# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Let $A$ be $3 \times 3$ matrix such that $\det(A) = 5$. If $\det(3\,\mathrm{adj}(2A\,\mathrm{adj}(2A))) = 2^{\alpha}\cdot3^{\beta}\cdot5^{\gamma}$, then $(\alpha + \beta + \gamma)$ is equal to

   (1) 25  (2) 26
   (3) 27  (4) 28

**Answer (3)**

**Sol.** $|3\mathrm{adj}\,(2A\,\mathrm{adj}(2A))| = 3^3|\,2A(\mathrm{adj}\,2A)|^2$

$= 3^3(2^3)^2|A|^2|\mathrm{adj}(2A)|^2$

$= 3^3\cdot2^6\cdot5^2(|(2A)|^2)^2$

$= 3^3\cdot2^6\cdot5^2\cdot\,|2A|^4$

$= 3^3\cdot2^6\cdot5^2\cdot\,(2^3)^4\cdot|A|^4$

$= 2^{18}\cdot3^3\cdot5^6 = 2^{\alpha}\cdot3^{\,\beta}\cdot5^{\gamma}$

$\Rightarrow \alpha = 18$

   $\beta = 3$

   $\gamma = 6$

$\Rightarrow \alpha + \beta + \gamma = 27$

2. The sum of all rational numbers in $\left(2+\sqrt{3}\right)^8$ is

   (1) 18117  (2) 18817
   (3) 17280  (4) 1800

**Answer (2)**

**Sol.** $S = {}^8C_0\left(2\right)^8 + {}^8C_1 2^7\left(\sqrt{3}\right) + \ldots + {}^8C_8\left(\sqrt{3}\right)^8$

Sum of rational terms

$= {}^8C_0\left(2\right)^8 + {}^8C_2 2^6\left(\sqrt{3}\right)^2 + {}^8C_4\left(2\right)^4\left(\sqrt{3}\right)^4 +$

$\qquad\qquad {}^8C_6\left(2\right)^2\left(\sqrt{3}\right)^2 + {}^8C_8\left(\sqrt{3}\right)^8$

$= 18,817$

3. If the sum $\sum_{r=1}^{9}\left(\dfrac{r+3}{2^r}\right)\cdot{}^9C_r = \alpha\cdot\left(\dfrac{3}{2}\right)^9 - \beta$, then the value of $(\alpha + \beta)^2$ is equal to

   (1) 9  (2) 81
   (3) 27  (4) 36

**Answer (2)**

**Sol.** $\sum_{r=1}^{9}\left(\dfrac{r+3}{2^r}\right)\cdot{}^9C_r = \sum_{r=1}^{9}\dfrac{r}{2^r}\cdot\dfrac{9}{r}\cdot{}^8C_{r-1} + \sum_{r=1}^{9}3\cdot{}^9C_r\left(\dfrac{1}{2}\right)^r$

$= \dfrac{9}{2}\sum_{r=1}^{9}{}^8C_{r-1}\left(\dfrac{1}{2}\right)^{r-1} + 3\sum_{r=1}^{9}{}^9C_r\left(\dfrac{1}{2}\right)^r$

$= \dfrac{9}{2}\sum_{r=0}^{8}{}^8C_{r-1}\left(\dfrac{1}{2}\right)^r + 3\sum_{r=1}^{9}{}^9C_r\left(\dfrac{1}{2}\right)^r$

$= \dfrac{9}{2}\left(1+\dfrac{1}{2}\right)^8 + 3\left[\left(1+\dfrac{1}{2}\right)^9 - {}^9C_0\left(\dfrac{1}{2}\right)^0\right]$

$= \dfrac{9}{2}\cdot\dfrac{3^8}{2^8} + 3\left[\dfrac{3^9}{2^9} - 1\right]$

$= \dfrac{3^{10}}{2^9} + \dfrac{3^{10}}{2^9} - 3 = 4\cdot\dfrac{3^{10}}{2^{10}} - 3$

$= 4\left(\dfrac{3}{2}\right)^{10} - 3$

$= 6\left(\dfrac{3}{2}\right)^9 - 3$

$\alpha = 6, \beta = 3 \Rightarrow (\alpha + \beta)^2 = 81$

4. Let $S_n = 1 + 3 + 11 + 25 + 45 + \ldots$ Then sum upto $20^{th}$ term equals

   (1) 6200  (2) 7200
   (3) 7240  (4) 6240

**Answer (3)**

**Sol.**
$$S = 1 + 3 + 11 + 25 + .... + T_n$$
$$\underline{S = 1 + 3 + 11 + .... T_{n-1} + T_n}$$
$$T_n = 1 + 2 + 8 + 14 + .... + (T_n - T_{n-1})$$

$$T_n = 1 + \frac{n-1}{2}[4 + (n-2)6]$$

$$= 1 + \left(\frac{n-1}{2}\right)[6n - 8]$$

$$= 1 + (n-1)(3n-4)$$

$$= 1 + 3n^2 - 4n - 3n + 4$$

$$T_n = 3n^2 - 7n + 5$$

$$S_n = \sum Tn = 3\sum n^2 - 7\sum n + \sum 5$$

$$= \frac{3n(n+1)(2n+1)}{6} - \frac{7n(n+2)}{2} + 5n$$

now $n = 20$

$$= \frac{3 \times 20 \times 21 \times 41}{6} - \frac{7 \times 20 \times 21}{2} + 5 \times 20 = 7240$$

5. Evaluate $\int x^3 \sqrt{1-x^2}\,dx$

(1) $\dfrac{-1}{15}\left(1-x^2\right)^{3/2}\left(3x^2+2\right)+C$

(2) $\dfrac{1}{3}\left(1+x^2\right)^{2/3} - \sqrt{1-x^2}+C$

(3) $\dfrac{2}{3}\left(1-x^2\right)^{3/2}\left(3x^2+2\right)+C$

(4) $\dfrac{1}{3}\left(1-x^2\right)^{2/3} + \sqrt{1-x^2}+C$

**Answer (1)**

**Sol.** $\int x^3 \sqrt{1-x^2}\,dx$

Put $1 - x^2 = t^2$

$-2x\,dx = 2t\,dt$

$$-\int t^2(1-t^2)dt = -\left[\frac{t^3}{3} - \frac{t^5}{5}\right] + C$$

$$\frac{t^5}{5} - \frac{t^3}{3} + C$$

$$= \frac{(1-x^2)^{5/2}}{5} - \frac{\left(1-x^2\right)^{3/2}}{3} + C$$

$$= \frac{\left(1-x^2\right)^{3/2}}{15}\left[3(1-x^2) - 5\right] + C$$

$$= \frac{-\left(1-x^2\right)^{3/2}}{15}\left(3x^2+2\right) + C$$

6. A relation $R = \{(x, y); x, y \in A = \{-3, -2, -1, 0, 1, 2, 3\}$ such that $x^2 + 2y \le 4\}$. If the number of ordered pairs in relation $R$ be $r$ and number of ordered pairs required to add in $R$ so that it becomes reflexive relations is $m$, then $r + m$ is equal to

(1) 26      (2) 28

(3) 24      (4) 23

**Answer (2)**

**Sol.** $x^2 + 2y \le 4$

$A = \{-3, -2, -1, 0, 1, 2, 3\}$

$x^2 \le 4 - 2y$

For $y = -3$

$x^2 \le 4 - (2(-3))$

$x^2 \le 10$

$\Rightarrow x^2 \in \{-3, -2, -1, 0, 1, 2, 3\}$

For $y = -2$

$x^2 \le 4 - 2(-2)$

$x^2 \le 8$

$\Rightarrow x \in \{-2, -1, 0, 1, 2\}$

For $y = -1$

$x^2 \le 4 - (2(-1))$

$x^2 \le 6$

$\Rightarrow x \in \{-2, -1, 0, 1, 2\}$

For $y = 0$

$x^2 \le 4$

$\Rightarrow x \in \{-2, -1, 0, 1, 2\}$

For $y = 1$

$x^2 \le 2$

$\Rightarrow$ $x \in \{-1, 0, 1\}$

For $y = 2$

$x^2 \leq 0$

$\Rightarrow$ $x \in \{0\}$

For $y = 3$

$x^2 \leq -2 \Rightarrow$ No value of $x$

Tota number of ordered pair in relation $R$ is, $r = 26$

For it to be reflexive we have to add $\{(3, 3), (2, 2)\}$

$\Rightarrow$ $m = 2$

$\Rightarrow$ $r + m = 28$

7. The radius of circle touching both parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

(1) $\dfrac{7\sqrt{2}}{2}$

(2) $\dfrac{7\sqrt{2}}{6}$

(3) $\dfrac{7\sqrt{2}}{8}$

(4) $\dfrac{7\sqrt{2}}{4}$

**Answer (4)**

**Sol.**
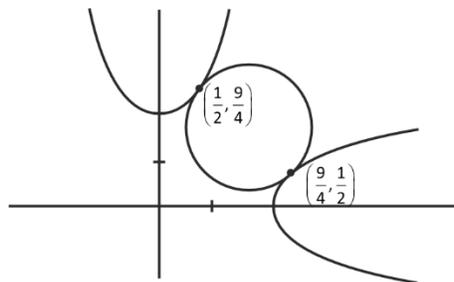


The circle will have its centre at $x = y$ line and since these parabolas are symmetric about the line $y = x$. The slope will be of tangents at closest points.

$\Rightarrow$ $y^2 = x - 2$  $\Rightarrow$ $2y\dfrac{dy}{dx} = 1$  $\Rightarrow$ $y = \dfrac{1}{2}$

$\Rightarrow$ Point will be $\left(\dfrac{9}{4}, \dfrac{1}{2}\right)$

Similarly on $x^2 = y - 2$

$\Rightarrow$ $2x = \dfrac{dy}{dx} = 1$  $\Rightarrow$ $x = \dfrac{1}{2}$

$\Rightarrow$ $\left(\dfrac{1}{2}, \dfrac{9}{4}\right)$

Circle's diameter will be equal to shortest distance

$2r = \sqrt{\left(\dfrac{9}{4} - \dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2} - \dfrac{9}{4}\right)^2} = \sqrt{\dfrac{7}{4} \times 2} = \sqrt{\dfrac{7}{2}}$

$\Rightarrow$ $r = \dfrac{1}{2}\sqrt{\dfrac{7}{2}} = \sqrt{\dfrac{7}{8}} = \dfrac{7\sqrt{2}}{4}$

8. Let $3x + 2\tan x = \pi$, $x \in [-2\pi, 2\pi] - \left\{\pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}\right\}$

Then number of values of $x$ satisfying the above condition is

(1) 4

(2) 5

(3) 6

(4) 7

**Answer (2)**

**Sol.** $3x + 2\tan x = \pi$

$2\tan x = \pi - 3x$

$\tan x = \dfrac{\pi - 3x}{2}$

5 solution

$\because$ in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ well get 1 solution as $\tan x$ is increasing in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

$\Rightarrow$ $\left.\begin{array}{l}\left(\dfrac{-3\pi}{2}, \dfrac{\pi}{2}\right) \rightarrow 1 \text{ solution} \\[2mm] \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right) \rightarrow 1 \text{ solution} \\[2mm] \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right) \rightarrow 1 \text{ solution} \\[2mm] \left(\dfrac{3\pi}{2}, 2\pi\right) \rightarrow 1 \text{ solution} \\[2mm] \left(-2\pi, \dfrac{-3\pi}{2}\right) \rightarrow 1 \text{ solution}\end{array}\right\} 5$

9. Let $\int_0^x g(t)dt = x - \int_0^x tg(t)dt$, $x \ge 0$ and

$\dfrac{dy}{dx} - y\tan x = 2(x+1)\sec x\, g(x)$ satisfying the

condition $y(0) = 0$. Then $y\left(\dfrac{\pi}{3}\right)$ is

(1) $\dfrac{2\pi}{3}$  (2) $\dfrac{4\pi}{3}$

(3) $\pi$  (4) $2\pi$

**Answer (2)**

**Sol.** Differentiate both side w.r.t $x$

$g(x) = 1 - xg(x)$

$g(x)(1 + x) = 1$

$g(x) = \dfrac{1}{1+x}$

Also $\dfrac{dy}{dx} - y\tan x = 2(x+1)\sec x\, g(x)$

$I.F = e^{-\int \tan x\, dn.}$

$I.F = e^{-(-\ln\cos x)}$

$I.F = \cos x$

$y\cos x = \int 2(x+1)\sec x \dfrac{1}{(1+x)}\cos x\, dx$

$y\cos x = \int 2\, dx$

$y\cos x = 2x + c$  ...(i)

$y(0) = 0$

$\Rightarrow c = 0$

from (i)

$y\cos x = dx$

Put $x = \dfrac{\pi}{3}$

$y \cdot \dfrac{1}{2} = \dfrac{2\pi}{3}$

$y = \dfrac{4\pi}{3}$

10. If $f(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$. Then, the

value of $f'(x) + f(x)$ is

(1) $-1$

(2) $28$

(3) $27$

(4) $1$

**Answer (1)**

**Sol.** $f(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$

$f(x) = \sin x(1) - \cos x(0) + (\sin x + \cos x + 1)(-1)$

$f(x) = \sin x - \sin x - \cos x - 1$

$f(x) = -\cos x - 1$

$f'(x) = \sin x$

$f''(x) - \cos x$

$f(x) + f''(x) = -\cos x - 1 + \cos x$

$\qquad\qquad = -1$

11. Let $\alpha$, $\beta$ are the roots of the equation $x^2 + \sqrt{3}x - 16 = 0$ and $\gamma$, $\delta$ are the roots of the equation $x^2 + 3x - 1 = 0$. If $Q_n = \alpha^n + \beta^n \ \forall \ n \in N$ and $P_n = \gamma^n + \delta^n \ \forall \ n \in N$ then the

value of $\dfrac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \left(\dfrac{P_{25} - P_{23}}{P_{24}}\right)$ is

(1) $5$  (2) $6$

(3) $7$  (4) $8$

**Answer (1)**

**Sol.** $x^2 + 3x - 1 = 0 \overset{\gamma}{\underset{\delta}{\diagdown}} \Rightarrow x^2 - 1 = -3x$

$\Rightarrow P^n = \gamma^n + \delta^n$

$P_{25} - P_{23} = (\gamma^{25} - \gamma^{23}) + (\delta^{25} - \delta^{23})$

$= \gamma^{23}(\gamma^2 - 1) + \delta^{23}(\delta^2 - 1)$

$= \gamma^{23}(-3\gamma) + \delta^{23}(-3\delta)$

$= -3[\gamma^{24} + \delta^{24}]$

$$\Rightarrow \frac{P_{25} - P_{23}}{P_{24}} = (-3)$$

Similarly

$$x^2 + \sqrt{3}x - 16 = 0 \underset{\beta}{\overset{\alpha}{\lessgtr}} \quad Q_n = \alpha^n + \beta^n$$

$$\Rightarrow Q_{25} + \sqrt{3}Q_{24} = (\alpha^{25} + \sqrt{3}\alpha^{24}) + (\beta^{25} + \sqrt{3}\beta^{24})$$

$$= \alpha^{23}(\alpha^2 + \sqrt{3}\alpha) + \beta^{23}(\beta^2 + \sqrt{3}\beta)$$

$$= \alpha^{23}(16) + 16\beta^{23}$$

$$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2 \cdot Q_{23}} = \frac{16(\alpha^{23} + \beta^{23})}{2(\alpha^{23} + \beta^{23})} = 8$$

$$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \frac{(P_{25} - P_{23})}{P_{24}} = 8 + (-3) = 5$$

12. If $y = \max\{|x|, x, |x-2|\}$, then the area under the curve from $x = -2$ to $x = 4$ is (in sq. units)

(1) 15  (2) 20
(3) 12  (4) 8

**Answer (1)**

**Sol.** $\max\{|x|, x, |x-2|\}$
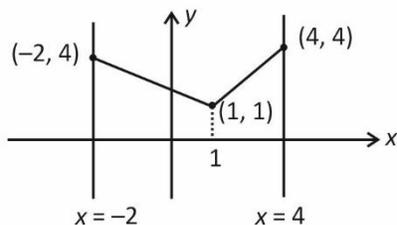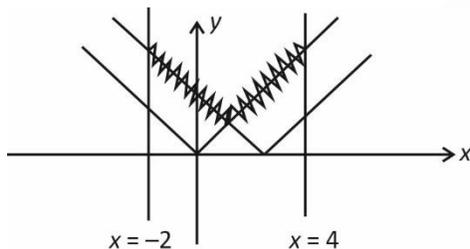




Area $= \frac{1}{2}\big[(1+4) \times 3\big] + \frac{1}{2}\big[1+4\big] \times 3$

$= \frac{15}{2} + \frac{15}{2} = 15$ sq. unit

13. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$. Let $\vec{c}$ is a unit vector such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. If $\vec{c} = \lambda\hat{i} + \mu\hat{j}$ and $\vec{d}$ is a vector perpendicular to $\vec{c}$ and $\vec{a}$, then $|\lambda\vec{c} + \mu\vec{d}|^2$ is equal to

(1) $\dfrac{6}{25}$

(2) $\dfrac{61}{25}$

(3) $\dfrac{41}{25}$

(4) $\dfrac{36}{25}$

**Answer (2)**

**Sol.** $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$

$$\Rightarrow (\vec{a} - \vec{b}) \times \vec{c} = 0$$

$$\Rightarrow \vec{c} || (\vec{a} - \vec{b}) = (-\hat{i} + 2\hat{j})$$

$$\Rightarrow \vec{c} = \frac{-i}{\sqrt{5}} + \frac{2j}{\sqrt{5}} = \lambda\hat{i} + \mu\hat{j}$$

$$\Rightarrow \lambda = \frac{-1}{\sqrt{5}}, \mu = \frac{2}{\sqrt{5}}$$

$$\vec{c} \cdot \vec{d} = 0$$

$$|\lambda\vec{c} + \mu\vec{d}|^2 = \lambda^2|\vec{c}|^2 + 2\lambda\mu(0) + \mu^2|\vec{d}|^2$$

$$\vec{d} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{vmatrix} = \hat{i}\left(\frac{-2}{\sqrt{5}}\right) + \left(\frac{-1}{\sqrt{5}}\right)\hat{j} + \left(\frac{3}{\sqrt{5}}\right)\hat{k}$$

$$|\vec{d}| = \sqrt{\frac{4}{5} + \frac{1}{5} + \frac{9}{5}} = \sqrt{\frac{14}{5}}$$

$$\left(\frac{1}{5}\right) \times (1) + 0 + \left(\frac{4}{5}\right) \cdot \left(\frac{14}{5}\right) = \frac{5}{25} + \frac{56}{25} = \left(\frac{61}{25}\right)$$
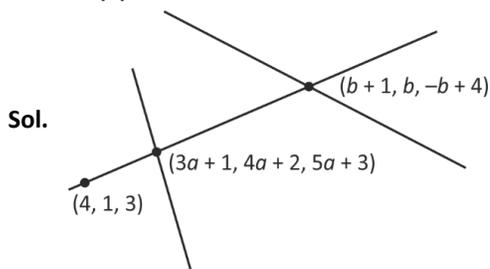
14. Let $a$ line passing through (4, 1, 3) intersects the line $l_1 : \dfrac{x-1}{3} = \dfrac{y-2}{4} = \dfrac{z-3}{5}$ at ($\alpha$, $\beta$, $\gamma$) and $l_2 : x - 1 = y = -z + 4$ at ($a$, $b$, $c$) then $\begin{vmatrix} 63 & 21 & -21 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ is equal to

(1) 102      (2) 204

(3) 63      (4) 21

**Answer (2)**

**Sol.**



Points: $(b+1, b, -b+4)$, $(3a+1, 4a+2, 5a+3)$, $(4, 1, 3)$

$\dfrac{3a+1-4}{b+1-4} = \dfrac{4a+2-1}{b-1} = \dfrac{5a+3-3}{-b+4-3}$

$\dfrac{3a-3}{b-3} = \dfrac{4a+1}{b-1} = \dfrac{-5a}{b-1}$

$\Rightarrow \; 4a+1 = -5a \Rightarrow a = \dfrac{-1}{9}$

$\Rightarrow \; \dfrac{3\left(\dfrac{-1}{9}-1\right)}{b-3} = \dfrac{4\left(-\dfrac{1}{9}\right)+1}{b-1} = \dfrac{\dfrac{-10}{3}}{b-3} = \dfrac{\dfrac{5}{9}}{b-1}$

$\Rightarrow \; b = \dfrac{9}{7}$

$\Rightarrow \; l_1(\alpha, \beta, \gamma) = \left(\dfrac{2}{3}, \dfrac{14}{7}, \dfrac{22}{9}\right) \equiv \left(\dfrac{6}{9}, \dfrac{14}{9}, \dfrac{22}{9}\right)$

$l_2(a, b, c) = \left(\dfrac{16}{7}, \dfrac{9}{7}, \dfrac{19}{7}\right)$

$\Rightarrow \; \begin{vmatrix} 63 & 21 & -21 \\ \dfrac{6}{9} & \dfrac{14}{9} & \dfrac{22}{9} \\ \dfrac{16}{7} & \dfrac{9}{7} & \dfrac{19}{7} \end{vmatrix} = 204$

15. Let $a_1$, $a_2$, $a_3$....be the terms of an increasing G.P. such that $a_3 \cdot a_5 = 729$ and $a_3 + a_5 = \dfrac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

(1) 139      (2) 129

(3) 125      (4) 119

**Answer (2)**

**Sol.** Let $a_3 + a_5 = \dfrac{111}{4}$

and $a_3 \cdot a_5 = 729$

$\Rightarrow (ar^2) \cdot (ar^4) = (27)^2$

$\Rightarrow ar^3 = 27$, $a_i > 0$

$a_4 = 27$

$\Rightarrow a_3 = \dfrac{27}{r}; \; a_5 = 27r$

$27r + \dfrac{27}{r} = \dfrac{111}{4} = \dfrac{37 \times 3}{4}$

$\Rightarrow r + \dfrac{1}{r} = \dfrac{37}{36} \Rightarrow r \Rightarrow \dfrac{1}{6}, 6 \Rightarrow r = 6$

$24(a_1 + a_2 + a_3) = 24\left[\dfrac{27}{216} + \dfrac{27}{36} + \dfrac{27}{6}\right]$

$= 24\left[\dfrac{1}{8} + \dfrac{3}{4} + \dfrac{9}{2}\right]$

$= 3 + 18 + 108 = 129$

16.

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21.

22.

23.

24.

25.

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