersection of

diagonals AC and $BD = \left(\dfrac{7+1}{2}, \dfrac{1+5}{2} \right)$

$= (4, 3)$ [2]

(ii) Length of diagonal $AC = \sqrt{(7-1)^2 + (5-1)^2}$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$= 2\sqrt{13} \text{ units} \quad [1]$$

(iii) (a) $AB = 6$ units $= CD$ [½]

$BC = 4$ units $= AD$ [½]

Area of campaign Board ABCD $= 6 \times 4$

$= 24$ sq. units [1]

OR

(b) Ratio of the length of side AB to the

length of the diagonal $AC = \dfrac{AB}{AC}$

$$= \dfrac{6}{\sqrt{(5-1)^2 + (7-1)^2}}$$ [1]

$$= \dfrac{6}{\sqrt{16 + 36}}$$

$$= \dfrac{6}{\sqrt{52}} = \dfrac{6}{2\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$$ [½]

$$= 3\sqrt{13} : 13$$ [½]

8 : Introduction to Trigonometry

1. Answer (d)

$\tan^2 45° - \cos2\ 60°$

$$= 1^2 - \left(\dfrac{1}{2} \right)^2 = \dfrac{3}{4}$$ [1]

2. Answer (c)

$\tan 90° = \dfrac{\sin 90°}{\cos 90°} = \dfrac{1}{0} = \text{Not defined}$ [1]

3. Answer (c)

∴ $\angle R = 180° - 90° - 45° = 45°$

$\tan P - \cos^2 R = \tan 45° - \cos^2 45°$

$$= 1 - \left(\dfrac{1}{\sqrt{2}} \right)^2$$

$$= \dfrac{1}{2}$$ [1]

4. Answer (a)

$\sec \theta = \sqrt{1 + \tan^2 \theta}$

$$= \dfrac{\sqrt{13}}{3} \qquad \left[\because \tan \theta = \dfrac{2}{3} \right]$$ [1]

5. Answer (b)

$\sin\theta - \cos\theta = 0$

⇒ $\sin\theta = \cos\theta$

⇒ $\tan\theta = 1$

$\theta = 45°$ [1]

6. Answer (d)

$$\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta} = \dfrac{1 - \sin\theta}{1 - \sin^2\theta} + \dfrac{1 + \sin\theta}{1 - \sin^2\theta}$$

$$= \dfrac{1 - \sin\theta}{\cos^2\theta} + \dfrac{1 + \sin\theta}{\cos^2\theta}$$

$$= \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$$ [1]

7. Answer (b)

    $(1 + \tan^2 A)(1 + \sin A)(1 - \sin A)$

    $= \sec^2 A \times \cos^2 A$

    $= 1$ [1]

8. Answer (d)

    $\sec^2\theta + \csc^2\theta$

    $= 1 + \tan^2\theta + 1 + \cot^2\theta$

    $= 2 + \dfrac{1}{3} + 3$

    $= 5\dfrac{1}{3}$ [1]

9. Answer (a)

    $\sin A = \dfrac{7}{25} = \dfrac{BC}{AC}$

    i.e., $BC = 7a$ and $AC = 25a$, where a is any non-zero positive constant.

    In $\triangle ABC$,

    $\cos C = \dfrac{BC}{AC} = \dfrac{7}{25}$ [1]

10. Answer (a)

    $\tan\theta = \pm\sqrt{\sec^2\theta - 1}$

    $\qquad = \pm\sqrt{2 - 1}$

    $\qquad = \pm 1$

    and $\sin\theta = \pm\sqrt{1 - \dfrac{1}{\sec^2\theta}}$

    $\qquad = \pm\sqrt{1 - \dfrac{1}{2}}$

    $\qquad = \pm\sqrt{\dfrac{1}{2}}$

    $\Rightarrow \dfrac{1 + \tan\theta}{\sin\theta} = \dfrac{(1 \pm 1)(\pm\sqrt{2})}{1}$

    $\qquad = 2\sqrt{2}$ or 0 [1]

11. Answer (c)

    $\tan\theta + \cot\theta = 2$

    $\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

    $\left[\because \cot\theta = \dfrac{1}{\tan\theta}\right]$

    $\Rightarrow (\tan\theta - 1)^2 = 0$

    $\Rightarrow \tan\theta - 1 = 0$

    $\Rightarrow \tan\theta = 1 = \tan 45°$

    $\qquad \theta = 45°$

    $\therefore \quad \sin^3\theta + \cos^3\theta = \sin^3 45° + \cos^3 45°$

    $\left[\because \sin 45° = \cos 45° = \dfrac{1}{\sqrt{2}}\right]$

    $\qquad\qquad = \dfrac{1}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$

    $\qquad\qquad = \dfrac{\sqrt{2}}{2}$ [1]

12. Answer (b)

    $a\cot\theta + b\csc\theta = p$ ...(i)

    $b\cot\theta + a\csc\theta = q$ ...(ii)

    Squaring both the equations and subtracting,

    $p^2 - q^2 = (a\cot\theta + b\csc\theta)^2 - (b\cot\theta + a\csc\theta)^2$

    $= (a^2\cot^2\theta + b^2\csc^2\theta + 2ab\cot\theta\csc\theta) - (b^2\cot^2\theta + a^2\csc^2\theta + 2ab\cot\theta\csc\theta)$

    $= (a^2 - b^2)(\cot^2\theta - \csc^2\theta)$

    $= b^2 - a^2$ $[\because \csc^2\theta - \cot^2\theta = 1]$ [1]

13. Answer (b)   [1]

    $\sec\theta + \tan\theta = p$

    $\Rightarrow \sec\theta - \tan\theta = \dfrac{1}{p}$ $[\because \sec^2\theta - \tan^2\theta = 1]$

    $\Rightarrow 2\tan\theta = p - \dfrac{1}{p}$

    $\Rightarrow \tan\theta = \dfrac{p^2 - 1}{2p}$

14. Answer (b)   [1]

    $\dfrac{2}{3} \times 0 - \dfrac{4}{5} \times 1 = \dfrac{-4}{5}$

15. Answer (c)   [1]

    $\sec^2\theta - \tan^2\theta = 1$

16. $\tan A = \dfrac{5}{12}$

$(\sin A + \cos A)\sec A = \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\cos A}$ [½]

$= \tan A + 1$

$= \dfrac{5}{12} + 1$

$= \dfrac{17}{12}$ [½]

17. $\sec^2\theta(1 + \sin\theta)(1 - \sin\theta) = k$

$\Rightarrow \sec^2\theta(1 - \sin^2\theta) = k$ [½]

$\Rightarrow \sec^2\theta \cdot \cos^2\theta = k$

$\Rightarrow \dfrac{\cos^2\theta}{\cos^2\theta} = k$

$\Rightarrow k = 1$ [½]

18. Given $3x = \mathrm{cosec}\,\theta$

$\dfrac{3}{x} = \cot\theta$

We know that $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow 9x^2 - \dfrac{9}{x^2} = 1$ [½]

$\Rightarrow 9\left(x^2 - \dfrac{1}{x^2}\right) = 1$

$\Rightarrow \boxed{3\left(x^2 - \dfrac{1}{x^2}\right) = \dfrac{1}{3}}$ [½]

19. $\tan(A + B) = \sqrt{3}$

$\Rightarrow A + B = 60°$ ...(i)

Also, $\tan(A - B) = \dfrac{1}{\sqrt{3}}$

$\Rightarrow A - B = 30°$ ...(ii) $[\because A > B]$

On adding (i) and (ii), we get

$2A = 90°$

$\Rightarrow A = 45°$ [1]

20. $\tan\theta = \dfrac{3}{5}$

Now, $\dfrac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \dfrac{5\tan\theta - 3}{4\tan\theta + 3}$ [½]

[Dividing numerator and denominator by $\cos\theta$]

$= \dfrac{3 - 3}{\dfrac{12 + 15}{5}}$

$= 0$ [½]

21. $\sin^2\theta + \dfrac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$ [1]

(using $\sec^2\theta - \tan^2\theta = 1$)

OR

$\left(1 + \tan^2\theta\right)\left(1 - \sin^2\theta\right)$

$\Rightarrow \sec^2\theta \times \cos^2\theta$

$\Rightarrow 1$ [1]

22. In $\triangle ABC$, $\angle C = 90°$

$\tan A = \dfrac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow A = 30°$

$\therefore \angle B = 90° - 30° = 60°$ [1]

$\sin A \cos B + \cos A \sin B = \sin 30° \cos 60° + \cos 30° \sin 60°$

$= \dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{1}{4} + \dfrac{3}{4} = \dfrac{4}{4} = 1$ [1]

23. $\cot\theta = \dfrac{15}{8}$ [Given]

$\dfrac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos)(2 - 2\cos\theta)} = \dfrac{2(1 - \sin^2\theta)}{2(1 - \cos^2\theta)}$ [½]

$= \dfrac{\cos^2\theta}{\sin^2\theta}$ [½]

$= \cot^2\theta$ [½]

$= \left(\dfrac{15}{8}\right)^2 = \dfrac{225}{64}$ [½]

24. Consider an equilateral $\triangle ABC$ of side a

Draw $AD \perp BC$.

$\therefore \triangle ABD \cong \triangle ACD$

$\therefore BD = DC$

$\Rightarrow BD = \dfrac{1}{2}BC$

$= \dfrac{1}{2}a$

and $\angle BAD = \angle CAD = \dfrac{60°}{2} = 30°$ [1]

Using Pythagoras

$AD^2 = AB^2 - BD^2$

$= a^2 - \dfrac{a^2}{4}$
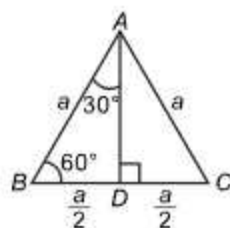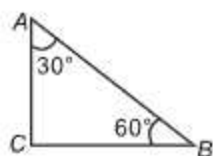
$$= \frac{3a^2}{4}$$

$$AD = \frac{\sqrt{3}a}{2}$$

$$\therefore \quad \tan 60° = \frac{AD}{BD} = \frac{\frac{\sqrt{3}a}{2}}{\frac{a}{2}} = \sqrt{3} \qquad \textbf{[1]}$$

25.
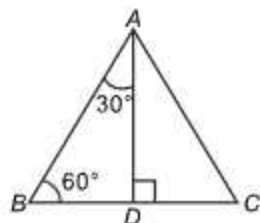
$\angle A = \angle B = \angle C = 60°$

Draw $AD \perp BC$

In $\triangle ABD$ and $\triangle ACD$,

$AD = AD$	(common)
$\angle ADB = \angle ADC$	(90°)
$AB = AC$	($\triangle ABC$ is equilateral $\triangle$)
$\therefore \quad \triangle ABD \equiv \triangle ACD$	(RHS congruence **[1]** criterion)
$BD = DC$	(C.P.C.T.)
$\angle BAD = \angle CAD$	(C.P.C.T.)

$BD = \dfrac{2a}{2} = a$ and $\angle BAD = \dfrac{60°}{2} = 30°$

In right $\triangle ABD$,

$$\sin 30° = \frac{BD}{AB} \qquad \left(\because \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} \right)$$

$$\Rightarrow \quad \sin 30° = \frac{a}{2a}$$

$$\Rightarrow \quad \sin 30° = \frac{1}{2} \Rightarrow \frac{1}{\sin 30°} = 2$$

$$\Rightarrow \quad \boxed{\text{cosec } 30° = 2} \qquad \textbf{[1]}$$

26. L.H.S.

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}} \qquad \text{[On rationalisation] [½]}$$

$$= \frac{1-\sin\theta}{\cos\theta} \qquad [\because 1 - \sin^2\theta = \cos^2\theta] \qquad \textbf{[½]}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \qquad \textbf{[½]}$$

$$= (\sec\theta - \tan\theta) \qquad \textbf{[½]}$$

L.H.S. = R.H.S.

OR

L.H.S.

$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\text{cosec}^2\theta}$$

$[\because \sec^2\theta = 1 + \tan^2\theta, \text{cosec}^2\theta = 1 + \cot^2\theta]$ **[½]**

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}} \qquad \textbf{[½]}$$

$$\left[\because \sec^2\theta = \frac{1}{\cos^2\theta}, \text{cosec}^2\theta = \frac{1}{\sin^2\theta} \right]$$

$$= \sin^2\theta + \cos^2\theta \qquad \textbf{[½]}$$

$$= 1 \qquad \textbf{[½]}$$

L.H.S. = R.H.S.

27. $AD = \sqrt{9 - 2.25}$

$$= \sqrt{6.75}$$
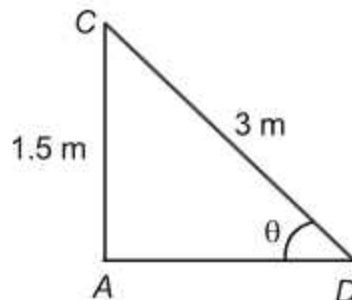
$$= \frac{3\sqrt{3}}{2} \qquad \textbf{[½]}$$

$$\therefore \quad \tan\theta = \frac{CA}{AD} = \frac{1.5}{3\sqrt{3}} \times \frac{2}{1} = \frac{1}{\sqrt{3}} \qquad \textbf{[½]}$$

$$\sec\theta + \text{cosec}\theta = \frac{CD}{AD} + \frac{CD}{CA}$$

$$= 3 \left[\frac{1 \times 2}{3\sqrt{3}} + \frac{1}{1.5} \right] \qquad \textbf{[½]}$$

$$= 3\left[\frac{2}{3\sqrt{3}} + \frac{2}{3}\right]$$

$$= 6\left[\frac{1+\sqrt{3}}{3\sqrt{3}}\right]$$

$$= \frac{2(\sqrt{3}+1)}{\sqrt{3}}$$

$$= \frac{2}{3}(3+\sqrt{3}) \quad [\frac{1}{2}]$$

28. $\sin = \dfrac{1}{2}$

$\Rightarrow \alpha = 30°$

$\Rightarrow \cos\alpha = \dfrac{\sqrt{3}}{2}$ [1]

$\Rightarrow 3\cos\alpha - 4\cos^3\alpha = 3\left(\dfrac{\sqrt{3}}{2}\right) - \left(4\left(\dfrac{\sqrt{3}}{2}\right)^3\right)$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - \left(4\times\frac{3\sqrt{3}}{8}\right) \quad [\frac{1}{2}]$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{3\sqrt{3}}{2}\right)$$

$$= 0 \quad [\frac{1}{2}]$$

29. (A) $\dfrac{5}{\cot^2 30°} + \dfrac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°$

$$= \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - (1)^2 + 2(1)^2 \quad [\frac{1}{2}]$$

$$= \frac{5}{3} + \frac{4}{3} - 1 + 2 \quad [\frac{1}{2}]$$

$$= 3 - 1 + 2 \quad [\frac{1}{2}]$$

$$= 4 \quad [\frac{1}{2}]$$

OR

(B) $\sin\theta = \cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \tan\theta = 1$ [½]

$\Rightarrow \theta = 45°$ [½]

$\therefore \tan^2 45° + \cot^2 45° - 2 = 1^2 + 1^2 - 2$ [½]

$$= 0$$

30. L.H.S. $= (1 + \cos A + \tan A)(\sin A - \cos A)$

$$= \left(1 + \frac{1}{\tan A} + \tan A\right)\left(\frac{\sin A}{\cos A} - 1\right)\cos A \quad [\frac{1}{2}]$$

$$= \frac{(1 + \tan^2 A + \tan A)(\tan A - 1)\cos A}{\tan A} \quad [\frac{1}{2}]$$

$$= \frac{(\tan^3 A - 1)\cos A}{\tan A}$$

$$[\because a^3 - b^3 = (a-b)(a^2+b^2+ab)] \quad [1]$$

$= \tan^2 A \cos A - \cot A \cos A$

$$= \tan A \cdot \frac{\sin A}{\cos A} \cdot \cos A - \cot A \cos A \quad [\frac{1}{2}]$$

$= \sin A \tan A - \cot A \cos A = $ R.H.S.; Proved [½]

31. L.H.S. $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \frac{(1 - \sin^2 A)(1 - \cos^2 A)}{\sin A \cos A}$$

$$= \frac{\cos^2 A \sin^2 A}{\sin A \cos A}$$

$$= \sin A \cdot \cos A \quad ...(i) \quad [1]$$

R.H.S. $= \dfrac{1}{\tan A + \cot A}$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}}$$

$$= \frac{\sin A \cdot \cos A}{1} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \sin A \cdot \cos A \quad ...(ii) \quad [1]$$

From (i) and (ii)

L.H.S. = R.H.S.; Hence Proved [1]

32. Given that,

$\tan\theta = \dfrac{3}{4}$ $\qquad \therefore \tan^2\theta = \dfrac{9}{16}$ [½]

We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$\therefore \sec^2\theta = 1 + \dfrac{9}{16} = \dfrac{25}{16}$

$\Rightarrow \ \sec\theta = \dfrac{5}{4}$ [½]

Now,

$\left(\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right) = \left(\dfrac{\dfrac{4\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta} + \dfrac{1}{\cos\theta}}{\dfrac{4\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta} - \dfrac{1}{\cos\theta}}\right)$ [½]

$= \dfrac{4\tan\theta - 1 + \sec\theta}{4\tan\theta + 1 - \sec\theta}$

$= \dfrac{3 - 1 + \dfrac{5}{4}}{3 + 1 - \dfrac{5}{4}}$ [½]

$= \dfrac{2 + \dfrac{5}{4}}{4 - \dfrac{5}{4}}$ [½]

$= \dfrac{\dfrac{(8+5)}{4}}{\dfrac{(16-5)}{4}}$

$= \dfrac{13}{11}$ [½]

33. L.H.S : $(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \csc^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$

$\left[\because \sin\theta = \dfrac{1}{\csc\theta} \text{ and } \cos\theta = \dfrac{1}{\sec\theta}\right]$ **[1]**

$= (\sin^2\theta + \cos^2\theta) + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$

$[\because \cos^2\theta + \sin^2\theta = 1]$ **[1]**

$= 1 + 1 + 1 + 4 + \tan^2\theta + \cot^2\theta$

$[\because \csc^2\theta + 1 + \cot^2\theta \text{ and } \sec^2\theta = 1 + \tan^2\theta]$ [½]

$= 7 + \tan^2\theta + \cot^2\theta = $ R.H.S.

Hence Proved [½]

34. L.H.S : $\left(1 + \dfrac{\cos A}{\sin A} - \dfrac{1}{\sin A}\right)\left(1 + \dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A}\right)$

$= \left(\dfrac{\sin A + \cos A - 1}{\sin A}\right)\left(\dfrac{\cos A + \sin A + 1}{\cos A}\right)$ [½]

$= \dfrac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cdot \cos A}$ [½]

$= \dfrac{\sin^2 A + \cos^2 A + 2\sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$ [½]

$= \dfrac{1 + 2\sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$ $[\because \sin^2 A + \cos^2 A = 1]$ [½]

$= 2 = $ R.H.S.

Hence Proved **[1]**

35. LHS = $(1 + \tan A - \sec A) \times (1 + \tan A + \sec A)$

$\because \ (x - y)(x + y) = x^2 - y^2$

here $x = 1 + \tan A$

$y = \sec A$

LHS = $(1 + \tan A)^2 - (\sec A)^2$ **[1]**

$= 1 + \tan^2 A + 2\tan A - \sec^2 A$ **[1]**

$= \sec^2 A + 2\tan A - \sec^2 A$ $(1 + \tan^2 A = \sec^2 A)$

$= 2\tan A = $ RHS **[1]**

Hence, proved.

OR

LHS = $\dfrac{\csc\theta}{\csc\theta - 1} + \dfrac{\csc\theta}{\csc\theta + 1}$

$= \csc\theta\left(\dfrac{1}{\csc\theta - 1} + \dfrac{1}{\csc\theta + 1}\right)$

$= \csc\theta\left(\dfrac{\csc\theta + 1 + \csc\theta - 1}{(\csc\theta - 1)(\csc\theta + 1)}\right)$ **[1]**

$= \csc\theta\left(\dfrac{2\csc\theta}{\csc^2\theta - 1}\right)$

$= \dfrac{2\csc^2\theta}{\cot^2\theta}$ $\left[\begin{array}{l}\because 1 + \cot^2\theta = \csc^2\theta \\ \Rightarrow \csc^2\theta - 1 = \cot^2\theta\end{array}\right]$ **[1]**

$= \dfrac{2 \times \dfrac{1}{\sin^2\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta}}$ $\left[\begin{array}{l}\because \csc\theta = \dfrac{1}{\sin\theta} \\ \cot\theta = \dfrac{\cos\theta}{\sin\theta}\end{array}\right]$

$= \dfrac{2}{\cos^2\theta} = 2\sec^2\theta = $ RHS **[1]**

Hence proved.

36. $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both sides, we get

$\Rightarrow \ \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$ [½]

$\Rightarrow \ 1 + 2\sin\theta\cos\theta = 3$ [½]

$\Rightarrow \quad \sin\theta\cos\theta = 1$ [½]

Now, $\tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ [½]

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ [½]

$= \dfrac{1}{1}$

$= 1 = \text{RHS}$ [½]

Hence proved.

37. Taking L.H.S

$\dfrac{1 + \tan^2 A}{1 + \cot^2 A}$

$= \dfrac{1 + \dfrac{\sin^2 A}{\cos^2 A}}{1 + \dfrac{\cos^2 A}{\sin^2 A}}$ [1]

$= \dfrac{\dfrac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A + \cos^2 A}{\sin^2 A}}$ [1]

$= \dfrac{\sin^2 A}{\cos^2 A} \quad [\because \sin^2 A + \cos^2 A = 1]$

$= \tan^2 A = \sec^2 A - 1 = \text{R.H.S}$ [1]

Hence, proved

38. (A) To prove : $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

LHS $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

Dividing numerator and denominator by $\cos^3 A$, we get

LHS $= \dfrac{\dfrac{\sin A}{\cos A \cdot \cos^2 A} - \dfrac{2\sin^3 A}{\cos^3 A}}{\dfrac{2\cos^3 A}{\cos^3 A} - \dfrac{\cos A}{\cos^3 A}}$ [1]

$= \dfrac{\sec^2 A \tan A - 2\tan^3 A}{2 - \sec^2 A}$ [½]

$= \dfrac{\tan A(\sec^2 A - 2\tan^2 A)}{2 - 1 - \tan^2 A}$ [½]

$\left[\because \ \sec^2 A - \tan^2 A = 1\right]$

$= \dfrac{\tan A (1 - \tan^2 A)}{1 - \tan^2 A}$ [½]

$= \tan A = \text{RHS}$

Hence proved.   [½]

OR

(B) To prove : secA(1 - sinA)(secA + tanA) = 1

LHS = secA(1 - sinA)(secA + tanA)

$= \sec A(1 - \sin A)\left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)$ [1]

$= \dfrac{\sec A}{\cos A}(1 - \sin A)(1 + \sin A)$

$= \dfrac{1}{\cos^2 A}(1 - \sin^2 A)$ [1]

$= \dfrac{1}{\cos^2 A} \cdot \cos^2 A$

$\left[\because \ \sin^2 A + \cos^2 A = 1\right]$

$= 1 = \text{RHS}$ [1]

Hence proved

39. L.H.S. $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$ [1]

$= \dfrac{\sin A}{\cos A}\left(\dfrac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - \sin^2 A - \cos^2 A}\right)$ [1]

$[\because \ \sin^2 A + \cos^2 A = 1]$

$= \tan A\left(\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right)$ [1]

$= \tan A = \text{R.H.S.}$

Hence proved.   [1]

40. LHS $= \dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$ **[½]**

$= \dfrac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$

(Dividing numerator & denominator by cos A) **[½]**

$= \dfrac{(\tan A + \sec A) - 1}{(\tan A - \sec A) + 1}$ **[½]**

$= \dfrac{\{(\tan A + \sec A) - 1\}(\tan A - \sec A)}{\{(\tan A - \sec A) + 1\}(\tan A - \sec A)}$ **[½]**

$= \dfrac{\left(\tan^2 A - \sec^2 A\right) - (\tan A - \sec A)}{\{\tan A - \sec A + 1\}(\tan A - \sec A)}$ **[½]**

$= \dfrac{-1 - \tan A + \sec A}{(\tan A - \sec A + 1)(\tan A - \sec A)}$ **[½]**

$= \dfrac{-1(\tan A - \sec A + 1)}{(\tan A - \sec A + 1)(\tan A - \sec A)}$ **[½]**

$= \dfrac{1}{\sec A - \tan A} = $ R.H.S.o **[½]**

Hence proved.

9 : Some Applications of Trigonometry

1. Answer (c) **[1]**
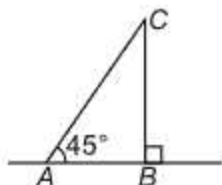
Given $AB = 25$ m

And angle of elevation of the top of the tower (BC) from $A = 45°$

$\because$ $\angle BAC = 45°$

In $\triangle ABC$, $\tan 45° = \dfrac{BC}{AB}$

$\Rightarrow BC = 25$ m

$\therefore$ Height of the tower = 25 m

2. Answer (b) **[1]**

Let AB be the tower and BC be its shadow. Let θ be the angle of elevation of the sun.

According to the given information,

$BC = \sqrt{3}\, AB$ ...(1)

In $\triangle ABC$,

$\tan\theta = \dfrac{AB}{BC} = \dfrac{AB}{\sqrt{3} AB} = \dfrac{1}{\sqrt{3}}$ [Using (1)]
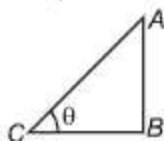
We know that $\tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore$ $\theta = 30°$

Hence, the angle of elevation of the sun is 30°.

3. Answer (c) **[1]**
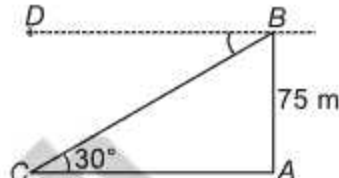
Let AB be the tower of height 75 m and C be the position of the car

In $\triangle ABC$,

$\cot 30° = \dfrac{AC}{AB}$

$\Rightarrow AC = AB\cot 30°$

$\Rightarrow AC = 75\, m \times \sqrt{3}$

$\Rightarrow AC = 75\sqrt{3}$ m

Thus, the distance of the car from the base of the tower is $75\sqrt{3}$ m.

4. Answer (d) **[1]**
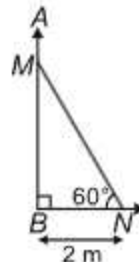
In the figure, MN is the length of the ladder, which is placed against the wall AB and makes an angle of 60° with the ground.

The foot of the ladder is at N, which is 2 m away from the wall.

$\therefore$ $BN = 2$ m

In right-angled triangle *MNB*:

$$\cos 60° = \frac{BN}{MN} = \frac{2}{MN}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{MN}$$

$$\Rightarrow MN = 4 \text{ m}$$

Therefore, the length of the ladder is 4 m.

Hence, the correct option is (d)

5. Answer (a)   **[1]**
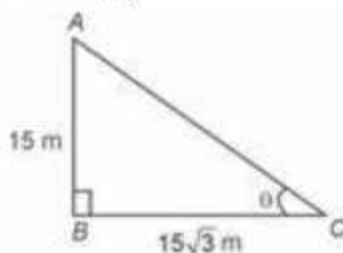
$$\tan \theta = \frac{15}{15\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30°$$

6. Answer (a)   **[1]**
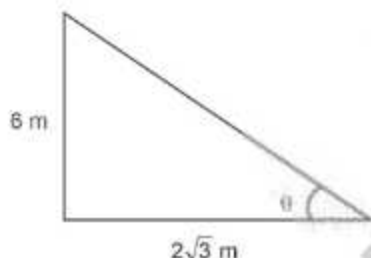
$$\tan \theta = \frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

7.
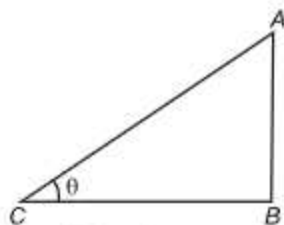
Let *AB* be the tower and *BC* be its shadow.

$$AB = 20, BC = 20\sqrt{3}$$

In $\triangle ABC$,

$$\tan \theta = \frac{AB}{BC}$$

$$\tan \theta = \frac{20}{20\sqrt{3}}$$ **[½]**

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30°$$

The Sun is at an altitude of 30°.   **[½]**

8.
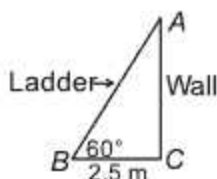
Let *AB* be the ladder and *CA* be the wall.

The ladder makes an angle of 60° with the horizontal.

$\therefore \triangle ABC$ is a 30° – 60° – 90°, right triangle. **[½]**

Given: *BC* = 2.5 m, $\angle ABC = 60°$

$$\therefore AB = 5 \text{ m}$$

Hence, length of the ladder is *AB* = 5 m.   **[½]**

9.
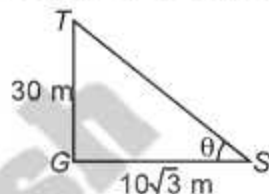
Angle of elevation of sun = $\angle GST = \theta$

Height of tower *TG* = 30 m

Length of shadow $GS = 10\sqrt{3}$ m   **[½]**

$\triangle TGS$ is a right angled triangle

$$\therefore \tan \theta = \frac{30}{10\sqrt{3}}$$

$$\tan \theta = \sqrt{3}$$ **[½]**

$$\theta = 60°$$

10. In $\triangle ABC$,

$$\tan \theta = \frac{AB}{BC}$$

$$\Rightarrow \tan \theta = \frac{x}{\sqrt{3}x}$$

[½]

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \boxed{\theta = 30°}$$ **[½]**
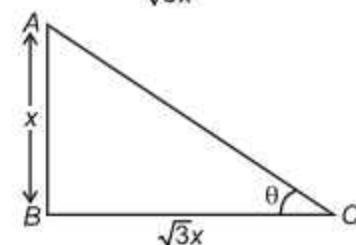
11.
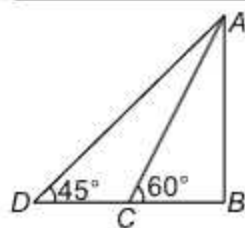
Given $CD = 100$ m, $AB = ?$

In $\triangle ABC$, $\tan 60^\circ = \dfrac{AB}{BC}$

$BC = \dfrac{AB}{\sqrt{3}}$ [1]

$BD = AB$ $[\because \tan 45^\circ = 1]$

$BD - BC = CD$

$AB - \dfrac{AB}{\sqrt{3}} = 100$ [1]

$AB\left(\dfrac{\sqrt{3}-1}{\sqrt{3}}\right) = 100$

$AB = \dfrac{100\sqrt{3}}{\sqrt{3}-1}$

$AB = 236.98$

$AB = 237$ m   [1]

12. Given: Position of kite is B.

Height of kite above ground = 45 m

Angle of inclination = 60°

Required length of string = AB
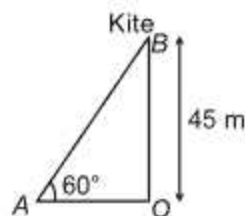
In right angled triangle AOB,

$\sin A = \dfrac{OB}{AB}$

$\Rightarrow \sin 60^\circ = \dfrac{45}{AB}$ [1]

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{45}{AB}$

$\Rightarrow AB = \dfrac{45 \times 2}{\sqrt{3}} = \dfrac{90}{\sqrt{3}} = 30\sqrt{3}$ m

Hence, the length of the string is $30\sqrt{3}$ m.   [1]

13.
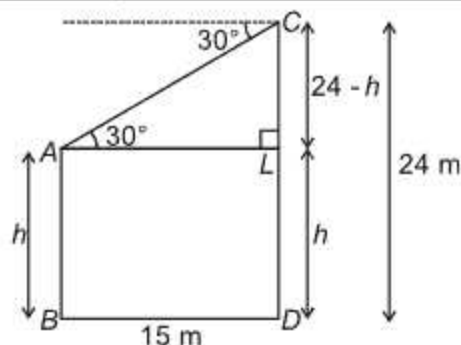
Let AB and CD be the two poles, where CD (the second pole) = 24 m.

$BD = 15$ m

Let the height of pole AB be h m.

$AL = BD = 15$ m and $AB = LD = h$

So, $CL = CD - LD = 24 - h$ [1]

In $\triangle ACL$,

$\tan 30^\circ = \dfrac{CL}{AL}$

$\Rightarrow \tan 30^\circ = \dfrac{24 - h}{15}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{24 - h}{15}$ [1]

$\Rightarrow 24 - h = \dfrac{15}{\sqrt{3}} = 5\sqrt{3}$

$\Rightarrow h = 24 - 5\sqrt{3}$

$\Rightarrow h = 24 - 5 \times 1.732$ [Taking $\sqrt{3} = 1.732$]

$\Rightarrow h = 15.34$

Thus, height of the first pole is 15.34 m.   [1]

14. Let d be the distance between the two ships. Suppose the distance of one of the ships from the light house is x meters, then the distance of the other ship from the light house is $(d - x)$ meter.
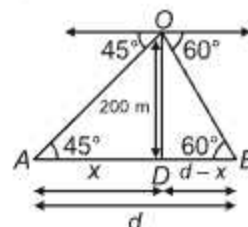
In right-angled $\triangle ADO$, we have.

$\tan 45^\circ = \dfrac{OD}{AD} = \dfrac{200}{x}$

$\Rightarrow \quad 1 = \dfrac{200}{x}$

$\Rightarrow \quad x = 200 \qquad \ldots\text{(i)}$ **[1]**

In right-angled $\triangle BDO$, we have

$\tan 60° = \dfrac{OD}{BD} = \dfrac{200}{d-x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{200}{d-x}$

$\Rightarrow \quad d - x = \dfrac{200}{\sqrt{3}}$ **[1]**

Putting $x = 200$. We have:

$d - 200 = \dfrac{200}{\sqrt{3}}$

$d = \dfrac{200}{\sqrt{3}} + 200$

$\Rightarrow \quad d = 200 \times 1.58$

$\Rightarrow \quad d = 316 \text{ m} \qquad \text{(approx.)}$ **[1]**

Thus, the distance between two ships is approximately 316 m.

15. Let BC be the height at watch the aeroplane is observed from point A.

Then, $BC = 1500\sqrt{3}$

In 15 seconds, the aeroplane moves from point B to D.

B and D are the points where the angles of elevation 60° and 30° are formed respectively. **[1]**

Let $AC = x$ metres and $CE = y$ metres

$AE = x + y$
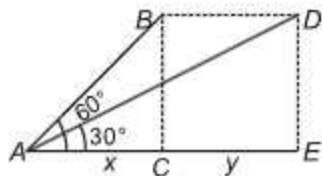
In $\triangle CBA$,

$\tan 60° = \dfrac{BC}{AC}$

$\sqrt{3} = \dfrac{1500\sqrt{3}}{x}$ **[1]**

$\therefore \quad x = 1500 \text{ m} \qquad \ldots\text{(i)}$

In $\triangle ADE$,

$\tan 30° = \dfrac{DE}{AE}$

$\dfrac{1}{\sqrt{3}} = \dfrac{1500\sqrt{3}}{x+y}$

$\therefore \quad x + y = 1500 \times (3) = 4500$

$\therefore \quad 1500 + y = 4500$

$\therefore \quad y = 3000 \text{ m} \qquad \ldots\text{(ii)}$

We know that, the aeroplane moves from point B to D in 15 seconds and the distance covered is 3000 metres.

$\text{Speed} = \dfrac{\text{distance}}{\text{time}}$

$\text{Speed} = \dfrac{3000}{15}$

Speed 200 m/s

Converting it to km/hr $= 200 \times \dfrac{18}{5} = 720 \text{ km/hr}$ **[1]**

16.
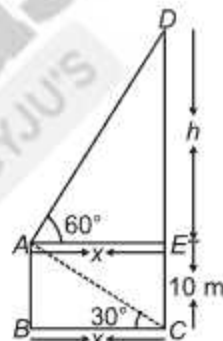
Let CD be the hill and suppose the man is standing on the deck of a ship at point A.

The angle of depression of the base C of the hill CD observed from A is 30° and the angle of elevation of the top D of the hill CD observed from A is 60°.

$\therefore \quad \triangle EAD = 60°$ and $\angle BCA = 30°$ **[1]**

In $\triangle AED$,

$\tan 60° = \dfrac{DE}{EA}$

$\sqrt{3} = \dfrac{h}{x}$

$h = \sqrt{3}x \qquad \ldots\text{(i)}$

In $\triangle ABC$,

$\tan 30° = \dfrac{AB}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{10}{x}$

$x = 10\sqrt{3}$ ...(ii)   **[1]**

Substituting $x = 10\sqrt{3}$ in equation (i), we get

$h = \sqrt{3} \times 10\sqrt{3} = 10 \times 3 = 30$

$DE = 30$ m

$CD = CE + ED = 10 + 30 = 40$ m

Thus, the distance of the hill from the ship is $10\sqrt{3}$ m and the height of the hill is 40 m.   **[1]**

17.
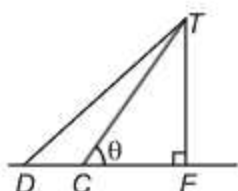
Given $CF = 4$ m

$DF = 16$ m

$\angle TCF + \angle TDF = 90°$

Let say $\angle TCF = \theta$ **[1]**

$\angle TDF = 90° - \theta$

In a right angled triangle TCF

$\tan\theta = \dfrac{TF}{CF} = \dfrac{TF}{4}$

$TF = 4\tan\theta$ ...(i)

In $\triangle TDF$

$\tan(90° - \theta) = \dfrac{TF}{16}$ **[1]**

$TF = 16\cot\theta$ ...(ii)

Multiply (i) and (ii), we get

$(TF)^2 = 64 \Rightarrow TF = 8$ m

$\Rightarrow$ Height of tower = 8 m   **[1]**

18. (a)
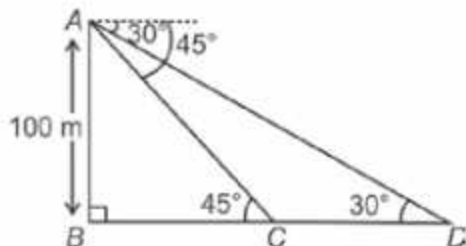
In $\triangle ABC$,

$\dfrac{AB}{BC} = \tan 45° = 1$ **[½]**

$\Rightarrow AB = BC = 100$ m   ...(i)   **[½]**

In $\triangle ABD$,

$\dfrac{AB}{BD} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow BD = AB \times \sqrt{3}$

          $= 100\sqrt{3}$ m   ...(ii)   **[½]**

$\therefore$ $CD = BD - BC$

$= \left(100\sqrt{3} - 100\right)$ m   [From (i) and (ii)]   **[½]**

$= 100\left(\sqrt{3} - 1\right)$ m

$= 100 (1.73 - 1)$ m

$= 100 \times 0.73$ m   **[½]**

$= 73$ m

$\therefore$ Ship will travel 73 m during the given time.   **[½]**

OR

(b)
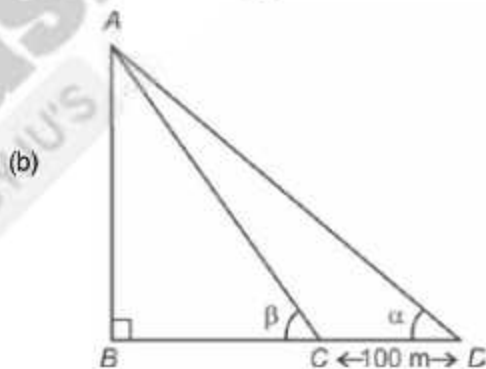
Let AB represents the tower. Observer is moving from D to C.

In $\triangle ABC$,

$\tan\beta = \dfrac{AB}{BC} = \dfrac{3}{4}$ ...(i)   **[½]**

and in $\triangle ABD$,

$\tan\alpha = \dfrac{AB}{BD} = \dfrac{1}{3}$ ...(ii)   **[½]**

From (i) and (ii), we get

$BC = \dfrac{4AB}{3}$ and $BD = 3AB$ **[½]**

$\Rightarrow CD = BD - BC$ **[½]**

$\Rightarrow 100 = 3AB - \dfrac{4AB}{3}$

$\Rightarrow 100 = \dfrac{9AB - 4AB}{3}$ [½]

$\Rightarrow 300 = 5AB$

$\Rightarrow AB = 60 \text{ m}$

$\therefore$ Height of tower is 60 m.   [½]

19.
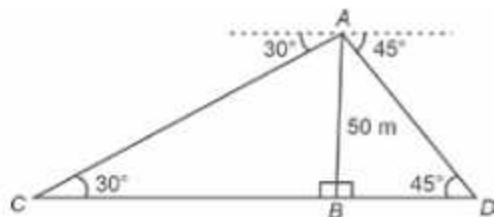
$\angle ACB = 30°$

and $\angle ADB = 45°$ [From figure]

Distance between two cars

$= CD = BC + BD$ [From figure] ...(i) [½]

Now,

In $\triangle ABC$,

$\tan 30° = \dfrac{AB}{BC} = \dfrac{50}{BC}$

or $BC = \dfrac{50}{\tan 30°} = 50\sqrt{3} \text{ m}$ [½]

and In $\triangle ABD$,

$\tan 45° = \dfrac{AB}{BD} = \dfrac{50}{BD}$

$BD = \dfrac{50}{1}$

$BD = 50 \text{ m}$ [1]

From equation (i), we get

$CD = BC + BD$

$= 50\sqrt{3} + 50$

$= 50\left(\sqrt{3} + 1\right) \text{ m}$ [1]

20. Let AC and BD be the two poles of the same height h m.
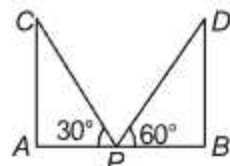
Given $AB = 80 \text{ m}$

Let $AP = x$ m, therefore, $PB = (80 - x)$ m

In $\triangle APC$,

$\tan 30° = \dfrac{AC}{AP}$ [1]

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{x}$ ...(i)

In $\triangle BPD$,

$\tan 60° = \dfrac{BD}{PB}$

$\sqrt{3} = \dfrac{h}{80 - x}$ ...(ii)   [1]

Dividing (i) by (ii), we get

$\dfrac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \dfrac{\frac{h}{x}}{\frac{h}{80 - x}}$

$\Rightarrow \dfrac{1}{3} = \dfrac{80 - x}{x}$

$\Rightarrow x = 240 - 3x$

$\Rightarrow 4x = 240$ [1]

$\Rightarrow x = 60 \text{ m}$

From (i),

$\dfrac{1}{3} = \dfrac{h}{x}$

$\Rightarrow h = \dfrac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$

Thus, the height of both the poles is $20\sqrt{3}$ m and the distances of the point from the poles are 60 m and 20 m.   [1]

21. Let AB be the building and CD be the tower.
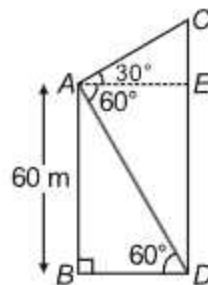
In right $\triangle ABD$,

$\dfrac{AB}{BD} = \tan 60°$

$$\Rightarrow \quad \frac{60}{BD} = \sqrt{3}$$

$$\Rightarrow \quad BD = \frac{60}{\sqrt{3}} \qquad\qquad [2]$$

$$\Rightarrow \quad BD = 20\sqrt{3}$$

In right $\triangle ACE$,

$$\frac{CE}{AE} = \tan 30°$$

$$\Rightarrow \quad \frac{CE}{AE} = \frac{1}{\sqrt{3}} \quad (\because AE = BD)$$

$$\Rightarrow \quad CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Height of the tower = $CE + ED = CE + AB = $ 20 m + 60 m = 80 m

Difference between the heights of the tower and the building = 80 m − 60 m = 20 m

Distance between the tower and the building

$$= BD = 20\sqrt{3} \text{ m} \qquad\qquad [2]$$

22.
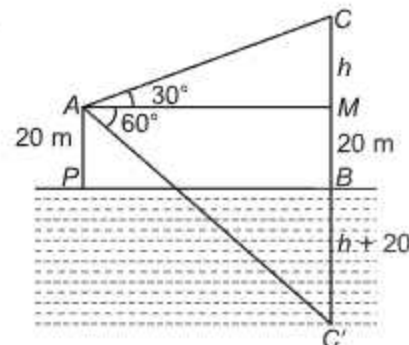
Let PB be the surface of the lake and A be the point of observation such that

$AP = 20$ metres. Let C be the position of the cloud and C' be its reflection in the lake.

Then $CB = C'B$. Let AM be perpendicular from A on CB. **[1]**

Then $m\angle CAM = 30°$ and $m\angle C'AM = 60°$

Let $CM = h$. Then, $CB = h + 20$ and $C'B = h + 20$.

In CMA we have,

$$\tan 30° = \frac{CM}{AM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{AM}$$

$$\Rightarrow \quad AM = \sqrt{3}h \qquad\qquad ...(i) \qquad [1]$$

In $\triangle AMC'$ we have,

$$\tan 60° = \frac{C'M}{AM}$$

$$\Rightarrow \quad \sqrt{3} = \frac{C'B + BM}{AM}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 20 + 20}{AM}$$

$$\Rightarrow \quad AM = \frac{h + 20 + 20}{\sqrt{3}} \qquad ...(ii) \qquad [1]$$

From equation (i) and (ii), we get

$$\sqrt{3}h = \frac{h + 20 + 20}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = h + 40$$

$$\Rightarrow \quad 2h = 40$$

$$\Rightarrow \quad h = 20 \text{ m}$$

In $\triangle CMA$, $\sin 30° = \dfrac{h}{CA} \Rightarrow CA = 40$ m

Hence, the distance of the cloud from the point A is 40 metres. **[1]**

23.
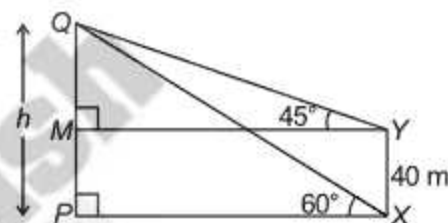
$MP = YX = 40$ m

$\therefore \quad QM = h - 40$

In right angled $\triangle QMY$,

$$\tan 45° = \frac{QM}{MY} \Rightarrow 1 = \frac{h-40}{PX} \quad ...(MY = PX) \quad [1]$$

$$\therefore \quad PX = h - 40 \qquad\qquad ...(i)$$

In right angled $\triangle QPX$,

$$\tan 60° = \frac{QP}{PX} \Rightarrow \sqrt{3} = \frac{QP}{PX}$$

$$PX = \frac{h}{\sqrt{3}} \qquad\qquad ...(ii) \qquad [1]$$

From (i) and (ii), we get

$$h - 40 = \frac{h}{\sqrt{3}}$$

$$\therefore \quad \sqrt{3}h - 40\sqrt{3} = h$$

$$\therefore \quad \sqrt{3}h - h = 40\sqrt{3} \qquad\qquad [1]$$

$$\therefore \quad 1.73h - h = 40(1.73) \Rightarrow h = 94.79 \text{ m}$$

Thus, PQ is 94.79 m and $PX = 94.79 + 1.73$ = 54.79 m **[1]**

24.
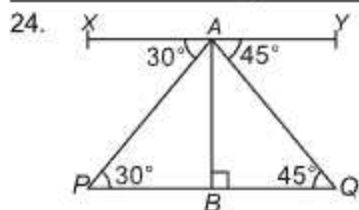
Given aeroplane is at height of 300 m

∴ $AB = 300$ m and $XY \parallel PQ$

Angles of depression of the two points P and Q are 30° and 45° respectively. **[1]**

$\angle XAP = 30°$ and $\angle YAQ = 45°$

$\angle XAP = \angle APB = 30°$

[Alternate interior angles]

$\angle YAQ = \angle AQB = 45°$ **[1]**

In $\triangle PAB$,

$\tan 30° = \dfrac{AB}{PB}$

$PB = 300\sqrt{3}$ m **[1]**

In $\triangle BAQ$,

$\tan 45° = \dfrac{AB}{BQ}$

$BQ = 300$ m

∴ Width of the river $= PB + BQ$

$= 300(1+\sqrt{3})$ m **[1]**

25. Let ships are at distance x from each other.
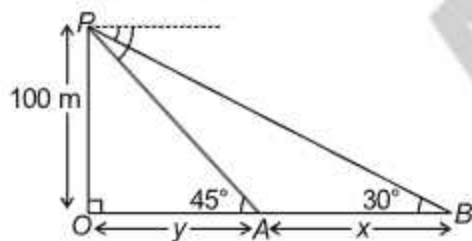
In $\triangle APO$

$\tan 45° = \dfrac{100}{y} = 1 \quad \therefore y = 100$ m ...(i) **[1]**

In $\triangle POB$

$\tan 30° = \dfrac{OP}{OB} = \dfrac{100}{x+y} = \dfrac{1}{\sqrt{3}}$ **[1]**

$\sqrt{3} = \dfrac{x+y}{100}$

$x + y = 100\sqrt{3}$ ...(ii) **[1]**

$x = 100\sqrt{3} - y = 100\sqrt{3} - 100 = 100(\sqrt{3} - 1)$

∴ $x = 100(1.732 - 1)$

$= 100 \times 0.732$

$= 73.2$ m

∴ Ships are 73.2 meters apart. **[1]**

26. Let the light house be PQ and the boat changes its position from R to S.

Here, $PQ = 100$ m, $\angle PRQ = 60°$ and $\angle PSR = 30°$.
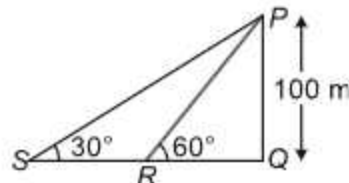
In $\triangle PQR$,

$\tan 60° = \dfrac{PQ}{QR} = \dfrac{100}{QR}$

$\Rightarrow QR = \dfrac{100\sqrt{3}}{3}$ m ...(i) **[1]**

In $\triangle PQS$,

$\tan 30° = \dfrac{PQ}{QS}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{100}{QS}$

$\Rightarrow QS = 100\sqrt{3}$ m **[1]**

∴ $RS = QS - QR =$

$100\sqrt{3} - \dfrac{100\sqrt{3}}{3} = \dfrac{200\sqrt{3}}{3}$ **[1]**

Speed $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{200\sqrt{3}}{3 \times 2} = \dfrac{100\sqrt{3}}{3}$

$= 57.73$ (approx.) (Using $\sqrt{3} = 1.732$)

$= 57.73$ m/min **[1]**

27. Let $AB = h$ m be the height of building and CD be height of tower.

∴ $CD = 60$ m
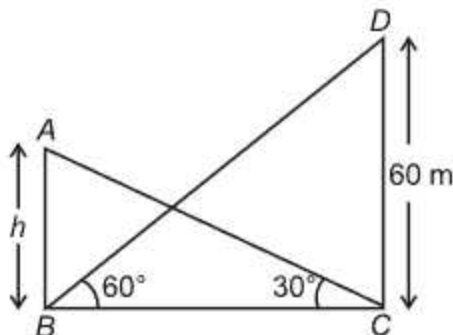
[1]

In $\triangle BDC$, $\tan 60° = \dfrac{CD}{BC}$

$\Rightarrow BC = \dfrac{60}{\sqrt{3}} = 20\sqrt{3}$ m ... (i)   [1]

In $\triangle ABC$,

$\tan 30° = \dfrac{AB}{BC}$ [1]

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{AB}{20\sqrt{3}}$ [From (i)]

$\Rightarrow AB = 20$ m

$\therefore$ Height of building = 20 m.   [1]

28. AB = height of flag-staff = 6 m
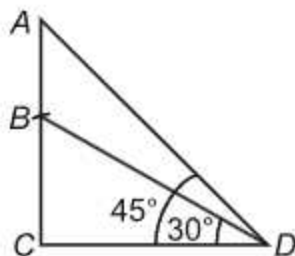
Let BC = height of tower = h m   [½]

In $\triangle BCD$

$\dfrac{BC}{CD} = \tan 30°$ [½]

$\Rightarrow \dfrac{h}{CD} = \dfrac{1}{\sqrt{3}} \Rightarrow CD = h\sqrt{3}$ ...(i)   [½]

In $\triangle ACD$, $\dfrac{AC}{CD} = \tan 45°$ [½]

$\Rightarrow \dfrac{h+6}{CD} = 1 \Rightarrow h = CD - 6$

$\Rightarrow h = h\sqrt{3} - 6$ [From (i)]   [½]

$\Rightarrow h(\sqrt{3} - 1) = 6$

$\Rightarrow h = \dfrac{6}{\sqrt{3}-1}$ [½]

$\Rightarrow h = 3(\sqrt{3} + 1)$ [½]

$h = 3 \times 2.73$

$h = 8.19$ m   [½]

$\therefore$ Height of the tower is 8.19 m

29. (i) Let positions of Charu and Daljeet be C and D respectively,

     [1]
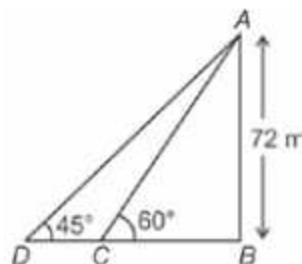
Charu is nearer to Qutub Minar as its angle of elevation is greater.

(ii) In $\triangle ABC$,

$\tan 60° = \dfrac{AB}{BC}$ [½]

$\Rightarrow \sqrt{3} = \dfrac{72}{BC}$ [½]

$\Rightarrow BC = 41.52$ m   [½]

In $\triangle ABD$,

$\tan 45° = \dfrac{AB}{BD}$ [½]

$\Rightarrow 1 = \dfrac{72}{BD}$

$\Rightarrow BD = 72$ m   [½]

$CD = BD - BC$

$CD = (72 - 41.52)$ m

$= 30.48$ m   [½]

30. (1) As from the figure, length of strings are AC and BC.

$AD = 50$ m

$BE = 60$ m
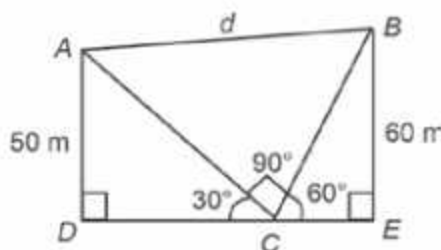
In $\triangle ADC$,

$\sin 30° = \dfrac{AD}{AC}$ [½]

$\Rightarrow \dfrac{1}{2} = \dfrac{50}{AC}$

$\Rightarrow AC = 100$ m   [½]

In $\triangle BCE$,

$$\sin 60° = \frac{BE}{BC}$$ [½]

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{BC}$$

$$\Rightarrow BC = 40\sqrt{3} \text{ m}$$ [½]

(2) As from the figure, we can see that $\angle ACB = 90°$ [½]

Applying Pythagoras theorem in $\triangle ACB$, we get

$$d = \sqrt{AC^2 + BC^2}$$ [½]

$$= \sqrt{(100)^2 + \left(40\sqrt{3}\right)^2}$$ [½]

$$= 20\sqrt{37} \text{ m}$$ [½]

31. (A) Let AB be tower of height h m
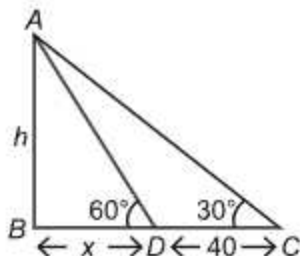
[1]

In $\triangle ABD$, [½]

$$\tan 60° = \frac{AB}{BD}$$ [½]

$$\sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \qquad \text{...(i)}$$ [½]

In $\triangle ABC$,

$$\tan 30° = \frac{AB}{BC}$$ [½]

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + 40}$$

$$\Rightarrow x + 40 = \sqrt{3}h$$ [½]

$$\therefore \frac{h}{\sqrt{3}} + 40 = \sqrt{3}h \quad \text{[From (i)]}$$ [½]

$$\Rightarrow h + 40\sqrt{3} = 3h$$ [½]

$$\Rightarrow 2h = 40\sqrt{3}$$

$$\Rightarrow h = 20\sqrt{3}$$

∴ The height of the tower is $20\sqrt{3}$ m [½]

OR

(B) $AB = CD = 7$ m [½]

[½]
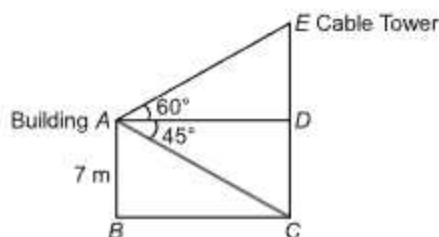
In $\triangle ADC$,

$$\tan 45° = \frac{CD}{AD}$$ [½]

$$\Rightarrow 1 = \frac{7}{AD}$$ [½]

$$\Rightarrow AD = 7 \text{ m}$$ [½]

In $\triangle ADE$

$$\tan 60° = \frac{ED}{AD}$$ [½]

$$\Rightarrow \sqrt{3} = \frac{ED}{7}$$ [½]

$$\Rightarrow ED = 7\sqrt{3} \text{ m}$$ [½]

∴ The total height of the cable tower

$$= CD + ED$$ [½]

$$= \left(7 + 7\sqrt{3}\right) \text{ m}$$

$$= 7\left(1 + \sqrt{3}\right) \text{ m}$$ [½]

32. (A)
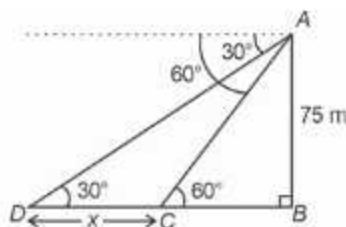
In ABC :

$$\tan 60° = \frac{AB}{BC}$$ [2]

$$\Rightarrow BC = \frac{75}{\sqrt{3}} \qquad \text{...(i)}$$

In ABD :

$$\tan 30° = \frac{AB}{BC + DC}$$ [1]

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC + x}$$

$$\Rightarrow BC + x = 75\sqrt{3}$$ [1]

$$\Rightarrow x = 75\sqrt{3} - \frac{75}{\sqrt{3}} \quad \text{(from (i))}$$

$$\Rightarrow x = \frac{75 \times 2}{\sqrt{3}} = \frac{75 \times 2 \times \sqrt{3}}{3}$$

$$\Rightarrow x = 86.5 \text{ m}$$ [1]

OR

(B)
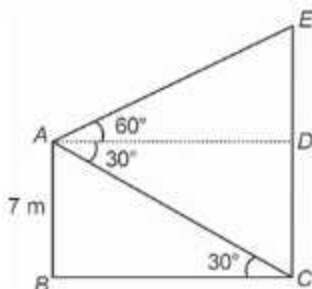
Let AB be the building of height 7 m and EC be tower.

A is the point from where angle of elevation of the top of tower is 60° and angle of depression of its foot is 30°. [1]

$EC = DE + CD$

Also, $CD = AB = 7$ m and $BC = AD$ [½]

To find : height of tower EC

In $\triangle ABC$ [½]

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{7}{BC}$$

$$\Rightarrow BC = 7\sqrt{3}$$ [½]

In ADE

$$\tan 60° = \frac{DE}{AD}$$ [½]

$$\Rightarrow \sqrt{3} = \frac{DE}{7\sqrt{3}} \quad \left[BC = AD \text{ and } BC = 7\sqrt{3}\right]$$ [½]

$$\Rightarrow DE = 7\sqrt{3} \times \sqrt{3} = 21$$

Now, $EC = DE + CD = 21 + 7$ [½]

$$= 28 \text{ m}$$

Height of cable tower is 28 m. [1]

33.
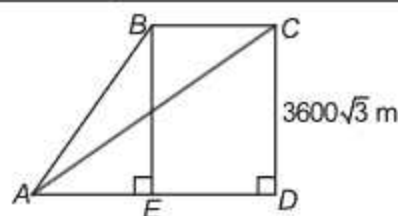
Height of aeroplane $(CD) = 3600\sqrt{3}$ m $= BE$

$\angle BAD = 60°$ and $\angle CAD = 30°$

In $\triangle ABE$

$$\tan 60° = \frac{BE}{AE}$$ [1]

$$AE = \frac{BE}{\tan 60°}$$

$$AE = 3600 \text{ m} \quad [\because BE = 3600\sqrt{3} \text{ m}]$$ [1]

In $\triangle ACD$

$$\tan 30° = \frac{CD}{AD}$$

$$AD = \frac{3600\sqrt{3}}{\frac{1}{\sqrt{3}}}$$

$$AD = 10800 \text{ m}$$ [1]

$$\therefore \quad BC = AD - AE = 10800 - 3600$$ [1]

$$BC = 7200 \text{ m}$$

$$\text{Speed of aeroplane} = \frac{\text{distance}}{\text{time}}$$ [1]

$$= \frac{7200}{30} = 240 \text{ m/s}$$

Speed (in km/hr) = 864 km/hour [1]

34.
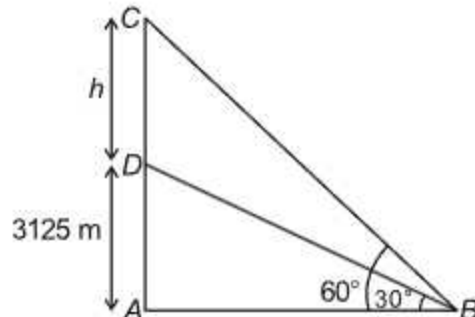
Let the distance between the two planes be h m.

Given that: $AD = 3125$ m and

$$\angle ABC = 60°$$ [1]

$$\angle ABD = 30°$$

In $\triangle ABD$,

$$\tan 30° = \frac{AD}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{3125}{AB}$$

$\Rightarrow \quad AB = 3125\sqrt{3} \qquad \text{...(i)}$ **[1]**

$\triangle ABC$

$$\tan 60° = \frac{AC}{AB}$$

$$\sqrt{3} = \frac{AD + DC}{AB} \qquad \textbf{[1]}$$

$$\sqrt{3} = \frac{3125 + h}{AB}$$

$\Rightarrow \quad AB = \frac{3125 + h}{\sqrt{3}} \qquad \text{...(ii)}$ **[1]**
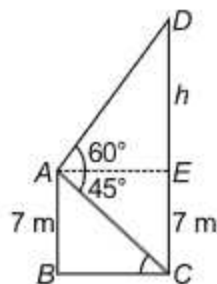
Equating equation (i) and (ii), we have

$$\frac{3125 + h}{\sqrt{3}} = 3125\sqrt{3}$$

$h = 3125 \times 3 - 3125$ **[1]**

$h = 6250$

Hence, distance between the two planes is 6250 m. **[1]**

35.

Let AB be the building and CD be the tower such that $\angle EAD = 60°$ and $\angle EAC = \angle ACB = 45°$ **[1]**

Now, in triangle ABC, $\tan 45° = 1 = AB/BC$

So, $AB = AE = 7$ m **[1]**

Again in triangle AED,

$\tan 60° = \sqrt{3} = DE/AE$ **[1]**

So, $DE = AE\sqrt{3} = 7\sqrt{3}$ **[1]**

$\Rightarrow h = 7\sqrt{3}$ m **[1]**

Height of tower $= h + 7 = 7(1 + \sqrt{3})$ m **[1]**

36.
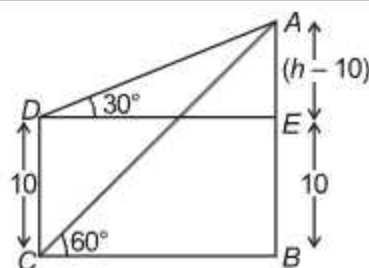
Height of the tower $(AB) = h$

Given $CD = 10$ m and $BC = ED$

$BE = CD = 10$ m **[1]**

In $\triangle ABC$, $\tan 60° = \frac{h}{BC}$ **[1]**

$BC = \frac{h}{\sqrt{3}}$ **[1]**

In $\triangle ADE$,

$\tan 30° = \frac{h-10}{ED}$ **[1]**

$ED = (h-10)\sqrt{3}$

$\therefore \quad \frac{h}{\sqrt{3}} = (h-10)\sqrt{3}$ **[1]**

$10 = \frac{2}{3}h$

$\boxed{h = 15 \text{ m}}$ **[1]**

37.
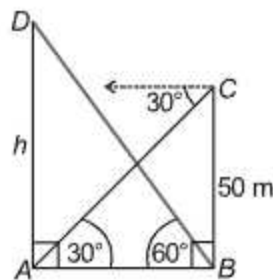
Let the height of hill be h.

In right triangle ABC,

$\frac{50}{AB} = \tan 30° \Rightarrow \frac{50}{AB} = \frac{1}{\sqrt{3}} \Rightarrow AB = 50\sqrt{3}$ **[2]**
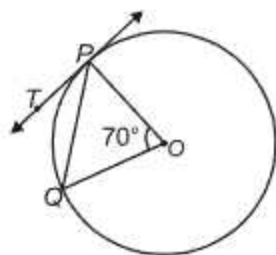
In right triangle BAD,

$\frac{h}{AB} = \tan 60° \Rightarrow \frac{h}{AB} = \sqrt{3} \Rightarrow h = \sqrt{3}AB$ **[2]**

$\Rightarrow h = \sqrt{3}(50\sqrt{3}) = 150$ m

Hence, the height of hill is 150 m. **[2]**

10 : Circles

1. Answer (d)

Given $\angle POQ = 70°$

In $\triangle POQ$, $OP = OQ$ (radii)

∴ It is an isosceles triangle

⇒ $\angle OPQ = \angle OQP$

In $\triangle POQ$,

$\angle POQ + \angle OPQ + \angle OQP = 180°$

$\angle POQ + 2\angle OPQ = 180°$

$\angle OPQ = 55°$ [½]

We know that $OP \perp PT$

∴ $\angle OPT = 90°$

$\angle OPT = \angle TPQ + \angle OPQ$

$90° = \angle TPQ + 55°$

$\angle TPQ = 35°$ [½]

2. Answer (c)
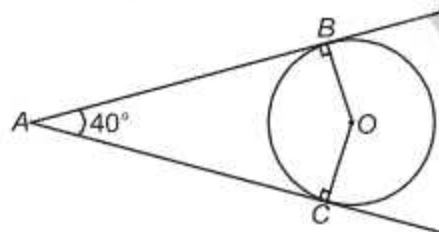
AB and AC are the tangents drawn from external point A to the circle.

∴ $OB \perp AB \Rightarrow \angle OBA = 90°$

$OC \perp AC \Rightarrow \angle OCA = 90°$

$ABCD$ is a quadrilateral in which sum of opposite angles is 180°

i.e.; $\angle OBA + \angle OCA = 180°$ [½]

∴ $ABCD$ is a cyclic quadrilateral

⇒ $\angle BAC + \angle BOC = 180°$

$\angle BOC = 180° - 40°$

$\boxed{\angle BOC = 140°}$ [½]

3. Answer (a)

It is known that the tangents from an external point to the circle are equal.

∴ $EK = EM$, $DK = DH$ and $FM = FH$ ...(i) [½]

Perimeter of $\triangle EDF = ED + DF + FE$

$= (EK - DK) + (DH + HF) + (EM - FM)$

$= (EK - DH) + (DH + HF) + (EM - FH)$

[Using (i)]
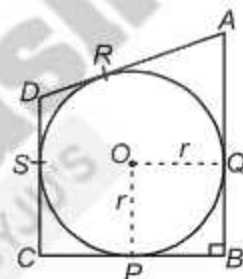
$= EK + EM$

$= 2 EK = 2 (9 \text{ cm}) = 18 \text{ cm}$

Hence, the perimeter of EDF is 18 cm.   [½]

4. Answer (a)

Given: AB, BC, CD and AD are tangents to the circle with centre O at Q, P, S and R respectively. $AB = 29$ cm, $AD = 23$, $DS = 5$ cm and $\angle B = 90°$

Construction: Join PQ.

We know that, the lengths of the tangents drawn from an external point to a circle are equal.

$DS = DR = 5$ cm

∴ $AR = AD - DR = 23 \text{ cm} - 5 \text{ cm} = 18 \text{ cm}$

$AQ = AR = 18$ cm

∴ $QB = AB - AQ = 29 \text{ cm} - 18 \text{ cm} = 11 \text{ cm}$

$QB = BP = 11$ cm

In $\triangle PQB$,

$PQ^2 = QB^2 + BP^2 = (11 \text{ cm})^2 + (11 \text{ cm})^2 = 2 \times (11 \text{ cm})^2$

$PQ = 11\sqrt{2}$ cm   ...(i)   [½]

In $\triangle OPQ$,

$PQ^2 = OQ^2 + OP^2 = r^2 + r^2 = 2r^2$

$(11\sqrt{2})^2 = 2r^2$

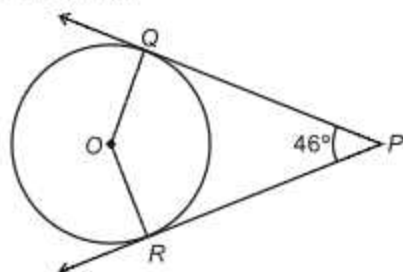$121 = r^2$

$r = 11$

Thus, the radius of the circle is 11 cm.   [½]

5. **Answer (b)**

    $AP \perp PB$ (Given)

    $CA \perp AP$, $CB \perp BP$ (Since radius is perpendicular to tangent)

    $AC = CB$ = radius of the circle   [½]

    Therefore, $APBC$ is a square having side equal to 4 cm.

    Therefore, length of each tangent is 4 cm.   [½]

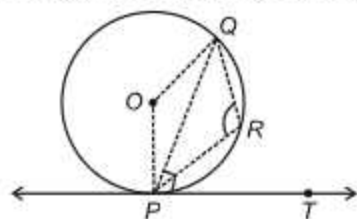6. **Answer (b)**

    Given : $\angle QPR = 46°$

    PQ and PR are tangents.

    Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

    So, we have $OQ \perp PQ$ and $OR \perp RP$.

    $\Rightarrow \angle OQP = \angle ORP = 90°$ [½]

    So, in quadrilateral $PQOR$, we have

    $\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360°$

    $\Rightarrow 90° + 46° + 90° + \angle ROQ = 360°$

    $\Rightarrow \angle ROQ = 360° - 226° = 134°$

    Hence, the correct option is (b).   [½]

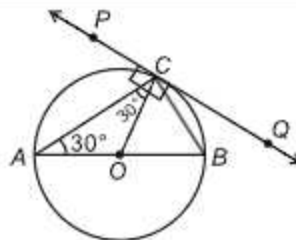7.

    $\angle OPT = 90°$

        (radius is perpendicular to the tangent)

    So, $\angle OPQ = \angle OPT - \angle QPT$

    $= 90° - 60°$

    $= 30°$

    $\angle POQ = 180° - 2\angle QPO = 180° - 60° = 120°$

    Reflex $\angle POQ = 360° - 120° = 240°$ [½]

    $\angle PRQ = \dfrac{1}{2}\text{reflex}\angle POQ$

    $= \dfrac{1}{2} \times 240°$

    $= 120°$

    $\therefore \quad PRQ = 120°$ [½]

8.

    In $\triangle ACO$,

    $OA = OC$ [Radii of the same circle]

    $\therefore \quad \triangle ACO$ is an isosceles triangle.

    $\angle CAB = 30°$ [Given]

    $\therefore \quad \angle CAO = \angle ACO = 30°$ [½]

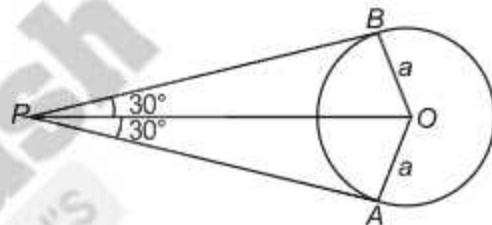   [angles opposite to equal sides of an isosceles triangle are equal]

    $\angle PCO = 90°$

        [radius drawn at the point of contact is perpendicular to the tangent]

    Now $\angle PCA = \angle PCO - \angle ACO$

    $\therefore \quad \angle PCA = 90° - 30° = 60°$ [½]

9.

    Given that $\angle BPA = 60°$

    $OB = OA = a$ [radii]

    $PA = PB$ [length of tangents are equal]

    $OP = OP$ [Common]

    $\therefore \quad \triangle PBO$ and $\triangle PAO$ are congruent.   [½]

        [By SSS criterion of congruency]

    $\therefore \quad \angle BPO = \angle OPA = \dfrac{60°}{2} = 30°$

    In $\triangle PBO$, $\sin 30° = \dfrac{a}{OP} = \dfrac{1}{2}$ $(\because OB \perp BP)$

    $OP = 2a$ units   [½]

10. **Answer (c)**

    In $\triangle POT$,

    $(OP)^2 = (OT)^2 + (PT)^2$

    $\Rightarrow OP^2 = (7)^2 + (24)^2$

    $\Rightarrow OP^2 = (25)^2$

    $\Rightarrow OP = 25$ cm

    $\therefore \quad PR = OP + OR = 25 + 7$

    $= 32$ cm

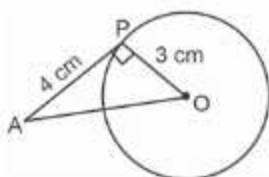   Hence, option (c) is correct.   [1]

11. Answer (a) [1]

One tangent can be drawn to a circle from a point on it.
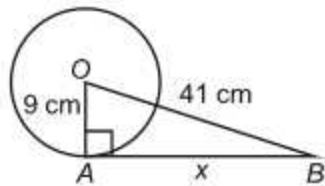
12. Answer (b) [1]

$\angle OPA = 90°$

$\Rightarrow OA^2 = 3^2 + 4^2$

$\Rightarrow OA = 5$ cm

13. Answer (a) [1]

In $\triangle OAB$,

$OA^2 + AB^2 = OB^2$

$\Rightarrow 9^2 + x^2 = 41^2$

$\Rightarrow x^2 = 41^2 - 9^2 = 1600$

$\Rightarrow x = 40$ cm

14. Answer (c) [1]

$\angle OPQ = \angle OPR - \angle QPR$
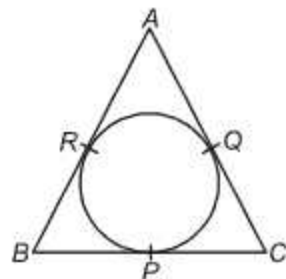
$= 90° - 50° = 40°$

Also, $\angle OPQ = \angle OQP = 40°$

$\angle POQ = 180° - 40° - 40°$

$= 100°$

15. Answer (b) [1]

Both Assertion (A) and Reason (R) are true but Reason (R) does not give the correct explanation of Assertion (A)

16.

Given $BR = 3$ cm, $AR = 4$ cm & $AC = 11$ cm

$BP = BR$

$AR = AQ$

$CP = CQ$

(Lengths of tangents to circle from external point will be equal)

$\therefore$ $AQ = 4$ cm and $BP = 3$ cm [½]

As $AC = 11$ cm

$QC + AQ = 11$ cm

$\Rightarrow QC = 7$ cm

$\therefore PC = 7$ cm

We know $BC = BP + PC$

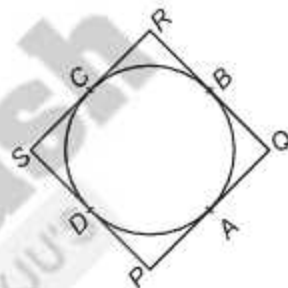$\therefore BC = 3 + 7$

$BC = 10$ cm [½]

17. $BP = BQ = 3$ cm

$AR = AP = 4$ cm

$RC = AC - AR = 7$ cm

$RC = QC = 7$ cm

$\therefore BC = 7 + 3 = 10$ cm [1]

18.

Given a parallelogram $PQRS$ in which a circle is inscribed

We know $PQ = RS$

$QR = PS$ [½]

$DP = PA$ ...(i)

(tangents to the circle from external point have equal length)

Similarly,

$QA = BQ$ ...(ii)

$BR = RC$ ...(iii)

$DS = CS$ ...(iv)

Adding above four equations, [½]

$DP + BQ + BR + DS = PA + QA + RC + CS$

$(DP + DS) + (BQ + BR) = (PA + QA) + (RC + CS)$ [½]

$2QR = 2(PQ)$

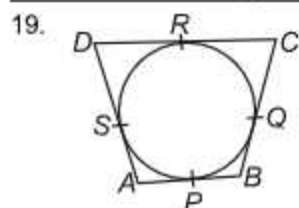$\therefore PQ = QR$

$\Rightarrow PQ = QR = RS = QS$

$\therefore PQRS$ is a rhombus [½]

19.

$AB = 6$ cm

$BC = 9$ cm

$CD = 8$ cm

AB, BC, CD, AD, are tangents to the circle

And $AP = AS$,　　　　$RD = DS$,

$BP = BQ$　　and

$CQ = CR$　　　　　　　　　　[½]

Also $AB = AP + BP$　　　...(i)

$BC = BQ + QC$　　　...(ii)

$CD = RC + DR$　　　...(iii)

$AD = AS + DS$　　　...(iv)　　[½]

Adding (i), (ii), (iii), (iv), we have

$6 + 9 + 8 + AD = AP + AS + BP + BQ + CQ + RC + RD + DS$　　　　[½]

$23 + AD = 2(AP) + 2(BP) + 2(RC) + 2(RD)$

$23 + AD = 2(AB) + 2(CD)$

$\boxed{AD = 5 \text{ cm}}$　　　　　　　　[½]

20. Given : ABC is an isosceles triangle, where $AB = AC$, circumscribing a circle.

To prove : The point of contact P bisects the base BC.

i.e. $BP = PC$

Proof : It can be observed that

BP and BR; CP and CQ; AR and AQ are pairs of tangents drawn to the circle from the external points B, C and A respectively.

So, applying the theorem that the tangents drawn from an external point to a circle are equal, we get

$BP = BR$　　　　　...(i)

$CP = CQ$　　　　　...(ii)

$AR = AQ$　　　...(iii)　　[½]

Given that $AB = AC$

$\Rightarrow AR + BR = AQ + CQ$　　　[½]

$\Rightarrow BR = CQ$ [from (iii)]

$\Rightarrow BP = CP$ [from (i) and (ii)]　　[½]

$\therefore$ P bisects BC.

Hence proved.　　　　　　　　[½]

21.
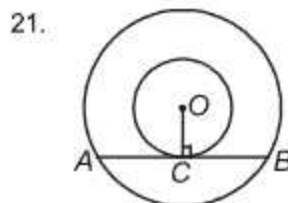
Given : AB is chord to larger circle and tangent to smaller circle at C concentric to it.

To prove : $AC = BC$

Construction : Join OC　　　　　[1]

Proof : $OC \perp AB$　　　　　　　[½]

($\because$ Radius is perpendicular to tangent at point of contact)

$\Rightarrow AC = BC$　　　　　　　[½]

($\because$ Perpendicular from centre bisects the chord)

22. Given : $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm.

Let, $AD = AF = x$ cm, $BD = BE = y$ cm and $CE = CF = z$ cm

(Tangents drawn from an external point to the circle are equal in length)

$\Rightarrow 2(x + y + z) = AB + BC + AC = AD + DB + BE + EC + AF + FC = 30$ cm　　[½]

$\therefore$ $x + y + z = 15$ cm

$AB = AD + DB = x + y = 12$ cm　　[½]

$\therefore$ $z = CF = 15 - 12 = 3$ cm

$AC = AF + FC = x + z = 10$ cm

$\therefore$ $y = BE = 15 - 10 = 5$ cm　　[½]

$\therefore$ $x = AD = x + y + z - z - y = 15 - 3 - 5$
$= 7$ cm　　　　　　　　　　[½]

23. Let XBY and PCQ be two parallel tangents to a circle with centre O.

Construction : Join OB and OC.

Draw $OA \parallel XY$
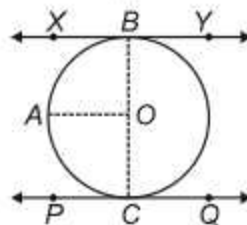
Now, $XB \parallel AO$

$\Rightarrow \angle XBO + \angle AOB = 180°$　　[½]

(Sum of adjacent interior angles is 180°)

Now, $\angle XBO = 90°$

(A tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow 90° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 90° = 90°$ [½]

Similarly , $\angle AOC = 90°$

$\angle AOB + \angle AOC = 90° + 90° = 180°$ [½]

Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.   [½]

24. Let us draw the circle with extent point P and two tangents PQ and PR.
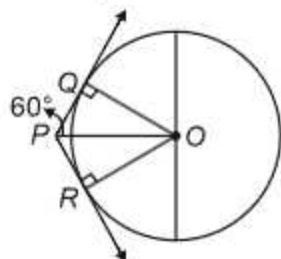
We know that the radius is perpendicular to the tangent at the point of contact.

$\therefore \angle OQP = 90°$ [½]

We also know that the tangents drawn to a circle from an external point are equally inclined to the line joining the centre to that point.
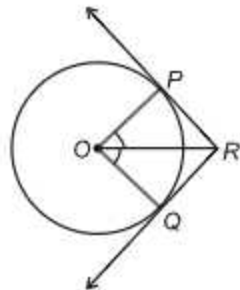
$\therefore \angle QPO = 60°$ [½]

Now, in $\triangle QPO$,

$\cos 60° = \dfrac{PQ}{PO}$ [½]

$\Rightarrow \dfrac{1}{2} = \dfrac{PQ}{PO}$

$\Rightarrow 2PQ = PO$ [½]

25.

Given that $\angle PRQ = 120°$

We know that the line joining the centre and the external point is the angle bisector of angle between the tangents.

Thus,

$\angle PRO = \angle QRO = \dfrac{120°}{2} = 60°$ [½]

Also we know that lengths of tangents from an external point are equal.

Thus, $PR = RQ$.

Join OP and OQ.

Since OP and OQ are the radii from the centre O,

$OP \perp PR$ and $OQ \perp RQ$.   [½]

Thus, $\triangle OPR$ and $\triangle OQR$ are right angled congruent triangles.

Hence, $\angle POR = 90° - \angle PRO = 90° - 60° = 30°$

$\angle QOR = 90° - \angle QRO = 90° - 60° = 30°$ [½]

$\sin \angle QRO = \sin 30° = \dfrac{1}{2}$

$\dfrac{PR}{OR} = \dfrac{1}{2}$

Thus, $\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + QR$ [½]

26.
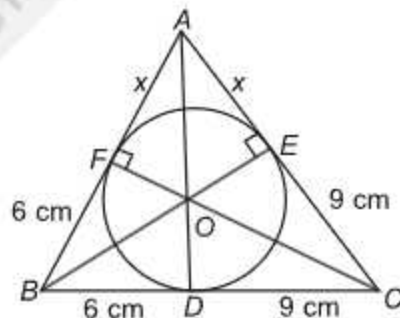
Let the given circle touch the sides AB and AC of the triangle at points F and E respectively and let the length of line segment AF be x.

Now, it can be observed that:

$BF = BD = 6$ cm (tangents from point B)

$CE = CD = 9$ cm (tangents from point C)

$AE = AF = x$ (tangents from point A)

$AB = AF + FB = x + 6$

$BC = BD + DC = 6 + 9 = 15$

$CA = CE + EA = 9 + x$ [½]

$2s = AB + BC + CA = x + 6 + 15 + 9 + x = 30 + 2x$

$s = 15 + x$

$s - a = 15 + x - 15 = x$

$s - b = 15 + x - (x + 9) = 6$

$s - c = 15 + x - (6 + x) = 9$

Area of $\triangle ABC = \sqrt{s(s-a)(a-b)(s-c)}$ [½]

$54 = \sqrt{(15+x)(x)(6)(9)}$

$54 = 3\sqrt{6(15x + x^2)}$

$18 = \sqrt{6(15x + x^2)}$

$324 = 6(15x + x^2)$

$54 = 15x + x^2$

$x^2 + 15x - 54 = 0$ [½]

$x^2 + 18x - 3x - 54 = 0$

$x(x + 18) - 3(x + 18)$

$(x + 18)(x - 3) = 0$

As distance cannot be negative, $x = 3$ cm

$AC = 3 + 9 = 12$ cm

$AB = AF + FB = 6 + x = 6 + 3 = 9$ cm   [½]

27. Since tangents drawn from an exterior point to a circle are equal in length,

$AP = AS$ ...(i)

$BP = BQ$ ...(ii)

$CR = CQ$ ...(iii)

$DR = DS$ ...(iv)   [½]

Adding equations (i), (ii), (iii) and (iv), we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$ [½]

∴ $(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$ [½]

∴ $AB + CD = AD + BC$

∴ $AB + CD = BC + DA$ [Proved]   [½]

28.
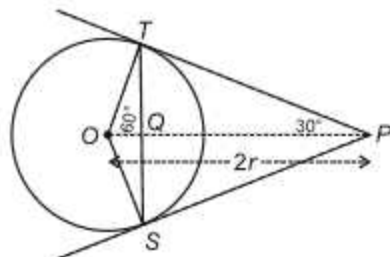
In the given figure,

$OP = 2r$ [Given]

$\angle OTP = 90°$

[radius drawn at the point of contact is perpendicular to the tangent]

In $\triangle OTP$,

$\sin \angle OPT = \dfrac{OT}{OP} = \dfrac{r}{2r} = \dfrac{1}{2} = \sin 30°$

$\angle OPT = 30°$

$\angle TOP = 60°$ [½]

∴ $\triangle OTP$ is a $30° - 60° - 90°$, right triangle.

In $\triangle OTS$,

$OT = OS$ [Radii of the same circle]

∴ $\triangle OTS$ is an isosceles triangle.

∴ $\angle OTS = \angle OST$ [½]

[Angles opposite to equal sides of an isosceles triangle are equal]

In $\triangle OTQ$ and $\triangle OSQ$

$OS = OT$ [Radii of the same circle]

$OQ = OQ$

[side common to both triangles]

$\angle OTQ = \angle OSQ$

[angles opposite to equal sides of an isosceles triangle are equal]

∴ $\triangle OTQ \cong \triangle OSQ$ [By S.A.S]   [½]

∴ $\angle TOQ = \angle SOQ = 60°$ [C.A.C.T]

∴ $\angle TOS = 120°$

[$\angle TOS = \angle TOQ + \angle SOQ$
$= 60° + 60° = 120°$]

∴ $\angle OTS + \angle OST = 180° - 120° = 60°$

∴ $\angle OTS = \angle OST = 60° \div 2 = 30°$ [½]

29.
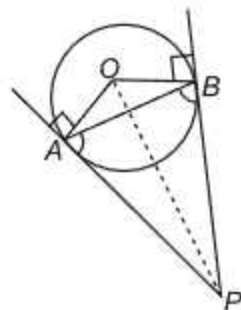
AB is the chord

We know that $OA = OB$ [radii]

$\angle OBP = \angle OAP = 90°$

Join OP and $OP = OP$ [Common]   [½]

By RHS congruency

$\triangle OBP \cong \triangle OAP$ [½]

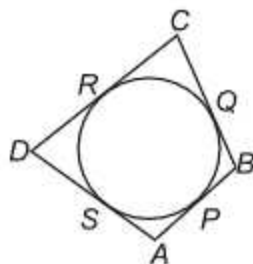$\therefore$ By $CPCT$, $BP = AP$ [½]

In $\triangle ABP$ $BP = AP$

Angles opposite to equal sides are equal

$\therefore$ $\angle BAP = \angle ABP$ [½]

Hence proved.

30.

$ABCD$ is the Quadrilateral

Circle touches the sides at P, Q, R, S

For the circle AS & AP are tangents

$\therefore$ $AS = AP$ ...(i)

Similarly,

$BP = BQ$ ...(ii)   [½]

$CQ = CR$ ...(iii)

$RD = DS$ ...(iv)   [½]

Now, $AB + CD = AP + PB + CR + RD$ ...(v)

and $BC + AD = BQ + QC + DS + AS$ ...(vi) [½]

$BC + AD = BP + CR + RD + AP$ using (i), (i), (iii), (iv)

$\therefore$ $AB + CD = BC + AD$ [Using (v)]

Hence proved   [½]

31. $\because$ Tangents from external point are equal in length.
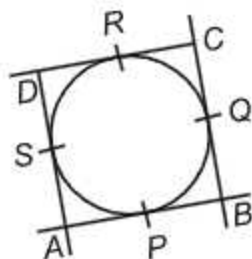
$\therefore$ $AP = AS$ ...(1)

$BP = BQ$ ...(2)

$CR = CQ$ ...(3)

$DR = DS$ ...(4)

Adding equations (1 + 2 + 3 + 4)

$AP + BP + CR + DR = AS + BQ + CQ + DS$
                                [1]

$AB + CD = AD + BC$

$6 + 8 = AD + 9$

$AD = 14 - 9 = 5$ cm   [1]

32. Join OQ.

$\angle OPQ = \angle OQP$ {$OP = OQ$}

$\Rightarrow$ $\angle OPQ + \angle OQP + \angle POQ = 180°$ [½]

{Angle sum property}

$\Rightarrow$ $2\angle OPQ = 180° - \angle POQ$ ...(i)

Also, $\angle PTQ + \angle POQ = 180°$

$\Rightarrow$ $\angle PTQ = 180° - \angle POQ$ ...(ii)   [½]

From (i) and (ii),   [½]

$\angle PTQ = 2\angle OPQ$ [½]

Hence Proved.

33. (a) $PA = PQ + QA$
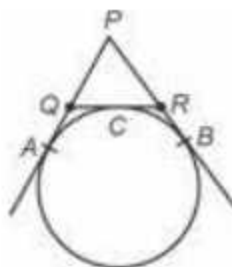
$= PQ + QC$ ...(i) [$\because$ $QA = QC$]   [½]

and $PB = PR + BR$ [½]

$= PR + CR$ ...(ii) [$\because$ $BR = CR$]

Adding (i) and (ii), we get

$PA + PB = PQ + QC + CR + PR$

$\Rightarrow$ $2PA = PQ + QR + PR$ [½]

[$\because$ $PA = PB$]

$\Rightarrow$ $PA = \dfrac{\text{Perimeter of } \triangle PQR}{2}$

$= \dfrac{20}{2}$ [½]

$= 10$ cm

OR

(b) $\angle ADB = 90°$ [Angle in semi-circle]　　[½]

$\angle ABD = 90° - \angle BAD$ 　[Angle sum property of $\triangle ABD$]

$= 90° - 55°$

$= 35°$ 　　　　　　　[½]

Now, $\angle DBC = 90° - \angle ABD$ 　　[½]

[∵ $AB \perp BC$]

$= 90° - 35°$

$= 55°$ 　　　　　　　[½]

34. In $\triangle OAB$,

$OA = OB$ 　　　　　[radius of circle]

∴ $\angle OAB = \angle OBA = 40°$ 　[∵ $OA = OB$]
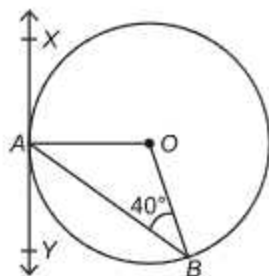
Since, XAY is tangent to the circle.

∴ $\angle OAY = 90°$

[∵ The tangent to a circle is perpendicular to the radius through the point of contact]

∴ $\angle BAY + \angle OAB = 90°$

$\angle BAY = 90° - 40°$

$\angle BAY = 50°$ 　　　　　[1]

Further in $\triangle ABO$,

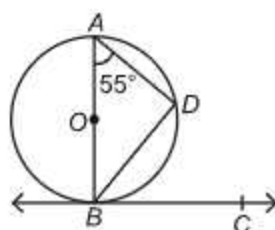$\angle OAB + \angle OBA + \angle AOB = 180°$

$\angle AOB = 180° - 80° = 100°$ 　[1]

35. Let $\angle OAB = \angle OBA = \theta$

[∵ $OA = OB = $ radius]

Also, $OA \perp PA$

∵ $\angle AOB = 180° - 2\theta$

[Angle sum property] 　　　　[½]
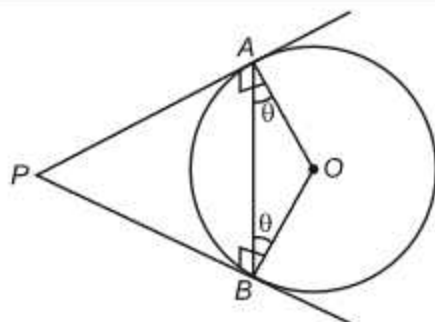
In $OAPB$,

By angle sum property

$90° + \angle APB + 90° + \angle AOB = 360°$ 　[½]

$90° + \angle APB + 90° + 180° - 2\theta = 360°$

⇒ $\angle APB = 2\theta$ 　　　　[½]

⇒ $\angle APB = 2\angle OAB$ 　　[½]

Hence proved.

36.
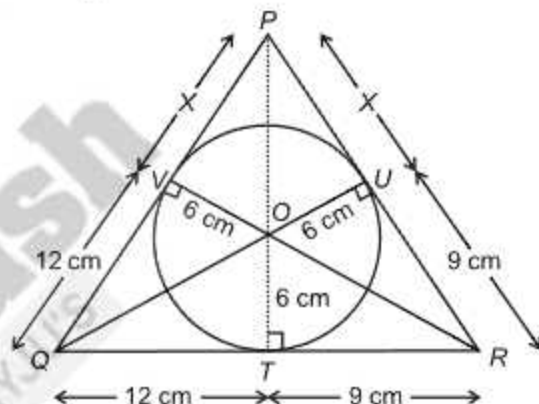
$ar(\triangle PQR) = ar(\triangle POQ) + ar(\triangle QOR) + ar(\triangle POR)$

⇒ $189 = \frac{1}{2} \times OV \times PQ + \frac{1}{2} \times OT \times QR + \frac{1}{2} \times OU \times PR$

[½]

$189 = \frac{1}{2} \times 6(PQ + QR + PR) = 3(PQ + QR + PR)$ 　[½]

(∵ $OT = OV = OU = 6$ cm)

⇒ $189 = 3(x + 12 + 12 + 9 + 9 + x)$

[∵ $PV = PU = x$, $QT = 12$ cm and $RT = RU$ $= 9$ cm as tangents from external point to a circle are equal] 　　　　　　[½]

⇒ $63 = 24 + 18 + 2x$

⇒ $2x = 21$

⇒ $x = \frac{21}{2} = PV = PU$ 　　[½]

∴ $PQ = PV + QV = 12 + \frac{21}{2} = \frac{45}{2}$ cm 　[½]

and $PR = PU + UR = 9 + \frac{21}{2} = \frac{39}{2}$ cm 　[½]

37. Join OT which bisects PQ at M and perpendicular to PQ
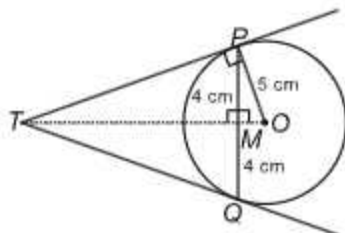
In $\triangle OPM$,

$OP^2 = PM^2 + OM^2$ [By Pythagoras Theorem]   [½]

$\Rightarrow$ $(5)^2 = (4)^2 + OM^2$

$\Rightarrow$ $OM = 3$ cm   [½]

In $\triangle OPT$ and $\triangle OPM$,

$\angle MOP = \angle TOP$ [Common angles]

$\angle OMP = \angle OPT$ [Each 90°]

$\therefore$ $\triangle POT \sim \triangle MOP$ [By AA similarity]   [½]

$\Rightarrow$ $\dfrac{TP}{MP} = \dfrac{OP}{OM}$ [½]

$\Rightarrow$ $TP = \dfrac{4 \times 5}{3}$ [½]

$[\because OP = 5$ cm, $PM = 4$ cm, $MO = 3$ cm$]$

$\Rightarrow$ $TP = \dfrac{20}{3} = 6\dfrac{2}{3}$ cm   [½]

38. (A) The lengths of two tangents drawn from an external point to a circle are equal.

Given : AB and AC are two tangents from a point A to a circle $C(O, r)$.

To Prove : $AB = AC$ [½]

Construction : Join OA, OB and OC.   [½]
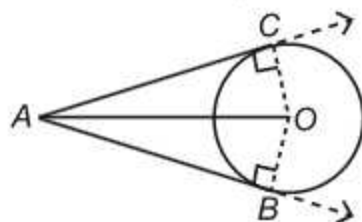
Proof : In $\triangle OBA$ and $\triangle OCA$,

$OB = OC$ Radii of the same circle]   [½]

$OA = OA$ [Common side]

$\angle OBA = \angle OCA = 90°$

[Each 90° because tangent is perpendicular to radius at the point of contact]   [½]

$\triangle OBA \equiv \triangle OCA$ [By R.H.S. congruency]   [½]

$\Rightarrow$ $AB = AC$ [CPCT]   [½]

Hence proved.

OR

(B) In $\triangle OAP$,
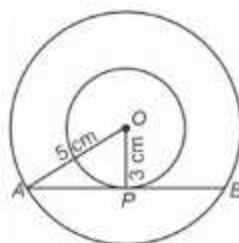
$OP \perp AB$

$OA^2 - OP^2 = AP^2$ [1]

$5^2 - 3^2 = AP^2$

$16 = AP^2$

$\therefore$ $AP = 4$ cm   [1]

Length of chord $AB = 2AP = 8$ cm   [1]

39. $\angle OBA = 90°$ [A tangent to the circle is perpendicular to the radius through the point of contact]   [½]
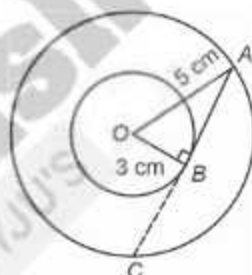
$OA^2 = OB^2 + AB^2$ [½]

$\Rightarrow$ $(5)^2 = (3)^2 + AB^2$

$\Rightarrow$ $AB^2 = 25 - 9$ [½]

$\Rightarrow$ $AB = 4$ [½]

$\Rightarrow$ Length of chord $AC = 2AB = 2(4) = 8$ cm   [½]

$\therefore$ The length of chord of the larger circle which touches the smaller circle is 8 cm.   [½]

40. **Given :** A circle with centre O. PA and PB are tangents to the circle at A and B.
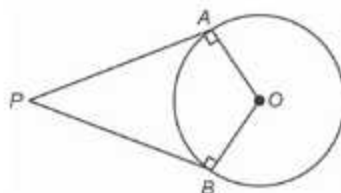
To prove : $\angle APB + \angle AOB = 180°$ [½]

Proof : We know that radius is perpendicular to tangent at point of contact.   [½]

⇒ $OA \perp PA$ and $OB \perp PB$

⇒ $\angle PAO = \angle PBO = 90°$ [½]

In quadrilateral *PBOA*,

$\angle APB + \angle PBO + \angle BOA + \angle OAP = 360°$

[Angle sum property of quadrilateral]

⇒ $\angle APB + 90° + \angle BOA + 90° = 360°$ [½]

⇒ $\angle APB + \angle BOA = 360° - 180° = 180°$ [½]

Hence proved.   [½]

41.
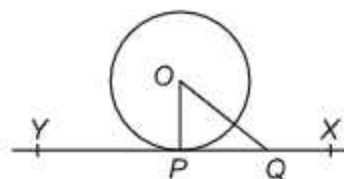
Given : A circle with centre *O* and a tangent *XY* to the circle at a point *P*   [½]

To Prove : *OP* is perpendicular to *XY*.

Construction : Take a point *Q* on *XY* other than *P* and join *OQ*.   [½]

Proof : Here the point *Q* must lie outside the circle as if it lies inside the tangent *XY* will become secant to the circle.   [½]

Therefore, *OQ* is longer than the radius *OP* of the circle, That is, *OQ* > *OP*.   [1]

This happens for every point on the line *XY* except the point *P*.   [½]

So *OP* is the shortest of all the distances of the point *O* to the points on *XY*.   [½]

And hence *OP* is perpendicular to *XY*.   [½]

Hence proved.

42.   Given : *l* and *m* are two parallel tangents to the circle with centre *O* touching the circle at *A* and *B* respectively. *DE* is a tangent at the point *C*, which intersects *l* at *D* and *m* at *E*.

To prove: $\angle DOE = 90°$

Construction: Join *OC*.

Proof:
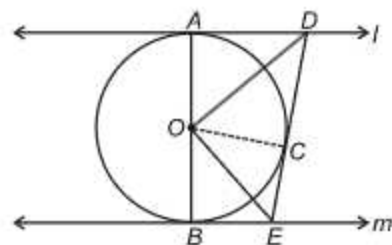
In $\triangle ODA$ and $\triangle ODC$,

$OA = OC$ [Radii of the same circle]

$AD = DC$

(Length of tangents drawn from an external point to a circle are equal]

$DO = OD$ [Common side]

$\triangle ODA \cong \angle ODC$ [SSS congruence criterion]
    [1]

∴ $\angle DOA = \angle COD$ ...(i)   [½]

Similarly, $\triangle OEB \cong \triangle OEC$ [½]

∴ $\angle EOB = \angle COE$ ...(ii)   [½]

Now, *AOB* is a diameter of the circle. Hence, it is a straight line.

$\angle DOA + \angle COD + \angle COE + \angle EOB = 180°$ [½]

From (i) and (ii), we have:

$2\angle COD + 2\angle COE = 180°$ [½]

⇒ $\angle COD + \angle COE = 90°$

⇒ $\angle DOE = 90°$

Hence, proved.   [½]

43.   Let *AP* and *BP* be the two tangents to the circle with centre O.
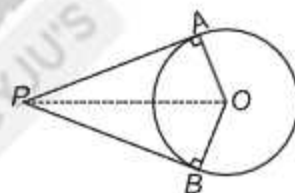
To Prove : *AP* = *BP*

Proof :   [½]

In $\triangle AOP$ and $\triangle BOP$,

$OA = OB$ [radii of the same circle]

$\angle OAP = \angle OBP = 90°$ [1]

[since tangent at any point of a circle is perpendicular to the radius through the point of contact]

$OP = OP$ [common]

∴ $\triangle AOP \cong \triangle BOP$ [1]

[by R.H.S. congruence criterion]

∴ $AP = BP$ [1]

[corresponding parts of congruent triangles]

Hence, the length of the tangents drawn from an external point to a circle are equal.   [½]

44. In the figure, C is the midpoint of the minor arc PQ, O is the centre of the circle and

    AB is tangent to the circle through point C.

    We have to show the tangent drawn at the midpoint of the arc PQ of a circle is parallel to the chord joining the end points of the arc PQ.

    We will show $PQ \parallel AB$.   [½]

    It is given that C is the midpoint point of the arc PQ.

    So, arc PC = arc CQ.   [½]

    $\Rightarrow PC = CQ$
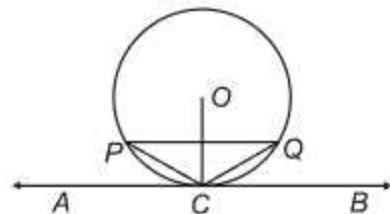
    This shows that $\triangle PQC$ is an isosceles triangle.

                    [½]

    Thus, the perpendicular bisector of the side PQ of $\triangle PQC$ passes through vertex C.

    The perpendicular bisector of a chord passes through the centre of the circle.   [½]

    So the perpendicular bisector of PQ passes through the centre O of the circle.   [½]

    Thus perpendicular bisector of PQ passes through the points O and C.

    $\Rightarrow PQ \perp OC$ [½]

    AB is the tangent to the circle through the point C on the circle.

    $\Rightarrow AB \perp OC$ [½]

    The chord PQ and the tangent AB of the circle are perpendicular to the same line OC.

    $\therefore PQ \parallel AB$.   [½]

45.
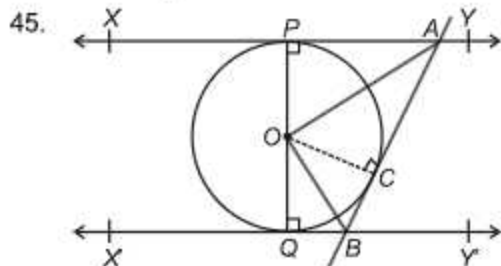
    To prove : $\angle AOB = 90°$

    In $\triangle AOC$ and $\triangle AOP$,

    $OA = OA$ [Common]

    $OP = OC$ [radii]   [½]

$\angle ACO = \angle APO$ [right angle]

$\therefore \triangle AOC \cong \triangle AOP$ (By RHS congruency)

                    [½]

By $CPCT$, $\angle AOC = \angle AOP$ ...(i)   [½]

Similarly In $\triangle BOC$ and $\triangle BOQ$

$OC = OQ$ [radii]

$OB = OB$ [Common]   [½]

and $\angle BCO = \angle BQO = 90°$

By RHS congruency, $\triangle BOC \cong \triangle BOQ$ [½]

By CPCT, $\angle BOC = \angle BOQ$ ...(ii)   [½]

PQ is a straight line

$\therefore \angle AOP + \angle AOC + \angle BOC + \angle BOQ = 180°$

From equations (i) and (ii), we have   [½]

$2(\angle AOC + \angle BOC) = 180°$

$\angle AOB = \dfrac{180°}{2}$

$\therefore \angle AOB = 90°$ [½]

46. (a) **Given :** A circle with centre O.

    A parallelogram $ABCD$ touching the circle at Points P, Q, R and S.
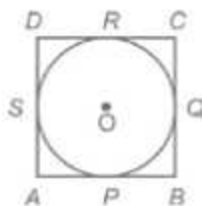
    To Prove: $ABCD$ is a rhombus

    Proof: A rhombus is a parallelogram with all sides equal

    In parallelogram $ABCD$

    $AB = CD$ and $BC = AD$ [1]

    We know that the lengths of tangents from an external point are equal

    $\therefore AP = AS$ ...(i)

    $BP = BQ$ ...(ii)

    $CQ = CR$ ...(iii)

    $DR = DS$ ...(iv)   [1]

Adding (i), (ii), (iii) and (iv), we get

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + (CR + DR) = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow CD + CD = BC + BC$ **[1]**

$[\because AB = CD \text{ and } AD = BC]$

$\Rightarrow CD = BC$

$\therefore AB = CD = BC = AD$

All sides are equal

$\Rightarrow$ Hence, $ABCD$ is a rhombus **[1]**

OR

(b) Let, O is the centre of the given circle. A segment PR has been drawn touching the circle at point P. **[½]**

Draw $QP \perp RP$ at point P, such that point Q lies on the circle. **[½]**
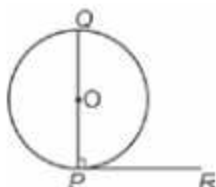
$\angle OPR = 90°$ [Radius $\perp$ Tangent] **[½]**

Also, $\angle QPR = 90°$ [given] **[½]**

$\therefore \angle OPR = \angle QPR$ **[½]**

Now, the above case is possible only when centre O lies on the line QP. **[1]**

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle. **[½]**

47.
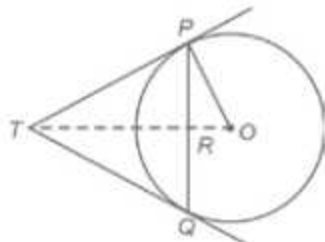
In $\triangle ORP$ and $\triangle OPT$,

$\angle ORP = \angle OPT$ [Each 90°]

$\angle POR = \angle POT$ [Common]

$\therefore \triangle ORP \sim \triangle OPT$ [By AA similarity] **[1]**

$\therefore \dfrac{OR}{OP} = \dfrac{PR}{PT}$ ...(i) **[1]**

In $\triangle POR$,

$OP^2 = OR^2 + PR^2$

[By Pythagoras theorem]

$\therefore (5)^2 = OR^2 + (4)^2$ $\left[\because PR = \dfrac{PQ}{2}\right]$

$\Rightarrow OR = 3$ cm **[1]**

From (i),

$\dfrac{3}{5} = \dfrac{4}{PT}$

$\therefore PT = \dfrac{20}{3}$ cm **[1]**

48.
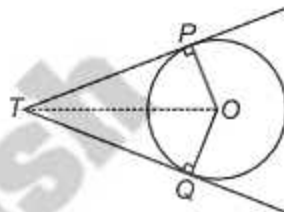
Given : PT and TQ are two tangents drawn from an external point T to the circle $C(O, r)$.

To prove : $PT = TQ$

Construction: Join OT. **[½]**

Proof : We know that a tangent to circle is perpendicular to the radius through the point of contact.

$\therefore \angle OPT = \angle OQT = 90°$

In $\triangle OPT$ and $\triangle OQT$,

$OT = OT$ [Common] **[½]**

$OP = OQ$ [Radius of the circle] **[½]**

$\angle OPT = \angle OQT = 90°$

$\therefore \triangle OPT \cong \triangle OQT$ [RHS congruence criterion] **[½]**

$\Rightarrow PT = TQ$ [CPCT] **[½]**

$\therefore$ The lengths of the tangents drawn from an external point to a circle are equal. **[½]**

Now,
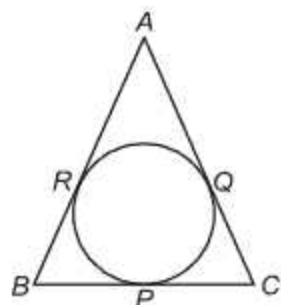
We know that the tangents drawn from an exterior point to a circle are equal in length.

∴ $AR = AQ$ (Tangents from A)   ...(i)   [½]

   $BP = BR$ (Tangents from B)   ...(ii)

   $CQ = CP$ (Tangents from C)   ...(iii)   [½]

Now, the given triangle is isosceles ($\because AB = AC$)

Subtract AR from both sides, we get

$AB - AR = AC - AR$ [½]

$\Rightarrow AB - AR = AC - AQ$ [Using (ii)]   [½]

$BR = CQ$

$\Rightarrow BP = CP$ (Using (ii), (iii)]   [½]

So $BP = CP$, shows that BC is bisected at the point of contact.   [½]

49. PT and TQ are two tangent drawn from an external part T to the circle $C(O, r)$
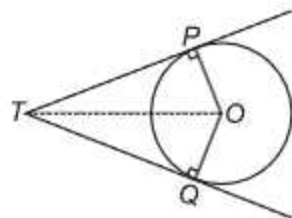
To prove : $PT = TQ$

Construction : Join OT [½]

Proof: We know that, a tangent to circle is perpendicular to the radius through the point of contact   [½]

∴ $\angle OPT = \angle OQT = 90°$ [½]

In $\triangle OPT$ and $\triangle OQT$,

$OT = OT$ [Common]

$OP = OQ$ [Radius of the circle]   [½]

$\angle OPT = OQT = 90°$

∴ $\triangle OPT \cong \triangle OQT$ [RHS congruence criterion]

    [½]

$\Rightarrow PT = TQ$ [CPCT]

∴   The lengths of the tangents drawn from an external point to a circle are equal.   [½]
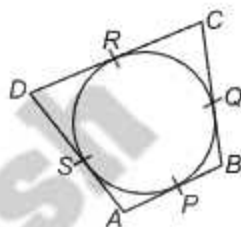
Let AB touches the circle at P. BC touches the circle at Q. DC touches the circle at $R.AD$. touches the circle at S.   [½]

Then, $PB = QB$ (Length of the tangents drawn from the external point are always equal)

Similarly, $QC = RC'$ [½]

$AP = AS$

$DS = DR$ [½]

Now,

$AB + CD$

$= AP + PB + DR + RC$ [½]

$= AS + QB + DS + CQ$ [½]

$= AS + DS + QB + CQ$

$= AD + BC$

Hence Proved   [½]

11 : Areas Related to Circles

1. Answer (c)

$$\frac{2\pi r \times \theta}{360°} = 22$$

$$\frac{2 \times 22}{7} \times \frac{21 \times \theta}{360°} = 22$$

∴ θ = 60°   [1]

2. Answer (d)

Perimeter of the sector

$$= 2\pi R \times \left(\frac{\theta}{360°}\right) + 2R$$

$$= \frac{45°}{360°} \times 2\pi R + 2R$$

= 39 cm   [1]

3. Answer (c)

Area of quadrant $= \dfrac{\pi r^2}{4}$

$$= \frac{22 \times 28 \times 28}{7 \times 4} \quad [\because\ 2\pi r = 176\ m]$$

$$= 616\ m^2 \qquad [1]$$

4. Answer (c)

Angle made by minute hand of a clock in 1 minute = 6°

Angle made by minute hand of the clock between 10:10 am to 10:25 am

(*i.e.* 15 minutes) = 15 × 6° = 90°

Distance covered (l) $= \dfrac{90°}{180°} \times \dfrac{22}{7} \times 84$

$$\left[\because\ l = \frac{\theta}{180°} \times \pi r\right]$$

= 132 cm   [1]

5. Answer (a)   [1]

length of arc $= \dfrac{2\pi \times 14}{6}$

$$= \frac{44}{3}\ cm$$

6. Answer (d)   [1]

Perimeter of protractor = πr + 2r

= 22 + 14

= 36 cm

7. Answer (c)   [1]

area of semicircle $= \dfrac{\pi\left(\dfrac{d}{2}\right)^2}{2} = \dfrac{1}{8}\pi d^2$

8. Length of arc = 22 cm

$$\Rightarrow \frac{2\pi r \theta}{360°} = 22 \qquad [\frac{1}{2}]$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times \frac{60°}{360°} = 22 \qquad [\frac{1}{2}]$$

$$\Rightarrow r = \frac{22 \times 7 \times 6}{2 \times 22} \qquad [\frac{1}{2}]$$

$$\Rightarrow r = 21\ cm \qquad [\frac{1}{2}]$$

9. Area of minor segment

= Area of sector AOB − Area of ΔAOB
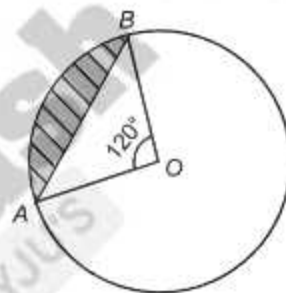
Given

∠AOB = 120°

OA = OB = 14 cm
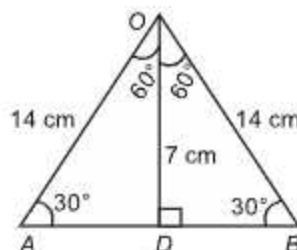
Area of sector $AOB = \dfrac{120°}{360°} \times \pi r^2$

$$= \frac{1}{3} \times \frac{22}{7} \times (14)^2 = \frac{616}{3} \qquad [1]$$

Draw OD ⊥ AB

In ΔODB,

∠O = 60° ∠B = 30°, ∠D = 90°

OD = 7 cm

$DB = 7\sqrt{3}$ cm

$\therefore \quad$ Area of $\Delta AOB = \dfrac{1}{2} \times AB \times OD$

$\qquad = \dfrac{1}{2} \times 14\sqrt{3} \times 7$

$\qquad = 49\sqrt{3}$ **[1]**

$\qquad = 84.77 \text{ cm}^2$

Area of minor segment $= \dfrac{616}{3} - 84.77$

$\qquad = 120.56 \text{ cm}^2$ **[1]**

10. The arc subtends an angle of $60°$ at the centre.

(i) $l = \dfrac{\theta}{360°} \times 2\pi r$ **[½]**

$\qquad = \dfrac{60°}{360°} \times 2 \times \dfrac{22}{7} \times 21$

$\qquad = 22 \text{ cm}$ **[1]**

(ii) Area of the sector $= \dfrac{\theta}{360°} \times \pi r^2$ **[½]**

$\qquad = \dfrac{60°}{360°} \times \dfrac{22}{7} \times 21 \times 21$

$\qquad = 231 \text{ cm}^2$ **[1]**

11. Radius of the circle = 14 cm

Central Angle, $\theta = 60°$,

Area of the minor segment

$\qquad = \dfrac{\theta}{360°} \times \pi r^2 - \dfrac{\sqrt{3}}{4} r^2$ **[1]**

$\qquad = \dfrac{60°}{360°} \times \pi (14)^2 - \dfrac{\sqrt{3}}{4} \times 14^2$

$\qquad = \dfrac{1}{6} \times \dfrac{22}{7} \times 14 \times 14 - \sqrt{3} \times (7)^2$

$\qquad = \dfrac{22 \times 14}{3} - 49\sqrt{3}$

$\qquad = \dfrac{22 \times 14}{3} - \dfrac{147\sqrt{3}}{3}$

$\qquad = \dfrac{308 - 147\sqrt{3}}{3} \text{ cm}^2$ **[1]**

$\therefore$ Area of the major segment

$\qquad = \pi (14)^2 - \left(\dfrac{308 - 147\sqrt{3}}{3} \right) \text{ cm}^2$

$\qquad = 616 - \dfrac{1}{3} \left[308 - 147\sqrt{3} \right]$

$\qquad = \left(1540 + 147\sqrt{3} \right) / 3 \text{ cm}^2$ **[1]**

12. Area of sector $= \dfrac{\pi r^2 \theta}{360°}$ **[½]**

$\qquad = \dfrac{22}{7} \times \dfrac{7 \times 7 \times 90°}{360°}$

$\qquad = 38.5 \text{ cm}^2$ **[½]**

Area of corresponding major sector = Area of circle − Area of minor sector **[½]**

$\qquad = \pi r^2 - 38.5$ **[½]**

$\qquad = \left(\dfrac{22}{7} \times 7 \times 7 \right) - 38.5$

$\qquad = (154 - 38.5) \text{ cm}^2$ **[½]**

$\qquad = 115.5 \text{ cm}^2$ **[½]**

13. *PQRS* is a square.

So each side is equal and angle between the adjacent sides is a right angle.

Also the diagonals perpendicularly bisect each other.

In ΔPQR using pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$PR^2 = (42)^2 + (42)^2$

$PR^2 = \sqrt{2} (42)$

$OR = \dfrac{1}{2} PR = \dfrac{42}{\sqrt{2}} = OQ$ **[1]**

From the figure we can see that the radius of flower bed ORQ is OR.

Area of sector $ORQ = \dfrac{1}{4} \pi r^2$

$\qquad = \dfrac{1}{4} \pi \left(\dfrac{42}{\sqrt{2}} \right)^2$

Area of the $\Delta ROQ = \dfrac{1}{2} \times RO \times OQ$

$\qquad = \dfrac{1}{2} \times \dfrac{42}{\sqrt{2}} \times \dfrac{42}{\sqrt{2}}$

$\qquad = \left(\dfrac{42}{2} \right)^2$ **[1]**

Area of the flower bed ORQ

= Area of sector ORQ − Area of the ROQ

$$= \frac{1}{2}\pi\left(\frac{42}{\sqrt{2}}\right)^2 - \left(\frac{42}{2}\right)^2$$

$$= \left(\frac{42}{2}\right)^2 \left[\frac{\pi}{2} - 1\right]$$

$= (441)\,[0.57]$

$= 251.37$ cm^2 **[1]**

Area of the flower bed ORQ = Area of the flower bed OPS

$= 251.37$ cm^2

Total area of the two flower beds

= Area of the flower bed ORQ + Area of the flower bed OPS

$= 251.37 + 251.37$

$= 502.74$ cm^2 **[1]**

14. (i) Here,

Radius of parking, $r = \frac{7}{2} = 3.5$ units

Perimeter of parking area $= 2r + r$ **[½]**

$$= 2\times\frac{7}{2} + \frac{22}{7}\times\frac{7}{2}$$

$= 7 + 11$

$= 18$ units **[½]**

(ii) (a) Radius of one quadrant $r = 2$ units

Radius of parking area, $r = \frac{7}{2}$ units

$\therefore$ Required area $= \frac{\pi r^2}{2} + 2\times\frac{\pi r'^2}{4}$ **[½]**

$$= \frac{\pi}{2}\left(r^2 + r'^2\right)$$

$$= \frac{22}{7\times2}\left(\left(\frac{7}{2}\right)^2 + (2)^2\right)$$

$$= \frac{11}{7}\left(\frac{49}{4} + 4\right)$$ **[½]**

$$= \frac{11\times65}{28}$$ **[½]**

$= 25.54$ square units (approx.) **[½]**

OR

(b) Area of playground $= \ell \times b$

$= 14 \times 7$

$= 98$ square units **[½]**

Area of parking area $= \frac{\pi r^2}{2}$

$$= \frac{\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}}{2}$$ **[½]**

$= \frac{77}{4}$ square units

Required ratio $= \dfrac{98}{\frac{77}{4}}$ **[½]**

$$= \frac{56}{11}$$

$= 56 : 11$ **[½]**

(iii) Length of fencing required $= 2\ell + b + r$

$$= 2(14) + 7 + \frac{22}{7}\times\frac{7}{2}$$

$= 28 + 7 + 11$

$= 46$ units **[½]**

Cost of Fencing $= 46 \times 2 = ₹92.$ **[½]**

12 : Surface Areas and Volumes

1. Answer (b)

    Largest cone that can be cut from a cube has the

    Diameter = side of cube   [½]

    Height = side of cube

    ∴ radius $= \dfrac{4.2}{2} = 2.1$ cm   [½]

2. Volume of cube = 27 cm³

    ∴ Volume of cube = (side)³ = 27 cm³

    Side $= \sqrt[3]{27}$ cm

    Side = 3 cm   [½]

    If two cubes are joined end to end the resulting figure is cuboid
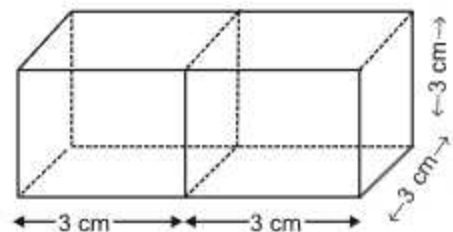
    i.e., length = l = 6 cm

    breadth = b = 3 cm   [½]

    height = h = 3 cm

    Surface area of resulting cuboid = 2($lb + bh + hl$)

    [½]

    = 2 × (6 × 3 + 3 × 3 + 3 × 6) cm²

    = 2 × (18 + 9 + 18)

    = 2 × 45 = 90 cm²   [½]

3.
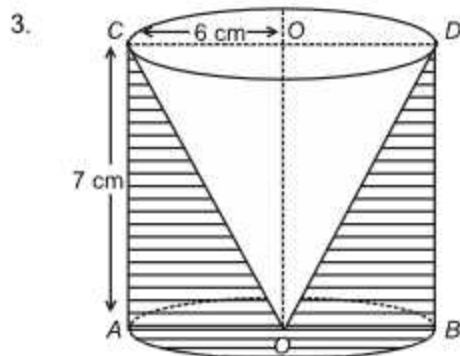
    Given: Radius of cylinder = radius of cone = r = 6 cm

Height of the cylinder = height of the cone

= h = 7 cm   [½]

Slant height of the cone = $l = \sqrt{7^2 + 6^2}$

$= \sqrt{85}$ cm   [½]

Total surface area of the remaining solid =

Curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

∴ Total surface area of the remaining solid

= ($2\pi rh + \pi r^2 + \pi rl$)   [1]

$= 2 \times \dfrac{22}{7} \times 6 \times 7 + \dfrac{22}{7} \times 6^2 + \dfrac{22}{7} \times 6\sqrt{85}$

$= 264 + \dfrac{792}{7} + \dfrac{132}{7}\sqrt{85}$

$= 377.1 + \dfrac{132}{7}\sqrt{85}$ cm²   [1]

4.
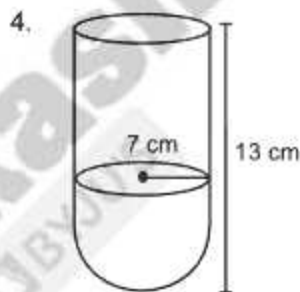
    Let the radius and height of cylinder be r cm and h cm respectively.

    Diameter of the hemispherical bowl = 14 cm

    ∴ Radius of the hemispherical bowl = Radius of the cylinder

    $= r = \dfrac{14}{2}$ cm = 7 cm   [1]

    Total height of the vessel = 13 cm

    ∴ Height of the cylinder, h = 13 cm − 7 cm = 6 cm   [1]

    Total surface area of the vessel = 2 (curved surface area of the cylinder + curved surface area of the hemisphere) (Since, the vessel is hollow)

    = 2($2\pi rh + 2\pi r^2$) = $4\pi r(h + r)$

    $= 4 \times \dfrac{22}{7} \times 7 \times (6 + 7)$ cm²

    = 1144 cm²   [1]

5.
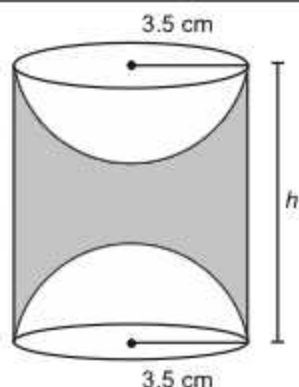
3.5 cm

Height of the cylinder, $h = 10$ cm

Radius of the cylinder = Radius of each hemisphere = $r = 3.5$ cm **[½]**

Volume of wood in the toy = Volume of the cylinder $-$ 2 × Volume of each hemisphere

$= \pi r^2 h - 2 \times \dfrac{2}{3}\pi r^3$ **[1]**

$= \pi r^2\left(h - \dfrac{4}{3}r\right)$

$= \dfrac{22}{7} \times (3.5)^2\left(10 - \dfrac{4}{3} \times 3.5\right)$

$= 38.5 \times (10 - 4.67)$ **[1]**

$= 38.5 \times 5.33$

$= 205.205$ cm^3 **[½]**

6. For the given tank

Diameter = 10 m

Radius, $R = 5$ m

Depth, $H = 2$ m **[½]**

Internal radius of the pipe

$= r = \dfrac{20}{2}$ cm $= 10$ cm $= \dfrac{1}{10}$ m **[½]**

Rate of flow of water = $v = 4$ km/h = 4000 m/h

Let t be the time taken to fill the tank. **[½]**

So, the volume of water flows through the pipe in t hours will equal to the volume of the tank.

$\therefore\ \pi r^2 \times v \times t = \pi R^2 H$ **[1]**

$\Rightarrow \left(\dfrac{1}{10}\right)^2 \times 4000 \times t = (5)^2 \times 2$

$\Rightarrow\ t = \dfrac{25 \times 2 \times 100}{4000} = 1\dfrac{1}{4}$

Hence, the time taken is $1\dfrac{1}{4}$ hours **[½]**

7. Diameter of the tent = 4.2 m

Radius of the tent, $r = 2.1$ m

Height of the cylindrical part of tent, $h_{\text{cylinder}} = 4$ m

Height of the conical part, $h_{\text{cone}} = 2.8$ m **[½]**

Slant height of the conical part, l

$= \sqrt{h^2_{\text{cone}} + r^2}$

$= \sqrt{2.8^2 + 2.1^2}$

$= \sqrt{2.8^2 + 2.1^2}$

$= 3.5$ m **[½]**

Curved surface area of the cylinder = $2\pi rh$

$= 2 \times \dfrac{22}{7} \times 2.1 \times 4$

$= 22 \times 0.3 \times 8 = 52.8$ m^2 **[½]**

Curved surface area of the conical tent

$= \pi rl = \dfrac{22}{7} \times 2.1 \times 3.5 = 23.1$ m^2 **[½]**

Total area of cloth required for building one tent = Curved surface area of the cylinder + Curved surface area of the conical tent

$= 52.8 + 23.1$

$= 75.9$ m^2 **[½]**

Cost of building one tent = 75.9 × 100 = ₹ 7590

Total cost of 100 tents = 7590 × 100

      = ₹ 7,59,000

Cost to be borne by the associations

  $= \dfrac{759000}{2} = 3,79,500$ **[½]**

It shows the helping nature, unity and cooperativeness of the associations.

8. Side of the cubical block, $a = 10$ cm

Largest diameter of a hemisphere = side of the cube

Since the cube is surmounted by a hemisphere,

Diameter of the hemisphere = 10 cm

Radius of the hemisphere, $r = 5$ cm **[1]**

Total surface area of the solid = Total surface area of the cube $-$ Inner cross-section area of the hemisphere + Curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2$ **[1]**

$= 6a^2 + \pi r^2$

$= 6 \times (10)^2 + 3.14 \times 5^2$

$= 600 + 78.5 = 678.5 \text{ cm}^2$

Total surface area of the solid = 678.5 cm² **[1]**

9. Radius of sphere = r = 6 cm

   Volume of sphere

   $$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (6)^3 = 288\pi \text{ cm}^3 \qquad [\tfrac{1}{2}]$$

   Let R be the radius of cylindrical vessel.

   Rise in the water level of cylindrical vessel

   $$= h = 3\frac{5}{9} \text{ cm} = \frac{32}{9} \text{ cm}$$

   Increase in volume of cylindrical vessel

   $$= \pi R^2 h = \pi R^2 \times \frac{32}{9} = \frac{32}{9}\pi R^2 \qquad [\tfrac{1}{2}]$$

   Now, volume of water displaced by the sphere is equal to volume of sphere

   $$\therefore \quad \frac{32}{9}\pi R^2 = 288\pi \qquad [1]$$

   $$\therefore \quad R^2 = \frac{288 \times 9}{32} = 81 \qquad [\tfrac{1}{2}]$$

   $$\therefore \quad R = 9 \text{ cm}$$

   $\therefore$ Diameter of the cylindrical vessel = 2 × R = 2 × 9 = 18 cm   **[½]**

10.
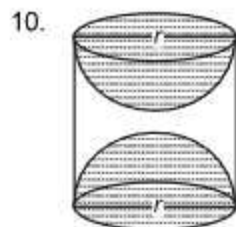
    Let r be the radius of the base of the cylinder and h be its height. Then,

    Total surface area of the article = curved surface area of the cylinder + 2 (Curved surface area of a hemisphere)   **[1]**

    $= 2\pi rh + 2 \times 2\pi r^2$

    $= 2\pi r(h + 2r)$ **[1]**

    $= 2 \times \frac{22}{7} \times 3.5(10 + 2 \times 3.5) \text{ cm}^2$

    $= 22 \times 17 \text{ cm}^2 = 374 \text{ cm}^2$ **[1]**

11.
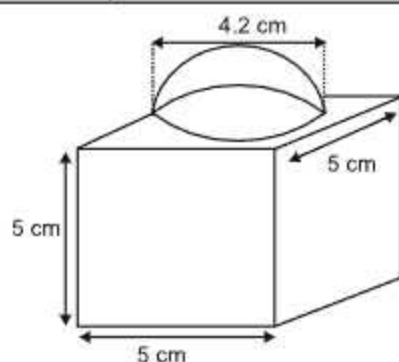
    The total surface area of the cube = 6 × (edge)²

    = 6 × 5 × 5 cm² = 150 cm²   **[1]**

    Note that the part of the cube where the hemisphere is attached is not included in the surface area.

    So, the surface area of the block = TSA of cube − base area of hemisphere + CSA of hemisphere   **[1]**

    $= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) \text{ cm}^2$ **[1]**

    $$= 150 \text{ cm}^2 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right) \text{ cm}^2$$

    $= (150 + 13.86) \text{ cm}^2 = 163.86 \text{ cm}^2$ **[1]**

12. The following figure shows the required cylinder and the conical cavity
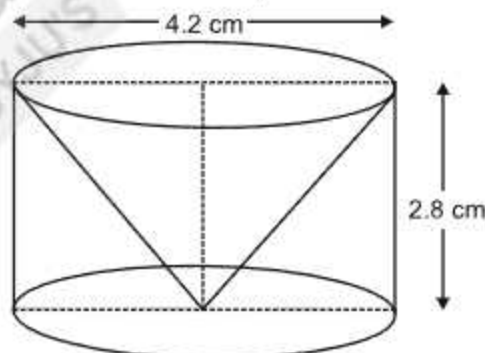
    Given Height (b) of the conical Part = Height (h) of the cylindrical part = 2.8 cm

    Diameter of the cylindrical part = Diameter of the conical part = 4.2 cm

    $\therefore$ Radius → of the cylindrical part = Radius → of the conical part = 2.1 cm   **[½]**

    Slant height (l) of the conical part

    $$= \sqrt{(2.1)^2 + (2.8)^2} \text{ cm}$$

    $$= \sqrt{4.41 + 7.81} \text{ cm}$$

    $$= \sqrt{12.25} \text{ cm} \qquad [\tfrac{1}{2}]$$

    $= 3.5 \text{ cm}$

Total surface area of the remaining solid = Curved surface area of the cylindrical part + Curved surface area of the conical part + Area of the cylindrical base

$= 2\pi rh + \pi rl + \pi r^2$ [1]

$= \left(2\times\dfrac{22}{7}\times2.1\times2.8 + \dfrac{22}{7}\times2.1\times3.5 + \dfrac{22}{7}\times2.1\times2.1\right)$ cm² [1]

$= (36.96 + 23.1 + 13.86)$ cm²

$= 73.92$ cm² [½]

Thus, the total surface area of the remaining solid is 73.92 cm² [½]

13. Height of conical upper part = 3.5 m, and radius = 2.8 m

(Slant height of cone)² = 2.1² + 2.8²

= 4.41 + 7.84

Slant height of cone $= \sqrt{12.25} = 3.5$ m [½]

The canvas used for each tent

Curved surface area of cylindrical base + curved surface area of conical upper part [½]

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \dfrac{22}{7}\times2.8(7 + 3.5)$ [½]

$= \dfrac{22}{7}\times2.8\times10.5$

$= 92.4$ m² [½]

So, the canvas used for one tent is 92.4 m²

Thus, the canvas used for 1500 tents

$= (92.4 \times 1500)$ m² [½]

Canvas used to make the tents cost ₹ 120 per sq. m

So, canvas used to make 1500 tents will cost ₹ 92.4 × 1500 × 120 [½]

The amount shared by each school to set up the tents

$= \dfrac{92.4\times1500\times120}{50} = ₹332640$ [½]

The amount shared by each school to set up the tents is ₹332640.

The value to help others in times of troubles is generated from the problem. [½]

14. According to the question, we get following figure.

[1]
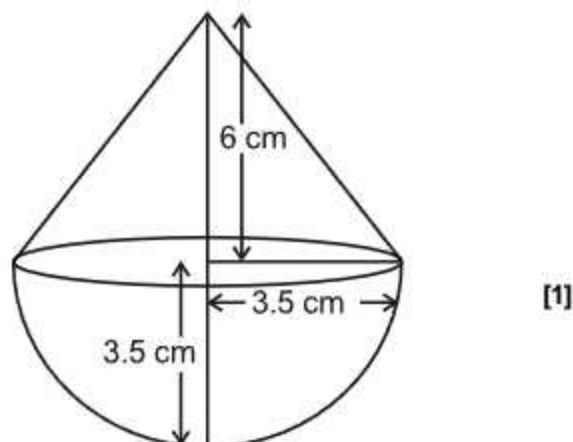
∴ Volume of solid = Volume of cone + volume of hemisphere

⇒ Volume $= \dfrac{1}{3}\pi r^2 h + \dfrac{2}{3}\pi r^3$ [1]

⇒ Volume $= \dfrac{1}{3}\pi(3.5)^2\times6 + \dfrac{2}{3}\pi(3.5)^3$

⇒ Volume $= \dfrac{1}{3}\pi(3.5)^2[6 + 3.5\times2]$

⇒ Volume $= \dfrac{1}{3}\times\dfrac{22}{7}\times\dfrac{7}{2}\times\dfrac{7}{2}[6 + 7]$ [1]

⇒ Volume $= \dfrac{1}{3}\times\dfrac{22}{7}\times\dfrac{49}{4}\times13$

⇒ Volume $= \dfrac{1}{3}\times\dfrac{2002}{4} = \dfrac{1001}{6}$ [½]

⇒ Volume $= 166\dfrac{5}{6}$ cm³

∴ Volume of solid $= 166\dfrac{5}{6}$ cm³ [½]

15. (i) Dimensions of cuboid = 10 cm × 10 cm × 8 cm
Dimensions of cone,
Radius, $R = 2.1$ cm
Height, $H = 6$ cm
Volume of wood carved out

= Volume of 5 cones $= \dfrac{1}{3}(\pi)R^2 H\times5$ [1]

$= 5\times\dfrac{1}{3}\times\dfrac{22}{7}\times(2.1)^2\times6 = 138.6$ cm³ [1]

(ii) Volume of the wood in the final product = Volume of cuboid − Volume of wood carved out [1]

$= (10 \times 10 \times 8 - 138.6)$ cm³ [½]

$= 661.4$ cm³ [½]

16. (1) For cylinder,

height, $H = 9$ m

radius, $R = 15$ m

For cone,

height, $h = 8$ m

radius, $R = 15$ m

Slant height, $l = \sqrt{8^2 + 15^2} = 17$ m   [½]

Area of canvas used in making the tent

= Curved surface area of cylinder

+ Curved surface area of cone   [½]

= $2\pi RH + \pi Rl = \pi R(2H + l)$ [½]

= $\dfrac{22}{7} \times 15\,(2 \times 9 + 17)$ [½]

= 1650 m²   [1]

(2) Total canvas used to make tent

= Curved surface area of tent

+ Canvas wasted during stitching

= 1650 + 30 = 1680 m²   [½]

Cost of canvas = ₹(1680 × 200)

= ₹3,36,000   [½]

17. Total surface area of the boiler = 2 × (curved surface area of hemisphere) + (curved surface area of cylindrical part)   [1]
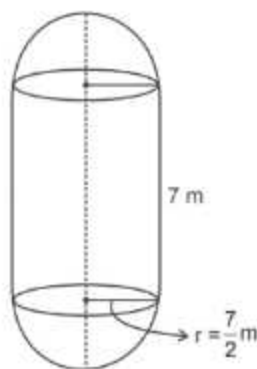
$$= \left[2\left\{ 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \right\} + \left\{ 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right) \times 7 \right\} \right]$$
    [½]

= 154 + 154 = 308 m²   [½]

Total volume of the boiler = 2 × volume of hemispherical part + volume of cylinder [1]

$$= 2 \times \frac{2}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 7$$ [½]

$$= \frac{11 \times 49}{3} + \frac{11 \times 49}{2}$$

$$= \frac{539}{3} + \frac{539}{2}$$ [½]

$$= \frac{1078 + 1617}{6} = \frac{2695}{6} = 449.16 \text{ m}^3$$

Required ratio

$$= \frac{\text{Volume of cylinder}}{\text{Volume of one hemispherical part}}$$

$$= \frac{\pi \left(\frac{7}{2}\right)^2 \times 7}{\frac{2}{3} \times \pi \times \left(\frac{7}{2}\right)^3}$$ [½]

= 3 : 1   [½]

18. Surface area of remaining solid = Total surface area of cylinder – Area of base + curved surface area of cone   [1]
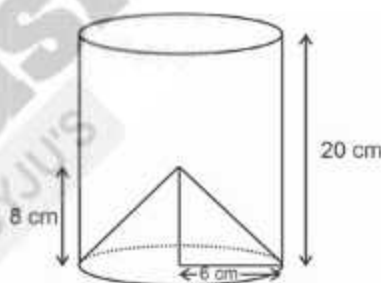
Height of cylinder (H) = 20 cm, Height of cone(h) = 8 cm, Radius of base of cylinder (r) = 6 cm and Radius of base of cone(r) = 6 cm   [1]

⇒ Surface Area of Remaining solid

= $2\pi rH + 2\pi r^2 - \pi r^2 + \pi rl$ [1]   [½]

where l = slant height of cone

$$= 2 \times \frac{22}{7} \times 6 \times 20 + \frac{22}{7} \times 6 \times 6 + \frac{22}{7} \times 6 \times 10 \text{ [1]}$$

$$\left[\because \ l = \sqrt{6^2 + 8^2} = 10 \text{ cm} \right]$$

$$= \frac{22}{7} \times 6 \ [40 + 6 + 10]$$ [½]

$$= \frac{22}{7} \times 6 \times 56$$ [½]

= 1056 cm²   [½]

19. Apparent capacity of the glass = Volume of cylinder [½]
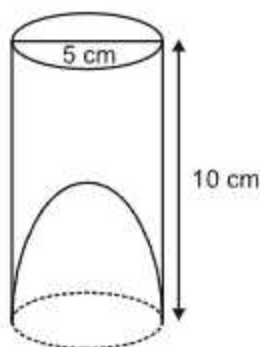
Actual capacity of the glass = Volume of cylinder – Volume of hemisphere [½]

Volume of the cylindrical glass = $\pi r^2 h$ [½]

$= 3.14 \times (2.5)^2 \times 10$

$= 3.14 \times 2.5 \times 2.5 \times 10$

$= 3.14 \times 6.25 \times 10$ [½]

$= 196.25 \text{ cm}^3$

Volume of hemisphere $= \dfrac{2}{3}\pi r^3$ [½]

$= \dfrac{2}{3}\pi (2.5)^3$

$= 32.7 \text{ cm}^3$ [½]

Apparent capacity of the glass = Volume of cylinder = 196.25 cm³

Actual capacity of the glass

= Total volume of cylinder – volume of hemisphere [1]

$= 196.25 - 32.7$ [½]

$= 163.54 \text{ cm}^3$ [½]

Hence, apparent capacity = 196.25 cm³ [½]

Actual capacity of the glass = 163.54 cm³ [½]

20.
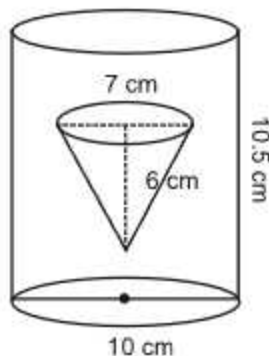
Given, internal diameter of the cylinder = 10 cm

Internal radius of the cylinder = 5 cm [½]

and height of the cylinder = 10.5 cm

Similarly, diameter of the cone = 7 cm [½]

Radius of the cone = 3.5 cm and Height of the cone = 6 cm

(i) Volume of water displaced out of cylindrical vessel = volume of cone [1]

$= \dfrac{1}{3}\pi r^2 h$ [½]

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 3.5 \times 3.5 \times 6 = 77 \text{ cm}^3$ [1]

(ii) Volume of water left In the cylindrical vessel = volume of cylinder – volume of cone [1]

$= \pi R^2 H - \text{Volume of cone}$ [½]

$= \dfrac{22}{7} \times 5 \times 5 \times 10.5 - 77$

$= 825 - 77 = 748 \text{ cm}^3$ [1]

21.
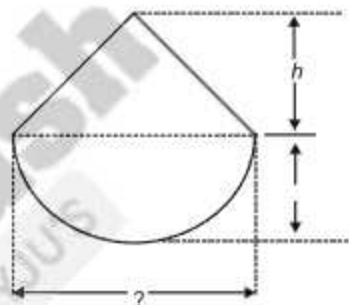
Radius of base of the cone = r = 21 cm [½]

Let the height of the cone be h cm

Volume of the cone = 2/3 volume of the hemisphere [½]

$\dfrac{1}{3}\pi r^2 h = \dfrac{2}{3} \times \dfrac{2}{3}\pi r^3$ [½]

$\Rightarrow h = \dfrac{4}{3}r = \dfrac{4}{3} \times 21 = 28 \text{ cm}$ [½]

Surface area of the toy = lateral surface area of cone + curved surface area of hemisphere [1]

$\pi r\sqrt{r^2 + h^2} + 2\pi r^2$ [1]

$= \dfrac{22}{7} \times 21 \times \sqrt{21^2 + 28^2} + 2 \times \dfrac{22}{7} \times 21 \times 21$ [1]

$= 66 \times \sqrt{441 + 784} + 2772$

$= 66 \times 35 + 2772$

$= 2310 + 2772 = 5082 \text{ cm}^2$ [1]

13 : Statistics

1. Answer (b)   **[1]**

Median class = 10-15

Modal class = 15-20

$\because$ Required sum = 10 + 15 = 25

2.

Class	Class marks	
10 – 25	$\dfrac{10+25}{2} = 17.5$	**[½]**
35 – 55	$\dfrac{35+55}{2} = 45$	**[½]**

3.

Class	Mid-value (x_i)	Frequency (f_i)	$f_i x_i$
3-5	4	5	20
5-7	6	10	60
7-9	8	10	80
9-11	10	7	70
11-13	12	8	96
Total		$\Sigma f_i = 40$	$\Sigma f_i x_i = 326$

[1]

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{326}{40}$ **[½]**

$= 8.15$ **[½]**

OR

Here, the maximum frequency is 12 and the corresponding class is 60-80. So, 60-80 is the modal class such that $l = 60$, $h = 20$, $f_0 = 12$, $f_1 = 10$ and $f_2 = 6$.   **[1]**

$\therefore$ Mode $= 60 + \left(\dfrac{12-10}{2\times12-10-6}\right)\times20$ **[½]**

$= 60 + \dfrac{2}{8}\times20$

$= 60 + 5$

$= 65$ **[½]**

4. Mode $= l + \dfrac{(f_m - f_1)}{(2f_m - f_1 - f_2)}\times h$ **[½]**

$\Rightarrow f_m = 45$

$f_1 = 30$

$f_2 = 42$

$h = 10$ **[½]**

$l = 40$

$\therefore$ Mode $= 40 + \left(\dfrac{45-30}{90-72}\right)\times10$ **[½]**

$= 40 + \left(\dfrac{15}{18}\times10\right) = 40 + \left(\dfrac{150}{18}\right) = 40 + 8.33 = 48.33$ **[½]**

5. Mode = 55 $\Rightarrow$ Modal class is 45 – 60

$\therefore l = 45$, $f_m = 15$, $f_1 = x$, $f_2 = 10$, $h = 15$

Mode $= \ell + \dfrac{(f_m - f_1)}{(2f_m - f_1 - f_2)}\times h$

$55 = 45 + \left(\dfrac{15-x}{30-x-10}\right)\times15$ **[1]**

$10 = \left(\dfrac{15-x}{20-x}\right)\times15$

$\Rightarrow 2(20-x) = 3(15-x)$

$\Rightarrow 40 - 2x = 45 - 3x$

$\Rightarrow x = 5$ **[1]**

6.

Class	Frequency	Cumulative frequency
5 – 10	49	49
10 – 15	133	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280

[1]

Let N = total frequency

$\therefore$ We have $N = 280$

$\therefore \dfrac{N}{2} = \dfrac{280}{2} = 140$ **[½]**

The cumulative frequency just greater than $\dfrac{N}{2}$ is 182 and the corresponding class is 10 – 15. Thus, 10 – 15 is the median class such that $l = 10$, $f = 133$, $F = 49$ and $h = 5$ **[½]**

Median $= l + \left(\dfrac{\dfrac{N}{2} - F}{f}\right)\times h = 10 + \left(\dfrac{140-49}{133}\right)\times5$

$= 13.42$ **[1]**

7.

Class	Frequency
0 - 10	8
10 - 20	10
20 - 30	10 $\to f_0$
30 - 40	16 $\to f_1$
40 - 50	12 $\to f_2$
50 - 60	6
60 - 70	7

[½]

Here, $30-40$ is the modal class, and $l = 30$, $h = 10$ [½]

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h \quad [1]$$

$$= 30 + \left(\frac{16 - 10}{2 \times 16 - 10 - 12}\right) \times 10 \quad [½]$$

$$= 30 + \frac{6}{10} \times 10 = 30 + 6 = 36 \quad [½]$$

8.

Class	Frequency (f)	Class Marks (x₁)	Product (f₁x₁)
10–15	4	12.5	50.00
15–20	10	17.5	175.00
20–25	5	22.5	112.50
25–30	6	27.5	165.00
30–35	5	32.5	162.50
Total	N = 30		$\sum f_i x_i = 665.00$

[1]

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{k} f_i x_i \quad [1]$$

$$= \frac{\sum_{i=1}^{5} f_i x_i}{N} = \frac{665.0}{30} \quad [1]$$

$$= 22.17 \text{ (approx.)}$$

9.

Class	Frequency	c.f.
0–10	6	6
10–20	9	15
20–30	10	25
30–40	8	33
40–50	x	33+x

[½]

Median = 25

$\Rightarrow$ Median class is 20–30

$\Rightarrow$ $f = 10$, c.f. $= 15$,

$N = 33 + x$, $h = 10$ and $l = 20$ [½]

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h \quad [½]$$

$$\Rightarrow 25 = 20 + \left(\frac{\frac{33+x}{2} - 15}{10} \times 10\right) \quad [½]$$

$$\Rightarrow 5 = \frac{33 + x - 30}{2} \quad [½]$$

$$\Rightarrow 10 = 3 + x$$

$$\therefore \quad x = 7 \quad [½]$$

10. (a)

Class	Class mark (x₁)	Frequency (f₁)	f₁x₁
0 - 10	5	5	25
10 - 20	15	18	270
20 - 30	25	15	375
30 - 40	35	f	35f
40 - 50	45	6	270
Total		$\sum f_i = 44 + f$	$\sum f_i x_i = 940 + 35f$

[1]

$$\text{Mean}(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{940 + 35f}{44 + f} \quad [1]$$

$$\Rightarrow 25 = \frac{940 + 35f}{44 + f}$$

$$\Rightarrow f = 16 \quad [1]$$

OR

(b)

Class	Frequency (f₁)	Class mark (x₁)	$d_i = x_i - a$	$f_i d_i$
0 - 5	8	2.5	–10	–80
5 - 10	7	7.5	–5	–30
10 - 15	10	12.5=a	0	0
15 - 20	13	17.5	5	65
20 - 25	12	22.5	10	120
Total	N = 50			$\sum f_i d_i = 70$

[2]

Let assumed mean be $a = 12.5$ and $N = 50$

$$\therefore \quad \bar{x} = a + \frac{1}{N}\sum_{i=1}^{5} f_i d_i$$

$$= 12.5 + \frac{1}{50} \times 70$$

$$= 12.5 + 1.4 = 13.9 \quad [1]$$

11.

Height (in cm)	Number of Students (f₁)	Cumulative frequency
130 – 135	4	4
135 – 140	11	15
140 – 145	12	27
145 – 150	7	34
150 – 155	10	44
155 – 160	6	50

[1]

$N = 50$, so $\frac{N}{2} = 25$. So, median class lies in the class $140 - 145$, then

$l = 140$

c.f. $= 15$

$f = 12$

$h = 5$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - \text{c.f.}\right)}{f} \times h \quad [1]$$

$$= 140 + \left(\frac{25 - 15}{12}\right)5 \quad [½]$$

$$= 144.166.....$$

Median height of students = 144.17 (approx.) [½]

12.

Class	Mid values x_i	Frequency f_i	$d_i = x_i -18$	$u_i = \dfrac{x_i - 18}{2}$	$f_i u_i$
11 – 13	12	3	–6	–3	–9
13 – 15	14	6	–4	–2	–12
15 – 17	16	9	–2	–1	–9
17 – 19	18	13	0	0	0
19 – 21	20	f	2	1	f
21 – 23	22	5	4	2	10
23 – 25	24	4	6	3	12
		$\Sigma f_i = 40 + f$			

[1]

$\Sigma f_i u_i = f - 8$

We have

$h = 2;\ A = 18,\ N = 40 + f, \Sigma f_i u_i = f - 8,\ \overline{X} = 18$

[½]

$\therefore$ Mean $= A + h\left\{\dfrac{1}{N}\sum f_i u_i\right\}$ **[1]**

$18 = 18 + 2\left\{\dfrac{1}{40 + f}(f - 8)\right\}$ [½]

$\dfrac{2(f - 8)}{40 + f} = 0$ [½]

$f - 8 = 0$

$f = 8$ [½]

13.

Daily income	Frequency	Income less than	Cumulative frequency
100 – 120	12	120	12
120 – 140	14	140	26
140 – 160	8	160	34
160 – 180	6	180	40
180 – 200	10	200	50

[1]

Using these values we plot the points (120, 12) (140, 26) (160, 34), (180, 40) (200, 50) on the axes to get less than ogive   **[1]**
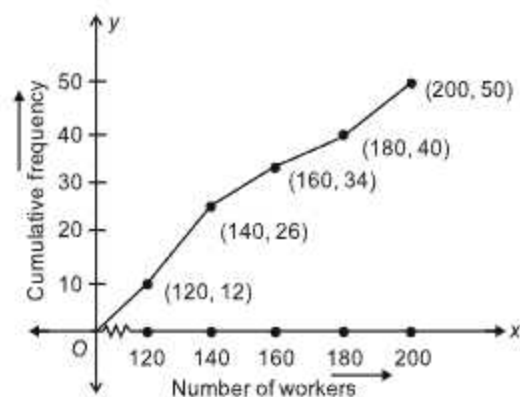
[2]

14.

Class	Frequency	Cumulative Frequency
0 – 10	f_1	f_1
10 – 20	5	$5 + f_1$
20 – 30	9	$14 + f_1$
30 – 40	12	$26 + f_1$
40 – 50	f_2	$26 + f_1 + f_2$
50 – 60	3	$29 + f_1 + f_2$
60 – 70	2	$31 + f_1 + f_2$
Total = 40 = n		

[1]

$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$f_1 + f_2 = 40 - 31 = 9$ ...(i)

Median = 32.5   [Given]

$\therefore$ Median Class is 30 – 40

$\ell = 30,\ h = 10,\ cf = 14 + f_1,\ f = 12$ **[1]**

Median $= \ell + \left(\dfrac{\dfrac{n}{2} - cf}{f}\right) \times h$ [½]

$32.5 = 30 + \left[\dfrac{\dfrac{40}{2} - (14 + f_1)}{12}\right] \times 10$ [½]

$2.5 = \dfrac{10}{12}(20 - 14 - f_1)$

$3 = 6 - f_1$

$f_1 = 3$ [½]

On putting in (i),

$f_1 + f_2 = 9$

$f_2 = 9 - 3$ $[\because f_1 = 3]$

$= 6$ [½]

15.

Classes	x_i	f_i	$A = 50$ $d_i = x_i - A$	$u_i = \dfrac{x_i - A}{h}$ $h = 20$	$f_i u_i$
0-20	10	20	10 – 50 = –40	–2	–40
20-40	30	35	30 – 50 = –20	–1	–35
40-60	50	52	50 – 50 = 0	0	0
60-80	70	44	70 – 50 = 20	1	44
80-100	90	38	90 – 50 = 40	2	76
100-120	110	31	110 – 50 = 60	3	93
		$\Sigma f_i = 220$			$\Sigma f_i u_i = 138$

[2]

$\overline{x} = A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h$ **[1]**

$= 50 + \dfrac{138}{220} \times 20$

$= 50 + 12.55$

$= 62.55$ **[1]**

16.

Production yield/hect	Number of farms	Production yield more than/hect	Cumulative frequency
40–45	4	40	100
45–50	6	45	96
50–55	16	50	90
55–60	20	55	74
60–65	30	60	54
65–70	24	65	24

[2]

[2]
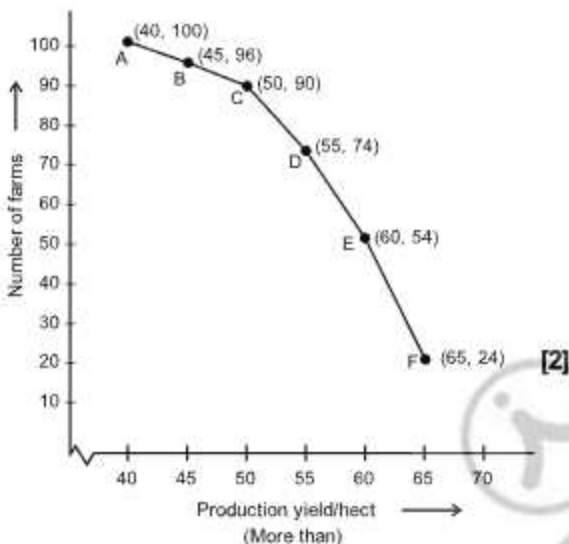
OR

Class	Frequency F_i	c.f.
0–100	2	2
100–200	5	7
200–300	x	7 + x
300–400	12	19 + x
400–500	17	36 + x
500–600	20	56 + x
600–700	y	56 + x + y
700–800	9	65 + x + y
800–900	7	72 + x + y
900–1000	4	76 + x + y = N

[1]

Here N = 100

$\Rightarrow$ 76 + x + y = 100

x + y = 24   ...(i)   [½]

Median = 525

Median class = 500 – 600

$I = 500, h = 100$

$f = 20$

c.f. = 36 + x   [½]

$$\text{Median} = I + \left[\dfrac{\dfrac{N}{2} - c.f.}{f}\right] \times h$$ [½]

$$\Rightarrow 525 = 500 + \left[\dfrac{50 - 36 - x}{20}\right] \times 100$$ [½]

$\Rightarrow$ 25 = (14 − x)5

$\Rightarrow$ 14 − x = 5

$\Rightarrow$ x = 9   [½]

Now from (i)

9 + y = 24

y = 15   [½]

17.

Weight (in kg)	No. of Students	Cumulative frequency
40-45	2	2
45-50	3	5
50-55	8	13 = Cf
55-60	6 = f	19
60-65	6	25
65-70	3	28
70-75	2	30

Total N = 30   [1]

Here, $\dfrac{N}{2} = \dfrac{30}{2} = 15$, which lies in the class 55-60   [1]

$I = 55, h = 60 - 55 = 5, Cf = 13, f = 6$ [1]

$$\text{Median} = I + \left\{\dfrac{\dfrac{N}{2} - Cf}{f}\right\} \times h$$ [1]

$$= 55 + \left\{\dfrac{15 - 13}{6}\right\} \times 5$$

$$= 55 + \left\{\dfrac{5}{3}\right\} = 56.66 \text{ (approx.)}$$ [1]

18.

Monthly Expenditure (in ₹)	Number of families (f)	Class mark (x)	$d_i = x_i - A$	$f_i d_i$	Cumulative frequency
1000 - 1500	24	1250	-1500	-36000	24
1500 - 2000	40	1750	-1000	-40000	64
2000 - 2500	33	2250	-500	-16500	97
2500 - 3000	x = 28	2750 = A	0	0	125
3000 - 3500	30	3250	500	15000	155
3500 - 4000	22	3750	1000	22000	177
4000 - 4500	16	4250	1500	24000	193
4500 - 5000	7	4750	2000	14000	200
Total	200			-17500	

Here,

$24 + 40 + 33 + x + 30 + 22 + 16 + 7 = 200$

$\Rightarrow \quad x + 172 = 200$

$\Rightarrow \quad x = 28$ **[1]**

Now,

Mean, $\bar{x} = A + \dfrac{\sum f_i d_i}{\sum f_i}$ **[½]**

$= 2750 + \dfrac{(-17500)}{200}$

$= 2750 - 87.5$

$= 2662.5$ **[1]**

Also, $\dfrac{N}{2} = \dfrac{200}{2} = 100$

Median class = 2500 - 3000 **[½]**

Here,

$l = 2500$

$cf = 97$

$f = 28$

$h = 500$

Median $= l + \left(\dfrac{\dfrac{N}{2} - cf}{f} \right) \times h$ **[½]**

$= 2500 + \dfrac{100 - 97}{28} \times 500$

$= 2500 + \dfrac{375}{7}$ **[½]**

$= 2500 + 53.57$

$= 2553.57$ (approx.) **[1]**

19. **[1]**

Class	Frequency	Class mark (x_i)	$x_i f_i$
0 – 20	6	10	60
20 – 40	8	30	240
40 – 60	10	50	500
60 – 80	12	70	840
80 – 100	6	90	540
100 – 120	5	110	550
120 – 140	3	130	390
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 3120$

Mean $= \dfrac{\sum x_i f_i}{\sum f_i}$

$= \dfrac{3120}{50}$

$= 62.4$ **[1]**

Class	f	Less than cumulative frequency
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50

$\therefore \quad n = \Sigma f_i = 50$

$\dfrac{n}{2} = 25$

$\therefore$ Median class = 60 – 80 **[1]**

Median $= l + \left(\dfrac{\dfrac{n}{2} - c.f}{f} \right) \times h$

Median $= 60 + \left(\dfrac{25 - 24}{12} \right) \times 20$

Median $= 61.66$ **[1]**

Mode :

Maximum class frequency = 12

$\therefore$ Model class = 60 – 80 **[1]**

Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$= 60 + \left(\dfrac{12 - 10}{2 \times 12 - 10 - 6} \right) \times 20$

$= 65$ **[1]**

20.

Class	f_i	Class mark(x_i)	F_ix_i
0 – 10	4	5	20
10 – 20	4	15	60
20 – 30	7	25	175
30 – 40	10	35	350
40 – 50	12	45	540
50 – 60	8	55	440
60 – 70	5	65	325
	$\Sigma f_i = 50$		$\Sigma f_ix_i = 1910$

[1]

mean $= \dfrac{1910}{50} = 38.2$ [1]

Class	Frequency	Cumulative frequency
0 – 10	4	4
10 – 20	4	8
20 – 30	7	15
30 – 40	10	25
40 – 50	12	37
50 – 60	8	45
60 – 70	5	50
	$N = 50$	

[1]

$\dfrac{N}{2} = 25$

Cumulative frequency just greater than 25 is 37.

$\therefore$ Median class 40–50

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - Cf}{f} \right) \times h$$

Here $\ell = 40$

$N = 50$

$Cf = 25$, $f = 12$, $h = 10$

$$\text{Median} = 40 + \left(\frac{25 - 25}{12} \right) 10 = 40 + 0$$

$\boxed{\text{Median} = 40}$ [1]

Mode :

Maximum frequency = 12 so modal class 40 – 50

$$\text{mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right)$$

Here $\ell = 40$, $h = 10$

$f_0 = 10$ $f_1 = 12$ $f_2 = 8$

$$\text{Mode} = 40 + \left(\frac{12 - 10}{2 \times 12 - 10 - 8} \right) \times 10$$

Mode = 40 + 3.33

= 43.33   [2]

14 : Probability

1.   Answer (c)

Number of aces in deck of cards = 4

Probability of drawing an ace card

$= \dfrac{\text{Number of ace}}{\text{Total cards}} = \dfrac{4}{52}$ [½]

Probability that the card is not an Ace

$= 1 - \dfrac{4}{52} = \dfrac{12}{13}$ [½]

2.   Answer (c)

When two dice are thrown together, the total number of outcomes is 36.

Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}   [½]

$\therefore$ Required probability

$= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{6}{36} = \dfrac{1}{6}$ [½]

3.   Answer (a)

$S = \{1, 2, 3, 4, 5, 6\}$

Let event E be defined as 'getting an even number'.

$n(E) = \{2, 4, 6\}$ [½]

$\therefore$ $P(E) = \dfrac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \dfrac{3}{6}$

$= \dfrac{1}{2}$ [½]

4. Answer (c)

$S = \{1, 2, 3, ..90\}$

$n(S) = 90$

The prime number less than 23 are 2, 3, 5, 7, 11, 13, 17, and 19.

Let event E be defined as 'getting a prime number less than 23'.   **[½]**

$n(E) = 8$

$\therefore \quad P(E) = \dfrac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

$= \dfrac{8}{90} = \dfrac{4}{45}$ **[½]**

5. Answer (d)

Possible outcomes on rolling the two dice are given below :

$\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$

$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$ **[½]**

Total number of outcomes = 36

Favourable outcomes are given below:

$\{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2),$
$(6, 4), (6, 6)\}$

Total number of favourable outcomes = 9

$\therefore$ Probability of getting an even number on both dice

$= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \dfrac{9}{36} = \dfrac{1}{4}$ **[½]**

6. Answer (c)

Total number of possible outcomes = 30

Prime numbers from 1 to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Total number of favourable outcomes = 10   **[½]**

$\therefore$ Probability of selecting a prime number from 1 to 30

$= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \dfrac{10}{30} = \dfrac{1}{3}$ **[½]**

7. Answer (d)

Favourable outcomes are 4, 8, 12, i.e., 3 outcomes and total number of outcomes = 15

$\therefore$ Required probability $= \dfrac{3}{15} = \dfrac{1}{5}$

Option (d) is correct.   **[1]**

8. Total outcomes = 36

Favourable outcomes $\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ **[½]**

Number of favourable outcomes = 5

$P(\text{sum } 8) = \dfrac{5}{36}$ **[½]**

9. $n(s) =$ Total number of alphabets in English = 26.

$n(E) =$ Total number of consonant in English alphabet = 21   **[½]**

$\therefore$ Probability (Chosen letter is a consonant)

$= \dfrac{n(E)}{n(s)}$

$= \dfrac{21}{26}$ **[½]**

10. Total number of outcomes = 6

Number of favourable outcomes = 2   **[½]**

P(getting a number less than 3) $= \dfrac{2}{6}$

$= \dfrac{1}{3}$ **[½]**

OR

Required probability

= 1 – Probability of winning a game   **[½]**

= 1 – 0.07

= 0.93   **[½]**

11. Answer (b)   **[1]**

Total possible outcomes = {HT, TH, HH, TT}

$\therefore$ Required probability $= \dfrac{1}{2}$

12. Answer (d)   **[1]**

$P(\overline{E})$ or P (not E) = 1 – P(E)

= 1 – 0.65

= 0.35

13. Answer (b)   [1]

$$\text{Probability} = \frac{\text{Number of favourable events in sample space}}{\text{Total number of events in sample space}}$$

$$P(\text{Blue balls}) = \frac{6}{16+8+6} = \frac{6}{30} = \frac{1}{5}$$

14. Answer (c)   [1]

P(Not happening of an event)

= 1 − P (Happening of the event)

= 1 − 0.02

= 0.98

15. Answer (a)   [1]

$x = 1$ $[\because P(E)+P(\overline{E})=1]$

$\Rightarrow x^3 - 3 = -2$

16. Answer (a)   [1]

P(Neither ace nor spade) = 1 − P (Ace or spade)

$$= 1 - \frac{16}{52}$$

$$= \frac{9}{13}$$

17. Answer (d)   [1]

Probability of any event always $0 \le P(E) \le 1$.

18. Answer (d)   [1]

(E) = Outcomes not possible are {(5, 5) (1, 5) (2, 5) (3, 5) (4, 5) (6, 5) (5, 1) (5, 2) (5, 3) (5, 4) (5, 6)}

n(E) = 11

Total outcomes = 36

∴ Number of possible outcomes = 36 − 11

= 25

∴ Probability $= \frac{25}{36}$

19. Answer (d)   [1]

$$P(\overline{E}) = \frac{1}{5}$$

$$P(E) = 1 - \frac{1}{5} = \frac{4}{5}$$

20. Answer (a)   [1]

There is only 1 king of hearts in a deck of 52 cards.

Required probability $= \frac{1}{52}$

21. Answer (d)   [1]

Probability (not Ace) = 1 − P(Ace)

$$= 1 - \frac{4}{52}$$

$$= \frac{12}{13}$$

22. Answer (b)   [1]

P(green ball) = 3P(red ball)

$$\Rightarrow \frac{n}{5+n} = 3 \times \frac{5}{5+n}$$

$$\Rightarrow n = 15$$

23. Answer (c)   [1]

$$P(\text{leap year having 53 Sundays}) = \frac{2}{7}$$

$$P(\text{getting 53 Sundays in a non-leap year}) = \frac{1}{7}$$

24. Total possible outcomes = 6

Outcomes which are less than 3 = 1, 2   [½]

Probability $= \frac{2}{6}$

$$= \frac{1}{3}$$ [½]

25. Two coins are tossed simultaneously

Total possible outcomes = {HH, HT, TH, TT}

Number of total outcomes = 4

Favourable outcomes for getting exactly

One head = {HT, TH}   [½]

Probability $= \frac{2}{4} = \frac{1}{2}$ [½]

26. A card is drawn from well shuffled 52 playing cards so total no of possible outcomes = 52

Number of face cards = 12

Number of Red face cards = 6   [½]

Probability of drawing $= \frac{6}{52}$

A red face card $= \frac{3}{26}$ [½]

27. Two dice are tossed

S = [(1, 1), (1, 2), (1, 3),(1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)]   [½]

Total number of outcomes when two dice are tossed = 6 × 6 = 36

Favourable events of getting product as 6 are:

(1 × 6 = 6), (6 × 1 = 6), (2 × 3 = 6), (3 × 2 = 6)

i.e. (1, 6), (6, 1), (2, 3), (3, 2)

Favourable events of getting product as 6 = 4

∴ $P(\text{getting product as } 6) = \dfrac{4}{36} = \dfrac{1}{9}$ [½]

28. There are 26 red cards including 2 red queens.

Two more queens along with 26 red cards will be 26 + 2 = 28

∴ $P(\text{getting a red card or a queen}) = \dfrac{28}{52}$ [½]

∴ $P(\text{getting neither a red card nor a queen})$

$$= 1 - \dfrac{28}{52} = \dfrac{24}{52} = \dfrac{6}{13}$$ [½]

29. Probability of selecting rotten apple

$$= \dfrac{\text{Number of rotten apples}}{\text{Total number of apple}}$$ [½]

∴ $0.18 = \dfrac{\text{Number of rotten apples}}{900}$

Number of rotten apples = 900 × 0.18 = 162   [½]

30. A ticket is drawn at random from 40 tickets

Total outcomes = 40

Out of the tickets numbered from 1 to 40 the number of tickets which is multiple of 5 = 5, 10, 15, 20, 25, 30, 35, 40

= 8 tickets

∴   Favorable outcomes = 8   [1]

∴   Probability $= \dfrac{8}{40}$

$$= \dfrac{1}{5}$$ [1]

31. The total number of outcomes is 50.

Favourable outcomes = {12, 24, 36, 48}   [1]

∴   Required probability

$$= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{4}{50} = \dfrac{2}{25}$$ [1]

32. Let *E* be the event that the drawn card is neither a king nor a queen.

Total number of possible outcomes = 52

Total number of kings and queens = 4 + 4 = 8

Therefore, there are 52 − 8 = 44 cards that are neither king nor queen.   [1]

Total number of favourable outcomes = 44

∴   Required probability = *P*(*E*)

$$= \dfrac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{44}{52} = \dfrac{11}{13}$$ [1]

33. Rahim tosses two coins simultaneously. The sample space of the experiment is {*HH*, *HT*, *TH*, and *TT*}.

Total number of outcomes = 4

Outcomes in favour of getting at least one tail on tossing the two coins = {*HT*, *TH*, *TT*}   [1]

Number of outcomes in favour of getting at least one tail = 3

∴   Probability of getting at least one tail on tossing the two coins

$$= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{3}{4}$$ [1]

34. Sample space = *S* = {(1, 1) (1, 2)...,(6, 6)}

$n(s) = 36$

(i)   *A* = getting a doublet

      A = {(1, 1), (2, 2) ...., (6, 6)}

      $n(A) = 6$

      ∴ $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$ [1]

(ii)   *B* = getting sum of numbers as 10

      B = {(6, 4), (4, 6), (5, 5)}

      $n(B) = 3$

      $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$ [1]

35. An integer is chosen at random from 1 to 100

Therefore $n(S) = 100$

(i)   Let *A* be the event that number chosen is divisible by 8

      ∴   *A* = {8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96}

      ∴ $n(A) = 12$

      Now, *P* (that number is divisible by 8)

$$= P(A) = \dfrac{n(A)}{n(S)}$$

$$= \dfrac{12}{100} = \dfrac{6}{50} = \dfrac{3}{25}$$ [1]

$$\boxed{P(A) = \dfrac{3}{25}}$$

(ii)   Let '*A*' be the event that number is not divisible by 8.

      ∴ $P(A') = 1 - P(A)$

$$= 1 - \dfrac{3}{25}$$ $\boxed{P(A') = \dfrac{22}{25}}$ [1]

36. Total possible outcomes are (HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT) i.e., 8.

    The favourable outcomes to the event E 'Same result in all the tosses' are TTT, HHH.   **[1]**

    So, the number of favourable outcomes = 2

    $\therefore \quad P(E) = \dfrac{2}{8} = \dfrac{1}{4}$

    Hence, probability of losing the game = $1 - P(E)$

    $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$ **[1]**

37. Total outcomes = 1, 2, 3, 4, 5, 6

    Prime numbers = 2, 3, 5

    Numbers lie between 2 and 6 = 3, 4, 5

    (i) P (Prime Numbers) $= \dfrac{3}{6} = \dfrac{1}{2}$ **[1]**

    (ii) P (Numbers lie between 2 and 6) $= \dfrac{3}{6} = \dfrac{1}{2}$ **[1]**

38. Let the number of blue balls be x.

    So, total number of balls in the bag = $(x + 5)$ **[½]**

    According to the question,

    $\dfrac{x}{x+5} = 3 \times \dfrac{5}{x+5}$ **[1]**

    $\Rightarrow x = 15$

    $\therefore$ Number of blue balls = 15   **[½]**

39. Total number of outcomes = $6 \times 6 = 36$ **[½]**

    Favourable outcomes = {(1, 1)(1, 2)(1, 3)(2, 1)(2, 2)(3, 1)}   **[½]**

    Number of favourable outcomes = 6   **[½]**

    $\therefore$ P(less than 5) = $\dfrac{6}{36} = \dfrac{1}{6}$ **[½]**

    OR

    In month of November 4 sundays are fixed.

    But there are two extra days. They may be {(Sun, Mon), (Mon, Tues), (Tues, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat), (Sat, Sun)}   **[1]**

    Number of favourable outcomes = 2   **[½]**

    $\therefore$ Required probability (5 sundays) = $\dfrac{2}{7}$ **[½]**

40. Let E be the event of getting square of a number less than or equal to 4.

    S be the sample space. Then,

    $S = \{-3, -2, -1, 0, 1, 2, 3\}$ **[½]**

    $\Rightarrow n(S) = 7$

    and, $E = \{-2, -1, 0, 1, 2\}$

    $\Rightarrow n(E) = 5$.   **[½]**

    $\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{5}{7}$ **[1]**

41. Total possible outcomes are HH, HT, TH, TT **[½]**

    And favourable outcomes are HT, TH, TT   **[½]**

    Required probability $= \dfrac{3}{4}$ **[1]**

42. Total outcomes = $6 \times 6 = 36$

    (i) Total outcomes when 5 comes up on either dice are (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 5) (4, 5) (3, 5) (2, 5) (1, 5)

    P (5 will come up on either side) $\dfrac{11}{36}$ **[1]**

    P (5 will not come up) $= 1 - \dfrac{11}{36}$

    $= \dfrac{25}{36}$

    (ii) P (5 will come at least once) $= \dfrac{11}{36}$ **[1]**

    (iii) P (5 will come up on both dice) $= \dfrac{1}{36}$ **[1]**

43. Total number of cards $= \dfrac{35-1}{2} + 1$

    $= 18$ **[1]**

    (i) Favourable outcomes = {3, 5, 7, 11, 13}

    P(prime number less than 15) $= \dfrac{5}{18}$ **[1]**

    (ii) Favourable outcomes = {15}

    P(a number divisible by 3 and 5) $= \dfrac{1}{18}$ **[1]**

44. Two dice are rolled once. So, total possible outcomes = $6 \times 6 = 36$ **[1]**

    Product of outcomes will be 12 for

    (2, 6), (6, 2), (3, 4) and (4, 3).   **[1]**

    Number of favourable cases = 4

    Probability $= \dfrac{4}{36} = \dfrac{1}{9}$ **[1]**

45. A disc drawn from a box containing 80 **[1]**

Total possible outcomes = 80

Number of cases where the disc will be numbered perfect square = 8

Perfect squares less than 80 **[1]**

= 1, 4, 9, 16, 25, 36, 49, 64

Probability $= \dfrac{8}{80} = \dfrac{1}{10}$ **[1]**

46. Total number of outcomes = 52

(i) Probability of getting a red king

Here the number of favourable outcomes = 2

Probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{2}{52}$

$= \dfrac{1}{26}$ **[1]**

(ii) Favourable outcomes = 12

Probability $= \dfrac{12}{52} = \dfrac{3}{13}$ **[1]**

(iii) Probability of queen of diamond.

Number of queens of diamond = 1, hence

Probability

$= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{1}{52}$ **[1]**

47. Here the jar contains red, blue and orange balls.

Let the number of red balls be x.

Let the number of blue balls be y.

Number of orange balls = 10

Total number of balls = $x + y + 10$

Now, let P be the probability of drawing a ball from the jar

$P(\text{a red ball}) = \dfrac{x}{x+y+10}$

$\Rightarrow \dfrac{1}{4} = \dfrac{x}{x+y+10}$

$\Rightarrow 4x = x + y + 10$

$\Rightarrow 3x - y = 10$ ...(i) **[1]**

Next,

$P(\text{a blue ball}) = \dfrac{y}{x+y+10}$

$\Rightarrow \dfrac{1}{3} = \dfrac{y}{x+y+10}$

$\Rightarrow 3y = x + y + 10$

$\Rightarrow 2y - x = 10$ ...(ii) **[1]**

Multiplying eq. (i) by 2 and adding to eq. (ii), we get

$$\begin{array}{rrcr} 6x & - & 2y & = & 20 \\ -x & + & 2y & = & 10 \\ \hline & & 5x & = & 30 \end{array}$$

$\Rightarrow x = 6$

Substitute $x = 6$ in eq. (i), we get $y = 8$

Total number of balls = $x + y + 10 = 6 + 8 + 10 = 24$

Hence, total number of balls in the jar is 24. **[1]**

48. Bag contains 15 white balls.

Let say there be x black balls.

Probability of drawing a black ball

$P(B) = \dfrac{x}{15+x}$ **[1]**

Probability of drawing a white ball

$P(W) = \dfrac{15}{15+x}$

Given that $P(B) = 3P(W)$ **[1]**

$\therefore \quad \dfrac{x}{15+x} = \dfrac{3 \times 15}{15+x}$

$x = 45$ **[1]**

Number of black balls = 45

49. (i) Probability of getting an even prime number

$= \dfrac{1}{6}$ **[1]**

(ii) P(a number greater than 4) $= \dfrac{2}{6} = \dfrac{1}{3}$ **[1]**

(iii) P(an odd number) $= \dfrac{3}{6} = \dfrac{1}{2}$ **[1]**

50. The group consists of 12 persons.

$\therefore$ Total number of possible outcomes = 12

Let A denote event of selecting persons who are extremely patient.

$\therefore$ Number of outcomes favourable to A is 3. **[1]**

Let B denote event of selecting persons who are extremely kind or honest. Number of persons who are extremely honest is 6. Number of persons who are extremely kind is $12 - (6 + 3)$ $= 3$ **[1]**

∴ Number of outcomes favourable to $B = 6 + 3 = 9$.

(i) $P(A) = \dfrac{\text{Number of outcomes favourable to } A}{\text{Total number of possible outcomes}}$

$= \dfrac{3}{12} = \dfrac{1}{4}$ **[1]**

(ii) $P(B) = \dfrac{\text{Number of outcomes favourable to } B}{\text{Total number of possible outcomes}}$

$= \dfrac{9}{12} = \dfrac{3}{4}$ **[1]**

Each of the three values, patience, honesty and kindness is important in one's life.

51. Total number of cards = 49

(i) Total number of outcomes = 49

The odd numbers from 1 to 49 are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47 and 49.

Total number of favourable outcomes = 25

∴ Required probability

$= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{25}{49}$ **[1]**

(ii) Total number of outcomes = 49

The number 5, 10, 15, 20, 25, 30, 35, 40 and 45 are multiples of 5.

The number of favourable outcomes = 9

∴ Required probability

$= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{9}{49}$ **[1]**

(iii) Total number of outcomes = 49

The number 1, 4, 9, 16, 25, 36 and 49 are perfect squares.

Total number of favourable outcomes = 7

∴ Required probability

$= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \dfrac{7}{49} = \dfrac{1}{7}$ **[1]**

(iv) Total number of outcomes = 49

We know that there is only one even prime number which is 2.

Total number of favourable outcomes = 1

∴ Required probability

$= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{1}{49}$ **[1]**

52. Let S be the sample space of drawing a card from a well-shuffled deck.

$n(S) = 52$

(i) There are 13 spade cards and 4 ace's in a deck. As ace of spade is included in 13 spade cards, so there are 13 spade cards and 3 ace's.

A card of spade or an ace can be drawn in

= 16 ways

Probability of drawing a card of spade or an

ace $= \dfrac{16}{52} = \dfrac{4}{13}$ **[1]**

(ii) There are 2 black king cards in a deck a card of black king can be drawn in = 2 ways

Probability of drawing a black king $= \dfrac{2}{52} = \dfrac{1}{26}$

[1]

(iii) There are 4 Jack and 4 King cards in a deck.

So there are $52 - 8 = 44$ cards which are neither Jacks nor Kings. A card which is neither a Jack nor a King.

Can be drawn in = 44 ways

Probability of drawing a card which is neither

a Jack nor a King $= \dfrac{44}{52} = \dfrac{11}{13}$ **[1]**

(iv) There are 4 King and 4 Queen cards in a deck.

So there are $4 + 4 = 8$ cards which are either King or Queen.

A card which is either a King or a Queen can be drawn in = 8 ways

So, probability of drawing a card which is

either a King or a Queen $= \dfrac{8}{52} = \dfrac{2}{13}$ **[1]**

53. x is selected from 1, 2, 3 and 4

    1, 2, 3, 4

    y is selected from 1, 4, 9 and 16

    Let $A = \{1, 4, 9, 16, 2, 8, 18, 32, 3, 12, 27, 48, 36, 64\}$ which consists of elements that are product of x and y.   **[2]**

    P(product of x and y is less than 16)

    $= \dfrac{\text{Number of outcomes less than 16}}{\text{Total number of outcomes}}$ **[1]**

    $= \dfrac{7}{14}$

    $= \dfrac{1}{2}$ **[1]**

54. Two dice are thrown together total possible outcomes $= 6 \times 6 = 36$

    (i) Sum of outcomes is even

    This can be possible when

    ⇒ Both outcomes are even

    ⇒ Both outcomes are odd

    For both outcomes to be even number of cases $= 3 \times 3 = 9$ **[1]**

    Similarly,

    Both outcomes odd $= 9$ cases

Total favourable cases $= 9 + 9 = 18$

Probability that $= \dfrac{18}{36}$

Sum of the even outcomes is $\dfrac{1}{2}$.   **[1]**

(ii) Product of outcomes is even

This is possible when

⇒ Both outcomes are even

⇒ First outcome even & the other odd

⇒ First outcome odd & the other even

Number of cases where both outcomes are even $= 9$ **[1]**

Number of cases for first outcome odd and the other even $= 9$

Number of cases for first outcome even and the other odd $= 9$

Total favourable cases $= 9 + 9 + 9 = 27$

Probability $= \dfrac{27}{36}$

$= \dfrac{3}{4}$ **[1]**

❏   ❏   ❏