23/05/2023

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Answers & Solutions

Time : 45 min. *for* M.M. : 200

CUET UG-2023

(Mathematics)

IMPORTANT INSTRUCTIONS:

1. The test is of 45 Minutes duration.
2. The test contains 50 Questions out of which 40 questions need to be attempted.
3. Marking Scheme of the test:
   a. Correct answer or the most appropriate answer: Five marks (+5)
   b. Any incorrect option marked will be given minus one mark (–1).
   c. Unanswered/Marked for Review will be given no mark (0).

Choose the correct answer :

SECTION-I (COMMON)

1. Let A be a square matrix of order 3 then $|3A|$ is equal to

(1) $3|A|$

(2) $3^2|A|$

(3) $|A|^3$

(4) $3^3|A|$

Answer (4)

Sol. $|3A| = 3^3 \times |A|$

Option (4) is correct.

2. In a LLP, let R be the feasible region.

A. If R is unbounded then a max./min. value of objective function may not exist.

B. If R is bounded then a max. and min. value of objective function will always exist.

C. If a solution exists, it must occur at a corner point.

D. If R is bounded then max. will exist but min. may or may not exist for an objective function.

Choose the correct answer from the options given below:

(1) A, B, C only (2) B only

(3) A, C only (4) D, C only

Answer (1)

Sol. If R is bounded, then the objective function Z has both a max. and a min. value on R and each of these occurs at a corner point of R.

If the feasible region R is unbounded, then the max. and min. value of the objective function may or may not exist.

Option (1) is correct.

3. The volume of a cube is increasing at the rate of 27 cm^3/s. How fast is the surface area increasing when the length of the cube is 12 cm?

(1) 9 cm^2/s (2) $\frac{9}{4}$ cm^2/s

(3) $\frac{4}{9}$ cm^2/s (4) $\frac{9}{2}$ cm^2/s

Answer (1)

Sol. $V = a^3$

$$\frac{dv}{dt} = 3a^2\frac{da}{dt}$$

$$\Rightarrow \frac{da}{dt} = \frac{27}{3a^2} = \frac{27}{3\times(12)^2} = \frac{1}{16}$$

$S = 6a^2$

$$\frac{ds}{dt} = 6\times 2a\frac{da}{dt}$$

$$= 12\times 12\times\frac{1}{16}$$

$= 9$ cm^2/s

Option (1) is correct.

4. If a fair coin is tossed 10 times, then the probability of obtaining at least one head is :

(1) $\frac{1}{1024}$

(2) $\frac{17}{1024}$

(3) $\frac{1023}{1024}$

(4) $\frac{23}{1024}$

Answer (3)

Sol. P (Obtaining at least one head)

$= 1 - P$ (Obtaining no head)

$$= 1 - \frac{1}{2^{10}}$$

$$= \frac{1023}{1024}$$

Option (3) is correct.

5. In the context of differential equation

Match List I with List II

	List I		List II
A.	$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$	I.	Not a differential equation
B.	$x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$	II.	Linear first order
C.	$\sin x + y = \cos(x + y)$	III.	Variable separable
D.	$(x+y)\frac{dy}{dx} = 1$	IV.	Homogenous

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-II, B-IV, C-III, D-I

(3) A-III, B-IV, C-I, D-II (4) A-IV, B-I, C-III, D-II

Answer (3)

Sol. A : $\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \rightarrow$ Variable separable type

B : $x^2\frac{dy}{dx} = x^2 - 2y^2x + xy$

$$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2} \rightarrow \text{Homogeneous}$$

C : $\sin x + y = \cos(x + y)$

$\rightarrow$ Not a differential equation

D : $(x+y)\frac{dy}{dx} = 1$

$$\Rightarrow \frac{dx}{dy} = x + y \rightarrow \text{Linear differential equation}$$

Option (3) is correct.

6. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Then variance of the number of kings is

(1) $\frac{680}{(221)^3}$ (2) $\frac{6080}{(221)^2}$

(3) $\frac{680}{221}$ (4) $\frac{6800}{(221)^2}$

Answer (4)

Sol. Let X denote the number of kings in a draw of two cards

$$\Rightarrow P(X=0)=\frac{48C_2}{52C_2}=\frac{188}{221}$$

$$\Rightarrow P(X=1)=\frac{4C_1\times 48C_1}{52C_2}=\frac{32}{221}$$

$$\Rightarrow P(X=2)=\frac{4C_2}{52C_2}=\frac{1}{221}$$

$$\Rightarrow \text{Mean of } X=\sum_{i=1}^{2}x_iP(x_i)$$

$$=0\times\frac{188}{221}+1\times\frac{32}{221}+\frac{2}{221}=\frac{34}{221}$$

$$\Rightarrow E(X^2)=\sum_{i=1}^{2}x_1^2P(x_i)$$

$$=0^2\times\frac{188}{221}+1^2\times\frac{32}{221}+2^2\times\frac{1}{221}$$

$$=\frac{36}{221}$$

$$\Rightarrow \mathrm{Var}(X)=E(X^2)-[E(X)]^2=\frac{36}{221}-\left(\frac{34}{221}\right)^2$$

$$=\frac{6800}{(221)^2}$$

Option (4) is correct

7. If $y=x^x,\frac{dy}{dx}$ will be

(1) x^x (2) $x^x(1+\log x)$

(3) x^{x-1} (4) x^{x+1}

Answer (2)

Sol. $y=x^x$

$$\frac{dy}{dx}=x^x(1+\log x)$$

8. The inverse of the matrix $A=\begin{bmatrix}2&3\\1&2\end{bmatrix}$ is

(1) $\begin{bmatrix}-2&3\\1&-2\end{bmatrix}$ (2) $\begin{bmatrix}2&-3\\-1&2\end{bmatrix}$

(3) $\begin{bmatrix}-2&-3\\-1&2\end{bmatrix}$ (4) $\begin{bmatrix}2&3\\1&2\end{bmatrix}$

Answer (2)

Sol. $A=\begin{bmatrix}2&3\\1&2\end{bmatrix}$

$$\text{adj } A=\begin{bmatrix}2&-1\\-3&2\end{bmatrix}^T=\begin{bmatrix}2&-3\\-1&2\end{bmatrix}$$

$$|A|=4-3=1$$

$$A^{-1}=\begin{bmatrix}2&-3\\-1&2\end{bmatrix}$$

9. The value of $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$ is

(1) $\frac{x^2}{2}+\log|x|+C$, (where C is constant of integration)

(2) $\frac{x^2}{2}+\log|x|+2x+C$, (where C is constant of integration)

(3) $\frac{3}{2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)+C$, (where C is constant of integration)

(4) $\frac{2}{3}x^{\frac{3}{2}}+2\sqrt{x}+C$, (where C is constant of integration)

Answer (2)

Sol. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$

$$=\int\left(x+\frac{1}{x}+2\right)dx$$

$$\frac{x^2}{2}+\log|x|+2x+C$$

10. The integral $\int e^x\left(\frac{x-1}{2x^2}\right)dx$ is equal to

(1) $\frac{e^x}{x}+C$, where C is constant of integration

(2) $\frac{e^x}{2x}+C$, where C is constant of integration

(3) e^xx+C, where C is constant of integration

(4) x^2e^x+C, where C is constant of integration

Answer (2)

Sol. $\int e^x\left(\frac{x-1}{2x^2}\right)dx$

$$\int e^x\left(\frac{1}{2x}-\frac{1}{2x^2}\right)dx$$

$$=\frac{e^x}{2x}+C$$

11. The order of a null matrix is

(1) 0 (2) 1

(3) 2 (4) any order

Answer (4)

Sol. Order of null matrix is any order

12. Which of the following differential equation represents the family of circles touching the x-axis at the origin?

(1) $(x^2 - y^2)dy - 2xydx = 0$

(2) $(x^2 + y^2)dy + 2xydx = 0$

(3) $(x^2 - y^2)dx + 2xydy = 0$

(4) $(x^2 + y^2)dy - 2xydx = 0$

Answer (1)

Sol. $x^2 + (y - a)^2 = a^2$

$r^2 + y^2 - 2ay = 0$

$$\frac{x^2+y^2}{y}=2a$$

$$\frac{y(2x+2yy')-(x^2+y^2)y'}{y^2}=0$$

$$2xy+2y^2\frac{dy}{dx}-(x^2+y^2)\frac{dy}{dx}=0$$

$$(x^2-y^2)\frac{dy}{dx}-2xy=0$$

$$(x^2-y^2)dy-2xydx=0$$

13. Let $A=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$, then adjoint ($A$) is

(1) $\begin{bmatrix}3 & 2\\ -1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}4 & -1\\ 2 & 3\end{bmatrix}$

(3) $\begin{bmatrix}-4 & 1\\ -2 & -3\end{bmatrix}$ (4) $\begin{bmatrix}4 & 1\\ -2 & 3\end{bmatrix}$

Answer (4)

Sol. $A=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$

$$\text{adj } A=\begin{bmatrix}4 & -2\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 1\\ -2 & 3\end{bmatrix}$$

14. The critical points of $f(x) = x^3 + x^2 + x + 1$ are

(1) 2, 1

(2) –2, –1

(3) 2, –1

(4) do not exist

Answer (4)

Sol. $f(x) = x^3 + x^2 + x + 1$

$f'(x) = 3x^2 + 2x + 1 = 0$

$\because$ $D < 0$

$\therefore$ No solution

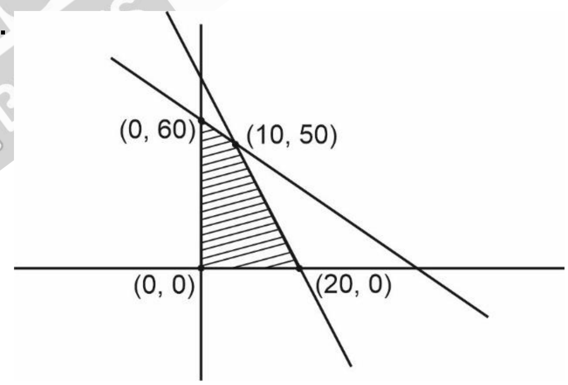
15. If $5x + y \le 100$, $x + y \le 60$, $x \ge 0$, $y \ge 0$. Then one of the corner points of the feasible region is

(1) (60, 0) (2) (0, 100)

(3) (10, 50) (4) (0, 20)

Answer (3)

Sol.



Corner point : {(0, 60), (20, 0), (10, 50), (0, 0)}

SECTION-II (CORE MATHEMATICS)

1. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\sqrt{\tan x}}$ is equal to :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{12}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

Answer (2)

Sol. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$I = \frac{\pi}{12}$$

Option (2) is correct.

2. The value of λ, so that lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular is :

(1) $-\frac{70}{11}$ (2) $\frac{70}{11}$

(3) $\frac{11}{70}$ (4) $-\frac{11}{70}$

Answer (2)

Sol. $\vec{v}_1 : -3\hat{i} + \frac{2\lambda}{7}\hat{j} + 2k$

$$\vec{v}_2 : \frac{-3\lambda}{7}\hat{i} + \hat{j} - 5k$$

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

$$\Rightarrow \frac{9\lambda}{7} + \frac{2\lambda}{7} - 10 = 0$$

$$\Rightarrow \lambda = \frac{+70}{11}$$

Option (2) is correct.

3. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(1) 0 (2) $(a + b + c)$

(3) $(a - b)(b - c)(c - a)$ (4) $a^2 + b^2 + c^2$

Answer (1)

Sol. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$C_2 \to C_2 + C_3$

$$\begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$$

$= 0$

4. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx} =$

(1) $-\frac{2}{1+x^2}$ (2) $\frac{2}{1+x^2}$

(3) $-\frac{1}{1+x^2}$ (4) $\frac{1}{1+x^2}$

Answer (1)

Sol. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$y = \frac{\pi}{2} - 2\tan^{-1} x$$

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

Option (1) is correct.

5. The intervals for which $f(x) = x^4 - 2x^2$ is increasing are :

(1) $(-\infty, 1)$

(2) $(-1, \infty)$

(3) $(-\infty, -1) \cup (0, 1)$

(4) $(-1, 0) \cup (1, \infty)$

Answer (4)

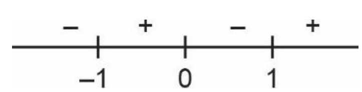
Sol. $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

$= 4(x^3 - x)$

$x^3 - x > 0$

$x(x^2 - 1) > 0$



$x \in (-1, 0) \cup (1, \infty)$

Option (4) is correct.

6. The simplest form of $\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}$ is, where $-a < x < a$.

(1) $\tan^{-1}\frac{x}{a}$ (2) $\tan^{-1}(ax)$

(3) $a\tan^{-1}\frac{x}{a}$ (4) $\sin^{-1}\frac{x}{a}$

Answer (4)

Sol. $x = a\sin\theta$

$$\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta = \sin^{-1}\left(\frac{x}{a}\right)$$

7. The maximum value of $\sin x + \cos x$, $x \in R$ is :

(1) 2 (2) $\sqrt{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) Not known

Answer (2)

Sol. $E = \sin x + \cos x$

$$E \in \left[-\sqrt{2}, \sqrt{2}\right]$$

$$E_{max} = \sqrt{2}$$

8. Which one of the following options is incorrect?

For a square matrix A in the matrix equation $AX = B$.

(1) If $|A| \neq 0$, then there exists a unique solution

(2) If $|A| = 0$ and (adj A)$B \neq 0$ then there is no solution

(3) If $|A| \neq 0$ and (adj A)$B \neq 0$ then there is no solution

(4) If $|A| = 0$ and (adj A)$B = 0$ then system has infinitely many solutions

Answer (3)

Sol. $AX = B$

$X = A^{-1}B$

$$X = \frac{(\text{adj } A)}{|A|}\cdot B$$

If $|A| \neq 0 \Rightarrow$ unique solution

If $|A| = 0$

and (adj A)$B \neq 0 \Rightarrow$ No solution

If $|A| \neq 0$

and (adj A)$B \neq 0 \Rightarrow$ unique solution

If $|A| = 0$

and (adj A)$B = 0 \Rightarrow$ Infinite solution.

9. The simplest form of $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$ is :

(1) $\tan^{-1}x$ (2) x

(3) $\frac{1}{2}\tan^{-1}x$ (4) $\sqrt{1+x^2}$

Answer (3)

Sol. $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Let $x = \tan\theta$

$$\tan\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= \tan\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$= \frac{1}{2}\tan^{-1}x$$

10. If x, y and z are non zero real numbers, the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

(1) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(2) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(3) $\frac{1}{xyz}\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(4) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer (1)

Sol. $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$$A^{-1} = \frac{\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}}{xyz}$$

$$A^{-1} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

11. Match List-I with List-II

List-I		List-II	
A.	$lx + my + nz = d$ is	I.	Equation of plane passing through a given point and normal to given vector
B.	$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	II.	Equation of plane in normal form
C.	$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$	III.	Plane passing through the intersection of two planes
D.	$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$	IV.	Intercept from of plane

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-IV, C-I, D-III

(4) A-I, B-II, C-III, D-IV

Answer (3)

Sol. (A) $lx + my + nz = d$ $\rightarrow$ Equation of plane in normal (II)

(B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ $\rightarrow$ Intercept from of plane (IV)

(C) $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ $\rightarrow$ Equation of plane passing through a given point and normal to given vector (I)

(D) $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$ $\rightarrow$ Plane passing through the intersection of two planes (III)

12. If $\begin{vmatrix} 2x & 2 \\ 4 & x \end{vmatrix} = 10$, then x is:

(1) ± 2 (2) ± 3

(3) ± 4 (4) 0

Answer (2)

Sol. $\begin{vmatrix} 2x & 2 \\ 4 & x \end{vmatrix} = 10$

$2x^2 - 8 = 10$

$x = \pm 3$

13. The minimum value of $f(x) = |2x - 1|$ is

(1) $-\infty$

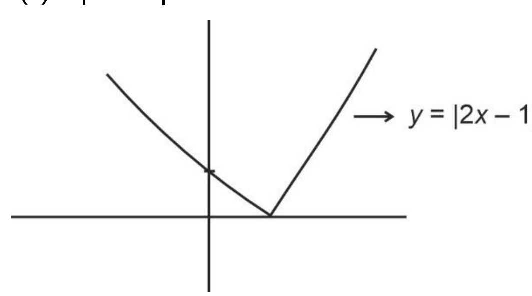
(2) 0

(3) $\frac{1}{2}$

(4) 1

Answer (2)

Sol. $f(x) = |2x - 1|$



$\therefore$ Range = $[0, \infty]$

Min value = 0

14. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is:

(1) 60°

(2) 90°

(3) 120°

(4) 30°

Answer (2)

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a} + \vec{b} = 6\hat{i} - 4\hat{j} + 2\hat{k} = \vec{c}$

$\vec{a} - \vec{b} = 4\hat{i} + 2\hat{j} - 8\hat{k} = \vec{d}$

$$\vec{c} \wedge \vec{d} \Rightarrow \cos\theta = \frac{\vec{c} \cdot \vec{d}}{|\vec{c}|\ |\vec{d}|}$$

$$= \left|\frac{24 - 8 - 16}{\sqrt{36 + 16 + 4}\sqrt{16 + 4 + 64}}\right|$$

= 0

θ = 90°

15. The relation $R = \{(a, b) : a \le b^2\}$ on the set of real numbers is:

(1) Reflexive and symmetric

(2) Neither reflexive nor symmetric

(3) Transitive

(4) Reflexive but not symmetric

Answer (2)

Sol. $R = \{(a, b) : a \le b^2\}$

For reflexive $(a, a) \in R$

If $a \in (0, 1)$ then $a > a^2$

$\Rightarrow$ R is not reflexive

For symmetric

if $(a, b) \in R \Rightarrow a \le b^2$

$\not\Rightarrow$ $(b, a) \in R \not\Rightarrow b \le a^2$

$\therefore$ R is not symmetric

$\therefore$ R is neither Reflexive nor symmetric

16. The solution of $y' - y'' = 2x$ is:

A. $y = x^2 + 2x + 2$

B. $y = x^2 + 2x + 1$

C. $y = x + 2$

D. $y = x^2 - 2x + 1$

Choose the correct answer from the options given below:

(1) A and B only (2) B only

(3) C only (4) A and D only

Answer (1)

Sol. $y' - y'' = 2x$

Now

(A) $y = x^2 + 2x + 2$

$y' = 2x + 2$ $\quad\therefore y' - y'' = 2x$

$y'' = 2$

(B) $y = x^2 + 2x + 1$

$y' = 2x + 2$ $\quad y' - y'' = 2x$

$y'' = 2$

(C) $y = x + 2$

$y' = 1$ $\quad y' - y'' = 1$

$y'' = 0$

(D) $y = x^2 - 2x + 1$

$y' = 2x - 2$ $\quad y' - y'' = 2x - 4$

$\therefore$ A is B only are solution

17. The slope of the normal to the curve $y = 2x^2 - 4$ at $P(1, -2)$ is

(1) 4 (2) –4

(3) $-\frac{1}{4}$ (4) 0

Answer (3)

Sol. $y = 2x^2 - 4$

$y' = 4x$

$y'_T = 4$

$y'_N = -\frac{1}{4}$

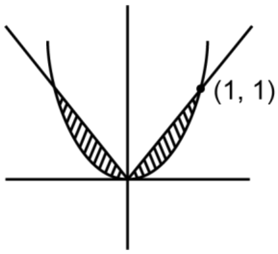
18. The area of the region $\{(x, y) : y \ge x^2 \text{ and } y \le |x|\}$ is

(1) 2 (2) 1

(3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Answer (4)

Sol. $\{(x,y): y \geq x^2 \text{ and } y \leq |x|\}$



$$A = 2\int_0^1 (x - x^2)dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{2}{6} = \frac{1}{3} \text{ sq unit.}$$

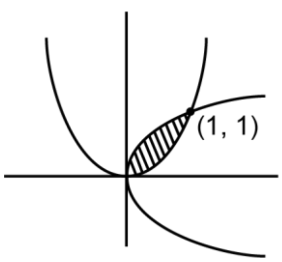
19. The area enclosed between the curves $y = x^2$ and $x = y^2$ is

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Answer (3)

Sol. $y = x^2$ and $x = y^2$



$$\text{Area} = \int_0^1 \left(x^2 - \sqrt{x}\right)dx$$

$$\left[\frac{x^3}{3} - \frac{2x^{\frac{3}{2}}}{3}\right]_0^1$$

$$\left|\frac{1}{3} - \frac{2}{3}\right| = \frac{1}{3}$$

20. The matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a

(1) Zero matrix (2) Identity matrix

(3) Scalar matrix (4) Diagonal matrix

Answer (3)

Sol. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

is scalar matrix

21. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(1) $\frac{\pi}{3}$ (2) $-\frac{\pi}{6}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2\pi}{3}$

Answer (4)

Sol. $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$$= \frac{2\pi}{3}$$

22. If $y = x^{(x \sin x)}$ then $\frac{dy}{dx} = ?$

(1) $x^{x \cos x}$

(2) $x^{(x \sin x)}[\sin x + \sin \log x]$

(3) $x^{(x \sin x)}[\sin(1 + \log x) + x \log x \cos x]$

(4) $x^{(x \cos x)}[\sin \log x) + x \log x \cos x]$

Answer (*)

Sol. $y = x^{x \sin x}$

$$\log y = x \sin x \log x$$

$$\frac{1}{y}y' = \sin x + (\log x)(\sin x + x \cos x)$$

$$y' = x^{x \sin x}(\sin x + (\sin x + x \cos x)(\log x))$$

$$y' = x^{x \sin x}((1 + \log x)\sin x + x \cos x \log x)$$

23. The angle between the lines

$\vec{r} = 3\hat{i} + 2\hat{j} + 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and

$\vec{r} = 5\hat{j} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ is

(1) $\sin^{-1}\left(\frac{19}{21}\right)$ (2) $\cos^{-1}\left(\frac{19}{23}\right)$

(3) $\cos^{-1}\left(\frac{19}{21}\right)$ (4) $\sin^{-1}\left(\frac{19}{23}\right)$

Answer (3)

Sol. $\cos\theta = \dfrac{(\hat{i}+2\hat{j}+2\hat{k})\cdot(3\hat{i}+2\hat{j}+6\hat{k})}{|\hat{i}+2\hat{j}+2\hat{k}||3\hat{i}+2\hat{j}+6\hat{k}|}$

$= \dfrac{19}{3.7}$

$\theta = \cos^{-1}\left(\dfrac{19}{21}\right)$

24. A doctor is to visit a patient. It is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he arrives late. The probability that he comes by bus is :

(1) $\frac{4}{9}$ (2) $\frac{1}{18}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Answer (1)

Sol. P(Comes by train) = $\frac{3}{10}$ P(late by train) = $\frac{1}{4}$

P(Comes by bus) = $\frac{1}{5}$ P(late by bus) = $\frac{1}{3}$

P(Comes by scooter) = $\frac{1}{10}$ P(late by scooter) = $\frac{1}{12}$

P(Comes by other means) = $\frac{2}{5}$ P(late by other) = 0

$$P\left(\text{by bus}/\text{late}\right) = \frac{\frac{1}{5}\times\frac{1}{3}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}$$

$$= \frac{\frac{1}{15}}{\frac{3}{40}+\frac{1}{15}+\frac{1}{120}}$$

$$= \frac{\frac{1}{15}}{\frac{9+8+1}{120}} = \frac{120}{15\times 18} = \frac{8}{18} = \frac{4}{9}$$

25. The number of all onto functions from the set $\{1, 2, \ldots n\}$ to itself is

(1) 2^n (2) n^2

(3) $n!$ (4) $(2n)!$

Answer (3)

Sol. $S : \{1, 2, \ldots.n\}$

Number of onto functions = $n!$

26. The unit vector in the direction of $\vec{a}+\vec{b}$ if $\vec{a} = 2\hat{i}-\hat{j}+2\hat{k}$ & $\vec{b} = -\hat{i}+\hat{j}+-\hat{k}$ is :

(1) $\hat{i}+0\hat{j}+\hat{k}$

(2) $\hat{i}-\hat{j}+\hat{k}$

(3) $\hat{i}+\hat{j}+\hat{k}$

(4) $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$

Answer (4)

Sol. $\dfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \dfrac{\hat{i}+\hat{k}}{\sqrt{2}} = \dfrac{\hat{i}}{\sqrt{2}}+\dfrac{\hat{k}}{\sqrt{2}}$

27. Points of discontinuity of the greatest integer function $f(x) = [x]$, where $[x]$ denotes integer less than or equal to x, are

(1) all natural numbers (2) all rational numbers

(3) all integers (4) all real numbers

Answer (3)

Sol. $[x]$ is discontinuous at all integers.

28. Let A be the square matrix of order 3, then $|kA|$, where k is a scalar, is equal to:

(1) $3k|A|$ (2) $k^3|A|$

(3) $k^2|A|$ (4) $k|A|$

Answer (2)

Sol. $|kA| = k^3\,|A|$

It is a standard property of determinants

29. If a fair coin is tossed 10 times the probability of atleast 6 heads is:

(1) $\frac{105}{512}$ (2) $\frac{53}{128}$

(3) $\frac{53}{64}$ (4) $\frac{193}{512}$

Answer (4)

Sol. P(atleast 6 heads) = $p(6H)$ + $p(7H)$ + $p(8H)$ + $p(9H)$ + $p(10H)$

$${}^{10}C_6\left(\frac{1}{2}\right)^{10}+{}^{10}C_7\left(\frac{1}{2}\right)^{10}+{}^{10}C_8\left(\frac{1}{2}\right)^{10}+{}^{10}C_9\left(\frac{1}{2}\right)^{10}+\left(\frac{1}{2}\right)^{10}$$

$$=\left(\frac{1}{2}\right)^{10}\left({}^{10}C_6+{}^{10}C_7+{}^{10}C_8+{}^{10}C_9+{}^{10}C_{10}\right)$$

$$=\left(\frac{1}{2}\right)^{10}\left(1+10+45+\frac{10\times9\times8}{6}+\frac{10\times9\times8\times7}{24}\right)$$

$$=\left(\frac{1}{2}\right)^{10}(1+10+45+120+210)$$

$$=\left(\frac{1}{2}\right)^{10}(386)$$

$$=\frac{193}{512}$$

30. The general solution of $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$ is :

(given that C is the constant of integration)

(1) $\tan^{-1}x=y+\frac{y^3}{3}+C$

(2) $\tan^{-1}y=x+\frac{x^3}{3}+C$

(3) $\tan^{-1}x=\tan^{-1}y+C$

(4) $\tan^{-1}x+\tan^{-1}y=C$

Answer (2)

Sol. $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$

$$\Rightarrow \frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$$

$$\Rightarrow \frac{dy}{1+y^2}=\left(1+x^2\right)dx$$

$$\Rightarrow \tan^{-1}y=x+\frac{x^3}{3}+C$$

31. The value of integral $\int\sqrt{4x^2+9}\,dx$ is

(1) $\frac{x}{2}\sqrt{4x^2+9}+\frac{9}{2}\log\left|2x+\sqrt{4x^2+9}\right|+C$

(2) $\frac{x}{2}\sqrt{4x^2+9}+\frac{3}{2}\log\left|2x+\sqrt{4x^2+9}\right|+C$

(3) $2x\sqrt{4x^2+9}+\frac{9}{2}\log\left|2x+\sqrt{4x^2+9}\right|+C$

(4) $x\sqrt{4x^2+9}+\frac{9}{4}\log\left|2x+\sqrt{4x^2+9}\right|+C$

Answer (Bonus*)

Sol. Using $\int\sqrt{(ax)^2+b^2}\,dx$

$$=\frac{\frac{ax}{2}\sqrt{(ax)^2+b^2}+\frac{b^2}{2}\log\left|ax+\sqrt{(ax)^2+b^2}\right|}{a}+C$$

$$\therefore\quad \int\sqrt{(2x)^2+(3)^2}\,dx$$

$$=\frac{\frac{2x}{2}\sqrt{4x^2+9}+\frac{9}{2}\log\left|2x+\sqrt{4x^2+9}\right|}{2}+C$$

$$=\frac{x}{2}\sqrt{4x^2+9}+\frac{9}{4}\log\left|2x+\sqrt{4x^2+9}\right|+C$$

32. Let $f(x) = x^3$ be a function with domain {0, 1, 2, 3} then domain of f^{-1} is :

(1) {3, 2, 1, 0}
(2) {0, –1, –2, –3}
(3) {0, 1, 8, 27}
(4) {0, –1, –8, –27}

Answer (3)

Sol. $f(x) = x^3$

$f(1) = 1$

$f(0) = 0$

$f(2) = 8$

$f(3) = 27$

$\therefore$ $f^{-1}(x)$ domain = $f(x)$ range = {0, 1, 8, 27}

$\therefore$ Domain of $f^{-1}(x)$ = {0, 1, 8, 27}

33. The area of the parallelogram determined by the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ is

(1) $8\sqrt{3}$
(2) $4\sqrt{3}$
(3) $16\sqrt{3}$
(4) $2\sqrt{3}$

Answer (1)

Sol. Area of a parallelogram whose adjacent edges are $\vec{a}$ and $\vec{b}$ is $|\vec{a}\times\vec{b}|$.

Given : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \Rightarrow |\vec{a}| = \sqrt{14}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k} \Rightarrow |\vec{b}| = \sqrt{14}$

$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$= 14.14 - (3 - 4 + 3)^2$

$= 196 - 4$

$= 192$

$|\vec{a}\times\vec{b}| = \sqrt{192}$

$8\sqrt{3}$

34. A manufacturing company makes two models M_1 and M_2 of a product. Each piece of M_1 requires 9 labour hours for fabricating and one labour hour for finishing. Each piece of M_2 require 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 800 on each piece of M_1 and Rs. 1200 on each piece of M_2.

The above Linear Programming Problem [LPP] is given by

(1) Maximize $Z = 800x + 1200y$

Subject to constraints,

$3x + 4y \le 60$

$x + 3y \le 30$

$x, y \ge 0$

(2) Maximize $Z = 800x + 1200y$

Subject to constraints,

$3x + 4y \ge 60$

$x + 3y \ge 30$

$x, y \ge 0$

(3) Minimize $Z = 800x + 1200y$

Subject to constraints,

$3x + 4y \le 60$

$x + 3y \ge 30$

$x, y \ge 0$

(4) Minimize $Z = 800x + 1200y$

Subject to constraints,

$3x + 4y \ge 60$

$x + 3y \le 30$

$x, y \ge 0$

Answer (1)

Sol.

	M_1 (x units)	M_2 (y units)	Available
Fabricating	9 labour hours	12 labour hours	180
Finishing	1 labour hours	3 labour hours	30
Unit profit	Rs. 800	Rs. 1200	

Maximize : $Z = 800x + 1200y$

Constraints : $9x + 12y \le 180$

$\Rightarrow 3x + 4y \le 60$ …(i)

$x + 3y \le 30$ …(ii)

$x, y \ge 0$ …(iii)

35. A manufacturing company makes two models M_1 and M_2 of a product. Each piece of M_1 requires 9 labour hours for fabricating and one labour hour for finishing. Each piece of M_2 require 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 800 on each piece of M_1 and Rs. 1200 on each piece of M_2.

The maximum profit will be at the point

(1) (0, 10) (2) (20, 0)

(3) (12, 6) (4) (0, 0)

Answer (3)

Sol.

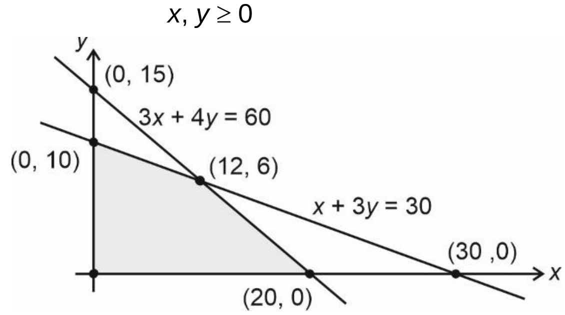
	M_1 (x units)	M_2 (y units)	Available
Fabricating	9 labour hours	12 labour hours	180
Finishing	1 labour hours	3 labour hours	30
Unit profit	Rs. 800	Rs. 1200	

Maximize : $Z = 800x + 1200y$

Constraints : $3x + 4y \le 60$

$x + 3y \le 30$

$x, y \ge 0$



Profit at (12, 6) is 16800

Profit at (20, 0) is 16000

$\therefore$ Maximum profit will be at (12, 6).