MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If for an arithmetic progression, if first term is 3 and sum of first four terms is equal to $\frac{1}{5}$ of the sum of next four terms, then the sum of first 20 terms is

(1) 1080 (2) 364

(3) –1080 (4) – 364

Answer (3)

Sol. Sum of first four term $=\frac{1}{5}$ sum of next four terms

$\Rightarrow \frac{4}{2}(2a+3d)=\frac{1}{5}(4a+22d)$

$\Rightarrow (4a+6d)\cdot 5 = 4a+22d$

$\Rightarrow 20a+30d = 4a+22d$

$\Rightarrow 16a = -8d \quad \Rightarrow \boxed{a=-\frac{d}{2}}$

$\Rightarrow \boxed{d=-6}\ \boxed{a=3}$

$\Rightarrow \frac{20}{2}[2(3)+19(-6)]=-10(18.6)$

$= -1080$

2. How many words can be formed from the word “DAUGHTER” such that any vowels are not together

(1) 34000 (2) 35000

(3) 36000 (4) 37000

Answer (3)

Sol. Total vowels together

$8! - 6! \times 3!$

$= 36,000$

3. Two biased dies are tossed. Die 1 has 1 on two faces, 2 on two faces, 3 and 4 on other faces, while die 2 has 2 on 2 faces, 4 on 2 faces and 1 and 3 on other faces. Then the probability that when throwing these dices we get sum of 4 or 5.

(1) $\frac{3}{7}$ (2) $\frac{2}{3}$

(3) $\frac{4}{9}$ (4) $\frac{8}{9}$

Answer (3)

Sol. Die 1 $\in \{1, 1, 2, 2, 3, 4\}$

Die 2 $\in \{2, 2, 4, 4, 1, 3\}$

P(Sum of faces is 4 or 5)

$= P(\text{sum}=4) + P(\text{sum}=5) - P(\text{sum}=4 \text{ and sum}=5)$

$=\begin{bmatrix}(D_1D_3)\\ D_2D_2\\ D_3D_1\end{bmatrix}+\begin{bmatrix}D_1D_4\\ D_2D_3\\ D_3D_2\\ D_4D_1\end{bmatrix}-$ (no cases)

$=\left[\left(\frac{2}{6}\times\frac{1}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)\right]+$

$\left[\frac{2}{6}\cdot\frac{2}{6}+\frac{2}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}\right]-0$

$=\frac{2}{36}+\frac{4}{36}+\frac{1}{36}+\frac{4}{36}+\frac{2}{36}+\frac{2}{36}+\frac{1}{36}=\frac{16}{36}=\frac{4}{9}$

4. Value of $\cos^{-1}\left[\frac{12}{13}\cos x+\frac{5}{13}\sin x\right]$ is

$\left(x\in\left[\frac{\pi}{2},\pi\right]\right)$

(1) $x+\tan^{-1}\frac{12}{13}$ (2) $x-\tan^{-1}\frac{12}{13}$

(3) $x-\tan^{-1}\frac{5}{12}$ (4) $x+\tan^{-1}\left(\frac{4}{5}\right)$

Answer (3)

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Sol. $\frac{12}{13}\cos x+\frac{5}{13}\sin x$; Let $\tan\alpha=\frac{5}{12},\ \alpha\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow\quad \sin\alpha=\frac{5}{13},\cos\alpha=\frac{12}{13}$

$\Rightarrow\quad \frac{12}{13}\cos x+\frac{5}{13}\sin x=\cos\alpha\cos x+\sin\alpha\sin x$

$=\cos(x-\alpha)$

$\Rightarrow\quad \cos^{-1}[\cos(x-\alpha)]=x-\alpha$

$=x-\tan^{-1}\left(\frac{5}{12}\right)$

5. A relation defined on set $A = \{1, 2, 3, 4\}$, then how many ordered pairs are added to $R = \{(1, 2), (2, 3), (3, 3)\}$ so that it becomes equivalence relation?

(1) 10

(2) 9

(3) 7

(4) 8

Answer (3)

Sol. Ordered pairs to be added be

$\{(1, 1), (2, 2), (4, 4), (2, 1), (3, 2), (3, 1), (1, 3)\}$

So total 7 ordered pairs to be added.

6. The sum of all rational terms in the expansion of $\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^6$ is

(1) 638

(2) 728

(3) 528

(4) 729

Answer (1)

Sol. The general term of multinomial expansion is $\frac{6!}{\alpha!\beta!\gamma!}(1)^{\alpha}\left(2^{1/3}\right)^{\beta}\left(3^{1/2}\right)^{\gamma}$

For terms to be rational $3|\beta$ and $2|\gamma$

$\Rightarrow$

β	γ	α	Term
0	0	6	$1\cdot3^3 = 27$
0	2	4	$15\cdot3 = 45$
0	4	2	$15\cdot3^2 = 135$
0	6	0	$1\cdot3^3 = 27$
3	0	3	$20\cdot2 = 40$
3	2	1	$60\cdot2\cdot3 = 360$
6	0	0	$1\cdot4 = 4$

$\Rightarrow$ Sum of the rational term

$= 27 + 45 + 135 + 27 + 40 + 360 + 4 = 638$

7. If $\left|\frac{z}{z+i}\right|=2$ represents a circle with centre P then distance of P from D is (where D : (1, 5) and $i=\sqrt{-1}$)

(1) $\sqrt{\frac{360}{9}}$

(2) $\sqrt{\frac{370}{9}}$

(3) $\frac{\sqrt{370}}{9}$

(4) $\frac{\sqrt{360}}{9}$

Answer (2)

Sol. Let $z = x + iy$

$|z|=2|z+i|$

$\sqrt{x^2+y^2}=2\sqrt{x^2+(y+1)^2}$

$x^2+y^2=4(x^2+(y+1)^2)$

$C: 3x^2+3y^2+8y+4=0$

$\therefore\quad P\left(0,\frac{-4}{3}\right)$

Now $PD:\sqrt{1^2+\left(5+\frac{4}{3}\right)^2}=\sqrt{1+\frac{361}{9}}=\sqrt{\frac{370}{9}}$

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8. Consider the set $S = \{1, 2, 3, \ldots, 1000\}$. Then the number of arithmetic progression that can be formed using elements of set S such that first term is 1 and last term is 1000 is

(1) 8
(2) 12
(3) 15
(4) 4

Answer (1)

Sol. Let n be the last term

$\Rightarrow T_n = a + (n - 1)d$

$\Rightarrow 1000 = 1 + (n - 1)d$

$\Rightarrow (n - 1)d = 999$

For all terms to be from S then

$d \mid 999 \Rightarrow d \mid 37 \times 27 = 37^1 \cdot 3^3$

Number of values of $d = (1 + 1)(3 + 1) = 8$

9. Let A and B are non-singular commutative matrices. Then $A[(\text{adj } A^{-1})\ (\text{adj}(B^{-1}))]^{-1} B$ is equal to

(1) $|A|\ |B|\ I_n$

(2) $\dfrac{I_n}{|A||B|}$

(3) $\dfrac{I_n}{|A|}$

(4) $\dfrac{I_n}{|B|}$

Answer (1)

Sol. $= \boxed{(\text{adj}(A))^{-1} = \dfrac{A}{|A|}}$

$\because\ A[(\text{adj } A^{-1})\cdot(\text{adj } B^{-1})]^{-1} B$

$= A\cdot(\text{adj } B^{-1})^{-1}\cdot(\text{adj } A^{-1})^{-1}\cdot B$

$= A\cdot\dfrac{B^{-1}}{|B^{-1}|}\cdot\dfrac{A^{-1}}{|A^{-1}|}\cdot B$

$= (A\cdot B^{-1}\cdot A^{-1}\cdot B)\cdot|A|\cdot|B|$

$= (A\cdot A^{-1})\,(B\cdot B^{-1})\,|A||B| = |A||B|\,I_n.$

10. Let $f(x) = \log_e x$ and $g(x) = \left(\dfrac{2x^4 - 2x^3 - x^2 + 2x - 1}{2x^2 - 2x + 1}\right)$, then domain of $f(g(x))$ for $x > 0$ is

(1) $(1, \infty)$
(2) $(0, \infty)$
(3) $\left(\dfrac{1}{2}, \infty\right)$
(4) $(0, 1)$

Answer (1)

Sol. Clearly $2x^2 - 2x + 1 > 0\ \forall\ x \in R$

also ± 1 are roots of equation

$2x^4 - 2x^3 - x^2 - 1 = 0$

$\Rightarrow 2x^4 - 2x^3 - x^2 + 2x - 1 = (2x^2 - 2x + 1)(x - 1)(x + 1)$

$\Rightarrow g(x) = (x - 1)(x + 1)$

$f(g(x)) = \log_e(x^2 - 1) \Rightarrow (x^2 - 1) > 0$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

11. If the curve satisfying the differential equation $\dfrac{dy}{dx} = \dfrac{6 - 2e^{2x}y}{1 + e^{2x}}$ passes through (0, 0) and (ln 2, k), then k is

(1) $\dfrac{3}{5}\ln 3$

(2) $\dfrac{6}{5}\ln 2$

(3) $\dfrac{8}{9}\ln 3$

(4) $\dfrac{7}{2}\ln 2$

Answer (2)

Sol. $\dfrac{dy}{dx} = \dfrac{6 - 2e^{2x}y}{1 + e^{2x}}$

$\dfrac{dy}{dx} = \left(\dfrac{2e^{2x}}{1 + e^{2x}}\right)y = \dfrac{6}{1 + e^{2x}}$

If $= e^{\int \frac{2e^{2x}}{1 + e^{2x}} dx}$

$= e^{\ln|1 + e^{ex}|} = 1 + e^{2x}$

$$y(1+e^{2x})=\int\frac{6}{(1+e^{2x})}\cdot(1+e^{2x})dx$$

$y(1+e^{2x}) = 6x + c$

Passes through (0, 0)

$\Rightarrow\ c = 0$

$$\therefore\quad y=\frac{6x}{1+e^{2x}}$$

Now if passes through (ln2, k)

$$k=\frac{6\ln 2}{1+4}=\frac{6}{5}\ln 2$$

12. Let $I=\int\frac{dx}{(x-1)^{\frac{11}{13}}\cdot(x+15)^{\frac{15}{13}}}$, then I is

(1) $\frac{13}{32}\left(\frac{x-1}{x+15}\right)^{\frac{2}{13}}+C$

(2) $\frac{32}{13}\left(\frac{x-1}{x+15}\right)^{\frac{2}{13}}+C$

(3) $\frac{1}{32}\left(\frac{x+15}{x-1}\right)^{\frac{2}{13}}+C$

(4) $\frac{13}{32}\left(\frac{x+15}{x-1}\right)^{\frac{15}{13}}+C$

Answer (1)

Sol. $I(x)=\int\frac{dx}{(x-1)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$

$$=\int\frac{dx}{(x-1)^2\left(\frac{x+15}{x-1}\right)^{\frac{15}{13}}}$$

Let $\frac{x+15}{x-1}=y$

$$\frac{(x-1)-(x+15)}{(x-1)^2}=\frac{dy}{dx}$$

$$\frac{-16dx}{(x-1)^2}=dy$$

$$I(x)=\int\frac{-\frac{1}{16}dy}{y^{\frac{15}{13}}}$$

$$=-\frac{1}{16}\left(\frac{y^{-\frac{15}{13}+1}}{\frac{-15}{13}+1}\right)+C$$

$$=\frac{13}{32}y^{-\frac{2}{13}}+C$$

$$=\frac{13}{32}\left(\frac{x-1}{x+15}\right)^{\frac{2}{13}}+C$$

13.

14.

15.

16.

17.

18.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $f(x)$ is continuous at $x = 0$, where

$$f(x)=\begin{cases}\frac{2}{x}(\sin(k_1+1)x+\sin(k_2+1)x) & x<0\\ 4 & x=0\\ \frac{2}{x}\log\left[\frac{k_2x+1}{k_1x+1}\right] & x>0\end{cases}$$

Then $k_1^2+k_2^2$ is

Answer (2)

Sol. $\because$ $f(x)$ is continuous at $x = 0$

Then $\lim_{x\to 0^-}f(x)=f(0)=\lim_{x\to 0^+}f(x)$

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$$\Rightarrow \lim_{x\to 0^-} \frac{2\left(\sin(k_1+1)x+\sin(x_2+1)x\right)}{x} = 4$$

$$= \lim_{x\to 0^+} \frac{2\log\left(\frac{k_2x+1}{k_1x+1}\right)}{x}$$

$$\Rightarrow \lim_{h\to 0} 2\left\{\frac{\sin(1+k_1)h}{(1+k_1)h}\cdot(1+k_1)+\frac{\sin(1+k_2)h}{(1+k_2)h}(1+k_2)\right\} = 4$$

$$= \lim_{h\to 0} \frac{2\log\left(1+\frac{(k_2-k_1)h}{1+k_1h}\right)}{\frac{(k_2-k_1)h}{1+k_1h}}\cdot\left(\frac{k_2-k_1}{1+k_1h}\right)$$

$\Rightarrow$ $2(2 + k_1 + k_2) = 4 = 2(k_2 + k_1)$

$\therefore$ $k_1 + k_2 = 0$ and $k_2 - k_1 = 2$

$\therefore$ $k_1 = -1$, $k_2 = 1$

$\therefore$ $k_1^2 + k_2^2 = 2$

22. If for the system of linear equations having infinite solutions

$(\lambda - 4)x + (\lambda - 2)y + \lambda z = 0$

$2x + 3y + 5z = 0$

$x + 2y + 6z = 0$

then $\lambda^2 + \lambda$ is

Answer (90)

Sol. For infinite solutions $\Delta = 0$

$$\begin{vmatrix} \lambda-4 & \lambda-2 & \lambda \\ 2 & 3 & 5 \\ 1 & 2 & 6 \end{vmatrix} = 0$$

$\Rightarrow$ $2\lambda - 18 = 0$

$\lambda = 9$

Now $\lambda^2 + \lambda = 9^2 + 9 = 81 + 9 = 90$

23. If the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ has equal roots and if $a + c = 5$ and $b = \frac{16}{5}$, then the value of $a^2 + c^2$ is equal to

Answer (9)

Sol. Clearly 1 satisfy $\Rightarrow$ other root is also 1.

$\Rightarrow$ $\frac{c(a-b)}{a(b-c)} = 1$ (using product of roots)

$\Rightarrow$ $c(a - b) = a(b - c)$

$\Rightarrow$ $2ac = b(a + c)$

$\Rightarrow$ $2ac = \left(\frac{16}{5}\right)(5)$

$\Rightarrow$ $2ac = 16$

Since $a^2 + c^2 = (a + c)^2 - 2ac$

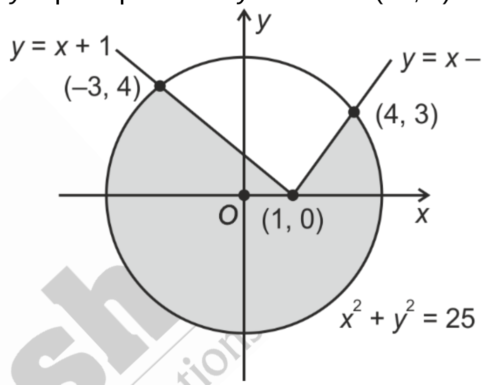
$= 25 - 16 = 9$

24. The area of larger portion enclosed by curves $y = |x - 1|$ and $x^2 + y^2 = 25$ is equal to $\frac{1}{4}(\alpha\pi + \beta)$ sq. units (where α, β are natural numbers), then $\alpha + \beta$ equals to

Answer (77)

Sol. Intersection points of

$y = |x - 1|$ and $x^2 + y^2 = 25$ are $(-3, 4)$ and $(4, 3)$



$$A = 25\pi - \int_{-3}^{4}\left(\sqrt{25-x^2} - |x-1|\right)dx$$

$$= 25\pi - \left[\frac{1}{2}x\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{3}^{4} + \left(8+\frac{9}{2}\right)$$

$$= 25\pi + \frac{25}{2} - \left(6 + \frac{25}{2}\sin^{-1}\frac{4}{5} + 6 + \frac{25}{2}\sin^{-1}\frac{3}{5}\right)$$

$$= 25\pi + \frac{25}{2} - 12 - \frac{25\pi}{4} = \frac{75\pi}{4} + \frac{1}{2}$$

$$= \frac{1}{4}(75\pi + 2)$$

$\Rightarrow$ $\alpha = 75$, $\beta = 2$

$\alpha + \beta = 77$

25.

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