MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

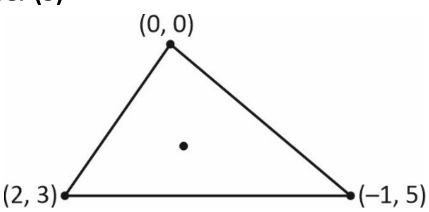
1. Let a circle C with radius r passes through four distinct points (0, 0), (K, 3k), (2, 3) and (–1, 5), such that $k \neq 0$, then $(10k + 2r^2)$ is equal to

(1) 35 (2) 34

(3) 27 (4) 32

Answer (3)

Sol.



Centre $\equiv \left(\frac{-1}{2}, \frac{5}{2}\right)$

$\Rightarrow$ Radius = $\frac{\sqrt{26}}{2}$

$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y-\frac{5}{2}\right)^2 = \frac{26}{4}$

$(2x + 1)^2 + (2y - 5)^2 = 26$

$\Rightarrow 4x^2 + 4y^2 + 4x - 20y = 0$

(K, 3k) lie on circle

$4k^2 + 36k^2 + 4k - 60k = 0$

$40k - 56 = 0 \Rightarrow k = \frac{7}{5}$

$\Rightarrow 10k + 2r^2 = 14 + 13 = 27$

2. $I = \int_0^{\pi} \frac{8x}{4\cos^2 x + \sin^2 x} dx$ equals to

(1) π^2 (2) $4\pi^2$

(3) $2\pi^2$ (4) $\frac{3}{2}\pi^2$

Answer (3)

Sol. $I = \int_0^{\pi} \frac{8x}{4\cos^2 x + \sin^2 x} dx$ ...(1)

$I = \int_0^{\pi} \frac{8(\pi - x)}{4\cos^2(\pi - x) + \sin^2(\pi - x)} dx$

$I = \int_0^{\pi} \frac{8(\pi - x)}{4\cos^2 x + \sin^2 x} dx$ ...(2)

Adding (1) and (2)

$2I = 8\pi \int_0^{\pi} \frac{1}{4\cos^2 x + \sin^2 x} dx$

$I = 4\pi \times 2 \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{4 \tan^2 x} dx$

Put $\tan x = t$

$\sec^2 x \, dx = dt$

$I = 8\pi \int_0^{\infty} \frac{dt}{4 + t^2}$

$I = 8\pi \frac{1}{2} \left(\tan^{-1} \frac{t}{2}\right)_0^{\infty}$

$I = 4\pi \left(\frac{\pi}{2}\right)$

$\boxed{I = 2\pi^2}$

3. $S = 1 + \frac{1+3}{1!} + \frac{1+3+5}{2!} + \ldots\infty$. The value of S is equal to

(1) $4e - 2$ (2) $4e$

(3) $5e$ (4) $7e$

Answer (3)

Sol. $S = \frac{1^2}{0!} + \frac{2^2}{1!} + \frac{3^2}{2!} + \ldots$

$$T_n = \frac{n^2}{(n-1)!}$$

$$= \frac{n^2-1}{(n-1)!} + \frac{1}{(n-1)!}$$

$$= \frac{(n-1)(n+1)}{(n-1)!} + \frac{1}{(n-1)!}$$

$$= \frac{n+1}{(n-2)!} + \frac{1}{(n-1)!}$$

$$= \frac{n-2+3}{(n-2)!} + \frac{1}{(n-1)!}$$

$$T_n = \frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}$$

$$\sum_{n=1}^{\infty} T_n = e + 3e + e = 5e$$

4. Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} + 3y\tan^2 x + 3y = \sec^2 x$ such that $f(0) = \frac{e^3}{3} + 1$, then $f\left(\frac{\pi}{4}\right)$ is equal to

(1) $(1+e^{-3})$ (2) $\frac{2}{3}\left(1+\frac{1}{e^3}\right)$

(3) $\frac{1}{3}\left(1-\frac{1}{e^3}\right)$ (4) $\frac{1}{3}\left(1+\frac{1}{e^3}\right)$

Answer (2)

Sol. $\frac{dy}{dx} + 3y(1+\tan^2 x) = \sec^2 x$

$$\Rightarrow \frac{dy}{dx} + y(3\sec^2 x) = \sec^2 x$$

I.F. = $e^{\int 3\sec^2 dx} = e^{3\tan x}$

$$\Rightarrow y(e^{3\tan x}) = \int e^{3\tan x} \cdot \sec^2 x dx + c$$

$$= \frac{e^{3\tan x}}{3} + c$$

$$f(0) = \frac{e^3}{3} + 1$$

$$y(e^0) = \frac{e^0}{3} + c = \frac{e^3}{3} + 1$$

$$\Rightarrow c = \frac{e^3}{3} + \frac{2}{3}$$

$$f\left(\frac{\pi}{4}\right) \Rightarrow y\left(\frac{\pi}{4}\right)e^3 = \frac{e^3}{3} + \frac{e^3}{3} + \frac{2}{3}$$

$$\Rightarrow y\left(\frac{\pi}{4}\right) = \frac{1}{e^3}\left[\frac{2e^3+2}{3}\right]$$

$$= \frac{2}{3}\left[1 + \frac{1}{e^3}\right]$$

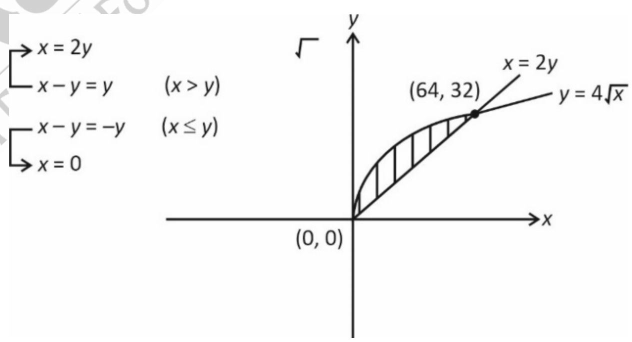
5. Area bounded by $|x-y| \le y \le 4\sqrt{x}$ is equal to (in square units)

(1) $\frac{2048}{3}$ (2) $\frac{1024}{3}$

(3) $\frac{512}{3}$ (4) $\frac{128}{3}$

Answer (2)

Sol. $|x-y| \le y \le 4\sqrt{x}$



$$\text{Area} = \int_0^{64}\left(4\sqrt{x} - \frac{x}{2}\right)dx$$

$$= \left.\frac{4x^{3/2}}{\frac{3}{2}} - \frac{x^2}{4}\right]_0^{64} = \frac{8}{3}(8)^3 - \frac{64^2}{4} = \frac{1024}{3}$$

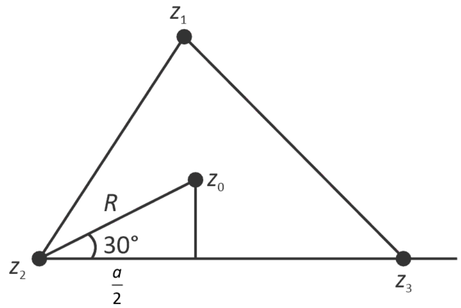
6. Let $A(z_1)$, $B(z_2)$ and $C(z_3)$ are the vertices of an equilateral triangle. If z_0 is the centroid of triangle ABC and $|z_1 - z_2| = 1$ then the value of $\sum_{i=1}^{3} |z_i - z_0|^2$ is equal to

(1) 1 (2) 2

(3) 3 (4) 9

Answer (1)

Sol.



$$\Rightarrow \cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{\frac{a}{2}}{R} \Rightarrow R = \frac{a}{\sqrt{3}}$$

$$\Rightarrow |z_1 - z_0| = |z_2 - z_0| = |z_3 - z_0| = \frac{a}{\sqrt{3}}$$

$$\Rightarrow \sum_{i=1}^{3} |z_i - z_0|^2 = 3 \cdot \left(\frac{a}{\sqrt{3}}\right)^2 = \frac{3.a^2}{3}$$

$= a^2 = |z_1 - z_2|^2 = 1$

7. If $f(x) = \left||x+2| - 2|x|\right|$, then the sum of number of points of local maxima and local minima is

(1) 5 (2) 3

(3) 2 (4) 7

Answer (2)

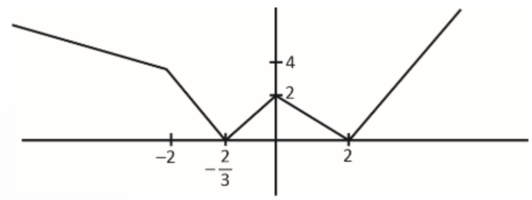
Sol. $$f(x) = \begin{cases} |-x-2+2x| & x \le -2 \\ |x+2+2x| & -2 \le x \le 0 \\ |x+2-2x| & x \ge 0 \end{cases}$$

$$f(x) = \begin{cases} |-x-2+2x| & x \le -2 \\ |x+2+2x| & -2 \le x \le 0 \\ |x+2-2x| & x \ge 0 \end{cases}$$

$$f(x) = \begin{cases} |x-2| & x \le -2 \\ |3x+2| & -2 < x \le 0 \\ |2-x| & x > 0 \end{cases}$$

$$f(x) = \begin{cases} 2-x & x \le -2 \\ -3x-2 & -2 < x \le -\frac{2}{3} \\ 3x+2 & -\frac{2}{3} < x \le 0 \end{cases}$$

$$f(x) = \begin{cases} 2-x & x \le -2 \\ -3x-2 & -2 < x \le -\frac{2}{3} \\ 3x+2 & -\frac{2}{3} < x \le 0 \\ 2-x & 0 < x < 2 \\ x-2 & x \ge 2 \end{cases}$$



No. of maxima = 1

No. of minima = 2

8. If $x(x-2)(12-k) = 2$ has both roots same. Then the distance of $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is

(1) 24 (2) 14

(3) 15 (4) 20

Answer (3)

Sol. $x^2 - 2x - \dfrac{2}{12-k} = 0$

$D = 0$

$$4 - 4 \cdot \left(-\frac{2}{12-k}\right) = 0$$

$$\Rightarrow 1 + \frac{2}{12-k} = 0$$

$$\Rightarrow k = 14$$

$$\therefore \left(k, \frac{k}{2}\right) \equiv (14, 7)$$

$$d = \left|\frac{3 \times 14 + 4 \times 7 + 5}{5}\right|$$

$= 15$

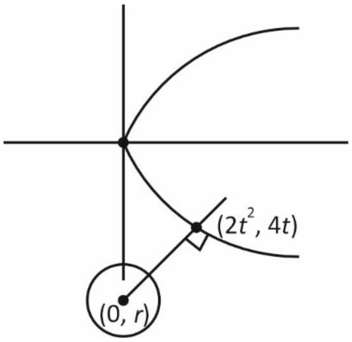
9. The shortest distance between the parabola $y^2 = 8x$ and the circle $x^2 + y^2 + 12y + 35 = 0$ is

(1) $(2\sqrt{2}-1)$ units (2) $(\sqrt{2}-1)$ units

(3) $(2\sqrt{2}+1)$ units (4) $(\sqrt{2}+1)$ units

Answer (1)

Sol.



The common normal passes through centre and on which shortest distance will lie.

$$y^2 = 8x \Rightarrow 2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$$

$\Rightarrow$ Slope of normal: $\frac{-y}{4} = \frac{-4t}{4} = -t$

$\Rightarrow$ $-t = \frac{4t+6}{2t^2-0} \Rightarrow 2t^3 + 4t + 6 = 0$

$\Rightarrow$ $(t+1)(2t^2 - 2t + 6) = 0$

$\Rightarrow$ $t = -1$ is only point

$\Rightarrow$ distance = distance between $(0, -6)$ to $(2, -4)$ – radius of circle = $= 2\sqrt{2} - 1$

10. Let $f(x) = \log_4(1 - \log_7(x^2 - 9x + 8))$. If the domain of $f(x)$ is $(\alpha, \beta) \cup (\gamma, \delta)$. Then $\alpha + \beta + \gamma + \delta$ equals to

(1) 18 (2) 27

(3) 21 (4) 9

Answer (1)

Sol. $1 - \log_7(x^2 - 9x + 8) > 0$

$\Rightarrow \log_7(x^2 - 9x + 8) < 1$

$\Rightarrow x^2 - 9x + 8 < 7$

$\Rightarrow x^2 - 9x + 1 < 0$

$$\Rightarrow x = \frac{9 \pm \sqrt{81-4}}{2}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{77}}{2}$$

$x^2 - 9x + 8 > 0$

$\Rightarrow x^2 - 8x - x - 8 > 0$

$\Rightarrow x(x-8) - 1(x-8) > 0$

$\Rightarrow (x-1)(x-8) > 0$

$$\therefore x \in \left(\frac{9-\sqrt{77}}{2}, 1\right) \cup \left(8, \frac{9+\sqrt{77}}{2}\right)$$

$$\therefore \alpha + \beta + \gamma + \delta = \frac{9-\sqrt{77}}{2} + 1 + 8 + \frac{9+\sqrt{77}}{2}$$

$\therefore \boxed{\alpha + \beta + \gamma + \delta = 18}$

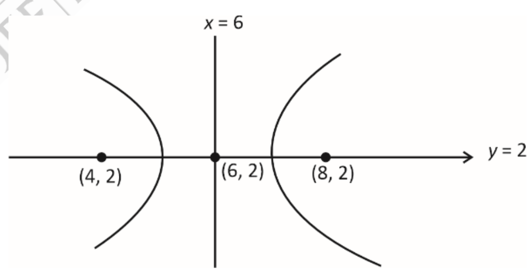
11. If the coordinates of foci of a hyperbola $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$ are (4, 2) and (8, 2). Then $(\alpha + \beta + \gamma)$ is equal to

(1) 81 (2) 137

(3) 121 (4) 141

Answer (4)

Sol.



Hyperbola:

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1$$

$$b^2x^2 - a^2y^2 - 12xb^2 + 4ya^2 + 4ya^2 + 36b^2 - 4a^2 - a^2b^2 = 0$$

Comparing: $\frac{b^2}{a^2} = 3 \Rightarrow e^2 = 1 + \frac{b^2}{a^2} = 4$

$\Rightarrow e = 2$

Similarly, $2ae = 4 \Rightarrow a = 1 \Rightarrow b = \sqrt{3}$

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 108 - 4 - 3 = 0$

$3x^2 - y^2 - 36x + 4y + 101 = 0$

$\Rightarrow \quad \alpha = 36$

$\beta = 4$

$\gamma = 101$

$\Rightarrow \quad \alpha + \beta + \gamma = 141$

12. Let the probability distribution is defined for a random variable x as $p(x) = k(1 - 3^{-x})$ for x = 0, 1, 2, 3. Then $P(x \geq 2)$ is

(1) $\frac{5}{17}$ (2) $\frac{25}{34}$

(3) $\frac{25}{68}$ (4) $\frac{7}{25}$

Answer (2)

Sol. $\Rightarrow \sum p(x) = 1$

$\Rightarrow k\left[1 - 3^{-0} + 1 - 3^{-1} + 1 - 3^{-2} + 1 - 3^{-3}\right] = 0$

$$k\left[4 - \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3}\right)\right] = 1$$

$\Rightarrow \; k = \frac{27}{68}$

Now $P(x \geq 2) = p(x = 3) + P(x = 2)$

$$= \frac{27}{68}\left(1 - \frac{1}{3^3}\right) + \frac{27}{68}\left(1 - \frac{1}{3^2}\right)$$

$$= \frac{26}{68} + \frac{3}{68}(8) = \frac{50}{68} = \frac{25}{34}$$

13. If the mean and variance of a data $x_1 = 1$, $x_2 = 4$, $x_3 = a$, $x_4 = 7$, $x_5 = b$ are 5 and 10 respectively. If new data is $r + x_r$, $r \in \{1, 2, 3, 4, 5\}$, then the new variance is

(1) 17.6 (2) 16.9

(3) 20.4 (4) 21.4

Answer (3)

Sol. $5 = \frac{1+4+a+7+b}{5} \Rightarrow a + b = 13$

$$10 = \frac{1 + 16 + a^2 + 49 + b^2}{5} - (5)^2$$

$a^2 + b^2 = 109$

$a = 3$, $b = 10$

New digits: $r + x_r$, $r \in [1, 5]$

$1 + x_1$, $2 + x_2$, $3 + x_3$, $4 + x_4$, $5 + x_5$

$\equiv 2, 6, 6, 11, 15$

Variance = $\frac{2^2 + 6^2 + 6^2 + 11^2 + 15^2}{5} - \left(\frac{2+6+6+11+15}{5}\right)^2$

= 20.4

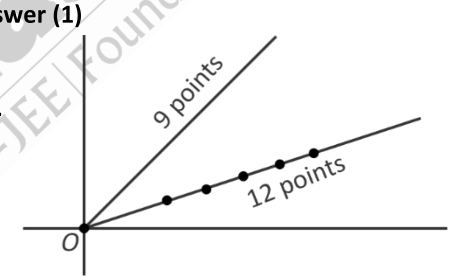
14. Let 9 points lie on the line $y = 2x$ and 12 points on the $y = \frac{x}{2}$ in the first quadrant. Find the number of triangles formed using these points and origin.

(1) 1134 (2) 1096

(3) 1120 (4) 1026

Answer (1)

Sol.



Total triangles : (two points of $y = 2x$, 1 point of $y = \frac{x}{2}$) + (two points on $y = \frac{x}{2}$, 1 point of $y = 2x$) + (1 point on $y = 2x$, 1 point of $y = \frac{x}{2}$ and origin)

$= {}^9C_2 \cdot {}^{12}C_1 + {}^9C_1 \cdot {}^{12}C_2 + {}^1C_1 \cdot {}^9C_1 \cdot {}^{12}C_1$

= 1134

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}} = P$,

then 96 ln P is

Answer (32)

Sol. $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$ 1^{∞} (form)

l^L

$$L = \lim_{x\to 0}\left(\frac{\tan x}{x} - 1\right)^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0}\left(\frac{\tan - x}{x^3}\right)$$

$$= \lim_{x\to 0}\left(\frac{x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots - x}{x^3}\right)$$

$$= \frac{1}{3}$$

$$\therefore \quad = \lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}} = e^{1/3} = P$$

$$96 \ln P = \frac{96}{3} = 32$$

22. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. A relation R is defined such that xRy iff $y = \max\{x, 1\}$.

Number of elements required to make it reflexive is l, number of elements required to make it symmetric is m and number of elements in the relation R is n. Then value of $l + m + n$ is equal to

Answer (15)

Sol. $R = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 2), (3, 3\}$

$\therefore$ $l = 4$ i.e., $(-3, -3), (-2, -2), (-1, -1), (0, 0)$

$m = 4$ i.e., $(1, -3), (1, -2), (1, -1), (1, 0)$

$n = 7$

$l + m + n = 15$

23. If $(1 + x + x^2)^{10} = 1 + a_1x + a_2x^2 + \ldots$, then $(a_1 + a_3 + a_5 + \ldots + a_{19}) - 11a_2$ equals to

Answer (28919)

Sol. $(1 + x + x^2)^{10} = 1 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$ ...(i)

$x = 1$

$3^{10} = 1 + a_1 + a_2 + \ldots + a_{20}$ ...(ii)

$x = -1$

$1 = 1 - a_1 + a_2 + \ldots + a_{20}$ ...(iii)

(ii) – (iii)

$$3^{10} - 1 = 2\left[a_1 + a_3 + \ldots + a_{19}\right]$$

$$\Rightarrow a_1 + a_3 + a_5 + \ldots + a_{19} = \frac{3^{10} - 1}{2}$$

Diff. (i) w.r.t. x

$$10(1 + x + x^2)^9(1 + 2x) = a_1 + 2a_2x + \ldots + 20a_{20}x^{19}$$

Again diff. w.r.t. x and substitute $x = 0$

$$10\left[9(1 + x + x^2)^8(1 + 2x)^2 + (1 + x + x^2)^9(2)\right] = 2a_2 + \ldots$$

$$10\left[9 + 2\right] = 2a_2$$

$55 = a_2$

Now

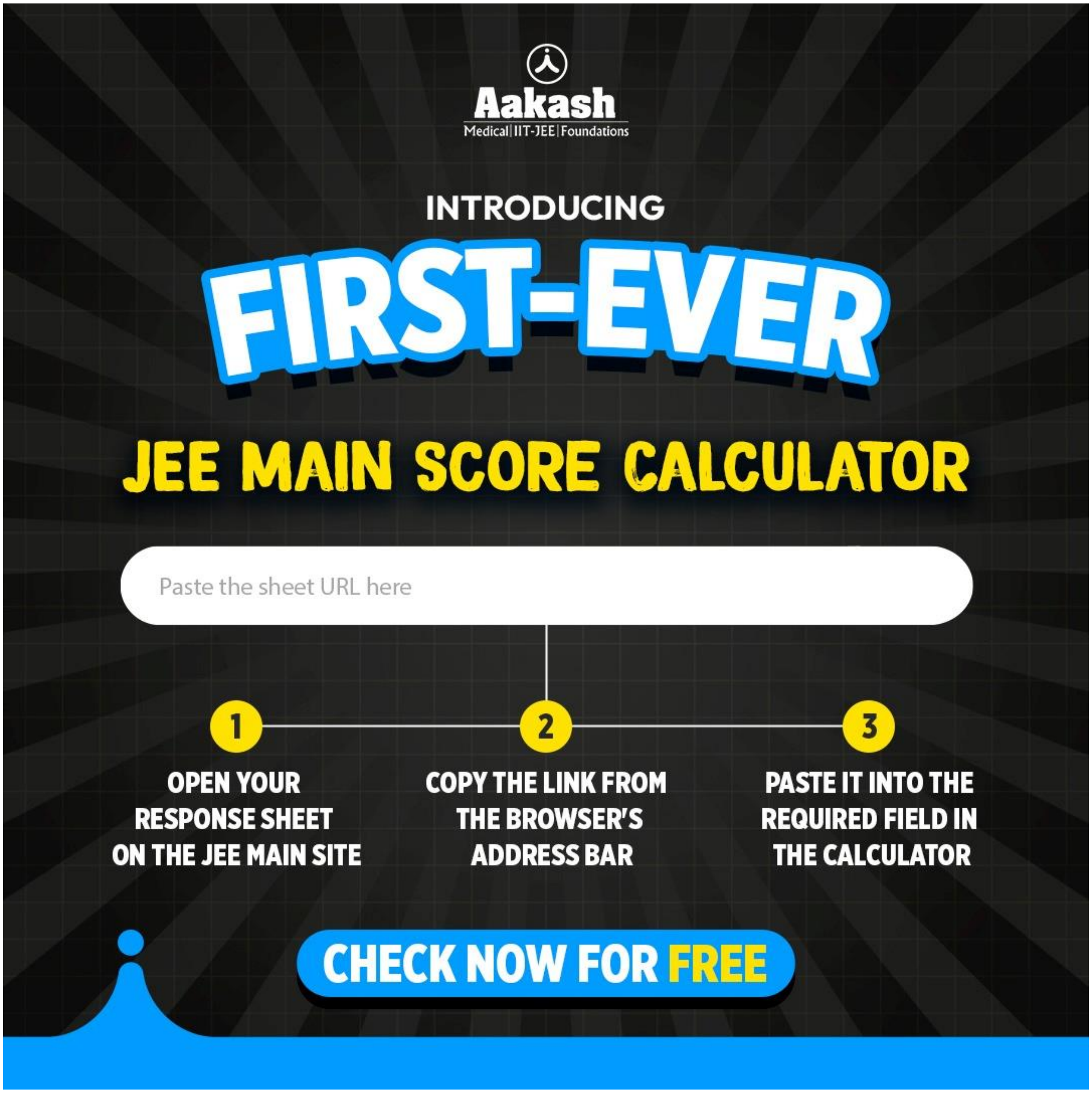
$$(a_1 + a_3 + \ldots + a_{19}) - 11a_2 = \frac{3^{10} - 1}{2} - 55 \times 11$$

$= 28919$

24.

25.

❑ ❑ ❑



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