

Date: 08/03/2025



Question Paper Code

65/1/3

# Aakash

Medical | IIT-JEE | Foundations

**Corporate Office:** Aakash Educational Services Limited,  
3rd Floor, Incuspaze Campus-2, Plot No. 13, Sector-18, Udyog Vihar, Gurugram, Haryana - 122015

Time: 3 hrs.

## Class-XII

Max. Marks: 80

# MATHEMATICS (Theory)

## (CBSE 2025)

### GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them:

- (i) This question paper contains **38** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) **In Section – A** : Question Numbers **1** to **18** are Multiple Choice Questions (MCQ) type and question number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) **In Section – B** : Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) **In Section – C** : Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) **In Section – D** : Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) **In Section – E** : Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculators is **NOT** allowed.

**SECTION-A**

This Section comprises of **20** Multiple Choice Questions (MCQs) of **1** mark each.

**20 × 1 = 20**

1. If the feasible region of a linear programming problem with objective function  $Z = ax + by$ , is bounded, then which of this following is correct?
- (A) It will only have a maximum value.  
 (B) It will only have a minimum value.  
 (C) It will have both maximum and minimum values.  
 (D) It will have neither maximum nor minimum value.

**Answer (C)**

**Sol.** If the feasible region of a linear programming problem with objective function  $Z = ax + by$ , is bounded then it will have both maximum and minimum values and each of these occur at corner point.

2. The unit vector perpendicular to the vectors  $\hat{i} - \hat{j}$  and  $\hat{i} + \hat{j}$  is
- (A)  $\hat{k}$  (B)  $-\hat{k} + \hat{j}$   
 (C)  $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$  (D)  $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

**Answer (A)**

**Sol.**  $\vec{a} = \hat{i} - \hat{j}$

$$\vec{b} = \hat{i} + \hat{j}$$

Let  $\hat{n}$  be the unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

$$\vec{a} \times \vec{b} = (\hat{i} - \hat{j}) \times (\hat{i} + \hat{j})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = 2$$

$$\Rightarrow \hat{n} = \frac{2\hat{k}}{2} = \hat{k}$$

3. If  $\int_0^1 \frac{e^x}{1+x} dx = \alpha$ , then  $\int_0^1 \frac{e^x}{(1+x)^2} dx$  is equal to

(A)  $\alpha - 1 + \frac{e}{2}$

(B)  $\alpha + 1 - \frac{e}{2}$

(C)  $\alpha - 1 - \frac{e}{2}$

(D)  $\alpha + 1 + \frac{e}{2}$

**Answer (B)**

**Sol.**  $\int_0^1 \frac{e^x}{1+x} dx = \alpha$

Here we will apply integration by parts formula

Which is  $\int uv dx = u \int v dx - \int (u' \int v dx) dx$

Here  $u = \frac{1}{1+x}$  and  $v = e^x$

$\Rightarrow u' = \frac{-1}{(1+x)^2}$

$\int_0^1 \frac{e^x}{1+x} dx = \left[ \frac{1}{(1+x)} e^x \right]_0^1 - \int_0^1 \left( \frac{-1}{(1+x)^2} \right) e^x dx$

$\alpha = \frac{e}{2} - 1 + \int_0^1 \frac{e^x}{1+x^2} dx$

$\Rightarrow \int_0^1 \frac{e^x}{(1+x)^2} dx = \alpha + 1 - \frac{e}{2}$

4. If  $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k \cdot 2^{\frac{1}{x}} + C$ , then  $k$  is equal to

(A)  $\frac{-1}{\log 2}$

(B)  $-\log 2$

(C)  $-1$

(D)  $\frac{1}{2}$

**Answer (A)**

**Sol.**  $\int \frac{2^{\frac{1}{x}}}{x^2} dx = K \cdot 2^{\frac{1}{x}} + C$

Let  $\frac{1}{x} = t$

Differentiating both sides

$\frac{-1}{x^2} dx = dt$

Putting  $\frac{1}{x} = t$  and  $\frac{dx}{x^2} = -dt$

$\Rightarrow \int \frac{2^{\frac{1}{x}}}{x^2} dx = \int 2^t (-dt)$

$= -\int 2^t dt$

$= \frac{-2^t}{\ln 2} + C$

$= \frac{-2^{\frac{1}{x}}}{\ln 2} + C$

$\Rightarrow k = \frac{-1}{\ln 2}$  or  $\frac{-1}{\log 2}$

5. If  $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ , then  $A^{-1}$  is

(A)  $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C)  $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D)  $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**Answer (D)**

**Sol.**  $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  as  $|A| = -1 \neq 0 \Rightarrow A^{-1}$  exists

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

where  $C_{ij} = (-1)^{i+j} M_{ij}$

where  $M_{ij}$  is the minor

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1$$

$$\text{adj } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = -1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. If  $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$ , then  $(x-y) = ?$

- (A) -7  
(B) -3  
(C) 3  
(D) 7

**Answer (B)**

**Sol.**  $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$

If matrices are equal, then their corresponding entries must be equal

So  $x+y=9$  ... (1),  $3y=4x+y$  ... (2)

$3x=x+6$  ... (3),  $x+3=y$  ... (4)

From (3)  $2x=6 \Rightarrow x=3$

Put  $x=3$  in (4)

$\Rightarrow 3+3=y$

$\Rightarrow y=6$

Now  $(x-y) = (3-6) = -3$

7. Let  $M$  and  $N$  be two events such that  $P(M) = 0.6$ ,  $P(N) = 0.2$  and  $P(M \cap N) = 0.5$ , then  $P(M/N)$  is

- (A)  $\frac{7}{8}$   
(B)  $\frac{2}{5}$   
(C)  $\frac{1}{2}$   
(D)  $\frac{2}{3}$

**Answer (A)**

**Sol.**  $P(M) = 0.6, P(N) = 0.2, P(M \cap N) = 0.5,$

$$P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

$$P(M' / N') = \frac{P(M' \cap N')}{P(N')} = \frac{P((M \cup N)')}{P(N')}$$

$$= \frac{1 - P(M \cup N)}{1 - P(N)}$$

$$= \frac{1 - [P(M) + P(N) - P(M \cap N)]}{1 - P(N)}$$

$$= \frac{1 - [0.6 + 0.2 - 0.5]}{1 - 0.2}$$

$$= \frac{1 - 0.3}{0.8} = \frac{0.7}{0.8}$$

$$= \frac{7}{8}$$

8. Which of the following is **not** a homogeneous function of  $x$  and  $y$ ?

(A)  $y^2 - xy$

(B)  $x - 3y$

(C)  $\sin^2 \frac{y}{x} + \frac{y}{x}$

(D)  $\tan x - \sec y$

**Answer (D)**

**Sol.** We know for homogeneous function

$$F(\lambda x, \lambda y) = \lambda f(x, y)$$

Option (A)  $f(x, y) = y^2 - xy$

$$f(\lambda x, \lambda y) = (\lambda y)^2 - (\lambda x)(\lambda y)$$

$$f(\lambda x, \lambda y) = \lambda^2(y^2 - xy)$$

Option (B)  $f(x, y) = x - 3y$

$$f(\lambda x, \lambda y) = \lambda x - 3\lambda y$$

$$f(\lambda x, \lambda y) = \lambda(x - 3y) = \lambda f(x, y)$$

Option (C)  $f(x, y) = \sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$

$$f(\lambda x, \lambda y) = \sin^2\left(\frac{\lambda y}{\lambda x}\right) + \frac{\lambda y}{\lambda x}$$

$$f(\lambda x, \lambda y) = \sin^2\left(\frac{y}{x}\right) + \frac{y}{x} \Rightarrow f(x, y)$$

Option (D)  $f(x, y) = \tan x - \sec y$

$$f(\lambda x, \lambda y) = \tan(\lambda x) - \sec(\lambda y) \neq \lambda f(x, y)$$

9. If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = \sqrt{37}, |\vec{b}| = 3$  and  $|\vec{c}| = 4$ , then angle between  $\vec{b}$  and  $\vec{c}$  is

(A)  $\frac{\pi}{6}$

(B)  $\frac{\pi}{4}$

(C)  $\frac{\pi}{3}$

(D)  $\frac{\pi}{2}$

**Answer (C)**

**Sol.** Given,  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ ,  $|\vec{a}| = \sqrt{37}$ ,  $|\vec{b}| = 3$ ,  $|\vec{c}| = 4$

$$\vec{b} + \vec{c} = -\vec{a}$$

Squaring both sides,

$$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |-\vec{a}|^2$$

$$\Rightarrow |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos\theta = |\vec{a}|^2$$

$$\Rightarrow 9 + 16 + 2(3)(4)\cos\theta = 37$$

$$\Rightarrow 24\cos\theta = 37 - 25$$

$$\Rightarrow 24\cos\theta = 12$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

10. If  $f(x) = |x| + |x - 1|$ , then which of the following is correct?

- (A)  $f(x)$  is both continuous and differentiable, at  $x = 0$  and  $x = 1$
- (B)  $f(x)$  is differentiable but not continuous, at  $x = 0$  and  $x = 1$
- (C)  $f(x)$  is continuous but not differentiable, at  $x = 0$  and  $x = 1$
- (D)  $f(x)$  is neither continuous nor differentiable, at  $x = 0$  and  $x = 1$ .

**Answer (C)**

$$\text{Sol. } f(x) = \begin{cases} -x - x + 1 & x < 0 \\ x - (x - 1) & 0 \leq x < 1 \\ x + x - 1 & x > 1 \end{cases}$$

$$f(x) = \begin{cases} -2x + 1 & ; \quad x < 0 \\ 1 & ; \quad 0 \leq x < 1 \\ 2x - 1 & ; \quad x \geq 1 \end{cases}$$

Now: If  $f(x)$  is continuous at  $x = 0$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$$

$$\Rightarrow 1 = \lim_{h \rightarrow 0} f(0 - h) = 1$$

$$\Rightarrow 1 = \lim_{h \rightarrow 0} (-2(-h) + 1) = 1$$

$$\Rightarrow 1 = 1 = 1$$

Which is true

$$\Rightarrow f(x) \text{ is continuous at } x = 0$$

Similarly for continuity at  $x = 1$

$$\lim_{h \rightarrow 0} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

$$\lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} f(1 + h) = f(1)$$

$$1 = \lim_{h \rightarrow 0} (2(1 + h) - 1) = 1$$

$$\Rightarrow 1 = 1 = 1$$

$\Rightarrow f(x)$  is continuous at  $x = 1$

When  $x > 0$

$$\lim_{x \rightarrow 0^+} f'(x) = \lim_{x \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1-1}{h} = 0$$

$$\lim_{x \rightarrow 0^-} f'(x) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{-2(-h) + 1 - 1}{-h} = -2$$

$$\therefore f'(0^+) \neq f'(0^-)$$

$\therefore f(x)$  is not differentiable at  $x = 0$

$$\text{Now } f'(1^+) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(1+h) - 1 - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h}{h} = 2$$

$$\therefore f'(1^+) \neq f'(1^-)$$

$\Rightarrow f(x)$  is not differentiable at  $x = 1$

$\therefore f(x)$  is continuous at  $x = 0, 1$  and non-differentiable at  $x = 0, 1$

$\therefore$  Option (C) is correct.

11. A system of linear equations is represented as  $AX = B$ , where  $A$  is coefficient matrix,  $X$  is variable matrix and  $B$  is the constant matrix. Then the system of equations is

- (A) Consistent, if  $|A| \neq 0$ , solution is given by  $X = BA^{-1}$ .
- (B) Inconsistent if  $|A| = 0$  and  $(\text{adj } A) B = 0$
- (C) Inconsistent if  $|A| \neq 0$
- (D) May or may not be consistent if  $|A| = 0$  and  $(\text{adj } A) B = 0$

**Answer (D)**

**Sol.** System of linear equations is represented as  $Ax = B$

If  $|A| \neq 0$ , then system of equations is consistent and solution is given by

$$X = A^{-1}B.$$

For inconsistent.  $|A| = 0$  and  $(\text{adj } A) B \neq 0$

If  $|A| = 0$  and  $(\text{adj } A) B = 0$ , then system may or may not consistent.

12. The absolute maximum value of function  $f(x) = x^3 - 3x + 2$  in  $[0, 2]$  is

- (A) 0
- (B) 2
- (C) 4
- (D) 5

**Answer (C)**

**Sol.**  $f(x) = x^3 - 3x + 2, x \in [0, 2]$

To check extremum, differentiate,

$$f'(x) = 3x^2 - 3$$

$$\Rightarrow f'(x) = 0 \Rightarrow 3(x^2 - 1) = x = \pm 1$$

$\Rightarrow$  Critical points are  $-1, 1$  and end points  $0, 2$

$$\Rightarrow f(0) = 2$$

$$f(1) = 0$$

$$f(2) = 2^3 - 3(2) + 2 = 8 - 6 + 2 = 4$$

absolute maximum value is 4

13. The order and degree of differential function  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$  are

(A) Order 1, degree 1

(B) Order 1, degree 2

(C) Order 2, degree 1

(D) Order 2, degree 2

**Answer (C)**

**Sol.** The equation is

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \left(\frac{d^2y}{dx^2}\right)^1$$

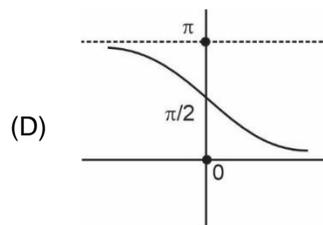
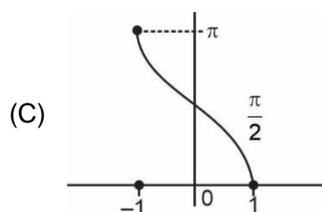
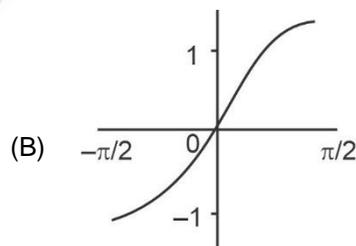
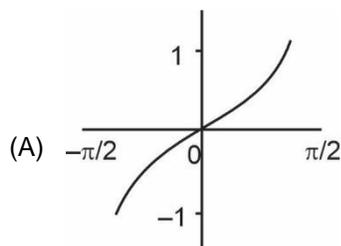
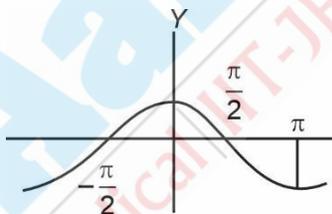
The highest derivative is  $\left(\frac{d^2y}{dx^2}\right)^1$

$\Rightarrow$  Order of differential equation is 2

$\Rightarrow$  The exponent of highest derivative is 1

Therefore, degree of differential equation is 1

14. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse?



**Answer (C)**

**Sol.** Let the given graph is of  $y = f(x)$

$$\text{Then } f\left(-\frac{\pi}{2}\right) = 0 = f\left(\frac{\pi}{2}\right)$$

Therefore, we need to choose an interval such that the function is one – one.

One such interval is  $[0, \pi]$

$\Rightarrow$  The range of  $f(x)$  will be domain of  $f^{-1}(x)$  and vice-versa.

Also, the  $f^{-1}(x)$  is image of  $f(x)$

About  $y = x$  line

$f^{-1}(x)$  domain will be  $[-1, 1]$  and range will be  $[0, \pi]$ , option (C) satisfy the criteria.

15. The corner points of the feasible region in graphical representation of a L.P.P. are  $(2, 72)$ ,  $(15, 20)$  and  $(40, 15)$ . If  $Z = 18x + 9y$  be the objective function, then
- (A)  $Z$  is maximum at  $(2, 72)$ , minimum at  $(15, 20)$
  - (B)  $Z$  is maximum at  $(15, 20)$  minimum at  $(40, 15)$
  - (C)  $Z$  is maximum at  $(40, 15)$ , minimum at  $(15, 20)$
  - (D)  $Z$  is maximum at  $(40, 15)$ , minimum at  $(2, 72)$

**Answer (C)**

**Sol.**  $Z = 18x + 9y$

The maximum or minimum value occurs at corner points.

Let  $A(2, 72)$ ,  $B(15, 20)$ ,  $C(40, 15)$

$$Z_A = 18 \times 2 + 9 \times 72 = 684$$

$$Z_B = 18 \times 15 + 9 \times 20 = 450$$

$$Z_C = 18 \times 40 + 9 \times 15 = 855$$

Maximum occurs at point  $C(40, 15)$  and minimum occurs at point  $B(15, 20)$

16. Let  $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$ ,  $C = [9, 8, 7]$ , which of the following is defined?

- (A) Only  $AB$
- (B) Only  $AC$
- (C) Only  $BA$
- (D) All  $AB$ ,  $AC$  and  $BA$

**Answer (A)**

**Sol.**  $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$ ,  $C = [9, 8, 7]$

Order of  $A$  is  $3 \times 3$

Order of  $B$  is  $3 \times 1$

Order of  $C$  is  $1 \times 3$

Product of two matrices  $A$  and  $B$  is defined if the number of columns in matrix  $A$  is equal to number of rows in matrix  $B$ .

No. of column of matrix  $A =$  no. of rows of matrix  $B$

$\therefore AB$  is defined

No. of column of matrix  $A = 3$

$\therefore$  no. of row of matrix  $C = 1$

$\therefore AC$  is not defined.

No. of column of matrix  $B$  is not equal to no. of rows matrix  $A$ .

So,  $BA$  is not defined.

No. of column of  $A$  is not equal to no. of row of  $C$

$\therefore AC$  is not defined

17. If  $A$  and  $B$  are invertible matrices, then which of the following is **not** correct?

(A)  $(A + B)^{-1} = B^{-1} + A^{-1}$

(B)  $(AB)^{-1} = B^{-1}A^{-1}$

(C)  $\text{adj}(A) = |A|A^{-1}$

(D)  $|A|^{-1} = |A^{-1}|$

**Answer (A)**

**Sol.** Since  $A$  &  $B$  are invertible matrix, so we can say that

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$\text{Also, } A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$\Rightarrow \text{adj}(A) = |A| \cdot A^{-1}$$

$$|A^{-1}| = \frac{1}{|A|}$$

$$|A^{-1}| = |A|^{-1}$$

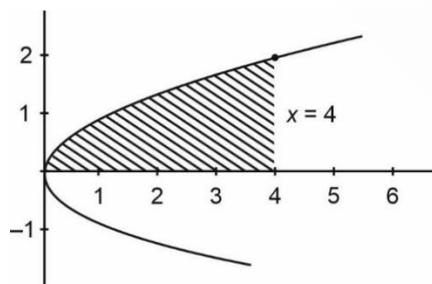
$$\text{Now, } (A + B)^{-1} = \frac{1}{|(A + B)|} \text{adj}(A + B)$$

$$\Rightarrow (A + B)^{-1} \neq A^{-1} + B^{-1}$$

This statement is not true. The inverse of a sum of matrices is not equal to the sum of their inverse.

$\therefore$  Option (A) is correct answer.

18. The area of shaded region by the curves  $y^2 = x$ ,  $x = 4$  and the  $x$ -axis is given by



(A)  $\int_0^4 x \, dx$

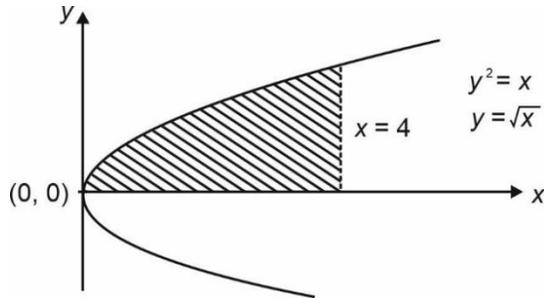
(B)  $\int_0^2 y^2 \, dy$

(C)  $2 \int_0^4 \sqrt{x} \, dx$

(D)  $\int_0^4 \sqrt{x} \, dx$

**Answer (D)**

Sol.



$$\text{Area} = \int_0^4 \sqrt{x} \, dx$$

∴ Option (D) is correct

### Assertion – Reason Based Questions

**Direction :** Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :**  $f(x) = \begin{cases} 3x-8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$  is continuous at  $x = 5$  for  $k = \frac{5}{2}$ .

**Reason (R) :** For a function  $f$  to be continuous at  $x = a$ ,  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$ .

**Answer (D)**

Sol. We have,  $f(x) = \begin{cases} 3x-8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$

since,  $f(x)$  is continuous at  $x = 5$

$$\therefore \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = f(5)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 5^-} (3x-8) = \lim_{h \rightarrow 0} [3(5-h)-8]$$

$$= 15 - 8$$

$$= 7$$

$$\text{RHL} = \lim_{x \rightarrow 5^+} 2k$$

$$= \lim_{h \rightarrow 0} 2k = 2k$$

$$\text{Also, } f(5) = 3(5) - 8 = 7$$

$$\therefore 2k = 7$$

$$k = \frac{7}{2}$$

Reason is correct as for continuity at  $x = a$ ,  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$

∴ Option (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A)** : Let  $Z$  be the set of integers. A function  $f : Z \rightarrow Z$  defined as  $f(x) = 3x - 5$ ,  $\forall x \in Z$  is a bijective.  
**Reason (R)** : A function is a bijective if it is both surjective and injective.

**Answer (D)**

**Sol.** We have  $f : Z \rightarrow Z$  defined by  $f(x) = 3x - 5$

Let us check if the function is injective

Assume,  $f(x_1) = f(x_2) \quad \forall \quad x_1, x_2 \in Z$

$$\Rightarrow 3x_1 - 5 = 3x_2 - 5$$

$$\Rightarrow 3x_1 = 3x_2$$

$$\Rightarrow x_1 = x_2$$

Thus,  $f$  is injective

Now, let us check if the function is surjective.

For  $f$  to be surjective, for every  $y \in Z$ , there must exist on  $x \in Z$  such that  $f(x) = y$

$$y = 3x - 5$$

$$\Rightarrow x = \frac{y+5}{3}$$

If  $y = 1$ , then  $x = \frac{1+5}{3} = 2$ , which is an integer.

If  $y = 2$ , then  $x = \frac{2+5}{3} = \frac{7}{2}$ , which is not an integer.

Since,  $x$  is not always an integer for every integer  $y$ ,  $f$  is not surjective.

$f$  is not bijective because it is not surjective.

The reason is correct, as a bijective function must be injective and surjective.

$\therefore$  Option (D) Assertion (A) is false, but Reason (R) is true.

### SECTION-B

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

**5 × 2 = 10**

21. The diagonals of a parallelogram are given by  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ . Find the area of the parallelogram.

**Sol.** Given:

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1)$$

$$= -2\hat{i} + 3\hat{j} + 7\hat{k}$$

$$\text{and } |\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + (3)^2 + (7)^2}$$

$$= \sqrt{4+9+49}$$

$$= \sqrt{62}$$

Area of parallelogram having diagonals

$$\vec{a} \text{ and } \vec{b} = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$$= \frac{1}{2} (\sqrt{62}) \text{ sq. units}$$

22. Find the value of 'a' for which  $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$  is decreasing on  $\mathbb{R}$ .

**Sol.** Given that :  $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$

Differentiating both sides with respect to x, we get

$$f'(x) = \sqrt{3} \cos x + \sin x - 2a$$

For decreasing function,  $f'(x) < 0$

$$\therefore \sqrt{3} \cos x + \sin x - 2a < 0$$

$$\Rightarrow 2 \left( \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right) - 2a < 0$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x - a < 0$$

$$\Rightarrow \cos \frac{\pi}{6} \cos x + \sin \frac{\pi}{6} \sin x - a < 0$$

$$\Rightarrow \cos \left( x - \frac{\pi}{6} \right) - a < 0$$

$$\text{Since, } \cos \left( x - \frac{\pi}{6} \right) \in [-1, 1]$$

For  $f(x)$  to be decreasing, a must be greater than or equal to the maximum value of  $\cos \left( x - \frac{\pi}{6} \right)$

$$\therefore a \geq 1$$

$$\text{or } a \in [1, \infty)$$

23. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors  $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ . Determine the angle formed between the kite strings. Assume there is no slack in the strings.

**OR**

(b) Find a vector of magnitude 21 units in the direction opposite to that of  $\overline{AB}$  where A and B are the points  $A(2, 1, 3)$  and  $B(8, -1, 0)$  respectively.

**Sol.** (a) Since there is no slack in the string, strings can be taken as straight lines with direction as given vectors.

The angles between two vectors  $v_1$  and  $v_2$  is given by  $\frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|} = \cos \theta$

$$\Rightarrow \text{cosine of Angle between } \vec{a} \text{ and } \vec{b} \text{ will be } \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$|\vec{a}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{4 + 4 + 16} = \sqrt{24}$$

$$\vec{a} \cdot \vec{b} = (3\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} + 4\hat{k}) = 2 \times 3 + (1)(-2) + (2)(4)$$

$$= 6 - 2 + 8 = 12$$

$$\cos \theta = \frac{12}{\sqrt{14}\sqrt{24}} = \sqrt{\frac{144}{24 \times 14}} = \sqrt{\frac{3}{7}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

(b)  $\overline{AB}$  is given by  $\overline{AB} = \overline{OB} - \overline{OA}$

$$\overline{OA} = (2\hat{i} + \hat{j} + 3\hat{k})$$

$$\overline{OB} = (8\hat{i} - \hat{j} + 0\hat{k})$$

$$\overline{AB} = \overline{OB} - \overline{OA} = (8 - 2)\hat{i} + (-1 - 1)\hat{j} + (0 - 3)\hat{k}$$

$$= 6\hat{i} - 2\hat{j} - 3\hat{k}$$

$$|\overline{AB}| = \sqrt{6^2 + (-2)^2 + (-3)^2} = \sqrt{36 + 4 + 9} = 7$$

Vector with magnitude 21 and direction opposite to  $\overline{AB}$  will be

$$-21(\overline{AB}) = (-21) \frac{\overline{AB}}{|\overline{AB}|} = (-21) \left[ \frac{6\hat{i} - 2\hat{j} - 3\hat{k}}{7} \right]$$

$$= (-3)[6\hat{i} - 2\hat{j} - 3\hat{k}] = (-18\hat{i} + 6\hat{j} + 9\hat{k})$$

24. Solve for  $x$ ,

$$2 \tan^{-1} x + \sin^{-1} \left( \frac{2x}{1+x^2} \right) = 4\sqrt{3}$$

**Sol.**

$$2 \tan^{-1} x + \sin^{-1} \left( \frac{2x}{1+x^2} \right) = 4\sqrt{3} ?$$

$$\text{Let } x = \tan \theta, \quad \theta \in \left( \frac{-\pi}{2}, \frac{\pi}{2} \right)$$

$$\Rightarrow x \in (-\infty, \infty)$$

$$\text{Clearly, } \Rightarrow x \in (-\infty, \infty) \Rightarrow \frac{2x}{1+x^2} = \frac{2}{\left(\frac{1}{x} + x\right)} \in [-1, 1]$$

$\Rightarrow$  The substitution is not going to change anything,

$$\Rightarrow 2 \tan^{-1}(\tan \theta) + \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = 4\sqrt{3}$$

$$\text{Since, } \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \sin \theta}{\cos \theta} = 2 \sin \theta \cos \theta$$

$$\left( \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} \right) = \sin 2\theta$$

$$\Rightarrow \forall \theta \in \left( \frac{-\pi}{2}, \frac{\pi}{2} \right) \Rightarrow 2 \tan^{-1}(\tan \theta) = 2\theta$$

$$\sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \sin^{-1}(\sin 2\theta), \theta \in \left( \frac{-\pi}{2}, \frac{\pi}{2} \right)$$

$$\Rightarrow 2\theta \in (-\pi, \pi)$$

$$\sin^{-1}(\sin 2\theta) = \begin{cases} 2\theta, & 2\theta \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \\ -2\theta + \pi, & 2\theta \in \left( \frac{\pi}{2}, \pi \right) \\ -2\theta - \pi, & 2\theta \in \left( -\pi, \frac{-\pi}{2} \right) \end{cases}$$

$$\Rightarrow 2 \tan^{-1}(\tan \theta) + \sin^{-1}(\sin 2\theta) = \begin{cases} 4\theta, & \theta \in \left[ \frac{-\pi}{4}, \frac{\pi}{4} \right] \\ \pi, & \theta \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right) \\ -\pi, & \theta \in \left( \frac{-\pi}{2}, \frac{-\pi}{4} \right) \end{cases}$$

$$\Rightarrow -\pi = 4\sqrt{3} \text{ is also, no solution,}$$

$$\pi = 4\sqrt{3}, \theta \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$4\theta \in \pi = (\pi, 2\pi)$$

$$-\pi \in (0, \pi), \text{ but } 4\sqrt{3} > \pi$$

$\Rightarrow$  no solution

Lastly,

$$4\theta = 4\sqrt{3}, 4\theta = 4\sqrt{3}, \theta \in \left( \frac{-\pi}{4}, \frac{\pi}{4} \right)$$

$$\Rightarrow 4\theta \in (-\pi, \pi), \text{ since } 4\sqrt{3} > \pi$$

$\Rightarrow$  no. solution for  $\theta$ .

$\Rightarrow$  Overall, no solution for  $x$ .

25. (a) Differentiate  $2^{\cos^2 x}$  w.r.t.  $\cos^2 x$ .

OR

(b) If  $\tan^{-1}(x^2 + y^2) = a^2$ , then find  $\frac{dy}{dx}$ .

**Sol.** (a) Let  $z = \cos^2 x$  and  $y = 2^{\cos^2 x}$

We need to find  $\frac{dy}{dz}$ ,

Using parametric differentiation,

$$\frac{dy}{dz} = \left(\frac{dy}{dx}\right) \times \left(\frac{dx}{dz}\right) = \frac{\left(\frac{dy}{dx}\right)}{\left(\frac{dz}{dx}\right)}$$

Lets determine,

$$\frac{dy}{dx} \text{ for } y = 2^{\cos^2 x}$$

taking log both side with natural base e.

$$\Rightarrow \ln y = \cos^2 x \ln 2$$

$\Rightarrow$  Differentiating both sides

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= 2 \cos x \cdot \frac{d(\cos x)}{dx} (\ln 2) \\ &= -2 \cos x \sin x (\ln 2) = -\ln 2 (\sin 2x) \\ \Rightarrow \frac{dy}{dx} &= \left(2^{\cos^2 x}\right) (-\ln 2 \sin 2x) \end{aligned}$$

Now, determine,  $\frac{dz}{dx}$

$\Rightarrow z = \cos^2 x$ , differentiating both sides

$$\begin{aligned} \frac{dz}{dx} &= \frac{d(\cos^2 x)}{dx} = \frac{2 \cos x d(\cos x)}{dx} = -2 \sin x \cos x \\ &= -\sin(2x) \\ \Rightarrow \frac{dz}{dx} &= \frac{2^{\cos^2 x} \cdot (-\ln 2 \sin 2x)}{(-\sin 2x)} = (\ln 2) 2^{\cos^2 x} \end{aligned}$$

$$\boxed{(\ln 2) 2^{\cos^2 x}}$$

(b)  $\tan^{-1}(x^2 + y^2) = a^2$

Let  $x^2 + y^2 = z$  ....(\*)

$$\Rightarrow \tan^{-1}(z) = a^2$$

differentiate both side with respect to x.

$$\Rightarrow \frac{d(\tan^{-1} z)}{dx} = \frac{d(a^2)}{dx} = 0 \text{ [as } a \text{ is constant with respect to } x]$$

$$\Rightarrow \frac{d(\tan^{-1} z)}{dz} \cdot \left(\frac{dz}{dx}\right) = 0$$

$$= \left(\frac{1}{1+z^2}\right) \frac{dz}{dx} = 0 \Rightarrow \frac{dz}{dx} = 0, \text{ as } 1+z^2 \neq 0$$

differentiate both sides of equation (\*)

with respect to x.

$$\begin{aligned} \frac{d(x^2 + y^2)}{dx} &= \frac{dz}{dx} \\ &= 2x \frac{dx}{dx} + 2y \frac{dy}{dx} = \frac{dz}{dx} = 0 \end{aligned}$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$\frac{-x}{y}$
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**SECTION-C**

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

**6 × 3 = 18**

26. Solve the following linear programming problem graphically:

Maximize  $z = 8x + 9y$

Subject to the constraints:

$$2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1$$

$$x \geq 0, y \geq 0$$

**Sol.** The give LPP is to maximize

$$z = 8x + 9y$$

with given constraints,

$$2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1$$

$$x \geq 0, y \geq 0$$

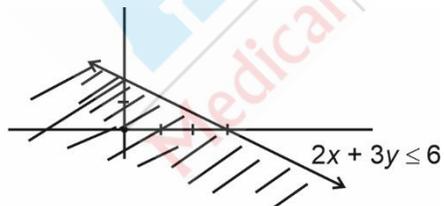
Let's draw graphs of  $2x + 3y \leq 6$ ,

$$0(2) + 0(3) \leq 6$$

$\Rightarrow (0, 0)$  lie in the region

Clearly,

x	0	3
y	2	0

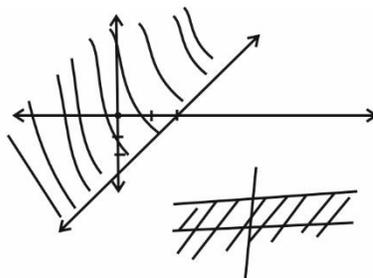


Now, draw graph  $3x - 2y \leq 6$ ,

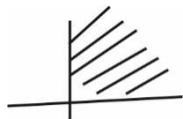
$(0, 0)$  lie inside as

$$3(0) - 2(0) = 0 \leq 6$$

x	0	2
y	-3	0

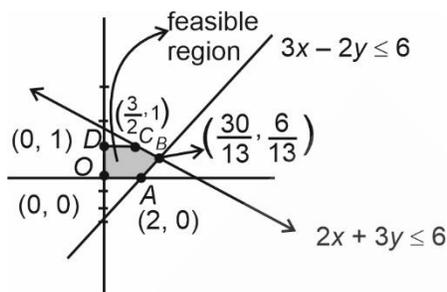


Now,  $y \leq 1$   
and  $x, y \geq 0$



Combining these graph

$B$  is given by intersection point of lines  $3x - 2y = 6$  and  $2x - 3y = 6$ .  $C$  is given by  $y = 1$  and  $2x + 3y - 6 = 0$  lines.



$O(0, 0)$ ,  $A(2, 0)$ ,  $B\left(\frac{30}{13}, \frac{6}{13}\right)$ ,  $C\left(\frac{3}{2}, 1\right)$ ,  $D(0, 1)$  are critical points.

Let's find value of  $z = 8x + 9y$  at  $A, B, C, D$  and  $O$ .

$$z_0 = 8(0) + 9(0) = 0$$

$$z_A = 8(2) + 9(0) = 16$$

$$z_B = 8\left(\frac{30}{13}\right) + 9\left(\frac{6}{13}\right) = \frac{240 + 54}{13} = \frac{294}{13} > 22$$

$$z_C = 8\left(\frac{3}{2}\right) + 9(1) = 12 + 9 = 21$$

$$z_D = 8(0) + 9(1) = 9$$

$\Rightarrow$  Clearly,  $z_B > z_C$  and other values

$z_B$  is maximum at  $\left(\frac{30}{13}, \frac{6}{13}\right)$  and maximum value is  $\frac{294}{13}$

27. (a) Find :  $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

OR

(b) Evaluate :  $\int_0^5 (|x-1| + |x-2| + |x-5|) dx$

**Sol.** (a) We can write the integrand as  $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)}{(x-1)(x+2)(x-3)}$$

By cancelling denominator

$$\Rightarrow 2x - 1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2) \dots (i)$$

Putting  $x = 1$  in equation (i)

$$1 = A(3)(-2) + 0 + 0$$

$$\Rightarrow A = -\frac{1}{6}$$

Putting  $x = -2$  in equation (i)

$$2(-2) - 1 = 0 + B(-2 - 1)(-2 - 3) + 0$$

$$\Rightarrow -5 = 15B$$

$$\Rightarrow B = -\frac{1}{3}$$

Putting  $x = 3$  in equation (i)

$$2(3) - 1 = 0 + 0 + C(2)(5)$$

$$\Rightarrow C = \frac{1}{2}$$

Hence we can write it as 
$$\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx = -\frac{1}{6} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{x+2} dx + \frac{1}{2} \int \frac{dx}{x-3}$$

Using the formula

$$\int \frac{dx}{x+a} = \ln |x+a| + c$$

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)} = -\frac{1}{6} \ln |x-1| - \frac{1}{3} \ln |x+2| + \frac{1}{2} \ln |x-3| + c$$

OR

(b) 
$$\int_0^5 (|x-1| + |x-2| + |x-5|) dx$$

Consider the integrand as  $f(x) = |x-1| + |x-2| + |x-5|$

We can redefine  $f$  as

$$f(x) = \begin{cases} -(x-1) - (x-2) - (x-5) = 8 - 3x & x \leq 1 \\ (x-1) - (x-2) - (x-5) = 6 - x & 1 < x \leq 2 \\ (x-1) + (x-2) - (x-5) = x + 2 & 2 < x \leq 5 \\ (x-1) + (x-2) + (x-5) = 3x - 8 & x > 5 \end{cases}$$

$$\therefore \int_0^5 (|x-1| + |x-2| + |x-5|) dx$$

$$= \int_0^1 (8 - 3x) dx + \int_1^2 (6 - x) dx + \int_2^5 (x + 2) dx$$

$$= \left( 8x - \frac{3x^2}{2} \right)_0^1 + \left( 6x - \frac{x^2}{2} \right)_1^2 + \left( \frac{x^2}{2} + 2x \right)_2^5$$

$$= \left[ \left( 8 - \frac{3}{2} \right) - 0 \right] + \left[ \left( 12 - \frac{4}{2} \right) - \left( 6 - \frac{1}{2} \right) \right] + \left[ \left( \frac{25}{2} + 10 \right) - (2 + 4) \right]$$

$$= \frac{13}{2} + \frac{9}{2} + \frac{33}{2}$$

$$\frac{55}{2}$$

28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.

**Sol.** Let the volume and radius of the spherical ball at any instant be  $V$  and  $r$ , respectively

$$\text{Given } \frac{dV}{dt} \propto -S,$$

Where  $S$  is the surface area of the ball

$$\text{To find : } \frac{dr}{dt}$$

$$\frac{dV}{dt} \propto -S$$

$$\Rightarrow \frac{dV}{dt} = -ks \dots (i) \text{ (where } k \text{ is the proportionality constant)}$$

$$V = \frac{4}{3} \pi r^3$$

Differentiating both sides w.r.t 't'

$$\frac{dV}{dt} = \frac{4}{3} \pi (3r^2) \frac{dr}{dt}$$

from (i)

$$-ks = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow -k(4\pi r^2) = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{-k \cdot 4\pi r^2}{4\pi r^2} = \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = -k$$

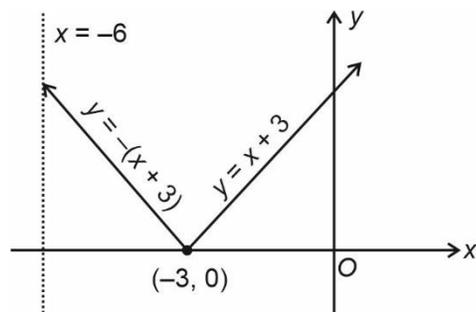
$\therefore$  The rate of decrease of radius is always constant

29. Sketch the graph of  $y = |x + 3|$  and find the area of the region enclosed by the curve, x-axis, between  $x = -6$  and  $x = 0$ , using integration.

**Sol.**  $y = |x + 3|$

$$y = |x + 3| = \begin{cases} x + 3 & x + 3 \geq 0 \\ -(x + 3) & x + 3 < 0 \end{cases}$$

$$= \begin{cases} x + 3 & \text{for } x \geq -3 \\ -(x + 3) & \text{for } x < -3 \end{cases}$$



Now, required area

$$\begin{aligned} & \int_{-6}^0 |x+3| dx \\ &= \int_{-6}^{-3} |x+3| dx + \int_{-3}^0 |x+3| dx \\ &= \int_{-6}^{-3} -(x+3) dx + \int_{-3}^0 (x+3) dx \\ &= \left[ -\frac{x^2}{2} - 3x \right]_{-6}^{-3} + \left[ \frac{x^2}{2} + 3x \right]_{-3}^0 \\ &= \left[ -\frac{(-3)^2}{2} - 3(-3) \right] - \left[ -\frac{(-6)^2}{2} - 3(-6) \right] + \left[ \frac{(0)^2}{2} + 3(0) \right] - \left[ \frac{(-3)^2}{2} + 3(-3) \right] \\ &= \left[ -\frac{9}{2} - (-9) \right] - \left[ -\frac{36}{2} - (-18) \right] + [0] - \left[ \frac{9}{2} - 9 \right] \\ &= -\frac{9}{2} + 9 + 0 - \frac{9}{2} + 9 \\ &= -9 + 18 \\ &= 9 \text{ square units} \end{aligned}$$

30. (a) Verify that lines given by  $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$  and  $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$  are skew lines. Hence, find shortest distance between the lines.

OR

- (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by  $\vec{B} = 2\hat{i} + 8\hat{j}$ ,  $\vec{W} = 6\hat{i} + 12\hat{j}$  and  $\vec{F} = 12\hat{i} + 18\hat{j}$  respectively. Calculate the ratio in which the wicketkeeper divided the line segment joining the bowler and the leg slip fielder.

**Sol.** (a)  $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$

$$= \hat{i} - 2\hat{j} + 3\hat{k} - \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$$

$$= \hat{i} - \hat{j} - \hat{k} + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\vec{r} = \vec{a} - \lambda\vec{b} = \vec{a} + \lambda'\vec{b}$$

$$\vec{r} = \vec{c} + \mu\vec{d}$$

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}; \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{c} = \hat{i} - \hat{j} - \hat{k}; \vec{d} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$\vec{b}$  is not a scalar multiple of  $\vec{d}$

$\therefore$  They are not parallel lines

Now to prove the lines are skew lines and not intersecting, we will find the shortest distance between the two lines. If the distance comes out to be non-zero, then lines will be non-intersecting and the lines will be skew lines.

For lines  $\vec{r} = \vec{a} + \lambda\vec{b}$ ,  $\vec{r} = \vec{c} + \mu\vec{d}$

Shortest distance 'd' between two skew lines is given by  $d = \frac{|(\vec{b} \times \vec{d}) \cdot [\vec{a} - \vec{c}]|}{|\vec{b} \times \vec{d}|}$

$$\vec{a} - \vec{c} = -\hat{j} + 4k$$

$$\vec{b} \times \vec{d} = (\hat{i} - \hat{j} + 2k) \times (\hat{i} \times 2\hat{j} - 2k)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -1 & 2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(-2) - \hat{j}(-4) + k(3)$$

$$= -2\hat{i} + 4\hat{j} + 3k$$

$$|\vec{b} \times \vec{d}| = \sqrt{4+16+9} = \sqrt{29}$$

Putting the values in the formula

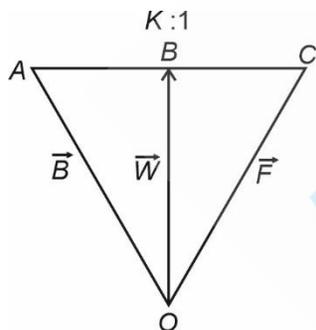
$$\therefore d = \frac{|(-2\hat{i} + 4\hat{j} + 3k) \cdot (-\hat{j} + 4k)|}{\sqrt{29}}$$

$$= \frac{|-4+12|}{\sqrt{29}} = \frac{8}{\sqrt{29}}$$

$\therefore$  Distance is non-zero means lines are skew lines & distance between them is  $\frac{8}{\sqrt{29}}$  units

OR

(b)



let ratio  $K : 1$

$$\vec{OW} = \frac{K\vec{OF} + 10\vec{B}}{K+1}$$

$$K\vec{OW} + \vec{OW} = K\vec{OF} + \vec{OB}$$

$$K(\vec{OW} - \vec{OF}) = \vec{OB} - \vec{OW}$$

$$K(6\hat{i} + 12\hat{j} - 12\hat{i} - 18\hat{j}) = 2\hat{i} + 8\hat{j} - 6\hat{i} - 12\hat{j}$$

$$K(-6\hat{i} - 6\hat{j}) = -4\hat{i} - 4\hat{j}$$

$$-6K(\hat{i} + \hat{j}) = -4(\hat{i} + \hat{j})$$

$$K = \frac{2}{3} \Rightarrow 2 : 3$$

31. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

<b>X</b>	0	2	4	5
<b>P(X)</b>	$p$	$2p$	$3p$	$p$

Where  $X$  is the number of students absent.

- (i) Calculate  $p$ .  
 (ii) Calculate the mean of the number of absent students on Saturday.

**OR**

- (b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

**Sol.** (a)

<b>X</b>	0	2	4	5
<b>P(X)</b>	$p$	$2p$	$3p$	$p$

(i)  $\sum P(x) = 1$   
 $p + 2p + 3p + p = 1$   
 $7p = 1$   
 $p = \frac{1}{7}$

(ii) Mean =  $\sum xP(x)$   
 $= 0(p) + 2(2p) + 4(3p) + 5(p)$   
 $= 0 + 4p + 12p + 5p$   
 $= 21p$   
 $= 21\left(\frac{1}{7}\right)$

Mean = 3

**OR**

(b)  $E_1 =$  Female candidate i.e.  $P(\varepsilon_1) = \frac{2}{3}$   
 $E_2 =$  Male candidate i.e.  $P(\varepsilon_2) = \frac{1}{3}$   
 $A =$  candidate have distinction.  
 $P\left(\frac{A}{\varepsilon_1}\right) = 0.35$  ;  $P\left(\frac{A}{\varepsilon_2}\right) = 0.4$   
 $P(A) = P(\varepsilon_1)P\left(\frac{A}{\varepsilon_1}\right) + P(\varepsilon_2)P\left(\frac{A}{\varepsilon_2}\right)$   
 $= \frac{2}{3} \times \frac{35}{100} + \frac{1}{3} \times \frac{4}{10}$

$$= \frac{70 + 40}{300}$$

$$= \frac{110}{33}$$

$$P(A) = \frac{1}{3}$$

### SECTION-D

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

**4 × 5 = 20**

32. A school wants to allocate students into three clubs: Sports, Music and Drama, under following conditions:

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

**Sol.** Let  $x$  be the number of students in sports club.

Let  $y$  be the number of students in the music club.

Let  $z$  be the number of students in the drama club.

Given Conditions:

1. The numbers of students in the sports club is equal to the sum of the number of students in the music and drama club.

$$x = y + z$$

2. The number of students in the Music club is 20 more than half of the number of students in sports class

$$\Rightarrow y = \frac{x}{2} + 20$$

3. Total students is 180

$$\Rightarrow x + y + z = 180$$

Now we given three equation

$$x - y - z = 0$$

$$2y - x = 0$$

$$x + y + z = 180$$

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \\ 180 \end{bmatrix}$$

$$A \quad X = B$$

$$\Rightarrow X = A^{-1} B$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$= \frac{\begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \\ -3 & -2 & 1 \end{pmatrix}}{4}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ -3/4 & -1/2 & 1/4 \end{pmatrix} \begin{pmatrix} 0 \\ 40 \\ 180 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 90 \\ 65 \\ 25 \end{pmatrix}$$

$\Rightarrow$  Sports club = 90

Music club = 65

Drama club = 2533.

33. Find :  $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$ .

**Sol.** We have to integrate

$$\sin^{-1} \sqrt{\frac{x}{a+x}} \text{ with respect to } x$$

Lets substitute

$$x = a \tan^2 t$$

$$dx = 2a \tan t \sec^2 t dt$$

After substitute value of  $x$  and  $dx$

We'll get

$$\begin{aligned} I_1 &= \int \sin^{-1} \sqrt{\frac{a \tan^2 t}{a + a \tan^2 t}} \times 2a \tan t \sec^2 t dt \\ &= \int \sin^{-1} \left( \sqrt{\frac{\tan^2 t}{\sec^2 t}} \right) \times 2a \tan t \sec^2 t dt \\ &= \int \sin^{-1} \left( \sqrt{\frac{\sin^2 t}{\cos^2 t \sec^2 t}} \right) \times 2a \tan t \sec^2 t dt \\ &= 2a \int \sin^{-1}(\sin t) \tan t \sec^2 t dt \\ &= 2a \int \underbrace{t}_{I} \underbrace{\tan t \sec^2 t dt}_{II} \\ &= 2a \left[ t \int \tan t \sec^2 t dt - \int \frac{d}{dt}(t) \int \tan t \sec^2 t dt \right] dt \\ &= 2a \left[ t \int \tan t \sec^2 t dt - \int \tan t \sec^2 t dt \right] \end{aligned}$$

Consider  $\int \tan t \sec^2 t \, dt = I$

Let  $\tan t = u$

$\sec^2 t \, dt = du.$

$= \int u \, du = \frac{u^2}{2}$

$\Rightarrow I = \frac{\tan^2 t}{2}$

Substitute into  $I_1$

$I_1 = 2a \left[ t \frac{\tan^2 t}{2} - \frac{1}{2} \int \tan^2 t \, dt \right]$

$= 2a \left[ t \frac{\tan^2 t}{2} - \frac{1}{2} \int (\sec^2 t - 1) \, dt \right]$

$= 2a \left[ t \frac{\tan^2 t}{2} - \frac{1}{2} \tan t + \frac{t}{2} \right] + C$

Substitutes value of  $t = \tan^{-1} \sqrt{\frac{x}{a}}$

$$I_1 = a \left[ \left( \tan^{-1} \sqrt{\frac{x}{a}} \right) \frac{x}{a} - \sqrt{\frac{x}{a}} + \tan^{-1} \sqrt{\frac{x}{a}} \right] + C$$

34. (a) If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , then prove that  $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ .

OR

(b) If  $x = a \left( \cos \theta + \log \tan \frac{\theta}{2} \right)$  and  $y = \sin \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .

**Sol.** (a) Given:  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

To prove:  $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Proof: Let's substitute

$x = \sin A$

$y = \sin B$

Now

$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

$= \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$= \cos A + \cos B = a(\sin A - \sin B)$

$$= 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) = 2a \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$= \cos\left(\frac{A-B}{2}\right) = a \sin\left(\frac{A-B}{2}\right)$$

$$= \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A-B}{2}\right)} = a$$

$$= \cot\left(\frac{A-B}{2}\right) = a$$

$$\Rightarrow \frac{A-B}{2} = \cot^{-1} a$$

Putting back the value of A & B

$$\sin^{-1}x - \sin^{-1}y = 2 \cot^{-1} a$$

Diff. wrt. x.

$$\frac{1}{\sqrt{1-x^2}} - \frac{y'}{\sqrt{1-y^2}} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Hence Proved

OR

(b) Given that

$$x = a \left( \cos \theta + \log \left( \tan \frac{\theta}{2} \right) \right)$$

$$y = \sin \theta$$

We have to find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$

We know that

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\frac{dy}{d\theta} = \cos \theta$$

$$\frac{dx}{d\theta} = a \left( -\sin \theta + \frac{1}{\left( \tan \frac{\theta}{2} \right)} \sec^2 \left( \frac{\theta}{2} \right) \times \frac{1}{2} \right)$$

$$= a \left( -\sin\theta + \frac{\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}} \right)$$

$$= a \left( -\sin\theta + \frac{1}{\sin\theta} \right)$$

$$\frac{dx}{d\theta} = a(-\sin\theta + \operatorname{cosec}\theta)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{\cos\theta}{a(\operatorname{cosec}\theta - \sin\theta)}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{1}{a} \left[ \frac{(\operatorname{cosec}\theta - \sin\theta)(-\sin\theta)}{(\operatorname{cosec}\theta - \sin\theta)^2} + \cos\theta(\operatorname{cosec}\theta \cot\theta + \cos\theta) \right] \frac{d\theta}{dx}$$

$$\therefore x = \frac{\pi}{4}$$

$$\left. \frac{dx}{d\theta} \right|_{x=\frac{\pi}{4}} = a \left[ \sqrt{2} - \frac{1}{\sqrt{2}} \right]$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=\frac{\pi}{4}} = \frac{1}{a} \left[ \frac{\left( \sqrt{2} - \frac{1}{\sqrt{2}} \right) \left( -\frac{1}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left( \sqrt{2} + \frac{1}{\sqrt{2}} \right)}{\left( \sqrt{2} - \frac{1}{\sqrt{2}} \right)^2} \right] \times \frac{1}{a \left( \sqrt{2} - \frac{1}{\sqrt{2}} \right)}$$

$$= \frac{1}{a^2} \left[ \frac{-1 + \frac{1}{2} + 1 + \frac{1}{2}}{\left( \sqrt{2} - \frac{1}{\sqrt{2}} \right)^3} \right]$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=\frac{\pi}{4}} = \frac{1}{a^2} (\sqrt{2})^3 = \frac{2\sqrt{2}}{a^2}$$

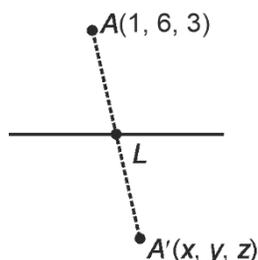
35. (a) Find the image  $A'$  of the point  $A(1, 6, 3)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ .

Also, find the equation of the line joining  $A$  and  $A'$ .

OR

- (b) Find a point  $P$  on the line  $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$  such that its distance from point  $Q(2, 4, -1)$  is 7 units. Also, find the equation of line joining  $P$  and  $Q$ .

**Sol.** (a) Let  $A(1, 6, 3)$  be the given point and set  $L$  be the foot of perpendicular from  $P$  to given line.



The co-ordinates of a general point on the line are

$$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

i.e.,  $x = \lambda, y = 2\lambda + 1, z = 3\lambda + 2$

If the co-ordinates of  $L$  are

$(\lambda, 2\lambda + 1, 3\lambda + 2)$ , then direction ratio of  $PL$  are  $\langle \lambda - 1, 2\lambda - 5, 3\lambda - 1 \rangle$

But the direction ratio of given line which is perpendicular to  $AL$  are  $\langle 1, 2, 3 \rangle$

$$\therefore (\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$$

$$\Rightarrow \lambda = 1.$$

$$\therefore L(1, 3, 5)$$

$\therefore L$  is the mid pointing  $AA'$

$$\therefore \frac{x_1+1}{2} = 1 \quad \Rightarrow x_1 = 1$$

$$\frac{y_1+6}{2} = 3 \quad \Rightarrow y_1 = 0$$

$$\frac{z_1+3}{2} = 5 \quad \Rightarrow z_1 = 7$$

$$\therefore \text{image of } (1, 6, 3) \text{ in given line is } \boxed{(1, 0, 7)}$$

Line joining  $A$  and  $A'$  will have direction ratio,  $\langle 0, 6, -4 \rangle$

$\therefore$  equation of line will be

$$\frac{x-1}{0} = \frac{y-6}{6} = \frac{z-3}{-4}$$

(b) Any point  $P$  on the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$$

is  $P(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$

$\therefore$  it is given that the distance between point  $P$  and  $Q$  is 7 units.

$$\Rightarrow |PQ| = 7$$

$$= |PQ|^2 = 49$$

$$\text{Now } \overline{PQ} = (\lambda - 7)\hat{i} + (4\lambda - 7)\hat{j} + (7 - 9\lambda)\hat{k}$$

$$\Rightarrow (\lambda - 7)^2 + (4\lambda - 7)^2 + (7 - 9\lambda)^2 = 49$$

$$\Rightarrow \lambda = 1$$

$\therefore$  point  $P$  will be  $(-4, 1, -3)$

The equation of line joining  $(-4, 1, -3)$  and  $(2, 4, -1)$  will have direction ratio  $\langle 6, 3, 2 \rangle$

$\therefore$  Line joining  $P$  &  $Q$  is

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$

### SECTION-E

This section comprises of 3 case study/passage based questions of 4 marks each.

**3 × 4 = 12**

36. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set  $A = \{1, 2, 3\}$ :

$$R_1 = \{(2, 3), (3, 2)\}$$

$$R_2 = \{(1, 2), (1, 3), (3, 2)\}$$

$$R_3 = \{(1, 2), (2, 1), (1, 1)\}$$

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

$$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$$

The students are asked to answer the following questions about the above relations:

- Identify the relation which is reflexive, transitive but not symmetric.
- Identify the relation which is reflexive and symmetric but not transitive.
- (a) Identify the relations which are symmetric but neither reflexive nor transitive.

**OR**

(b) What pairs should be added to the relation  $R_2$  to make it an equivalence relation?

- Sol.** (i)  $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$(1, 1), (2, 2), (3, 3)$  are reflexive symmetric and transitive

$(1, 2), (2, 2)$  are transitive

But  $(1, 2) \in R$

$(2, 1) \notin R$

$\therefore$  Not symmetric

- (ii)  $R_1 = \{(2, 3), (3, 2)\}$

Symmetric

$$R_2 = \{(1, 2), (1, 3), (3, 2)\}$$

Transitive

$$R_3 = \{(1, 2), (2, 1), (1, 1)\}$$

Symmetric and transitive

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

Reflexive but not symmetric

$$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$$

This is reflexive, symmetric and transitive

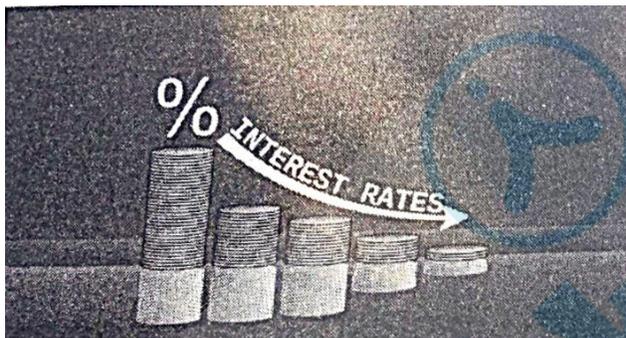
Hence No, reflection which is "Reflexive and symmetric but not transitive".

- (iii) (a)  $R_1 = \{(2, 3), (3, 2)\}$   
 $(2, 2) \notin R$   
 $(3, 3) \notin R$  Not reflexive  
 $(2, 3) \in R$   
 $(3, 2) \in R$   
 But  $(2, 2) \notin R$   
 $\therefore$  Not transitive

OR

- (b)  $R_2 = \{(1, 2) (1, 3) (3, 2)\}$   
 To make  $R_2$  equivalence we required  
 $(1, 1) (2, 2) (3, 3)$  to make it reflexive  
 And  $(2, 1) (3, 1) (2, 3)$  to make it symmetric  
 Hence, it because reflexive, symmetric and transitive  
 $\therefore R_2 = \{(1, 1) (2, 2) (3, 3) (1, 2) (2, 1) (1, 3) (3, 1) (3, 2) (2, 3)\}$   
 Will be equivalence.

37.



A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following:

- (i) What is the probability that a customer after availing the loan will default on the loan repayment?
- (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

**Sol.**  $E_1$  : Loan at fixed rate

$E_2$  : Loan at floating rate

$E_3$  : Loan at variable rate

$A$  : A person defaults on loan

$$P(E_1) = \frac{10}{100}; \quad P(E_2) = \frac{20}{100}; \quad P(E_3) = \frac{70}{100}$$

$$P(A/E_1) = \frac{5}{100}; \quad P(A/E_2) = \frac{3}{100}; \quad P(A/E_3) = \frac{1}{100}$$

$$(i) P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$

$$= \frac{10}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{3}{100} + \frac{70}{100} \times \frac{1}{100}$$

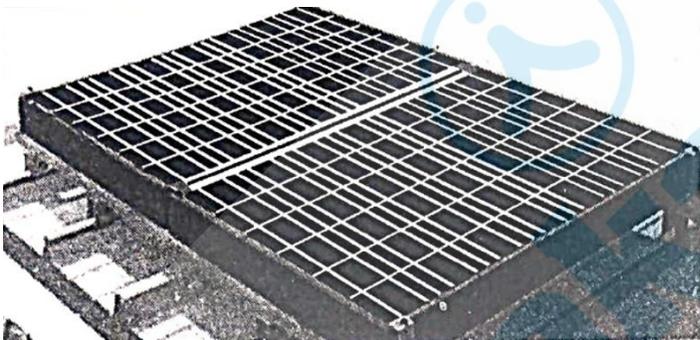
$$= \frac{50 + 60 + 70}{10000}$$

$$= \frac{180}{10000} = 0.018$$

$$(ii) P(E_3/A) = \frac{P(E_3)(P(A/E_3))}{P(A)}$$

$$= \frac{\frac{70}{100} \times \frac{1}{100}}{\frac{180}{10000}} = \frac{70}{10000 \times \frac{180}{10000}} = \frac{7}{18}$$

38.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be  $x$  metres and with parallel to the partition be  $y$  metres.

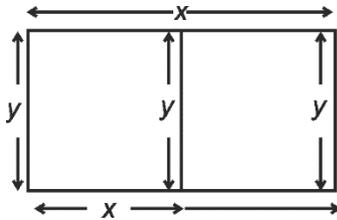
Based on this information, answer the following questions:

- (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of  $x$  and  $y$ .
- (ii) Write the area of the solar panel as a function of  $x$ .
- (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

**OR**

- (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

Sol.



(i) Total boundary length including partition is  $2x + 3y = 300$

(ii) Area =  $l \times b$

$$= xy$$

$$A = x \left( \frac{300 - 2x}{3} \right) = x \left( 100 - \frac{2}{3}x \right)$$

(iii) (a)  $A' = x \left( -\frac{2}{3} \right) + \left( 100 - \frac{2}{3}x \right)$

$$A' = 100 - \frac{4}{3}x \quad \& \quad A'' = -\frac{4}{3} < 0 \text{ Maximum area}$$

For critical point  $A' = 0$

$$100 - \frac{4}{3}x = 0$$

$$x = 75 \Rightarrow y = 50$$

$$\text{Area} = x \times y$$

$$\text{Area} = 75 \times 50$$

$$\text{Area} = 3750 \text{ sq. unit}$$

OR

(b) As  $A' = 100 - \frac{4}{3}x$

For  $x < 75 \Rightarrow A' > 0$

For  $x > 75 \Rightarrow A' < 0$

Function changes from increasing to decreasing at  $x = 75$

$\therefore$  Max. area is at  $x = 75$  only

Hence area is 3750 sq. unit

□ □ □