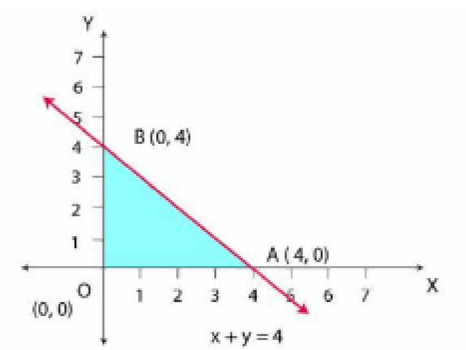
NCERT solutions for class 12 maths chapter 12 linear programming-Exercise: 12.1

Question:1 Solve the following Linear Programming Problems graphically:
Maximise $Z = 3x + 4y$ Subject to the constraints $x + y \leq 4, x \geq 0, y \geq 0.$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints, $x + y \leq 4, x \geq 0, y \geq 0.$ is as follows,



The region A0B represents the feasible region

The corner points of the feasible region are $B(4,0), C(0,0), D(0,4)$

Maximize $Z = 3x + 4y$

The value of these points at these corner points are :

Corner points	$Z = 3x + 4y$	
$B(4,0)$	12	

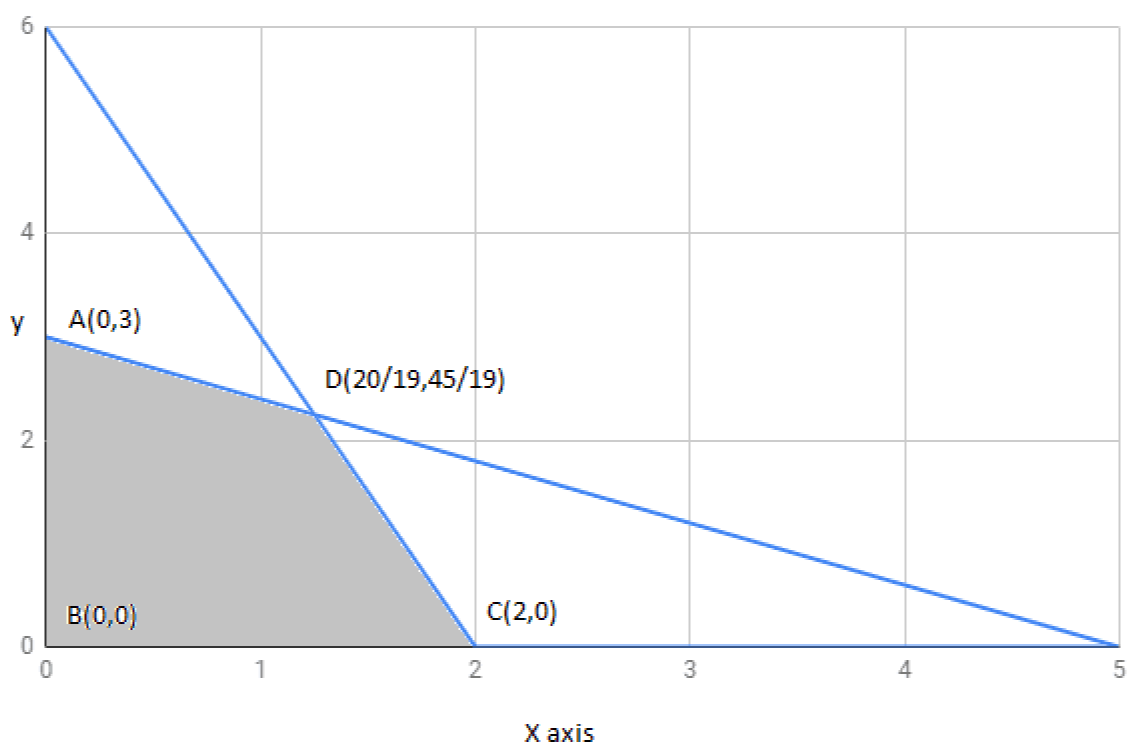
$C(0,0)$	0	
$D(0,4)$	16	maximum

The maximum value of Z is 16 at $D(0,4)$

Question:2 Solve the following Linear Programming Problems graphically: Minimise $z=-3x+4y$ Subject to . $x+2y\leq 8, 3x+2y\leq 12, x\geq 0, y\geq 0.$ Show that the minimum of Z occurs at more than two points

Answer:

The region determined by constraints, $x+2y\leq 8, 3x+2y\leq 12, x\geq 0, y\geq 0.$ is as follows,



The corner points of feasible region are $A(2,3), B(4,0), C(0,0), D(0,4)$

The value of these points at these corner points are :

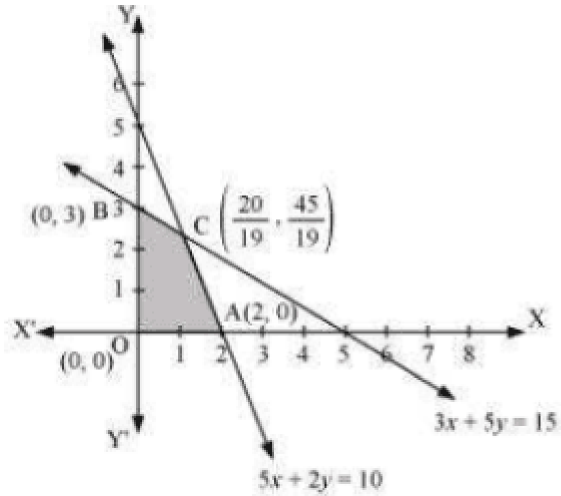
Corner points	$z=-3x+4y$	
$A(2,3)$	6	
$B(4,0)$	-12	Minimum
$C(0,0)$	0	
$D(0,4)$	16	

The minimum value of Z is -12 at $B(4,0)$

Question:3 Solve the following Linear Programming Problems graphically: Maximise $Z=5x+3y$ Subject to $3x+5y\leq 15$, $5x+2y\leq 10$, $x\geq 0, y\geq 0$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints, $3x+5y\leq 15$, $5x+2y\leq 10$, $x\geq 0, y\geq 0$ is as follows :



The corner points of feasible region are $A(0,3), B(0,0), C(2,0), D(\frac{20}{19}, \frac{45}{19})$

The value of these points at these corner points are :

Corner points	$Z = 5x + 3y$	
$A(0,3)$	9	
$B(0,0)$	0	
$C(2,0)$	10	
$D(\frac{20}{19}, \frac{45}{19})$	$\frac{235}{19}$	Maximum

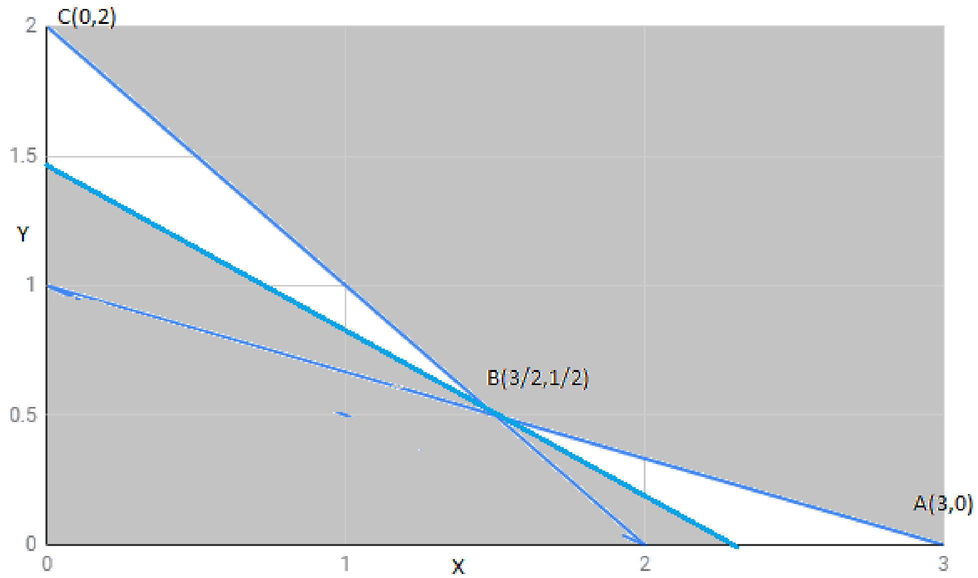
The maximum value of Z is $\frac{235}{19}$ at $D(\frac{20}{19}, \frac{45}{19})$

Question:4 Solve the following Linear Programming Problems graphically:

Minimise $Z = 3x + 5y$ Such that $x + 3y \geq 3, x + y \geq 2, x, y \geq 0.$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints $x + 3y \geq 3, x + y \geq 2, x, y \geq 0.$ is as follows,



The feasible region is unbounded as shown.

The corner points of the feasible region are $A(3,0), B(\frac{3}{2},\frac{1}{2}), C(0,2)$

The value of these points at these corner points are :

Corner points	$Z = 3x + 5y$	
$A(3,0)$	9	

$B(\frac{3}{2}, \frac{1}{2})$	7	Minimum
$C(0, 2)$	10	

The feasible region is unbounded, therefore 7 may or may not be the minimum value of Z .

For this, we draw $3x + 5y < 7$ and check whether resulting half plane has a point in common with the feasible region or not.

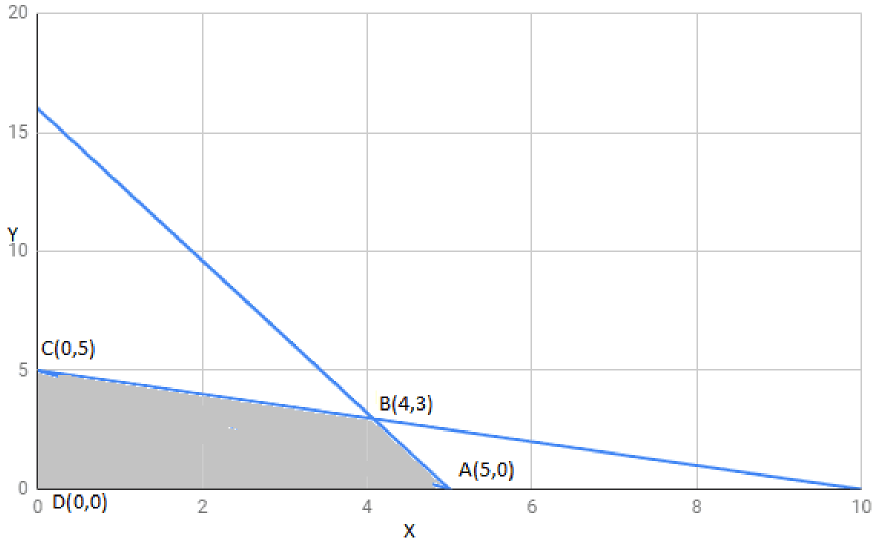
We can see a feasible region has no common point with. $Z = 3x + 5y$

Hence, Z has a minimum value of 7 at $B(\frac{3}{2}, \frac{1}{2})$

Question:5 Solve the following Linear Programming Problems graphically:
Maximise $Z = 3x + 2y$Subject to $x + 2y \leq 10, 3x + y \leq 15, x, y \geq 0$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints, $x + 2y \leq 10, 3x + y \leq 15, x, y \geq 0$ is as follows,



The corner points of feasible region are $A(5,0), B(4,3), C(0,5)$

The value of these points at these corner points are :

Corner points	$Z=3x+2y$	
$A(5,0)$	15	
$B(4,3)$	18	Maximum
$C(0,5)$	10	

The maximum value of Z is 18 at $B(4,3)$

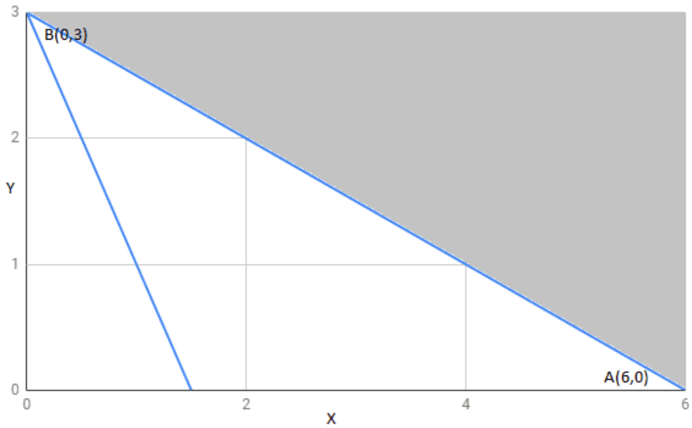
Question:6 Solve the following Linear Programming Problems graphically:

Minimise $Z = x + 2y$ Subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0.$

Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0.$ is as follows,



The corner points of the feasible region are $A(6,0), B(0,3)$

The value of these points at these corner points are :

Corner points	$Z = x + 2y$
$A(6,0)$	6
$B(0,3)$	6

Value of Z is the same at both points. $A(6,0), B(0,3)$

If we take any other point like $(2,2)$ on line $Z = x + 2y$, then Z=6.

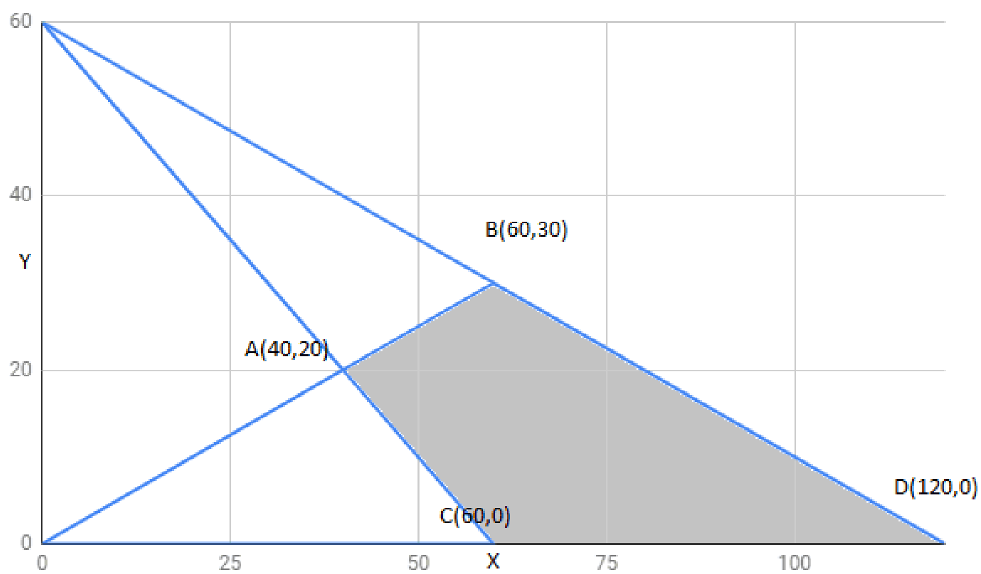
Thus the minimum value of Z occurs at more than 2 points .

Therefore, the value of Z is minimum at every point on the line $Z = x + 2y$.

Question:7 Solve the following Linear Programming Problems graphically: Minimise and Maximise $z = 5x + 10y$ Subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints, $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$ is as follows,



The corner points of feasible region are $A(40,20), B(60,30), C(60,0), D(120,0)$

The value of these points at these corner points are :

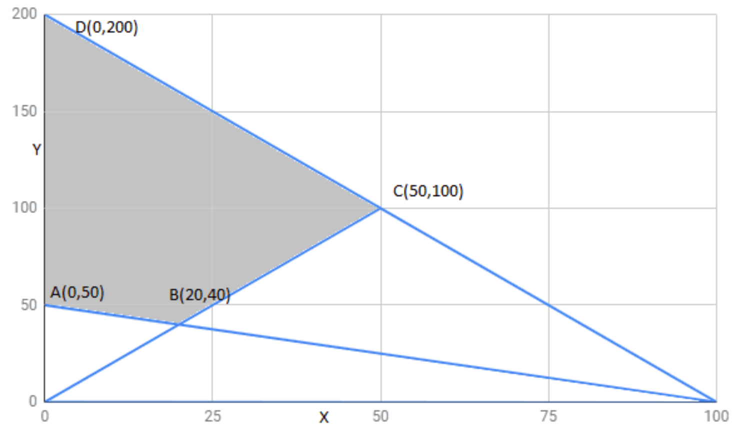
Corner points	$z = 5x + 10y$	
$A(40, 20)$	400	
$B(60, 30)$	600	Maximum
$C(60, 0)$	300	Minimum
$D(120, 0)$	600	maximum

The minimum value of Z is 300 at $C(60, 0)$ and maximum value is 600 at all points joing line segment $B(60, 30)$ and $D(120, 0)$

Question:8 Solve the following Linear Programming Problems graphically: Minimise and Maximise $z = x + 2y$ Subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y, \geq 0$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y, \geq 0$ is as follows,



The corner points of the feasible region are $A(0,50), B(20,40), C(50,100), D(0,200)$

The value of these points at these corner points are :

Corner points	$z = x + 2y$	
$A(0,50)$	100	Minimum
$B(20,40)$	100	Minimum
$C(50,100)$	250	
$D(0,200)$	400	Maximum

The minimum value of Z is 100 at all points on the line segment joining points $A(0,50)$ and $B(20,40)$.

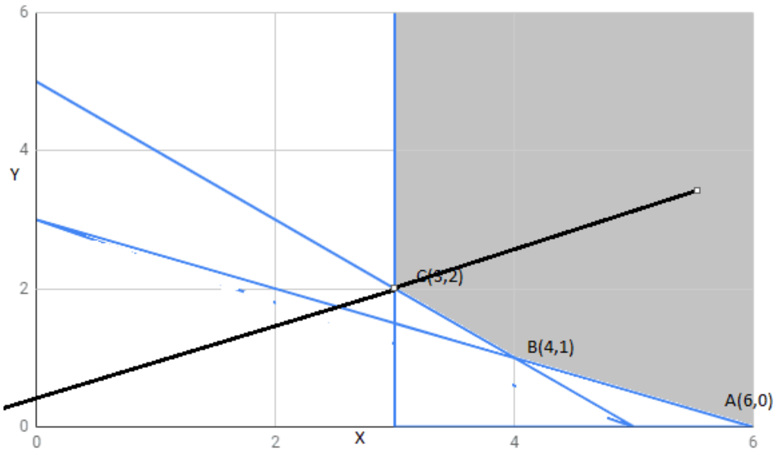
The maximum value of Z is 400 at $D(0,200)$.

Question:9 Solve the following Linear Programming Problems graphically:

Maximise $Z = -x + 2y$ Subject to the constraints: $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$ is as follows,



The corner points of the feasible region are $A(6, 0), B(4, 1), C(3, 2)$

The value of these points at these corner points are :

Corner points	$Z = -x + 2y$	
$A(6, 0)$	- 6	minimum
$B(4, 1)$	-2	

$C(3,2)$	1	maximum

The feasible region is unbounded, therefore 1 may or may not be the maximum value of Z.

For this, we draw $-x+2y>1$ and check whether resulting half plane has a point in common with a feasible region or not.

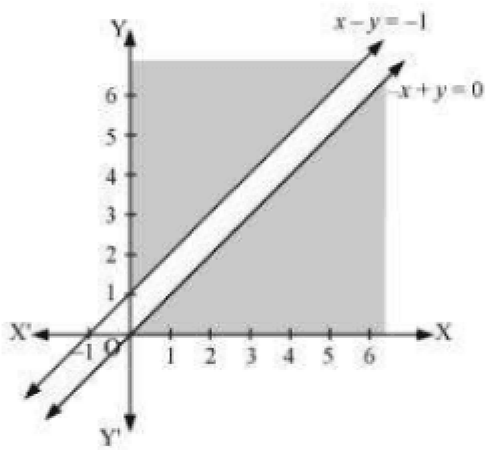
We can see the resulting feasible region has a common point with a feasible region.

Hence , Z =1 is not maximum value , Z has no maximum value.

Question:10 Solve the following Linear Programming Problems graphically: Maximise $Z = x + y$; Subject to $x - y \leq -1, -x + y \leq 0, x, y, \geq 0.$ Show that the minimum of Z occurs at more than two points.

Answer:

The region determined by constraints $x - y \leq -1, -x + y \leq 0, x, y, \geq 0.$ is as follows,



There is no feasible region and thus, Z has no maximum value.

NCERT solutions for class 12 maths chapter 12 linear programming-Exercise: 12.2

Question:1 Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs.80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Answer:

Let mixture contain x kg of food P and y kg of food Q. Thus, $x \geq 0, y \geq 0$.

The given information can be represented in the table as :

	Vitamin A	Vitamin B	Cost

Food P	3	5	60
Food Q	4	2	80
requirement	8	11	

The mixture must contain 8 units of Vitamin A and 11 units of Vitamin B.

Therefore, we have

$3x + 4y \geq 8$

$5x + 2y \geq 11$

Total cost is Z. $Z = 60x + 80y$

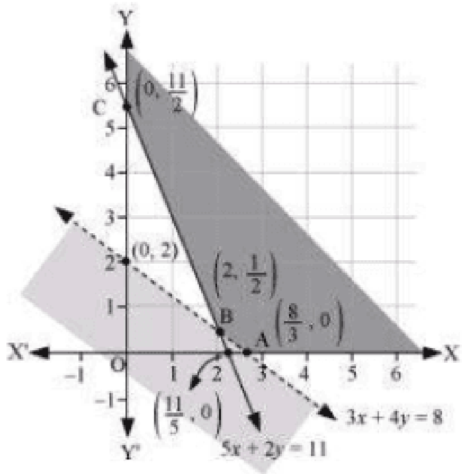
Subject to constraint,

$3x + 4y \geq 8$

$5x + 2y \geq 11$

$x \geq 0, y \geq 0$

The feasible region determined by constraints is as follows:



It can be seen that a feasible region is unbounded.

The corner points of the feasible region are $A(\frac{8}{3}, 0), B(2, \frac{1}{2}), C(0, \frac{11}{2})$

The value of Z at corner points is as shown :

corner points	$Z = 60x + 80y$	
$A(\frac{8}{3}, 0)$	160	MINIMUM
$B(2, \frac{1}{2})$	160	minimum
$C(0, \frac{11}{2})$	440	

Feasible region is unbounded, therefore 160 may or may not be the minimum value of Z.

For this, we draw $60x + 80y < 160 \ or \ \ 3x + 4y < 8$ and check whether resulting half plane has a point in common with the feasible region or not.

We can see a feasible region has no common point with. $3x + 4y < 8$

Hence, Z has a minimum value 160 at line segment joining points $A(\frac{8}{3}, 0)$ and $B(2, \frac{1}{2})$.

Question:2 One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

Answer:

Let there be x cakes of first kind and y cakes of the second kind.Thus, $x \geq 0, y \geq 0$.

The given information can be represented in the table as :

	Flour(g)	fat(g)
Cake of kind x	200	25
Cake of kind y	100	50
Availability	5000	1000

Therefore,

$200x + 100y \leq 5000$

$\Rightarrow \quad 2x + y \leq 50$

. $25x + 50y \leq 10000$

$\Rightarrow \quad x + 2y \leq 400$

The total number of cakes, Z. Z=X+Y

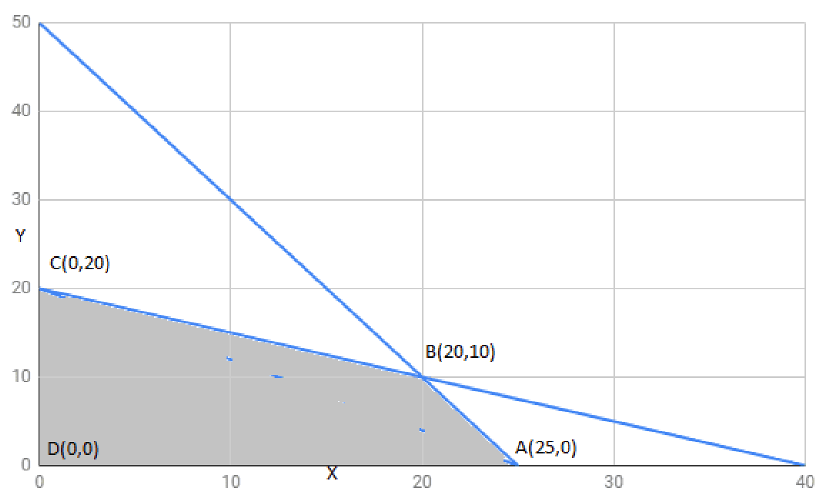
Subject to constraint,

$\Rightarrow \quad 2x + y \leq 50$

$\Rightarrow \quad x + 2y \leq 400$

$x \geq 0, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of the feasible region are $A(25,0), B(20,10), C(0,20), D(0,0)$

The value of Z at corner points is as shown :

corner points	Z=X+Y	
$A(25,0)$	25	
$B(20,10)$	30	maximum
$C(0,20)$ $D(0,0)$	20 0	 minimum

The maximum cake can be made 30 (20 of the first kind and 10 of the second kind).

Question:3 A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

Answer:

Let number of rackets be x and number of bats be y.

the machine time availability is not more than 42 hours.

i.e. $1.5x + 3y \leq 42$

craftsman's time availability is 24 hours

i.e. $3x + y \leq 24$

The factory has to work at full capacity.

Hence, $1.5x + 3y = 42..............1$

$3x + y = 24..............2$

Solving equation 1 and 2, we have

$x = 4 \; and \;\; y = 12$

Thus, 4 rackets and 12 bats are to be made .

Question:3 A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

Answer:

Let the number of rackets is x and the number of bats is y.

the machine time availability is not more than 42 hours.

craftsman's time availability is 24 hours

The given information can be repreented in table as shown :

	racket	bat	availability
machine time	1.5	3	42
craftman's time	3	1	24

$1.5x + 3y \leq 42$

$3x + y \leq 24$

$x, y \geq 0$

The profit on the bat is 10 and on the racket is 20.

$Z = 20x + 10y$

The mathematical formulation is :

maximise $Z = 20x + 10y$

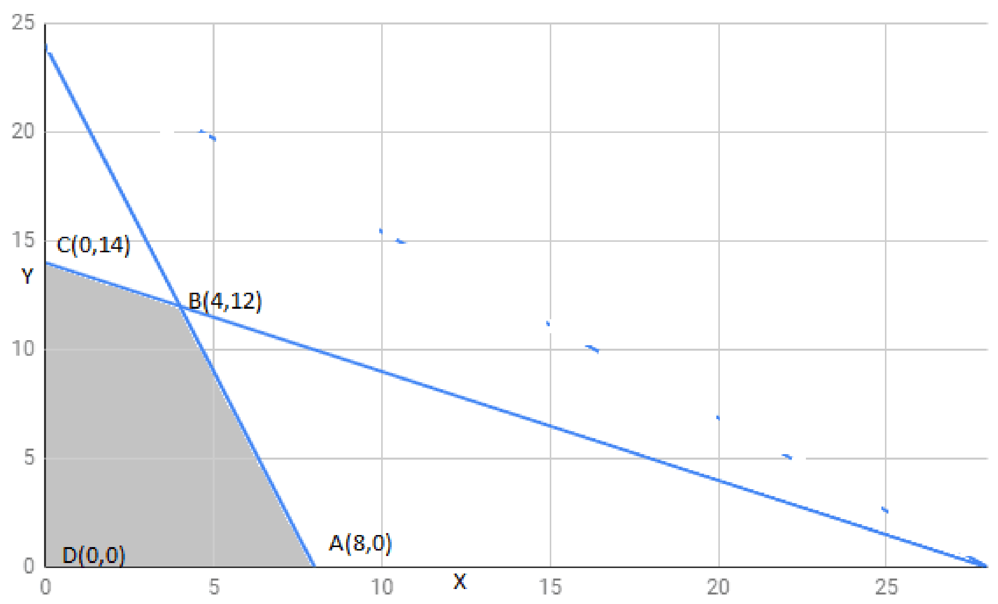
subject to constraints,

$1.5x + 3y \leq 42$

$3x + y \leq 24$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points are $A(8,0), B(4,12), C(0,14), D(0,0)$

The value of Z at corner points is as shown :

CORNER POINTS	$Z=20x+10y$	
$A(8,0)$	160	
$B(4,12)$	200	maximum
$C(0,14)$	140	
$D(0,0)$	0	

Thus, the maximum profit of the factory when it works at full capacity is 200.

Question:4 A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

Answer:

Let packages of nuts be x and packages of bolts be y .Thus, $x \geq 0, y \geq 0$.

The given information can be represented in table as :

	bolts	nuts	availability
machine A	1	3	12
machine B	3	1	12

Profit on a package of nuts is Rs. 17.5 and on package of bolt is 7.

Therefore, constraint are

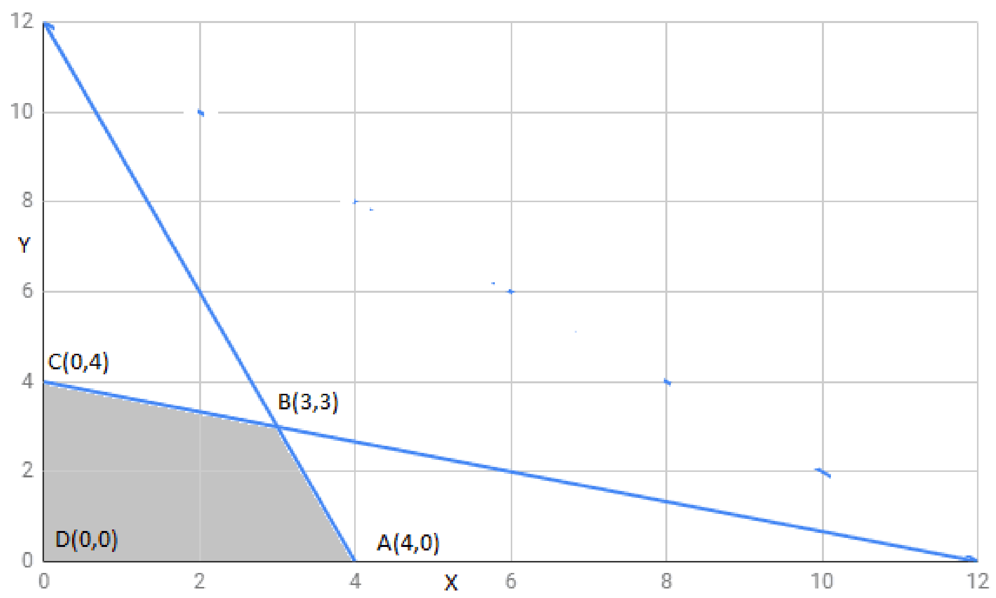
$x + 3y \leq 12$

$3x + y \leq 12$

$x \geq 0, y \geq 0$

$Z = 17.5x + 7y$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(4,0), B(3,3), C(0,4), D(0,0)$

The value of Z at corner points is as shown :

Corner points	$Z = 17.5x + 7y$	
$A(4,0)$	70	
$B(3,3)$	73.5	maximum

$C(0,4)$	28	
$D(0,0)$	0	

The maximum value of z is 73.5 at $B(3,3)$.

Thus, 3 packages of nuts and 3 packages of bolts should be manufactured everyday to get maximum profit.

Question:5 A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

Answer:

Let factory manufactures screws of type A and factory manufactures screws of type B. Thus, $x \geq 0, y \geq 0$.

The given information can be represented in the table as :

	screw A	screw B	availability

Automatic machine	4	6	$4 \times 60 = 240$
hand operated machine	6	3	$4 \times 60 = 240$

Profit on a package of screw A is Rs.7 and on the package of screw B is 10.

Therefore, the constraint is

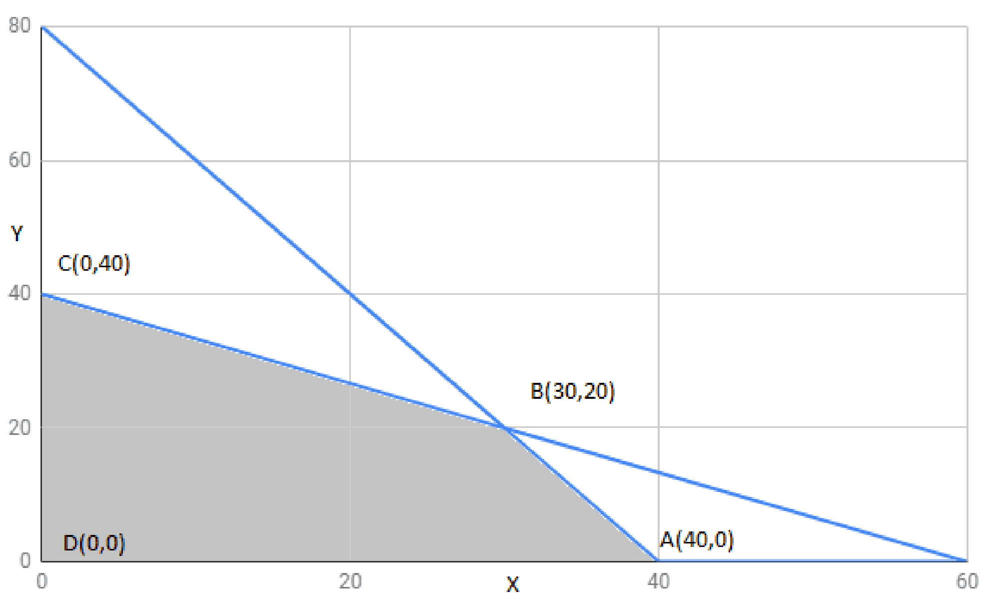
$4x + 6y \leq 240$

$6x + 3y \leq 240$

$x \geq 0, y \geq 0$

$Z = 7x + 10y$

The feasible region determined by constraints is as follows:



The corner points of the feasible region are $A(40,0), B(30,20), C(0,40), D(0,0)$

The value of Z at corner points is as shown :

Corner points	$Z=7x+10y$	
$A(40,0)$	280	
$B(30,20)$	410	maximum
$C(0,40)$	400	
$D(0,0)$	0	

The maximum value of z is 410 at $B(30,20)$.

Thus, 30 packages of screw A and 20 packages of screw B should be manufactured every day to get maximum profit.

Question:6 A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

Answer:

Let the cottage industry manufactures x pedestal lamps and y wooden shades.

Thus, $x \geq 0, y \geq 0$.

The given information can be represented in the table as :

	lamps	shades	availability
machine (h)	2	1	12
sprayer (h)	3	2	20

Profit on a lamp is Rs. 5 and on the shade is 3.

Therefore, constraint is

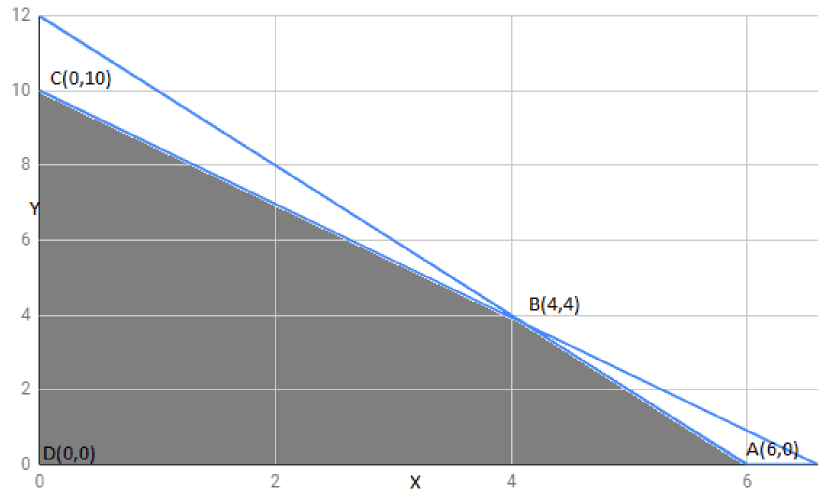
$2x + y \leq 12$

$3x + 2y \leq 20$

$x \geq 0, y \geq 0$

$Z = 5x + 3y$

The feasible region determined by constraints is as follows:



The corner points of the feasible region are $A(6,0), B(4,4), C(0,10), D(0,0)$

The value of Z at corner points is as shown :

Corner points	$Z=5x+3y$	
$A(6,0)$	30	
$B(4,4)$	32	maximum
$C(0,10)$	30	
$D(0,0)$	0	

The maximum value of z is 32 at $B(4,4)$.

Thus, 4 shades and 4 pedestals lamps should be manufactured every day to get the maximum profit.

Question:7 A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

Answer:

Let x be Souvenirs of type A and y be Souvenirs of type B .Thus, $x \geq 0, y \geq 0$.

The given information can be represented in table as :

	Type A	Type B	availability
cutting	5	8	$(3 \times 60) + 20 = 200$
asembling	10	8	$4 \times 60 = 240$

Profit on type A Souvenirs is Rs. 5 and on type B Souvenirs is 6.

Therefore, constraint are

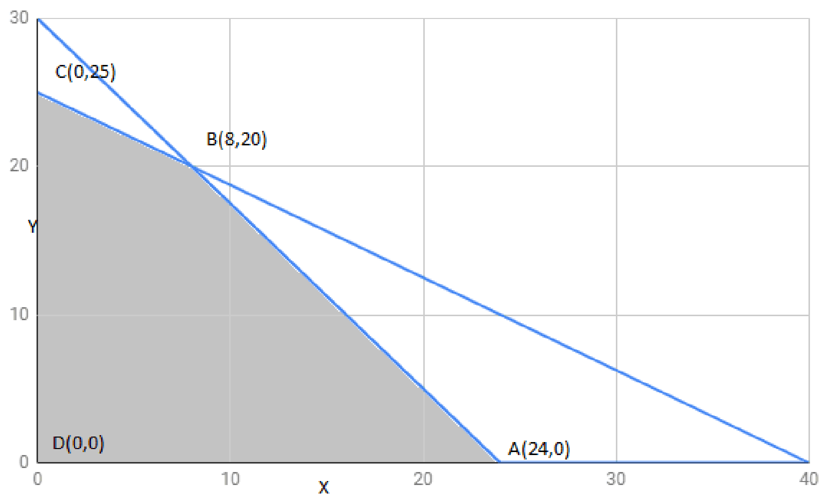
$5x + 8y \leq 200$

$10x + 8y \leq 240$

$x \geq 0, y \geq 0$

$Z = 5x + 6y$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(24, 0), B(8, 20), C(0, 25), D(0, 0)$

The value of Z at corner points is as shown :

Corner points	$Z = 5x + 6y$	
$A(24, 0)$	120	
$B(8, 20)$	160	maximum
$C(0, 25)$	150	
$D(0, 0)$	0	

The maximum value of z is 160 at $B(8,20)$.

Thus,8 Souvenirs of type A and 20 Souvenirs of type B should be manufactured everyday to get maximum profit.

Question:8 A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

Answer:

Let merchant plans has personal computers x desktop model and y portable model

.Thus, $x \geq 0, y \geq 0$.

The cost of desktop model is cost Rs 25000 and portable model is Rs 40000.

Merchant can invest Rs 70 lakhs maximum.

$25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

the total monthly demand of computers will not exceed 250 units.

$x + y \leq 250$

profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

Total profit = Z , $Z = 4500x + 5000y$

The mathematical formulation of given problem is :

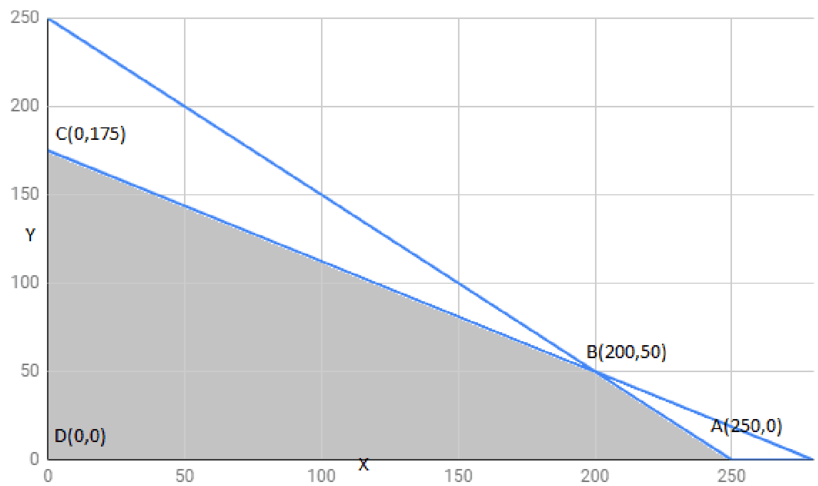
$5x + 8y \leq 1400$

$x + y \leq 250$

$x \geq 0, y \geq 0$

$Z = 4500x + 5000y$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(250, 0), B(200, 50), C(0, 175), D(0, 0)$

The value of Z at corner points is as shown :

Corner points	$Z = 4500x + 5000y$	
$A(250, 0)$	1125000	

$B(200,50)$	1150000	maximum
$C(0,175)$	875000	
$D(0,0)$	0	

The maximum value of z is 1150000 at $B(200,50)$.

Thus, merchant should stock 200 desktop models and 50 portable models to get maximum profit.

Question:9 A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F1 and F2 are available. Food F1 costs Rs 4 per unit food and F2 costs Rs 6 per unit. One unit of food F1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Answer:

Let diet contain x unit of food F1 and y unit of foof F2 .Thus, $x \geq 0, y \geq 0$.

The given information can be represented in table as :

	Vitamin	minerals	cost per unit
foof F1	3	4	4

food F2	6	3	6
	80	100	

Cost of food F1 is Rs 4 per unit and Cost of food F2 is Rs 6 per unit

Therefore, constraint are

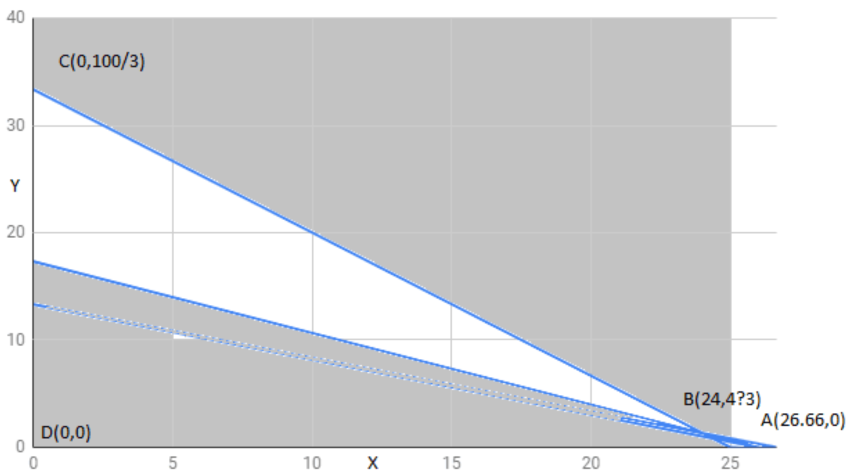
$3x + 4y \geq 4$

$6x + 3y \geq 6$

$x \geq 0, y \geq 0$

$Z = 4x + 6y$

The feasible region determined by constraints is as follows:



We can see feseable region is unbounded.

The corner points of feasible region are $A(\frac{80}{3},0), B(24,\frac{4}{3}), C(0,\frac{100}{3})$

The value of Z at corner points is as shown :

Corner points	$Z = 4x + 6y$	
$A(\frac{80}{3}, 0)$	106.67	
$B(24, \frac{4}{3})$,	104	minimum
$C(0, \frac{100}{3})$	200	maximum

Feasible region is unbounded , therefore 104 may or may not be minimum value of Z .

For this we draw $4x + 6y < 104$ or $2x + 3y < 52$ and check whether resulting half plane has point in common with feasible region or not.

We can see feasible region has no common point with $2x + 3y < 52$.

Hence , Z has minimum value 104.

Question:10 There are two types of fertilisers F1 and F2 . F1 consists of 10% nitrogen and 6% phosphoric acid and F2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F1 costs Rs 6/kg and F2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Answer:

Let farmer buy x kg of fertilizer F1 and y kg of F2 .Thus, $x \geq 0, y \geq 0$.

The given information can be represented in table as :

	Nitrogen	phosphoric acid	Cost
F1	10	6	6
F2	5	10	5
requirement	14	14	

F1 contain 10% nitrogen and F2 contain 5% nitrogen .Farmer requires atleast 14 kg of nitrogen

$$10\%x + 5\%y \geq 14$$

$$\frac{x}{10} + \frac{y}{20} \geq 14$$

$$2x + y \geq 280$$

F1 contain 6% phophoric acid and F2 contain 10% phosphoric acid .Farmer requires atleast 14 kg of nitrogen

$$6\%x + 10\%y \geq 14$$

$$\frac{6x}{100} + \frac{y}{20} \geq 14$$

$3x + 56y \geq 700$

Total cost is Z . $Z = 6x + 5y$

Subject to constraint ,

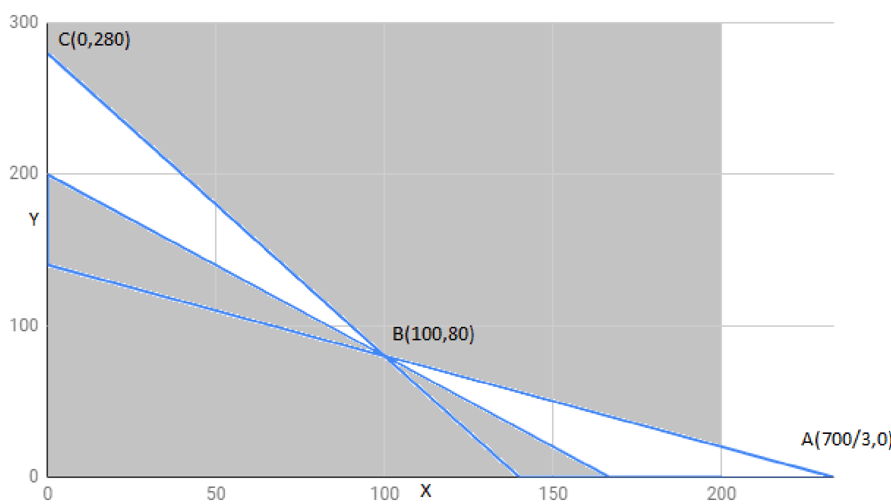
$2x + y \geq 280$

$3x + 56y \geq 700$

$x \geq 0, y \geq 0$

$Z = 6x + 5y$

The feasible region determined by constraints is as follows:



It can be seen that feasible region is unbounded.

The corner points of feasible region are $A(\frac{700}{3}, 0), B(100, 80), C(0, 280)$

The value of Z at corner points is as shown :

corner points	$Z = 6x + 5y$	
$A(\frac{700}{3}, 0)$	1400	
$, B(100, 80)$	1000	minimum
$C(0, 280)$	1400	

Feasible region is unbounded , therefore 1000 may or may not be minimum value of Z .

For this we draw $6x + 5y < 1000$ and check whether resulting half plane has point in common with feasible region or not.

We can see feasible region has no common point with $6x + 5y < 1000$.

Hence , Z has minimum value 1000 at point $, B(100, 80)$

Question:11 The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \leq 10, x + 3y \leq 15, x, y \geq 0$ are $(0, 0), (5, 0), (3, 4)$ and $(0, 5)$.

Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both $(3, 4)$ and $(0, 5)$ is

$(A) p = q$

$(B) p = 2q$

$(C) p = 3q$

$(D) q = 3p$

Answer:

The maximum value of Z is unique.

It is given that maximum value of Z occurs at two points $(3,4)\ and\ \ (0,5)$.

$\therefore$ Value of Z at $(3,4)$ =value of Z at $(0,5)$

$\Rightarrow \quad p(3)+q(4)=p(0)+q(5)$

$\Rightarrow \quad 3p+4q=5q$

$\Rightarrow \quad q=3p$

Hence, D is correct option.

NCERT solutions for class 12 maths chapter 12 linear programming-Miscellaneous Exercise

Question:1 **Reference of Example 9 (Diet problem):** A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol.

How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Answer:

Let diet contain x packets of food P and y packets of food Q. Thus, $x \geq 0, y \geq 0$.

The mathematical formulation of the given problem is as follows:

Total cost is Z . $Z = 6x + 3y$

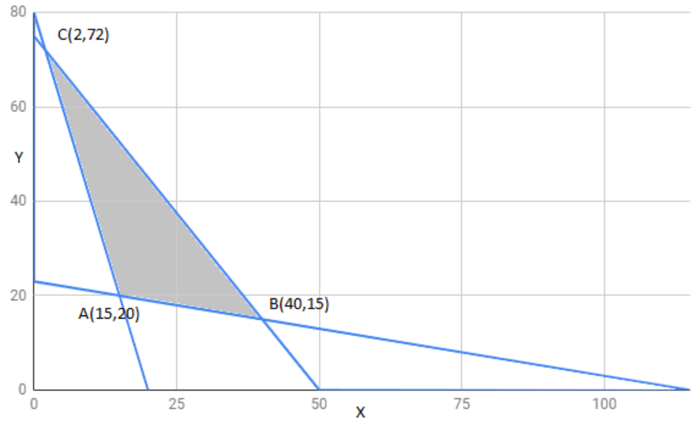
Subject to constraint,

$4x + y \geq 80$

$x + 5y \geq 115$

$x \geq 0, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(15,20), B(40,15), C(2,72)$

The value of Z at corner points is as shown :

corner points	$Z = 6x + 3y$	

$A(15, 20)$	150	MINIMUM
$B(40, 15)$	285	maximum
$C(2, 72)$	228	

Hence, Z has a maximum value of 285 at the point $B(40, 15)$.

to maximise the amount of vitamin A in the diet, 40 packets of food P and 15 packets of food Q should be used. The maximum amount of vitamin A is 285 units.

Question:2 A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Answer:

Let farmer mix x bags of brand P and y bags of brand Q. Thus, $x \geq 0, y \geq 0$.

The given information can be represented in the table as :

	Vitamin A	Vitamin B	Cost
Food P	3	5	60

Food Q	4	2	80
requirement	8	11	

The given problem can be formulated as follows:

Therefore, we have

$3x + 1.5y \geq 18$

$2.5x + 11.25y \geq 45$

$2x + 3y \geq 24$

$Z = 250x + 200y$

Subject to constraint,

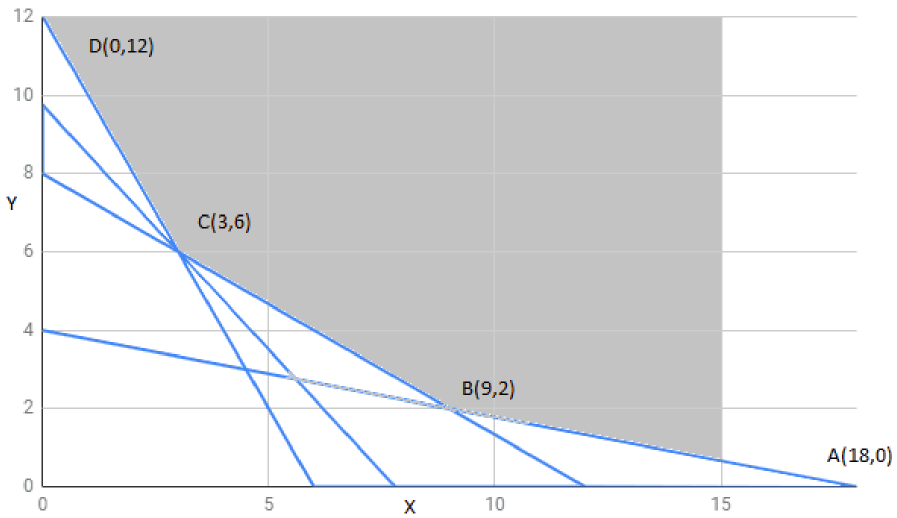
$3x + 1.5y \geq 18$

$2.5x + 11.25y \geq 45$

$2x + 3y \geq 24$

$x \geq 0, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of the feasible region are $A(18,0), B(9,2), C(3,6), D(0,12)$

The value of Z at corner points is as shown :

corner points	$Z=250x+200y$	
$A(18,0)$	4500	
$B(9,2)$	2650	
$C(3,6)$	1950	minimum
$D(0,12)$	2400	

Feasible region is unbounded, therefore 1950 may or may not be a minimum value of Z. For this, we draw $250x+200y<1950$ and check whether resulting half plane has a point in common with the feasible region or not.

We can see a feasible region has no common point with $250x+200y<1950$.

Hence, Z has a minimum value 1950 at point $C(3,6)$.

Question:3 A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

Food	**Vitamin A**	**Vitamin B**	**Vitamin C**
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

Answer:

Let mixture contain x kg of food X and y kg of food Y.

Mathematical formulation of given problem is as follows:

Minimize : $z = 16x + 20y$

Subject to constraint ,

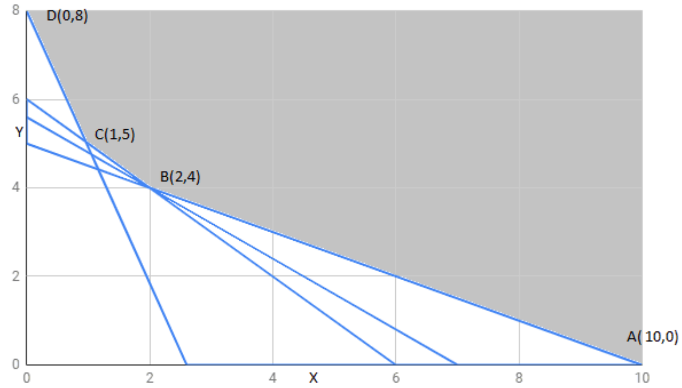
$x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(10,0), B(2,4), C(1,5), D(0,8)$

The value of Z at corner points is as shown :

corner points	$z = 16x + 20y$	
$A(10,0)$	160	
$B(2,4)$	112	minimum
$C(1,5)$	116	
$D(0,8)$	160	

The feasible region is unbounded , therefore 112 may or may not be minimum value of Z .

For this we draw $16x + 20y < 112$ and check whether resulting half plane has point in common with feasible region or not.

We can see feasible region has no common point with $16x + 20y < 112$.

Hence , Z has minimum value 112 at point $B(2, 4)$

Question:4 A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Types of toys	**Machines**		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Answer:

Let x and y toys of type A and type B.

Mathematical formulation of given problem is as follows:

Minimize : $z = 7.5x + 5y$

Subject to constraint ,

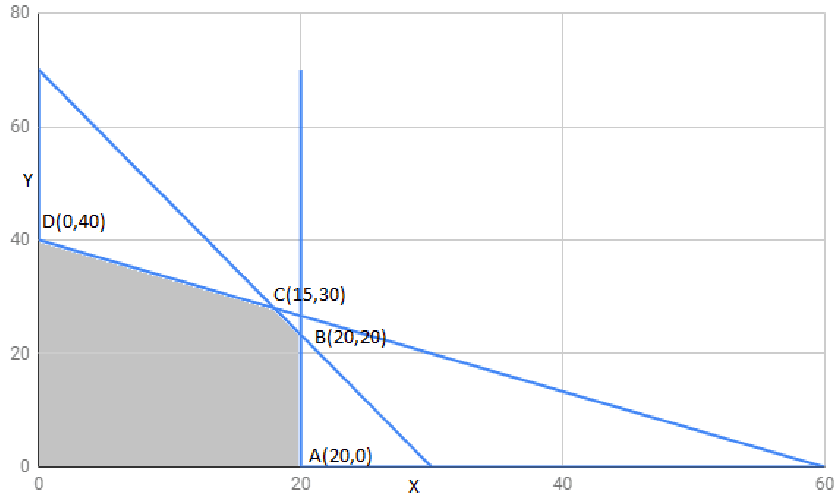
$2x + y \leq 60$

$x \leq 20$

$2x + 3y \leq 120$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(20,0), B(20,20), C(15,30), D(0,40)$

The value of Z at corner points is as shown :

corner points	$z = 7.5x + 5y$	
$A(20,0)$	150	
$B(20,20)$	250	
$C(15,30)$	262.5	maximum

$D(0,40)$	200	

Therefore 262.5 may or may not be maximum value of Z .

Hence , Z has maximum value 262.5 at point $C(15,30)$

Question:5 An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

Answer:

Let airline sell x tickets of executive class and y tickets of economy class.

Mathematical formulation of given problem is as follows:

Minimize : $z = 1000x + 600y$

Subject to constraint ,

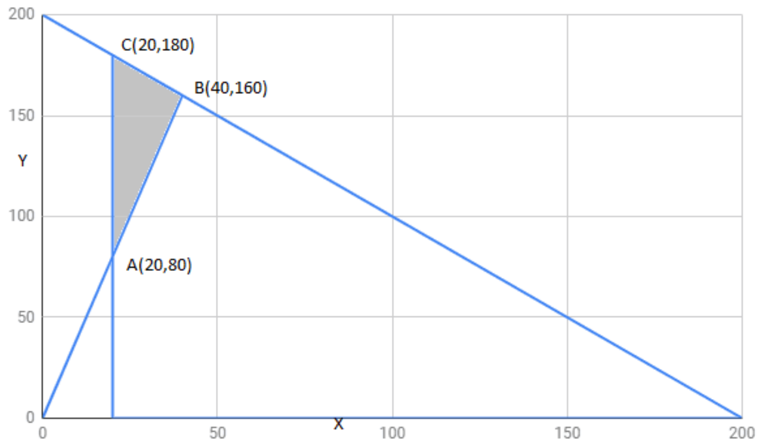
$x + y \leq 200$

$x \geq 20$

$y - 4x \geq 0$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(20, 80), B(40, 160), C(20, 180)$

The value of Z at corner points is as shown :

corner points	$z = 1000x + 600y$	
$A(20, 80)$	68000	
$B(40, 160)$	136000	maximum
$C(20, 180)$	128000	

therefore 136000 is maximum value of Z .

Hence , Z has maximum value 136000 at point $B(40, 160)$

Question:6 Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

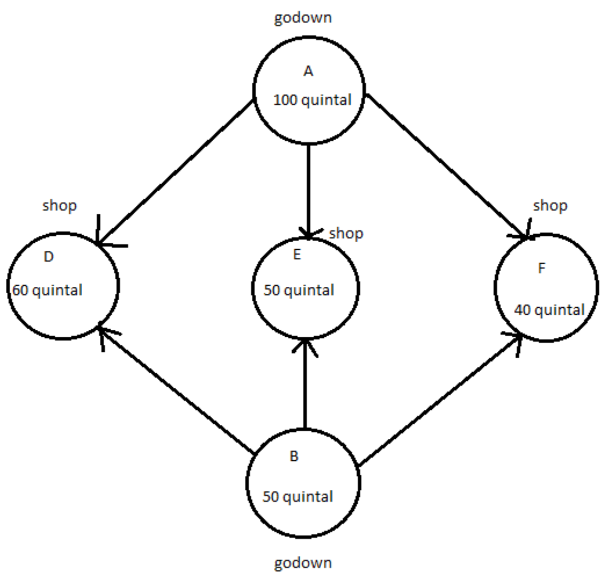
Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Answer:

Let godown A supply x and y quintals of grain to shops D and E respectively. Then , (100-x-y) will be supplied to shop F. Requirements at shop D is 60 since godown A supply x .Therefore remaining (60-x) quintals of grain will be transported from godown B.

Similarly, (50-y) quintals and 40-(100-x-y)=(x+y-60) will be transported from godown B to shop E and F respectively. The problem can be represented diagrammatically as follows:



$x, y \geq 0$ and $100 - x - y \geq 0$

$x, y \geq 0$ and $x + y \leq 100$

$60 - x \geq 0, 50 - y \geq 0 \ \ and\ x + y - 60 \geq 0$

$\Rightarrow \quad x \leq 60, y \leq 50, x + y \geq 60$

Total transportation cost z is given by ,

$z = 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$

$z = 2.5x + 1.5y + 410$

Mathematical formulation of given problem is as follows:

Minimize : $z = 2.5x + 1.5y + 410$

Subject to constraint ,

$x + y \leq 100$

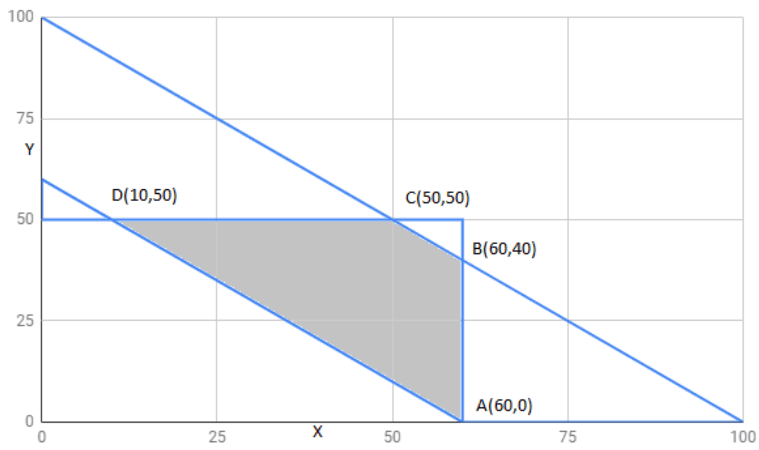
$x \leq 60$

$y \leq 50$

$x + y \geq 60$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(60,0), B(60,40), C(50,50), D(10,50)$

The value of Z at corner points is as shown :

corner points	$z = 2.5x + 1.5y + 410$	
$A(60,0)$	560	
$B(60,40)$	620	
$C(50,50)$	610	

$D(10,50)$	510	minimum

therefore 510 may or may not be minimum value of Z .

Hence , Z has miniimum value 510 at point $D(10,50)$

Question:7 An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km.)		
From/To	**A**	**B**
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

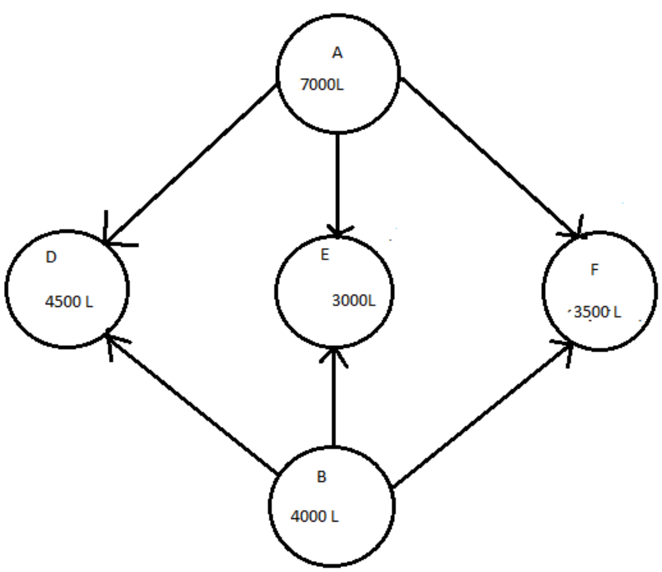
Answer:

Let x and y litres of oil be supplied from A to petrol pump,D and E. Then , (7000-x-y) will be supplied from A to petrol pump F.

Requirements at petrol pump D is 4500 L. since x L A are transported from depot A,remaining 4500-x L will be transported from petrol pump B

Similarly, (3000-y)L and 3500-(7000-x-y)=(x+y-3500) L will be transported from depot B to petrol E and F respectively.

The problem can be represented diagrammatically as follows:



$x, y \geq 0$ and $7000 - x - y \geq 0$

$x, y \geq 0$ and $x + y \leq 7000$

$$4500 - x \geq 0, 3000 - y \geq 0 \;\; and \; x + y - 3500 \geq 0$$

$$\Rightarrow \quad x \leq 4500, y \leq 3000, x + y \geq 3500$$

Cost of transporting 10 L petrol =Re 1

Cost of transporting 1 L petrol $= \frac{1}{10}$

Total transportation cost z is given by ,

$$z = \frac{7}{10}x + \frac{6}{10}y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$

$$z = 0.3x + 0.1y + 3950$$

Mathematical formulation of given problem is as follows:

Minimize : $z = 0.3x + 0.1y + 3950$

Subject to constraint ,

$$x + y \leq 7000$$

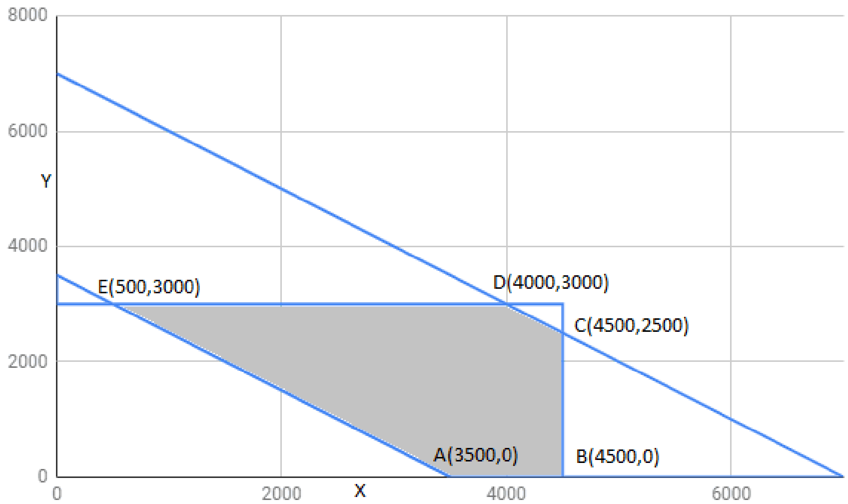
$$x \leq 4500$$

$$y \leq 3000$$

$$x + y \geq 3500$$

$$x, y \geq 0$$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(3500,0), B(4500,0), C(4500,2500), D(4000,3000), E(500,3000)$

The value of Z at corner points is as shown :

corner points	$z = 0.3x + 0.1y + 3950$	
$A(3500,0)$	5000	
$B(4500,0)$	5300	
$C(4500,2500)$	5550	
$E(500,3000)$	4400	minimum
$D(4000,3000)$	5450	

Hence , Z has miniimum value 4400 at point $E(500,3000)$

Question:8 A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

Kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric Acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Answer:

Let fruit grower use x bags of brand P and y bags of brand Q.

Mathematical formulation of given problem is as follows:

Minimize : $z = 3x + 3.5y$

Subject to constraint ,

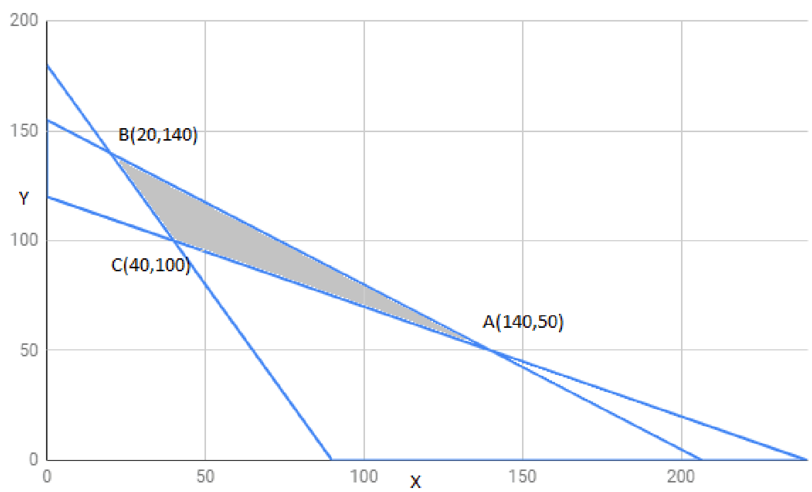
$x + 2y \geq 240$

$x + 0.5y \geq 90$

$1.5x + 2y \geq 310$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(140, 50), C(40, 100), B(20, 140)$

corner points	$z = 3x + 3.5y$	
$A(140, 50)$	595	
$B(20, 140)$	550	

$C(40, 100)$	470	minimum

The value of Z at corner points is as shown :

Therefore 470 is minimum value of Z .

Hence , Z has minimum value 470 at point $C(40, 100)$

Question:9 **Reference of Que 8 :** A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

Kg per bag		
	Brand A	**Brand P**
Nitrogen	3	3.5
Phosphoric Acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Answer:

Let fruit grower use x bags of brand P and y bags of brand Q.

Mathematical formulation of given problem is as follows:

Maximize : $z = 3x + 3.5y$

Subject to constraint ,

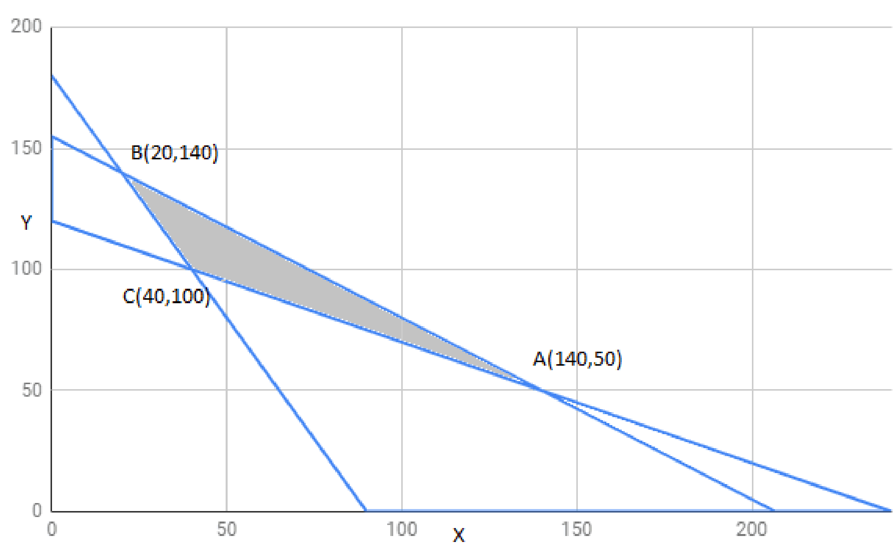
$x + 2y \geq 240$

$x + 0.5y \geq 90$

$1.5x + 2y \geq 310$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $B(20,140), A(140,50), C(40,100)$

The value of Z at corner points is as shown :

corner points	$z=3x+3.5y$	
$A(140,50)$	595	maximum
$B(20,140)$	550	
$C(40,100)$	470	minimum

therefore 595 is maximum value of Z .

Hence , Z has minimum value 595 at point $A(140,50)$

Question:10 A toy company manufactures two types of dolls, A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

Answer:

Let x and y be number of dolls of type A abd B respectively that are produced per week.

Mathematical formulation of given problem is as follows:

Maximize : $z = 12x + 16y$

Subject to constraint ,

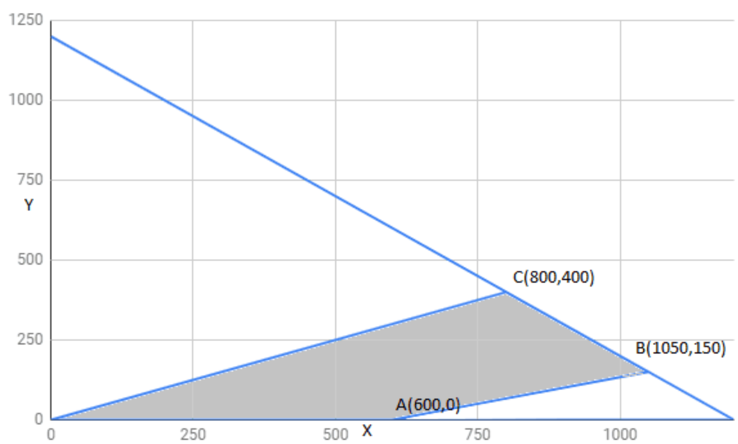
$x + y \leq 1200$

$y \leq \dfrac{x}{2} \Rightarrow x \geq 2y$

$x - 3y \leq 600$

$x, y \geq 0$

The feasible region determined by constraints is as follows:



The corner points of feasible region are $A(600, 0), B(1050, 150), C(800, 400)$

The value of Z at corner points is as shown :

corner points	$z = 12x + 16y$	

$A(600, 0)$	7200	
$B(1050, 150)$	15000	
$C(800, 400)$	16000	Maximum

Therefore 16000 is maximum value of Z .

Hence , Z has minimum value 16000 at point $C(800, 400)$