Date: 29/01/2023

Max. Marks : 60

Time: 3 Hrs.

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Answers & Solutions

for

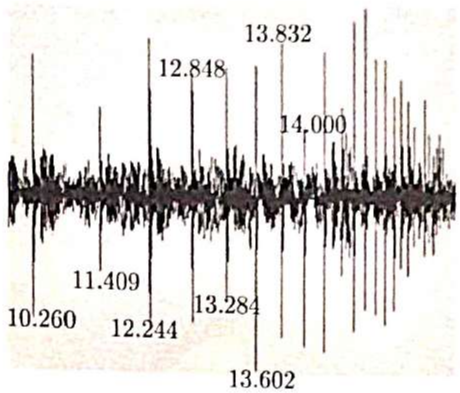
Indian National Physics Olympiad (INPhO) (2023)

INSTRUCTIONS TO CANDIDATES

1. This booklet consists of 5 questions.
2. Maximum marks are indicated in front of each question.
3. Booklet to write the answers is provided separately. Instructions to write the answers are on the Answer Booklet.
4. Marks will be awarded on the basis of what you write on both the Summary Answer Sheet and the Detailed Answer Sheets in the Answer Booklet. Simple short answer and plots may be directly entered in the Summary Answer Sheet. Marks may be deducted for absence of detailed work in questions involving longer calculations.
5. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.
6. **Please submit the Answer Booklet at the end of the examination**. You may retain the Question Paper.

1. **[6 marks] Dancing on the floor**

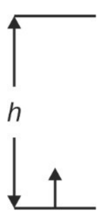
There are various apps that record the intensity of an audio signal. An app (WaveEditor™ here) displays the audio signal as a wave, whose amplitude is proportional to the audio signal's loudness. A smartphone with this app recording the sound signal is kept on a uniformly built flat floor of a classroom.



A perfectly small spherical steel ball is thrown up such that it almost touches the ceiling and comes back without hitting. The ball hits the floor and thereafter it keeps bouncing. The app records the sound signal produced when the ball hits the floor on every bounce. A screenshot of the recording is shown. The timestamps (in seconds) of the first eight consecutive bounces are also shown next to the peak. For example, the app records a peak at 10.260 s when the first time the ball hits the floor.

Make reasonable assumptions, when the ball hits the floor and calculate the height of the classroom from the given data. State your assumptions clearly.

Sol.



Suppose t_1 is time instance at which the ball hits the floor at first time shown here as 10.26 second.

At t_2, the ball hits the floor for second time, where t_2 = 11.409 seconds.

If h is the height of the hall and t_0 is the time of throwing the ball

$$\Rightarrow \quad t_1 - t_0 = 2\sqrt{\frac{2h}{g}} \qquad \text{...(i)}$$

$$t_2 - t_1 = 2\sqrt{\frac{2h}{g}}e \qquad \text{...(ii)}$$

$$t_3 - t_2 = 2\sqrt{\frac{2h}{g}}e^2 \qquad \text{...(iii)}$$

From equation (iii) and (ii),

$e = 0.726$

On repeating the same calculations for other sets of data average value of e (coefficient of restitution) comes out to be 0.723 and using this value of e, the value of height of ceiling (h) comes out 3.1 m.

2. Knock it off!

Consider a 100 W small isotropic source of blue light of wavelength 4500 Å. A metallic surface of 1.00 cm^2 and work function 2.20 eV is kept at a distance of 1.00 m from the source and oriented to receive normal radiation.

(a) **[2 marks]** Assume that all the energy is uniformly absorbed by atoms on the top layer of the surface. Also, all the energy absorbed by an atom on the surface is taken up by one electron. The radius of the atom is 1.00 Å. Estimate the time τ_e needed by the electron to receive 1.00 eV of energy.

(b) **[1 marks]** According to the above classical model, how many electrons are emitted by the metallic surface in time τ_e?

(c) **[2 marks]** In quantum theory, photons are emitted and absorbed as quanta. Assuming photoelectric efficiency of 1%, calculate the rate of emission of electrons (N_e) from the surface.

(d) **[1 marks]** Assuming further that all the emitted photoelectrons move normal to the surface what would be the maximum current density (J_{max}) one may expect?

Sol. (a) Intensity at a point r_0 distance away from the source is

$$I = \frac{P}{4\pi r_0^2}$$

Let A : area of metallic surface

r : radius of one atom

$\Rightarrow$ Power received by 1 atom $= \frac{P}{4\pi r_0^2} \cdot \pi r^2$

$$= \frac{100}{4(1)^2}(10^{-10})^2 \text{ W}$$

$$= 2.5 \times 10^{-19} \text{ J/s}$$

$\Rightarrow$ For time τ_e :

$1.6 \times 10^{-19} = 2.5 \times 10^{-19} \times \tau_e$

$\Rightarrow$ $\tau_e = \frac{16}{25}$ seconds

$\tau_e = 0.64$ s

(b) Since upto time τ_e, each atom has received 1 eV (< 2.2 eV)

$\Rightarrow$ Number of electrons emitted = 0

(c) Number of photons emitted by source per second $= \frac{P}{\frac{hc}{\lambda}}$

$$= \frac{100}{\frac{12420}{4500} \times 1.6 \times 10^{-19}}$$

$$\simeq 2.26 \times 10^{20}$$

$\Rightarrow$ Number of photons received $= \dfrac{2.26 \times 10^{20}}{4\pi r_0^2} \times 1\ \text{cm}^2$

$$= \frac{2.26 \times 10^{20}}{4\pi(1)^2} \times 10^{-4}$$

$$= 1.8 \times 10^{15}\ /\text{s}$$

Since efficiency is 1%

$\Rightarrow$ $N_e = \dfrac{1}{100} \times 1.8 \times 10^{15}\ /\text{s}$

$N_e = 1.8 \times 10^{13}\ /\text{s}$

(d) Current $= N_e \times e$

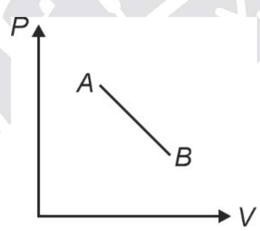
$\Rightarrow$ $J_{\text{max}} = \dfrac{\text{Current}}{\text{Area}}$

$$= \frac{1.8 \times 10^{13} \times 1.6 \times 10^{-19}}{10^{-4}}\ \text{A/m}^2$$

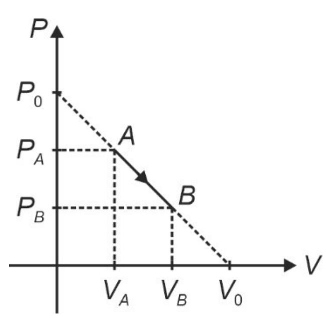
$J_{\text{max}} = 2.88 \times 10^{-2}\ \text{A/m}^2$

3. [16 marks] Work in progress

One mole of an ideal monoatomic gas goes through a linear process from A to B as shown in the pressure-volume (P-V) diagram. The temperature at A is $T_A = 227°\text{C}$. The process is such that, the temperature decreases and the heat is continuously supplied to the gas. The ratio of the specific heat at the constant pressure to that at the constant volume is 5/3. Obtain the expression for the maximum work (W_{max}) the gas can perform in such a process. Calculate W_{max}.



Sol. $n = 1$, $T_A = 227°\text{C} = 500\ \text{K}$, monoatomic, slope $= -\dfrac{P_0}{V_0}$



In the process, where PV indicator diagram is a straight line as shown, the temperature first increases and reaches to maximum and then decreases. And, the process turns from endothermic to exothermic at certain point.

Therefore, for maximum work done by the gas, the point A should be point of maximum temperature and point B should be point where endothermic turns exothermic.

$\therefore \quad V_A$ = Volume where $T = T_{max}$

and V_B = Volume where ΔQ changes its sign

$\therefore \quad W_{max} = \dfrac{(P_A + P_B)\times(V_B - V_A)}{2}$ is the required expression for W_{max}

Now, calculation of V_A :

$$\because \quad P = -\frac{P_0}{V_0}V + P_0$$

$$\Rightarrow \quad \frac{nRT}{V} = -\frac{P_0}{V_0}V + P_0$$

$$\Rightarrow \quad T = \frac{1}{nR}\left[-\frac{P_0}{V_0}V^2 + P_0 \times V\right]$$

$$\therefore \quad \frac{dT}{dV} = \frac{1}{nR}\left[-\frac{P_0}{V_0}\times 2V + P_0\right] = 0$$

$$\Rightarrow \quad V_A = \frac{V_0}{2} \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad P_A = \frac{P_0}{V_0}\times\left(\frac{V_0}{2}\right) = \frac{P_0}{2} \qquad \ldots\text{(ii)}$$

For calculation of V_B :

At B, $\dfrac{dQ}{dV} = 0$ as dQ changes its sign.

Now, $dQ = dU + dW$

$$\Rightarrow \quad dQ = nC_v dT + PdV$$

$$\Rightarrow \quad dQ = 1\times\frac{3}{2}R\,dT + PdV$$

$$\Rightarrow \quad dQ = \frac{3}{2}(PdV + VdP) + PdV$$

$$\Rightarrow \quad dQ = \frac{5}{2}PdV + \frac{3}{2}VdP = 0$$

$$\therefore \quad \frac{dP}{dV} = -\frac{5}{3}\frac{P}{V}$$

$$\Rightarrow \quad -\frac{P_0}{V_0} = -\frac{5}{3}\frac{P}{V}$$

$$\Rightarrow \quad -\frac{P_0}{V_0} = -\frac{5}{3}\times\frac{P_B}{V_B}$$

$$\Rightarrow \quad P_B = \frac{3}{5}\frac{P_0}{V_0}\times V_B \qquad \ldots\text{(iii)}$$

Now, $\frac{P_A - P_B}{V_A - V_B} = -\frac{P_0}{V_0}$

$$\Rightarrow \quad \frac{\frac{P_0}{2} - P_B}{\frac{V_0}{2} - V_B} = -\frac{P_0}{V_0} \qquad \text{...(iv)}$$

From (iii) and (iv),

$$V_B = \frac{5}{8}V_0 \text{ and } P_B = \frac{3}{8}P_0$$

$$\therefore \quad W_{max} = \frac{P_A + P_B}{2} \times (V_B - V_A)$$

$$= \frac{\left[\frac{P_0}{2} + \frac{3P_0}{8}\right]}{2} \times \left(\frac{5V_0}{8} - \frac{V_0}{2}\right)$$

$$= \frac{1}{2} \times \frac{7P_0}{8} \times \left(\frac{V_0}{8}\right)$$

$$\Rightarrow \quad W_{max} = \frac{7P_0V_0}{128} \qquad \text{...(v)}$$

Now, $\frac{P_0}{2} \times \frac{V_0}{2} = 1 \times R \times T_A$

$$\Rightarrow \quad P_0V_0 = 4RT_A$$

$$= 4R \times 500$$

$$= 2000R$$

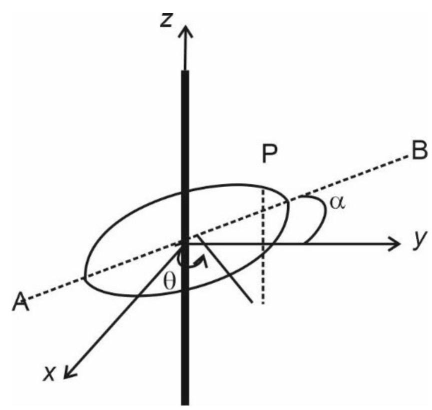
$$\therefore \quad W_{max} = \frac{7}{128} \times (2000R)$$

$$\Rightarrow \quad W_{max} = \frac{875R}{8}$$

4. Electrostatic TikTok

Consider a fixed infinite vertical thin rod (shown by the red color in the figure below) of linear charge density λ along the *z*-axis at the origin (see figure below). A uniformly charged ring of total charge *Q*, mass *M*, and radius *a* is placed with its centre at the origin in the *x*-*y* plane. Point P is an arbitrary point on the ring. The projection of point P on *x*-*y* plane makes an angle θ with respect to the *x*-axis in the anticlockwise direction as seen from the top.

The ring is now given an initial angular velocity ω_0 about the *x*-axis. We define the angle α which the plane of the ring makes with the *x*-*y* plane. This is illustrated by drawing line segment AB in the plane of the ring. Initially $\alpha = 0$. Ignore gravity.



You may find the following differentiation useful

$$D = \frac{d}{d\theta}[\tan^{-1}(q\tan\theta)] = \frac{1}{1+(q\tan\theta)^2}[q(\sec^2\theta)] \quad (4.1)$$

(a) **[1 marks]** State an expression for the electric field $(\vec{E}_0)$ due to the infinite rod at a point on the ring when $\alpha = 0$ in terms of *x, y* and θ, and related quantities.

(b) **[2 marks]** At some instant the ring makes an angle α. Derive an expression for the electric field $\vec{E}$ due to the infinite rod at a point on the ring in terms of θ, and α.

(c) **[1 marks]** Find the net force $\vec{F}$ acting on the ring.

(d) **[5 marks]** Find the net torque $\vec{\tau}$ acting on the ring in terms of α and the constants only. Qualitatively plot torque as a function of α.

(e) **[2 marks]** Let the ring is in equilibrium with respect to $\alpha = 0$. Derive an expression for the time period *T* of small oscillations of the ring in terms of λ, and *Q*. Take $\lambda = 0.1$ μC/m, $Q = 2.0$ μC, $M = 50.0$ g, radius $a = 5.0$ cm, and $\omega_0 = 1.0$ rad/s. Calculate *T*.

(f) **[2.5 marks]** Find an expression for the potential energy *U* of the ring in terms of α. Qualitatively plot *U* as a function of α. Take the zero of potential energy to be at $\alpha = 0$.

(g) **[2.5 marks]** Obtain the expression of maximum value of α (α_{max}) in terms of ω_0. Calculate α_{max}.

Sol. (a) As $\alpha = 0$, i.e., the ring lies in *x-y* plane. At point in *x-y* plane as the wire is placed along *z*-axis so the electric field will be radially outwards.

Coordinates of a general point on the ring is given as

$(a\cos\theta, a\sin\theta)$

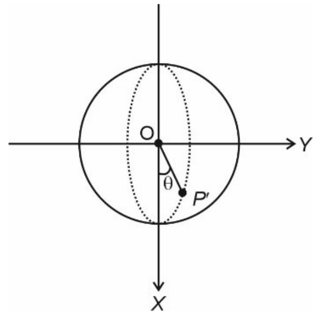
magnitude of electric field at this point is given as $\frac{2k\lambda}{r}$

$$\Rightarrow \vec{E}_0 = \frac{2k\lambda}{r}\hat{r} = \frac{2k\lambda\vec{r}}{r^2}$$

$$= \frac{2\left(\frac{1}{4\pi\varepsilon_0}\right)\lambda\left(a\cos\theta\hat{i} + a\sin\theta\hat{j}\right)}{\left(\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}\right)^2}$$

$$\vec{E}_0 = \frac{\lambda\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)}{2\pi\varepsilon_0 a}$$

(b) Looking in top view from z-axis the projection of ring seems like a projection of circle on x-y plane shown by dotted line in figure below



Let us take a random point P' on this dotted line structure which is projection of point P such that $OP' = l$ (variable)

Then coordinates of point P can be written at ($l\cos\theta$, $l\sin\theta$, $l\sin\theta\tan\alpha$)

Now, $OP = a$ (radius)

$\Rightarrow l^2\cos^2\theta + l^2\sin^2\theta + l^2\sin^2\theta\tan^2\alpha = a^2$

$$\Rightarrow l = \frac{a}{\sqrt{1+\sin^2\theta\tan^2\alpha}}$$

$\Rightarrow$ coordinates of point P will be

$$\left(\frac{a\cos\theta}{\sqrt{1+\sin^2\theta\tan^2\alpha}}, \frac{a\sin\theta}{\sqrt{1+\sin^2\theta\tan^2\alpha}}, \frac{a\sin\theta\tan\alpha}{\sqrt{1+\sin^2\theta\tan^2\alpha}}\right)$$

Electric field at point P will be equal to

$$= \frac{\lambda}{2\pi\varepsilon_0}\frac{\dfrac{a\cos\theta\hat{i}}{\sqrt{1+\sin^2\theta\tan^2\alpha}} + \dfrac{a\sin\theta\hat{j}}{\sqrt{1+\sin^2\theta\tan^2\alpha}}}{\dfrac{a^2\cos^2\theta + a^2\sin^2\theta}{1+\sin^2\theta\tan^2\alpha}}$$

$$= \frac{\lambda(\cos\theta\hat{i} + \sin\theta\hat{j})\left(\sqrt{1+\sin^2\theta\tan^2\alpha}\right)}{2\pi\varepsilon_0 a}$$

(c) As the ring is placed symmetric about origin with x-axis as central axis, its each part will find a diametrically opposite point with which the force experienced by this part will get balanced and on whole the ring will experience net zero force.

(d) $$d\tau = \frac{\left(1+\tan^2\alpha\right)^{1/2}\lambda d\theta}{\left(1+\tan^2\alpha\sin^2\theta\right)} \times \frac{\lambda'\sin\theta}{2\pi\varepsilon_0} \times 2a\sin\theta\tan\alpha$$

$$d\tau = \frac{\sin^2\theta d\theta}{\left(1+\tan^2\alpha\sin^2\theta\right)}\left(\frac{\lambda'\lambda 2a\tan\alpha(1+\tan^2\alpha)}{2\pi\varepsilon_0}\right)$$

Integrating with limits from 0 to π

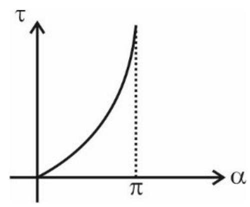
$$\tau = \frac{a\lambda'\lambda\tan\alpha\left(1+\tan^2\alpha\right)^{1/2}}{\pi\varepsilon_0} \times \frac{\pi}{\tan^2\alpha}\left(\frac{-1+\sec\alpha}{\sec\alpha}\right)$$

$$= \frac{a\lambda'\lambda}{\varepsilon_0}\frac{(-1+\sec\alpha)}{\tan\alpha}$$

$$= \frac{a\lambda}{\varepsilon_0} \times \frac{Q}{2\pi a} \cot\alpha\left(-1+\sec\alpha\right)$$

$$\tau = \frac{a\lambda Q}{2\varepsilon_0 \pi a}\left(\frac{1-\cos\alpha}{\sin\alpha}\right)$$

$$\tau = \frac{\lambda Q}{2\pi\varepsilon_0} \times \frac{\alpha}{2} \qquad \text{near } \alpha = 0$$



(e) $\tau = -I\dfrac{d^2\alpha}{dt^2}$

$$I = \frac{ma^2}{2}$$

$$\frac{ma^2}{2}\frac{d^2\alpha}{dt^2} = -\frac{\lambda Q\alpha}{4\pi\varepsilon_0}$$

$$\Rightarrow \quad \omega^2 = \frac{\lambda Q \times 2}{4\pi\varepsilon_0 ma^2}$$

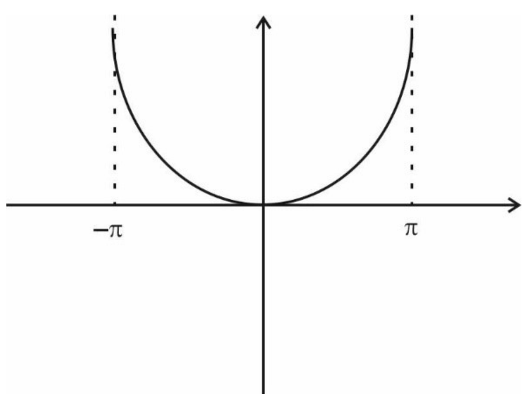
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2\pi\varepsilon_0 ma^2}{\lambda Q}} = 1.6557 \text{ seconds}$$

(f) $\tau = \dfrac{a\lambda Q}{2\varepsilon_0 \pi a}\left(\dfrac{1-\cos\alpha}{\sin\alpha}\right)$

$$U = +\int \tau d\theta$$

$$= -2\log\left(\cos\frac{\alpha}{2}\right)\frac{\lambda Q}{2\varepsilon_0\pi}$$

$$= -\frac{\lambda Q}{\varepsilon_0\pi}\log\left[\cos\left(\frac{\alpha}{2}\right)\right]$$



(g) $U_{max} = -2\log\left(\cos\frac{\alpha_{max}}{2}\right)\frac{\lambda Q}{2\varepsilon_0\pi}$

At mean position $U = 0$

$$\Rightarrow \frac{1}{2}I\,\omega_0^2 = -\log\left(\cos\frac{\alpha_{max}}{2}\right)\frac{\lambda Q}{\pi\varepsilon_0}$$

$$\cos\frac{\alpha_{max}}{2} = e^{\left(-\frac{\pi\varepsilon_0 I\,\omega_0^2}{2\lambda Q}\right)}$$

$$\alpha_{max} = 2\cos^{-1}\left(e^{-\frac{\pi\varepsilon_0 I\,\omega_0^2}{2\lambda Q}}\right)$$

$$\alpha_{max} \cong 10°40'25''$$

5. If Prof. Snell had a smartphone

A typical smartphone screen is made up of mainly two components: a sheet of touch-sensitive glass (where you move your finger to operate the phone) of thickness t at the top and a LCD screen below it consisting of a regular array of "RGB elements" that emit light. These elements have a separation of d between them. There is a thin air gap of depth h between the touch-sensitive glass and the LCD screen (see Fig. (1) for a cross sectional view). We estimate the value of h from the following experiment.

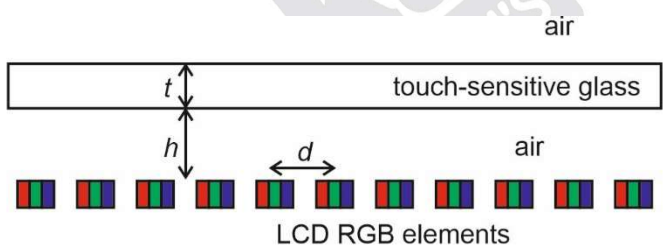


Figure 1

We use two smartphones (S-I and S-II) in this exercise – S-I is the target instrument in which we want to estimate h, and S-II is the measuring instrument that can capture photos of the screen of S-I which we then analyse using an image-processing software.

A digital image captured by the camera of a smartphone (S-II here) consists of discrete picture elements called pixels. The image captured by S-II is processed through a software. A red color reference line is drawn on the image (see Fig. 3(a)). The software plots the "brightness value" at every point of the reference line as a function of the number of pixels from the left end of the line. Thus, pixel number is a marker for distance here. First, we need to calibrate distance in terms of pixel number.

The phone S-I is kept horizontal and the display is kept ON. A ruler is placed on its screen. S-II is fixed above S-I to capture images. The image of the screen captured is shown in Fig. (2).

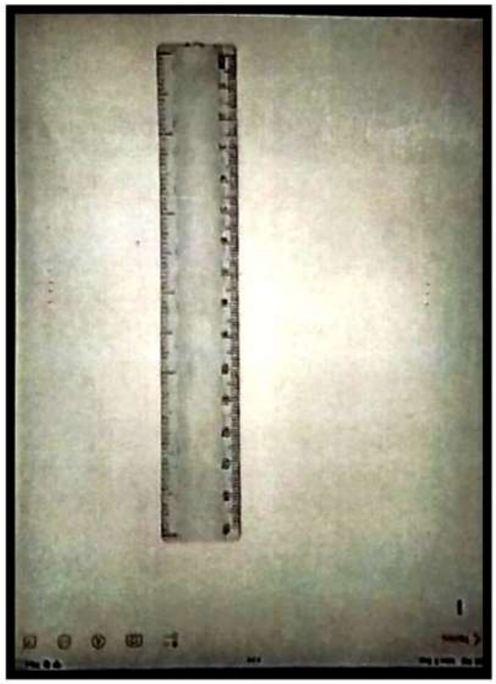
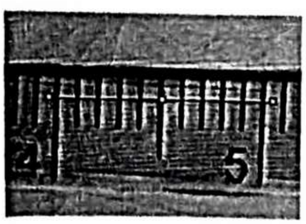
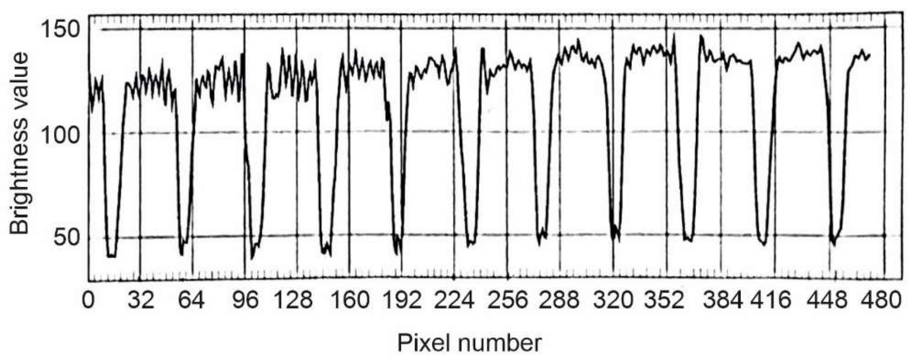


Figure 2

Figure 3(a) shows a part of the image of the ruler and its brightness value profile along the red reference line in Fig. 3(b).



(a)



(b)

Figure 3

(a) **[2 marks]** State the number of pixels used by the camera of S-II to capture one centimeter of the screen of S-I.

(b) **[5 marks]** We keep the setup the same as the last part. Next, a few small water drops are placed on the glass screen of S-I beside the ruler (see Figs. 4(a) and 4(b) for a top and side view, respectively). We model every drop as a hemispherical lens of radius R that magnifies the array of RGB elements of the LCD screen of S-I (see Fig. 4(c); the figure is not to scale).

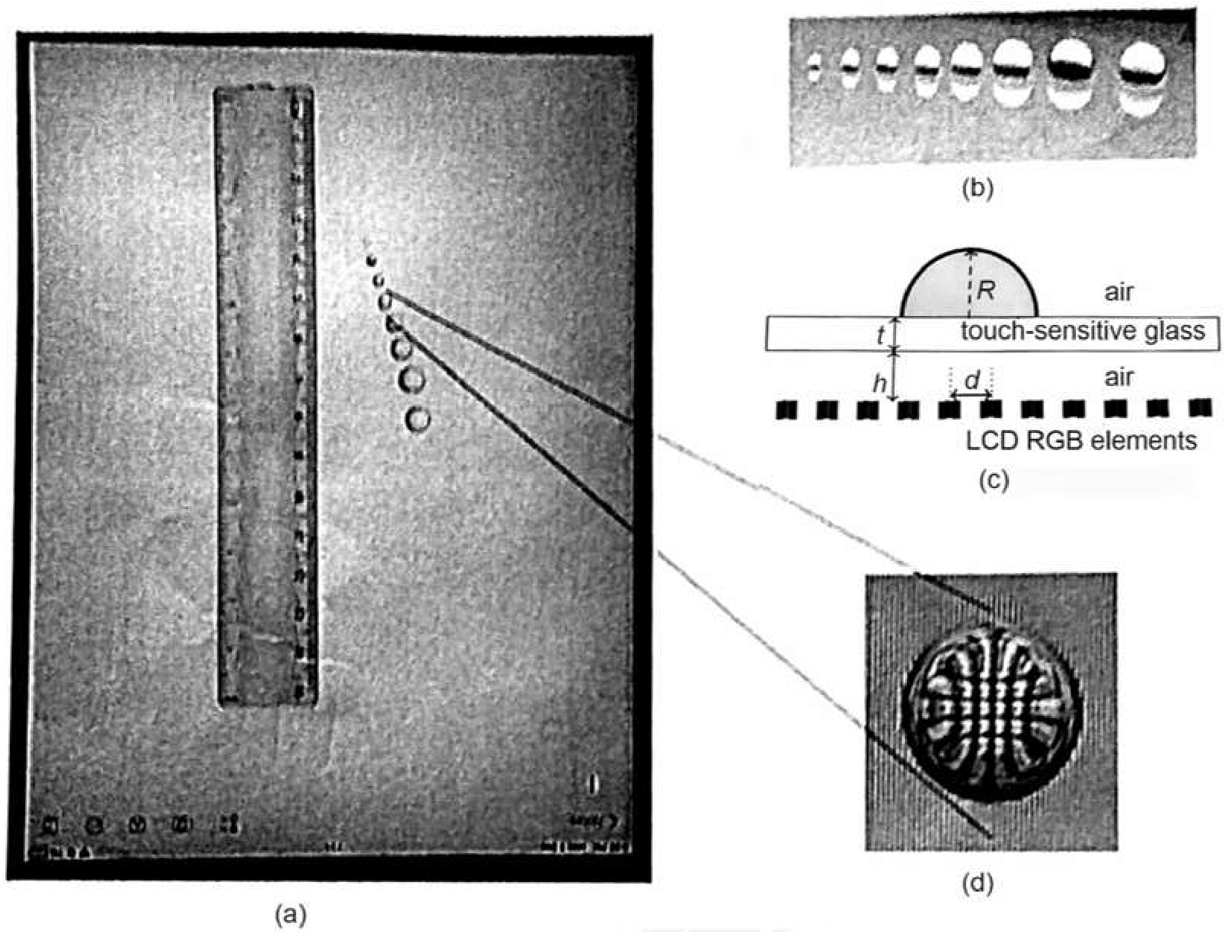
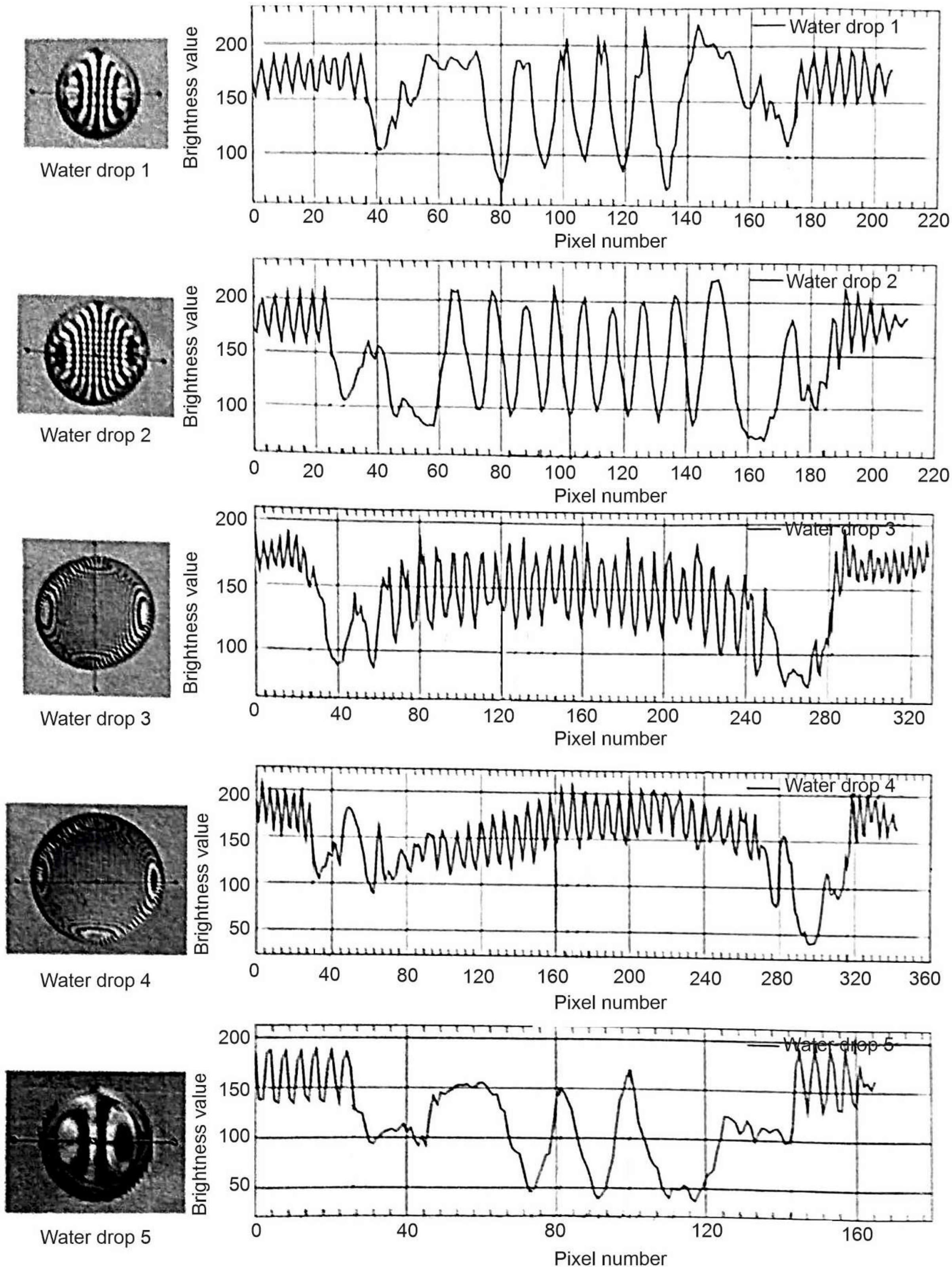


Figure 4

Figure 4(d) shows the magnified image of the array of the RGB elements of the screen as viewed from the top through one of the drops. This image is captured by S-II keeping the camera settings and distance same as in the previous part. The brightness value profiles of the images of the five chosen drops along the reference lines are shown in Fig. (5) on the next page.

Using the profile plots, write the radius of the water drop (R in mm) and the corresponding magnification (M) of the separation d between the array of RGB elements of S-I for each water drop lens. Use the table in the Summary Answer sheet to report your data. Describe the method you have used and the calculations in the Detailed Answer sheet.

(c) **[9 marks]** For the given smartphone, t = 0.50 mm, the refractive indices of the touch-sensitive glass, water drop, and the air to be 3/2, 4/3 and 1 respectively. Using the data table of the previous part, plot a suitable linear graph to obtain the distance (h) of the RGB elements from the touch-sensitive glass. Use the table given in the summary answer sheet to enter the data used to plot the graph. Show your detailed theoretical calculation in the Detailed Answer Sheet.



Sol. (a) From the figure 3(b), 11 dips in brightness value are given over 1 cm length. The corresponding dip values are, for first dip is 12th pixel and for 11th dip is 448th pixel.

$\therefore$ Number of pixels per cm comes out to be 448 – 12 = 436 pixels/cm

(b) (i) From figure 5 water drop 1

The diameter of the drop (in pixels) = 176 – 36 pixels

= 140 pixels

$\therefore$ Radius of drop 1 = $\frac{140}{2}\times\frac{1}{436}$cm

$= \frac{140}{2}\times\frac{1}{436}\times 10$ mm

= 1.60 mm

And, from the graph given, $3d_m$ = 120 – 80 pixel

$$d_m = \frac{40}{3}\text{pixel} = \frac{40}{3}\times\frac{1}{436}\times 10 \text{ mm}$$

d_m = 0.30 mm

where d_m is magnified separation between array.

$\therefore$ Magnification, $M_1 = \frac{d_m}{d} = \frac{0.30}{d}$

From graph, d = 4 pixels = $4\times\frac{10}{436}$ mm = 0.09 mm

$\therefore$ $M_1 = \frac{0.30}{0.09} = 3.3$

(ii) Similarly, radius of drop 2

$$r_2 = \frac{184-24}{2}\times\frac{1}{436}\times 10 = 1.83 \text{ mm}$$

and $M_2 = \frac{120-80}{4d}\times\frac{1}{436}\times 10 = \frac{0.23}{d} = \frac{0.23}{0.09}$

M_2 = 2.5

(iii) Radius of drop 3

$$r_3 = \frac{282-27}{2}\times\frac{10}{436} = 2.92 \text{ mm}$$

and $M_3 = \frac{147-120}{4d}\times\frac{10}{436} = \frac{0.15}{d} = \frac{0.15}{0.09}$

$\Rightarrow$ M_3 = 1.66

(iv) Radius of drop 4

$$r_4 = \frac{316-31}{2}\times\frac{10}{436} = 3.27 \text{ mm}$$

and $M_4 = \frac{178-160}{3d}\times\frac{10}{436} = \frac{0.14}{d} = \frac{0.14}{0.09}$

$\Rightarrow$ M_4 = 1.6

(v) Radius of drop 5

$$r_5 = \frac{143-27}{2} \times \frac{10}{436} = 1.33 \text{ mm}$$

and $M_5 = \frac{91-74}{d} \times \frac{10}{436} = \frac{0.34}{d} = \frac{0.34}{0.09}$

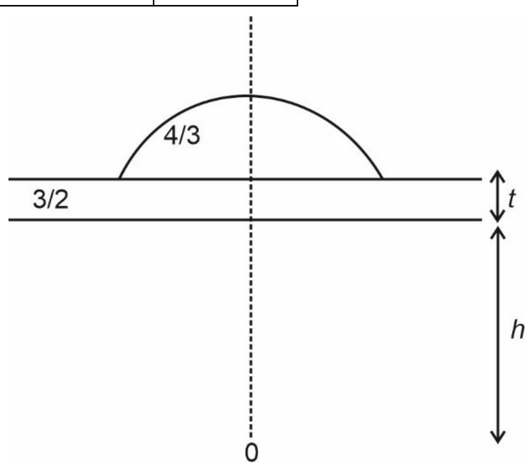
$\Rightarrow$ $M_5 = 3.8$

Table of radius and magnification of drops

	Radius	**Magnification**
Drop-1	1.60 mm	3.3
Drop-2	1.83 mm	2.5
Drop-3	2.92 mm	1.66
Drop-4	3.27 mm	1.6
Drop-5	1.33 mm	3.8

(c)

	$\frac{1}{r}$	$\frac{1}{M}$
Drop - 1	0.625	0.3
Drop - 2	0.546	0.4
Drop - 3	0.342	0.6
Drop - 4	0.306	0.62
Drop - 5	0.751	0.26



For first refraction :

$$d' = \frac{3h}{2}$$

For second refraction :

$$d'' = \frac{\frac{3h}{2}+t}{\frac{9}{8}} = \frac{8}{9}\left(\frac{3h}{2}+t\right)$$

$$d'' = \frac{4(3h+2t)}{9}$$

For spherical surface :- (3rd refraction)

$$u = -\left[\frac{4(3h+2t)}{9}+R\right] = -\frac{1}{9}\left[4(3h+2t)+9R\right]$$

$$n_1 = \frac{4}{3}$$

$$n_2 = 1$$

$$R = -R$$

$$\frac{n_2}{v}-\frac{n_1}{u} = \frac{n_2-n_1}{R}$$

$$\frac{1}{v}+\frac{\frac{4}{3}}{\frac{1}{9}[4(3h+2t)+9R]} = \frac{1-\frac{4}{3}}{-R}$$

$$\frac{1}{v}+\frac{12}{4(3h+2t)+9R} = \frac{1}{3R}$$

$$\frac{1}{v} = \frac{1}{3R}-\frac{12}{4(3h+2t)+9R}$$

$$\Rightarrow \quad \frac{1}{M} = \frac{n_2}{n_1}\frac{u}{v}$$

$$\Rightarrow \quad \frac{1}{M} = \frac{-[4(3h+2t)-27R]}{36R}$$

For drop 1 :

$$\frac{1}{3.3} = \frac{-[12h+4-27(1.6)]}{36(1.6)}$$

$\Rightarrow \quad h$ = 1.81 mm

For drop 2 :

$$\frac{1}{2.5} = \frac{27(1.83)-12h-4}{36(1.83)}$$

$\Rightarrow \quad h$ = 1.59 mm

For drop 3 :

$$\frac{1}{1.66} = \frac{27(2.92)-12h-4}{36(2.92)}$$

$\Rightarrow \quad h$ = 0.96 mm

For drop 4 :

$$\frac{1}{1.6}=\frac{27(3.27)-12h-4}{36(3.27)}$$

$\Rightarrow$ h = 0.9 mm

For drop 5 :

$$\frac{1}{3.8}=\frac{27(1.33)-12h-4}{36(1.33)}$$

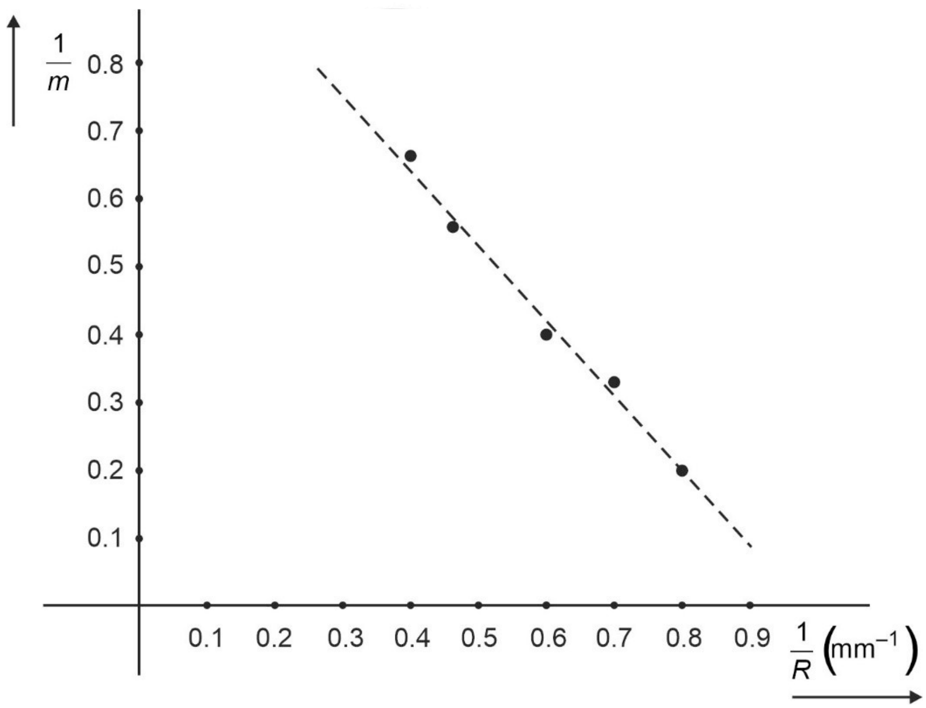
$\Rightarrow$ h = 1.61 mm

For graph :

$$\frac{1}{m}=\frac{3}{4}-\frac{3h+2t}{9}\cdot\frac{1}{R}$$

$\Rightarrow$ Graph between $\frac{1}{m}$ and $\frac{1}{R}$ would be linear.

Slope of the graph = $-\frac{3h+1}{9}$ $\left(\because\ t=\frac{1}{2}\text{ mm}\right)$



❑ ❑ ❑