Date: February 02, 2021 **Time: 3 Hours**

Number of Questions: 30 **Max Marks: 100**

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Questions & Answers

for

IOQM - 2020-21

INSTRUCTIONS

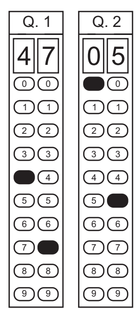
1. Use of mobile phones, smartphones, ipads, calculators, programmable wrist watches is **Strictly Prohibited**. Only ordinary pens and pencils are allowed inside the examination hall.
2. The correction is done by machines through scanning. On the OMR sheet, darken bubbles completely with a **black or blue ball pen**. Please **DO NOT use a pencil or a gel pen.** Darken the bubbles completely, only after you are sure of your answer; else, erasing may lead to the OMR sheet getting damaged and the machine may not be able to read the answer.
3. The name, email address, and date of birth entered on the OMR sheet will be your login credentials for accessing your score.
4. Incompletely, incorrectly or carelessly filled information may disqualify your candidature.
5. Each question has a one or two digit number as answer. The first diagram below shows improper and proper way of darkening the bubbles with detailed instructions. The second diagram shows how to mark a 2-digit number and a 1-digit number.

INSTRUCTIONS

1. "Think before your ink".
2. Marking should be done with Blue/Black Ball Point Pen only.
3. Darken only one circle for each question as shown in Example Below.

WRONG METHODS	CORRECT METHOD
⊗ ● ⊙ ⌀	Ⓐ ● Ⓒ Ⓓ

4. If more than one circle is darkened or if the response is marked in any other way as shown "WRONG" above, it shall be treated as wrong way of marking.
5. Make the marks only in the spaces provided.
6. Carefully tear off the duplicate copy of the OMR without tampering the Original.
7. Please do not make any stray marks on the answer sheet.



6. The answer you write on OMR sheet is irrelevant. The darkened bubble will be considered as your final answer.
7. Questions 1 to 8 carry 2 marks each; questions 9 to 21 carry 3 marks each; questions 22 to 30 carry 5 marks each.
8. All questions are compulsory.
9. There are no negative marks.
10. Do all rough work in the space provided below for it. You also have blank pages at the end of the question paper to continue with rough work.
11. After the exam, you may take away the Candidate's copy of the OMR sheet.
12. Preserve your copy of OMR sheet till the end of current Olympiad season. You will need it later for verification purposes.
13. You may take away the question paper after the examination.

1. If a, b, c are real numbers and

$(a + b - 5)^2 + (b + 2c + 3)^2 + (c + 3a - 10)^2 = 0$

find the integer nearest to $a^3 + b^3 + c^3$.

Answer (57)

2. If $ABCD$ is a rectangle and P is a point inside it such that $AP = 33$, $BP = 16$, $DP = 63$. Find CP.

Answer (56)

3. Sita and Geeta are two sisters. If Sita's age is written after Geeta's age a four digit perfect square (number) is obtained. If the same exercise is repeated after 13 years another four digit perfect square (number) will be obtained. What is the sum of the present ages of Sita and Geeta?

Answer (55)

4. Let ABC be an isosceles triangle with $AB = AC$ and incentre I. If $AI = 3$ and the distance from I to BC is 2, what is the square of the length of BC?

Answer (80)

5. Find the number of positive integers n such that the highest power of 7 dividing $n!$ is 8.

Answer (07)

6. Let $ABCD$ be a square with side length 100. A circle with center C and radius CD is drawn. Another circle of radius r, lying inside $ABCD$, is drawn to touch this circle externally and such that the circle also touches AB and AD. If $r = m + n\sqrt{k}$, where m, n are integers and k is a prime number, find the value of $\frac{m+n}{k}$.

Answer (50)

7. a, b, c are positive real numbers such that $a^2 + b^2 = c^2$ and $ab = c$. Determine the value of $\left|\frac{(a+b+c)(a+b-c)(b+c-a)(c+a-b)}{c^2}\right|$.

Answer (04)

8. Find the largest 2-digit number N which is divisible by 4, such that all integral powers of N end with N.

Answer (76)

9. Find the number of ordered triples (x, y, z) of real numbers that satisfy the system of equation :

$x + y + z = 7$; $x^2 + y^2 + z^2 = 27$; $xyz = 5$.

Answer (03)

10. Let A and B be two finite sets such that there are exactly 144 sets which are subsets of A or subsets of B. Find the number of elements in $A \cup B$.

Answer (08)

11. The prime numbers a, b and c are such that $a + b^2 = 4c^2$. Determine the sum of all possible values of $a + b + c$.

Answer (31)

12. Let $A = \{m : m$ an integer and the roots of $x^2 + mx + 2020 = 0$ are positive integers) and

$B = (n : n$ an integer and the roots of $x^2 + 2020x + n = 0$ are negative integers).

Suppose a is the largest element of A and b is the smallest element of B. Find the sum of digits of $a + b$.

Answer (26)

13. The sides of a triangle are x, $2x + 1$ and $x + 2$ for some positive rational number x. If one angle of the triangle is 60°, what is the perimeter of the triangle?

Answer (9)

14. Let ABC be an equilateral triangle with side length 10. A square $PQRS$ is inscribed in it, with P on AB, Q, R on BC and S on AC. If the area of the square $PQRS$ is $m + n\sqrt{k}$ where m, n are integers and k is a prime number then determine the value of $\sqrt{\frac{m+n}{k^2}}$.

Answer (10)

15. Ria has 4 green marbles and 8 red marbles. She arranges them in a circle randomly. If the probability that no two green marbles are adjacent is $\frac{p}{q}$ where the positive integers p, q have no common factors other than 1, what is $p + q$?

Answer (40)

16. If x and y are positive integers such that $(x - 4)(x - 10) = 2^y$, find the maximum possible value of $x + y$.

Answer (16)

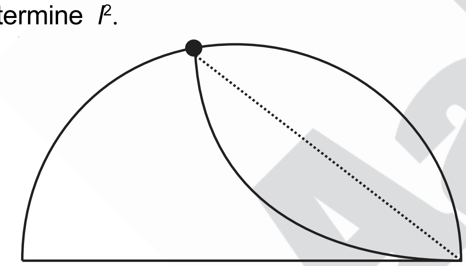
17. Two sides of a regular polygon with n sides, when extended, meet at an angle of 28°. What is the smallest possible value of n ?

Answer (45)

18. Let D, E, F be points on the sides BC, CA, AB of a triangle ABC, respectively. Suppose AD, BE, CF are concurrent at P. If $PF/PC = 2/3$, $PE/PB = 2/7$ and $PD/PA = m/n$, where m, n are positive integers with $\gcd(m, n) = 1$, find $m + n$.

Answer (45)

19. A semicircular paper is folded along a chord such that the folded circular arc is tangent to the diameter of the semicircle. The radius of the semicircle is 4 units and the point of tangency divides the diameter in the ratio 7 : 1. If the length of the crease (the dotted line segment in the figure) is l then determine l^2.



Answer (39)

20. Two people A and B start from the same place at the same time to travel around a circular track of length 100m in opposite directions. First B goes more slowly than A until they meet, then by doubling his rate he next meets A at the starting point. Let d m be the distance travelled by B before he met A for the first time after leaving the starting point. Find the integer closest to d.

Answer (41)

21. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{9, 10, 11, 12, 13, 14, 15, 16\}$ and $C = \{17, 18, 19, 20, 21, 22, 23, 24\}$. Find the number of triples (x, y, z) such that $x \in A$, $y \in B$, $z \in C$ and $x + y + z = 36$.

Answer (46)

22. Let ABC be a triangle with $\angle BAC = 90°$ and D be the point on the side BC such that $AD \perp BC$. Let r, r_1, and r_2 be the inradii of triangles ABC, ABD and ACD, respectively. If r, r_1, and r_2 are positive integers and one of them is 5, find the largest possible value of $r + r_1 + r_2$.

Answer (30)

23. Find the largest positive integer N such that the number of integers in the set $\{1, 2, 3, ..., N\}$ which are divisible by 3 is equal to the number of integers which are divisible by 5 or 7 (or both).

Answer (35)

24. Two circles, S_1 and S_2, of radii 6 units and 3 units respectively, are tangent to each other externally. Let AC and BD be their direct common tangents with A and B on S_1, and C and D on S_2. Find the area of quadrilateral $ABDC$ to the nearest integer.

Answer (68)

25. A five digit number $n = \overline{abcde}$ is such that when divided respectively by 2, 3, 4, 5, 6 the remainders are a, b, c, d, e. What is the remainder when n is divided by 100?

Answer (11)

26. Let a, b, c be three distinct positive integers such that the sum of any two of them is a perfect square and having minimal sum, $a + b + c$. Find this sum.

Answer (55)

27. Let ABC be an acute-angled triangle and P be a point in its interior. Let P_A, P_B, and P_C be the images of P under reflection in the sides BC, CA and AB, respectively. If P is the orthocentre of the triangle $P_AP_BP_C$ and if the largest angle of the triangle that can be formed by the line segments PA, PB, and PC is $x°$; determine the value of x.

Answer (60)

28. For a natural number n, let n' denote the number obtained by deleting zero digits, if any. (For example, if $n = 260$, $n' = 26$; if $n = 2020$, $n' = 22$.) Find the number of 3-digit numbers n for which n' is a divisor of n, different from n.

Answer (93)

29. Consider a permutation $(a_1, a_2, a_3, a_4, a_5)$ of $\{1, 2, 3, 4, 5\}$. We say the 5-tuple $(a_1, a_2, a_3, a_4, a_5)$ is flawless if for all $1 \le i < j < k \le 5$, the sequence (a_i, a_j, a_k) is not an arithmetic progression (in that order). Find the number of flawless 5-tuples.

Answer (19)

30. Ari chooses 7 balls at random from n balls numbered 1 to n. If the probability that no two of the drawn balls have consecutive numbers equals the probability of exactly one pair of consecutive numbers in the chosen balls, find n.

Answer (54)