Date: 31/01/2026

Medical | IIT-JEE | Foundations

Corporate Office : AESL, 3rd Floor, Incuspaze Campus-2, Plot-13, Sector-18,
Udyog Vihar, Gurugram, Haryana-122018

Answers & Solutions

Time : 3 hrs. | Max. Marks : 100

for

Indian National Astronomy Olympiad
(INAO) 2026

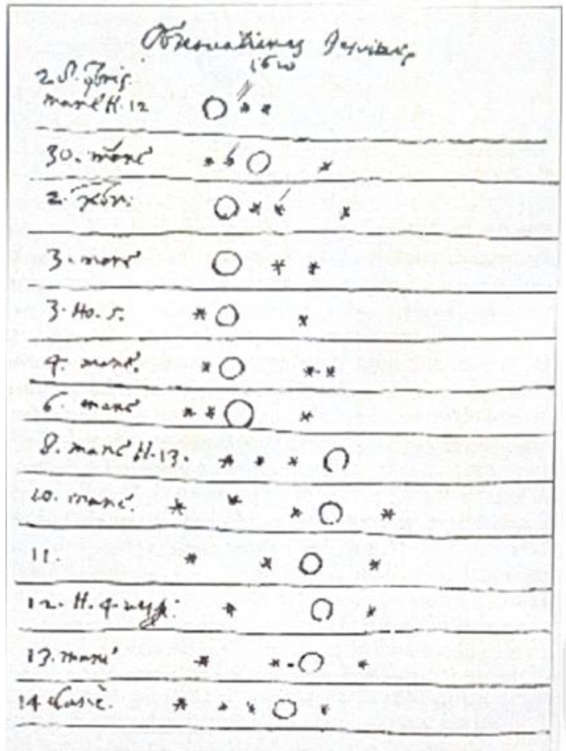
(For Class XI & XII Students)

(HBCSE-TIFR) & (IISER-Mohali)

INSTRUCTIONS TO CANDIDATES

- Before starting, please ensure that you have received a copy of the question paper containing total 8 pages (4 sheets).
- Please write your roll number in the space provided above.
- There are total 6 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- The answersheet must be returned to the invigilator. You can take this question paper back with you.

1. One of Galileo's discoveries was the satellites of Jupiter and in his notebook he drew sketches showing the position of the satellites every night. In this picture of his original hand-written notes, the big circle denotes Jupiter and small stars represent the positions of four satellites. The writing on the left is the date.



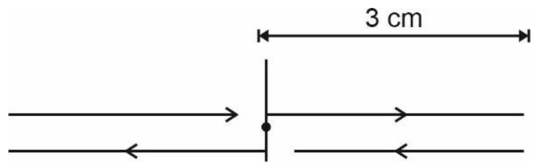
(a) (10 marks) The following table gives recent measurements of the position of one of the satellites, Europa, with respect to Jupiter at various times. Here, *x* represents the magnitude of distance between Jupiter and Europa, as measured in a similar image/sketch. Obtain the time period (*T*) of Europa through a suitable linear plot. (You may assume that the maximum magnitude of *x* is 3 cm).

***T* {in hour)**	***x* (in cm)**
0	not seen
1	0.49
2	0.65
3	0.84
4	1.06
5	1.26
6	1.45
7	1.65
8	1.82

***T* {in hour)**	***x* (in cm)**
9	2.02
10	2.18
11	2.32
12	2.45
13	2.56
14	2.67
15	2.78
16	2.84
17	2.91

(b) (2 marks) Daw a rough sketch of the same plot as in part (a), for a full period Europa.

Sol. (a) Europa will almost be in circular motion about Jupiter. So its projection (linear) will be in SHM.



We can see in table that, starting from $t = 0$ at $x = 0$ it reaches $x = 2.91$ at $t = 17$

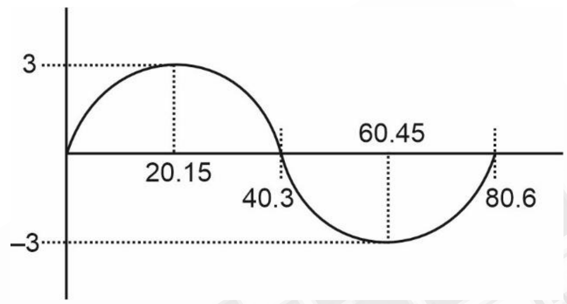
So lets say eqn. is $x = A\sin(\omega t)$

$$\Rightarrow \quad \sin^{-1}\left(\frac{2.91}{3}\right) = \omega(17)$$

$$\Rightarrow \quad \omega = 0.0779 \text{ rad/hr.}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 80.6 \text{ Hours}$$

(b) This distance with respect to time will be a sinusoidal curve.



2. (a) (6 marks) An observer is located in the city of Nashik (latitude ≈ 20°N and longitude ≈ 73°E). She observes the rising of the Sun on different days of the year.

The figure in the answersheet depicts the eastern horizon (approximated as a straight line) for the city of Nashik with East cardinal point marked as E. The azimuth range is given to be 60° to 120° with markings at every 10°. In the table below, you are given certain dates alongside alphabets. Mark the approximate rising points of the Sun as seen by the observer for these dates on the image of the horizon given in the answersheet and label them with the corresponding alphabets. Precise calculations are not expected.

Note: For definition of Azimuth refer Appendix.

Letter	Date
A	01 Jan 2025
B	01 Apr 2025
C	15 May 2025

Letter	Date
D	01 Jun 2025
E	01 Jul 2025
F	01 Oct 2025

(b) (4 marks) Now consider an observer who is located in a city which lies on the equator of Earth. The figure given in the answersheet is of the horizon (approximated as a straight line) as seen from the equator in the azimuth range 180° to 360°. The cardinal west point is marked with letter W. The azimuths are marked at a separation of 30°.

Mark the approximate setting points of the following stars (by writing their corresponding Sr. No.), if applicable, on the figure of horizon given in your answersheet.

Sr. No.	Star Common Name	Bayer Name
1	Pollux	α Gem
2	Polaris	α UMi
3	Canopus	α Car
4	Vega	α Lyr
5	Revati	ζ Psc
6	Rigel	β Ori
7	Dubhe	α UMa
8	Kaus Borealis	λ Sgr

Sol. (a) Refer the appendix for Azimuth reference.

Sun rise due east (A_z = 90°) on equinoxes for Northern hemisphere.

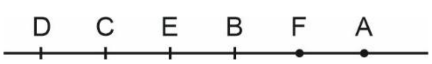
Summer : Sun rises North of East (A_z < 90°)

Winter : Sun rises North of East (A_z > 90°)

Maximum shift is limited by sun's declination ±23.5°

Letter (relative placement)	Date	Season	Rising position
A (110°)	01 Jan	Winter	South of East
B (95-100)	01 Apr	After equinox	North of East
C (85-88)	15 may	Late spring	Further North of East
D (75-80)	01 Jun	Near summer solistic	Farthest North of East
E (70-75)	01 Jul	After solistic	Slightly less North
F (78-82)	01 Oct	After summer equinox	Slightly South of East

So DCEBFA



(b) Setting point of stars as seen from equator.

$\Rightarrow$ A star : sets if (δ) < 90°

Does not set if |δ| = 90°

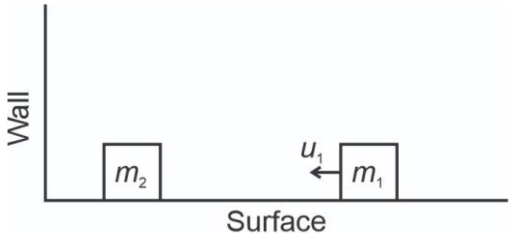
$\Rightarrow$ Setting Azimuth depends on declination

$\delta > 0 \rightarrow$ Sets North of West (Az > 270°)

$\delta < 0 \rightarrow$ Sets South of West (Az < 270°)

$\delta = 0 \rightarrow$ Sets exactly West (Az = 270°)

3. Two blocks, m_1 and m_2, are placed on a frictionless horizontal surface next to a fixed rigid wall. Block m_2 is at rest close to the wall and block m_1 moves towards it with velocity $\upsilon_1 = -1\ ms^{-1}$. All collisions (between the blocks and with the wall) are perfectly elastic.



(a) (5 marks) In the first case, we consider two identical blocks, each of mass 1 kg. Calculate total number of collisions (block-block or block-wall), n_1, that will occur in this system.

(b) (2 marks) Now, we will attempt to describe this motion in velocity space by drawing an appropriate figure but for a more general problem.

In the coordinate grid given in the answersheet, we redefine the two axes as, $\alpha = v_1\sqrt{m_1}$ and $\beta = v_2\sqrt{m_2}$.

Plot the values of α and β that depict the velocities between successive collisions during each phase and connect them with straight line arrows depicting the transition at the instance of each collision.

(c) (3 marks) On this phase diagram, we can plot several constant energy contours. Draw the constant energy contour, on the same grid given in part (b), that passes through the phase points that you have plotted.

(d) (6 marks) Consider a general case with arbitrary values of m_1 and m_2. In the phase diagram for this general case, let points a, b, c be the first three phase points. Let ∠abc = θ. Find expression for θ in terms of m_1 and m_2.

(e) (3 marks) Using this information from part (d) draw a complete phase diagram for $m_1 = 4m_2$.

(f) (3 marks) You may have realised, in a general case, i.e. for an arbitrary ratio m_1/m_2, you will be able to define a region of this curve in which the last phase point must lie. If *p* and *q* are the end points of that region on the curve and *o* is the origin, find equations of lines *op* and *oq*.

Draw these lines for $m_1/m_2 = 4$ case in the diagram of part (e).

Hint: In one of the two equations, you will be using the ratio $\frac{m_2}{m_1}$.

(g) (5 marks) Write an inequality showing bounds on n, where n is the total number of collisions, in terms of m_1 and m_2.

Hint: Find an inequality between θ and n.

(h) (3 marks) Lastly, use the information above to estimate the total number of collisions, n_{total}, for the case $m_1 = 10^{10}\ m_2$.

Sol. (a) For identical point masses getting into elastic collision of 1-dimension we know their velocities get exchanged

1st collision → Between blocks $v_1 \to 0$ $v_2 = -u_1$

2nd collision → Between blocks and wall $v_1' \to 0$ $v_2' \to +u_1$

3rd collision → Between blocks $v_1 \to +u_1$ $v_2 \to 0$

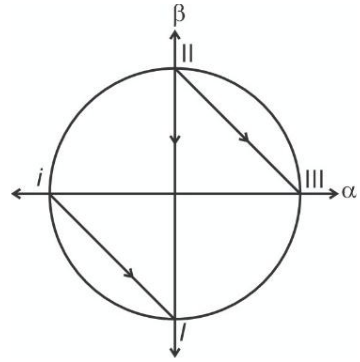
So, no collision after that so total no. of collision as = 3

(b) Before collision $\alpha = -u_1\sqrt{m_1}$ β = 0

1st Collision α = 0 $\beta = -u_1\sqrt{m_1}$ (assuming $m_1 = m_2$)

2nd Collision α = 0 $\beta = +u_1\sqrt{m_1}$

3rd Collision (phase) $\alpha = +u_1\sqrt{m_1}$ β = 0



(c) $\frac{\alpha^2}{2}+\frac{\beta^2}{2}=\text{T.E.}$

$\Rightarrow$ $\alpha^2 + \beta^2 = 2(\text{T.E.})$

$\therefore$ a circle centered at origin with radius $= \sqrt{2\left(\frac{1}{2}m_1 u_1^2\right)}$

(d) for $m_1 \neq m_2$ initially $\alpha = -u_1\sqrt{m_1}$ & $\beta = 0$

1st Collision

$-m_1u_1 = m_2v_2 + m_1v_1$, $v_2 - v_1 = -u_1$ (assuming e = 1)

Since energy is conserved the new α & β would still lie on same energy contour as described in part (C)

$v_2 = \frac{-2m_1 u}{m_1+m_2}$ $v_1 = \frac{-(m_1-m_2)u_1}{m_1+m_2}$

2nd Collision

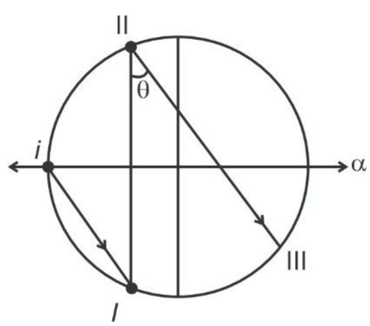
$v_2' = \frac{+2m_1u_1}{m_1+m_2}$ $v_1' = -\frac{(m_1-m_2)u_1}{m_1+m_2}$

3rd Collision

$^-v_1 = \frac{4m_1m_2-(m_1-m_2)^2}{(m_1+m_2)^2}u$ $^-v_2 = \frac{-4m_1(m_1-m_2)}{(m_1+m_2)^2}u$

$\alpha_3^2 = m_1{}^-v_1^2$ $\beta_3^2 = m_2{}^-v_2^2$

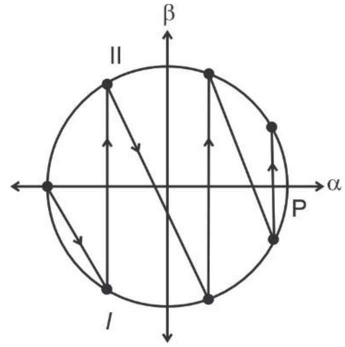
$\alpha_3 = \sqrt{m_1}{}^-v_1$ $\beta_3 = \sqrt{m_2}{}^-v_2$



$\tan\theta = \frac{\alpha_3-\alpha_2}{\beta_2-\beta_3} = +\sqrt{\frac{m_2}{m_1}}$ (after simplification)

Even every

(e) for $m_1 = 4m_2$ $\tan\theta = \frac{1}{2}$



(f) for no further collision $V_2 \geq 0$ $V_1 \geq 0$ and $V_1 > V_2$ or $\alpha \geq 0, \beta \geq 0$ and $\frac{\alpha}{\sqrt{m_1}} \geq \frac{\beta}{\sqrt{m_2}}$ so, equation of line OP is

simply $\beta = 0$

& equation of line 0q $\Rightarrow$ $\beta = \sqrt{\frac{m_2}{m_1}}\alpha$

(g) No. of collisions between block-block rotates direction vector by 2θ & starting from $\theta_0 = \pi$ collision continue untill $\phi < \frac{\pi}{2}$ where ϕ is angle of velocity vector with α-axis & $\tan\theta = \sqrt{\frac{m_2}{m_1}}$

Thus total number of block-block collision = $\frac{\pi}{2\theta}$ and almost same number of wall to wall collision such that we may conclude for N collision

$N < \frac{\pi}{0} \leq N + 1$

(h) for $m_1 = m_2 \times 10^{10}$

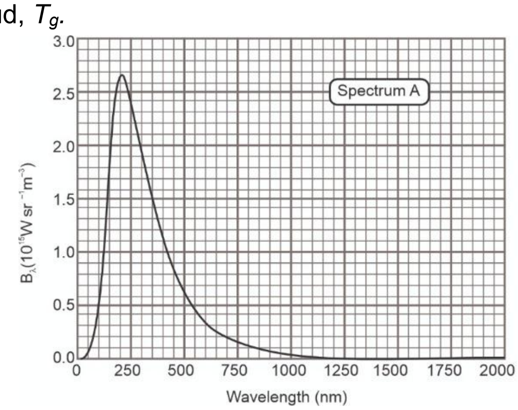
$\tan\theta = \frac{1}{10^5}$ $q \approx 10^{-5}$

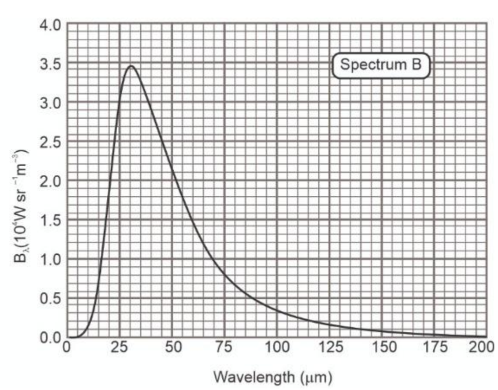
$N \leq \frac{\pi}{10^{-5}} = 314159.27$

N = 314159

4. Blackbody Radiation

(a) (4 marks) Consider a star of radius $R_s = 3R_\odot$ and a gas cloud of radius R_g at a distance d from the star. Assume that the gas cloud doesn't contain any source of radiation and there are no other stars in the vicinity of this system. Both the star and the gas cloud can be assumed to be blackbodies. You are given two spectra (A and B) below. Determine which spectra corresponds to the star and which corresponds to the gas cloud. Justify your answer. Also, calculate the effective temperature for the star, T_s, and for the gas cloud, T_g.





(b) (4 marks) Calculate the distance, d, of the gas cloud from the star.

Sol. (a) As per the graph given, for star, maximum energy radiated corresponds to wavelength 200nm.

$\therefore \quad \lambda T = b$ (Wien's Law)

$\Rightarrow \quad T_s = \dfrac{b}{\lambda_s}$

$\Rightarrow \quad T_s = \dfrac{2.897\times10^{-3}}{200\times10^{-9}}$

$\Rightarrow \quad T_s = 14485$ K

Similarly,

$T_g = \dfrac{2.897\times10^{-3}}{30\times10^{-6}}$

$\Rightarrow \quad T_g = 96.57$ K

(b) In this part we have to calculate d, so power absorbed by cloud = Power emitted

$$\Rightarrow \quad \frac{4\pi(3R_\odot)^2\sigma T_s^4}{4\pi d^2}\times\pi R_g^2 = 4\pi(R_g)^2\sigma T_g^4$$

$\Rightarrow \quad d = 33747.67\ R_\odot,\ (R_\odot = 695700\ \text{km})$

$\Rightarrow \quad d = 2.347\times10^{13}$ m

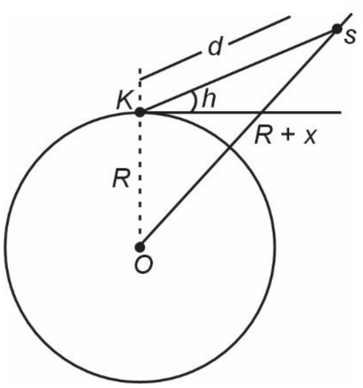
5. (10 marks) **Observing the ISS**

The International Space Station (ISS) is sometimes visible in the sky, during the morning and evening twilight, as a bright moving object. One day, Kundan sees the ISS rising at the horizon, then passing near the zenith, and then setting in the opposite direction.

Consider the ISS to be w metre across and orbiting the Earth in a circular orbit at a height x metre above the Earth's surface.

Derive an expression for its apparent angular size, ϕ, measured by Kundan as a function of its altitude from Kundan's location.

Sol.



$(R + x)^2 = R^2 + d^2 - 2Rd\cos(90 + h)$

$\rightarrow \quad R^2 + d^2 - 2R\sin h.d - R^2 - x^2 - 2Rx$

$\rightarrow \quad d^2 - 2R\sin h.d - (x^2 + 2Rx) = 0$

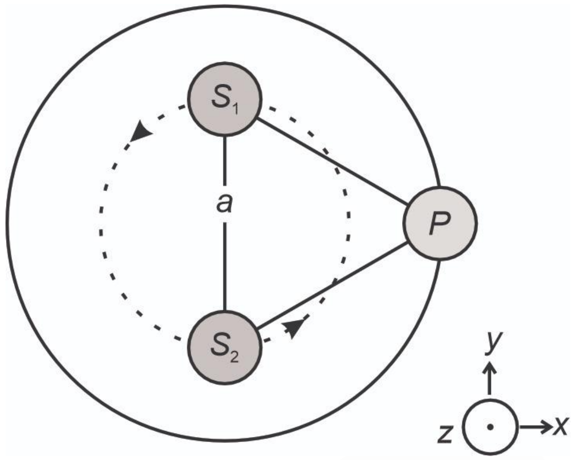
$$\rightarrow \quad d = \frac{2R\sin h \pm \sqrt{(2R\sin h)^2 + 4(x^2 + 2Rx)}}{2}$$

$$d = R\sin h + \sqrt{(R\sin h)^2 + (x^2 + 2Rx)}$$

So Angular size $\phi = \dfrac{w}{R\sin h + \sqrt{(R\sin h)^2 + (x^2 + 2Rx)}}$

6. **Planet's Orbit around a Binary Star System:**

Dhananjay, an exoplanet researcher, wanted to explore the prospects of life on an Earth like planet in a binary star system. More specifically he wanted to look into the following configuration: two identical stars S_1, S_2 and a planet P have circular orbits about the centre of mass of the system and at every point in time, the three bodies form a equilateral triangle with side of length $a = 2au$ as shown in the figure below.



(Note that at the instance shown in this figure, the radius vector from the centre of mass of the system to P is exactly along the x-axis).

The period t_P of such a system is known to be:

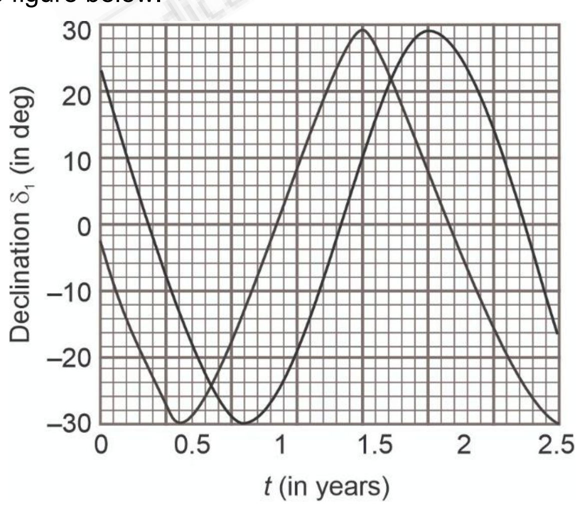
$$t_P = \sqrt{\frac{4\pi^2 a^3}{G(m_1 + m_2 + m_3)}}$$

You can take the mass and radius of the planet P to be exactly that of Earth and the two stars are sun like. The sidereal period of rotation of P is 1 d and the angle made by the P's axis of rotation with the z-axis in the diagram above, called the axial tilt, is $\theta = 30°$.

Help Dhananjay answer the following questions about the planet:

(a) (3 marks) Taking the albedo of the planet P to be $A = 0.15$, estimate its average surface temperature (T_P).

Dhananjay calculated the variation of declination, δ_1, of star 1 with time as seen from the planet. This variation is plotted in the figure below.



Note that declination is defined in the usual sense - the angle made by the object with the celestial equator of the planet.

(b) (4 marks) Plot the declination, δ_2, of star S_2 *vs* time in the plot provided in answer sheet.

(c) (6 marks) The configuration of the two stars and planet system shown in the figure given in the preamble of the question is the orientation of the system corresponding to time $t = 0$ in the declination *vs* time plot.

In the figure given in the answer sheet, mark the direction of the equinoxes, as seen from the planet, with respect to the x-axis on the orbit of planet. Mark the equinox after which the days get longer, in the northern hemisphere of the planet, with a × and write VE beside it and the other one with Δ and write AE beside it.

Note that the equinox is the day corresponding to the median day length.

(d) (10 marks) We define the effective flux at a point on the planet as the energy per unit area per unit time falling on a tangential plane at that point. Plot the effective flux on the planet's North Pole from $t = 0$ yr to $t = 2.5$ yr.

(e) (4 marks) What will be the typical day length at the equator, $t_{day,eq}$, and the pole, $t_{day,pole}$? Explain your answer.

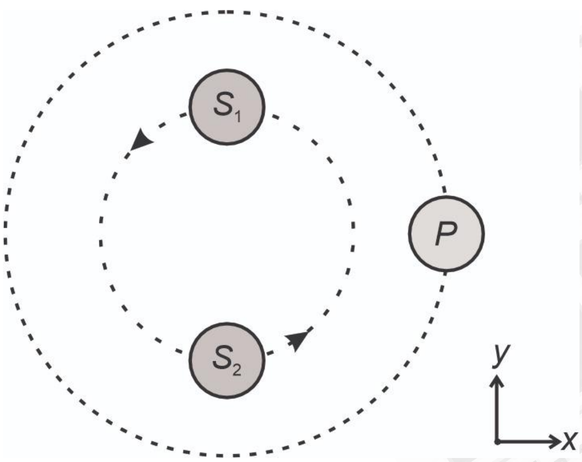
(f) (3 marks) What is/are the number of zero shadow days, N_{ZSD} for an observer on the planet in one period of the system for the following two cases?

Case 1 : Latitude > 30°

Case 2 : Latitude < 30°

Explain your answer in brief.

Sol. (a) Given $t_P = \sqrt{\dfrac{4\pi^2 a^3}{G(m_1+m_2+m_3)}}$



Power received = $\left(\dfrac{L}{4\pi a^2}\right)\times 2\times \pi R_P^2(1-0.15)$

$$= \frac{1}{2}L\left(\frac{R_P}{a}\right)^2\times 0.85 \qquad \text{… (i)}$$

Let at equilibrium the power radiated by plant is P_P and temperature be T_P.

So $\qquad P_P = 4\pi R_P^2.\sigma.T_P^4 \qquad$ …(ii)

From (i) and (ii)

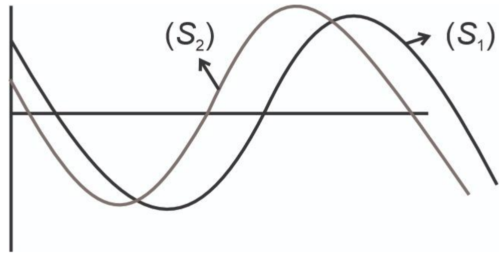
$$4\pi\sigma.R_P^2.T_P^4 = \frac{LR_P^2}{2a^2}0.85$$

$$\rightarrow \quad T_P^4 = \frac{L_R^2}{2a^2}\frac{0.85}{4\pi r\times R_P^2} = \frac{3.828\times 10^{26}\times 0.85}{(2\times 1.496\times 10^{11})^2 8\pi R}$$

$$\rightarrow \quad T_P^4 = 2550614110$$

$$\rightarrow \quad T_P = 224.7\ \text{K}$$

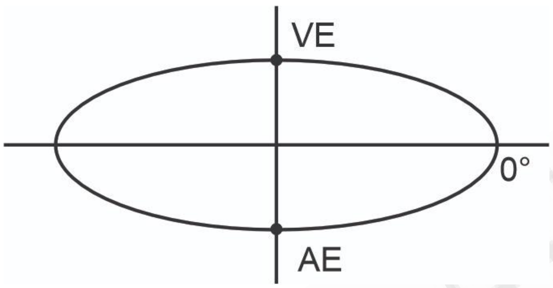
(b) As per the given declination δ_1 of Star 1 from the plant. The declination δ_2 of star 2 will be forming the same pattern but lagging w.r.t. the plot for star 1.



$$\delta_1(t) = \varepsilon \sin\left(\frac{2\pi t}{t_P}\right)$$

$$\delta_2(t) = \varepsilon \sin\left(\frac{2\pi t}{t_P} - \frac{2\pi}{3}\right)$$

(c)



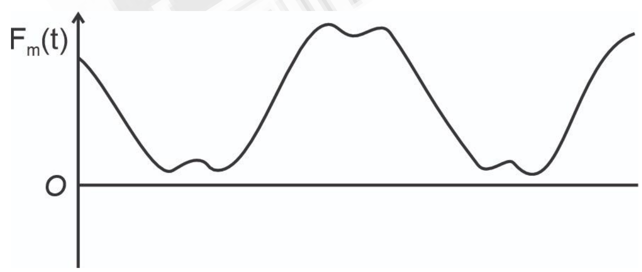
(d) Effective flux at North pole $t = 0 \rightarrow 2.5$ yr

At latitude ϕ = 90°, flux from one year

$F_{eff} \propto \max(0,\ \sin\delta)$

Total effective flux

$F_N(t) \propto \max(0, \sin\delta_1) + \max(0, \sin\delta_2)$



(e) Each star rises and sets daily with two stars offset by 120°.

At equator length of day = $12 + \left(\frac{60}{360} \times 24\right)$

= 16 hrs

At pole length of the day 1.33 years

But no continuous night

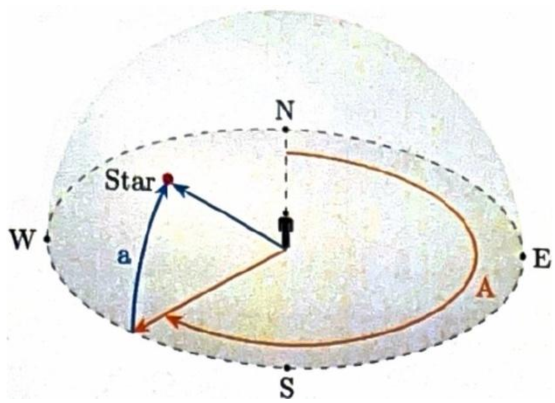
(f) No. of zero shadow day for

Latitude > 30° $\rightarrow$ No zero shadow day

Latitude < 30° $\rightarrow$ 4 days of zero shadow day

Appendix

Altitude – Azimuth System



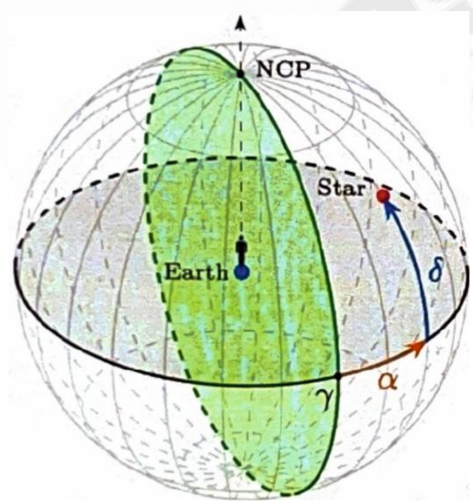
Azimuth (A):

Azimuth is the angle measured along the horizon to indicate the position of a given star. It is measured from the north (0°), towards East (90°). The south will correspond to 180°, and the west will correspond to 270°. The angle of elevation of the star does not matter; e.g. all objects exactly due east will have same 90° azimuth, irrespective of elevation.

Altitude (a):

Altitude is the angle of elevation of the star measured from the horizon (0°). A star exactly overhead, i.e. at Zenith, will have the highest altitude (90°).

Right Ascension – Declination System



Right Ascension (α):

Right Ascension (RA) is the celestial analogue of the terrestrial longitude. The zero of RA passes through the Vernal Equinox point (γ), which marks the position of the Sun when it crosses the Celestial Equator into the northern part of the sky around 21 March every year. RA, measured in hours-minutes-seconds along the celestial equator, increases as the Sun moves ahead from γ for the full year.

Declination (δ):

Declination is the celestial counterpart of the terrestrial latitude. It is the angular distance in degrees of the star from the celestial equator along the meridian (i.e. given line of RA). It is measured from the celestial equator (0°) to the poles (±90°).

Sol. Approximate declination

Star	Declination	Result
Polaris (α UMi)	+90°	Does not set
Dubhe (α UMa)	+62°	Sets North of West
Pollux (α Gem)	+28°	Sets North of West
Vega (α Lyr)	+39°	Sets North of West
Kaus Borealis (α Sgr)	–25°	Sets South of West
Rigel (β ori)	–8°	Sets South of West
Canopus (α car)	–53°	Sets South of West
Revati (z Psc)	~0°	Almost exactly West

So placement of horizon (cardinal West = 270°)

North $\rightarrow$ South

Dubhe $\rightarrow$ Vega $\rightarrow$ Pollux $\rightarrow$ Revati $\rightarrow$ Rigel $\rightarrow$ Kaus Borealis $\rightarrow$ canopus

❑ ❑ ❑