

24/01/2025

Morning



Aakash

Medical | IIT-JEE | Foundations

Corporate Office : AESL, 3rd Floor, Incuspaze Campus-2, Plot-13, Sector-18, Udyog Vihar,
Gurugram, Haryana-122018

Memory Based Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2025 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.
- (4) **Section - A** : Attempt all questions.
- (5) **Section - B** : Attempt all questions.
- (6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.
- (7) **Section - B (21 – 25)** contains 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

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JEE (Advanced) 2024

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| 25 | Rishi Shekher Shukla | 2 Year Classroom |
| 67 | Krishna Sai Shishir | 2 Year Classroom |
| 78 | Abhishek Jain | 2 Year Classroom |
| 93 | Hardik Aggarwal | 2 Year Classroom |
| 95 | Ujjwal Singh | 2 Year Classroom |
| 98 | Rachit Aggarwal | 2 Year Classroom |

JEE (Main) 2024

| AIR | Name | Classroom | State |
|-----|------------------------|------------------|-----------|
| 1 | Sarvvi Jais | 2 Year Classroom | Karnataka |
| 15 | M Sai Divya Teja Reddy | 2 Year Classroom | Telangana |
| 19 | Rishi Shekher Shukla | 2 Year Classroom | Telangana |

$$\Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = 0$$

$$(-5\hat{i} + 10\hat{j} - 5\hat{k}) \cdot (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) = 0$$

$$\Rightarrow -5a_1 + 10a_2 - 5a_3 = 0$$

$$\Rightarrow \boxed{a_1 + a_3 = 2a_2} \quad \dots(i)$$

$$\vec{c} \cdot \vec{b} = 0 \text{ and } \vec{a} \cdot \vec{c} = 5$$

$$\boxed{3a_1 + a_2 - a_3 = 0} \quad \dots(ii)$$

$$(a_1 + 2a_2 + 3a_3) = 5 \quad \dots(iii)$$

Solving (i), (ii) and (iii)

$$a_1 = \frac{1}{6}, a_2 = \frac{2}{3}, a_3 = \frac{7}{6}$$

$$\Rightarrow \vec{c} = \frac{1}{6}\hat{i} + \frac{2}{3}\hat{j} + \frac{7}{6}\hat{k}$$

$$\Rightarrow |\vec{c}| = \sqrt{\frac{11}{6}}$$

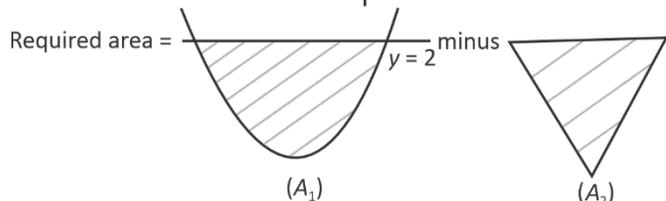
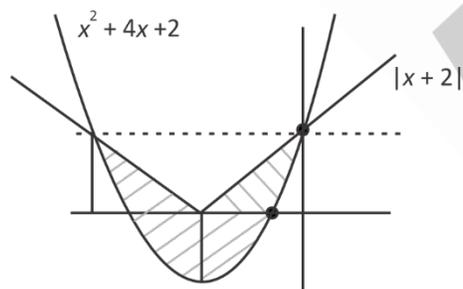
5. The area of the region bounded by $S(x, y)$ such that $S = \{(x, y) : x^2 + 4x + 2 \leq y \leq |x + 2|\}$ is (in sq. units)

(1) $\frac{24}{5}$ (2) 5

(3) $\frac{20}{3}$ (4) 7

Answer (3)

Sol. $x^2 + 4x + 2 \leq y \leq |x + 2|$



$$A_1 = \int_{-4}^0 [2 - (x^2 + 4x + 2)] dx - \frac{1}{2} \times 4 \times 2$$

$$= \left(\frac{-x^3}{3} - 2x^2 \right) \Big|_{-4}^0 - 4$$

$$= 0 - \left(\frac{64}{3} - 32 \right) - 4$$

$$= 32 - \frac{64}{3} - 4 = \frac{20}{3}$$

6. If $\frac{dy}{dx} + \left(\frac{x}{1+x^2} \right) y = \frac{\sqrt{x}}{\sqrt{1+x^2}}$; $y(0) = 0$, then $y(1)$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{\sqrt{2}}{3}$ (4) $\sqrt{\frac{2}{3}}$

Answer (3)

Sol. $\frac{dy}{dx} + \left(\frac{x}{1+x^2} \right) y = \frac{\sqrt{x}}{\sqrt{1+x^2}}$

$$\text{If } = e^{\int \frac{x}{1+x^2} dx} = e^{\frac{1}{2} \ln(1+x^2)} = \sqrt{1+x^2}$$

Solution will be $y\sqrt{1+x^2} = \int \frac{\sqrt{x}}{\sqrt{1+x^2}} \cdot \sqrt{1+x^2} dx$

$$y\sqrt{1+x^2} = \frac{2x^{3/2}}{3} + c$$

$$y(0) = 0 \Rightarrow c = 0$$

$$y = \frac{2}{3} \frac{x^{3/2}}{\sqrt{1+x^2}}$$

$$\text{Now } y(1) = \frac{\sqrt{2}}{3}$$

7. If α and β are real numbers such that $\sec^2(\tan^{-1}(\alpha)) + \text{cosec}^2(\cot^{-1}(\beta)) = 36$ and $\alpha + \beta = 8$, then $(\alpha^2 + \beta)$ is ($\alpha < \beta$)

(1) 23 (2) 14

(3) 24 (4) 27

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JEE (Advanced) 2024

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| AIR 25 Rishj Shekher Shukla 2-Year Classroom | AIR 67 Krishna Sai Shishir 2-Year Classroom | AIR 78 Abhishek Jain 2-Year Classroom | AIR 93 Harshik Aggarwal 2-Year Classroom | AIR 95 Ujjwal Singh 2-Year Classroom | AIR 98 Rishit Aggarwal 2-Year Classroom |
|---|--|--|---|---|--|

JEE (Main) 2024

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|---|--|--|
| Karnataka Topper AIR 1 Sarvi Jain 2-Year Classroom | Telangana Topper AIR 15 M Sai Divya Taja Reddy 2-Year Classroom | Telangana Topper AIR 19 Rishi Shekher Shukla 2-Year Classroom |
|---|--|--|

Answer (2)

Sol. Let $\tan^{-1} \alpha = A \Rightarrow \tan A = \alpha$

$$\cot^{-1} \beta = B \Rightarrow \cot B = \beta$$

$$\sec^2 A + \operatorname{cosec}^2 B = 36$$

$$(1 + \tan^2 A) + (1 + \cot^2 B) = 36$$

$$\Rightarrow 1 + \alpha^2 + 1 + \beta^2 = 36$$

$$\Rightarrow \alpha^2 + \beta^2 = 34$$

$$\alpha + \beta = 8$$

$$(\alpha + \beta)^2 = 34 + 2\alpha\beta = 64$$

$$\Rightarrow \alpha\beta = 15$$

$\Rightarrow \alpha, \beta$ are roots of equation

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$\Rightarrow x = 3, 5$$

$$\Rightarrow \alpha = 3, \beta = 5$$

$$\Rightarrow \alpha^2 + \beta^2 = 34$$

8. Two persons A and B throws a pair of dice alternatively. For A to win he should throw sum of 5 before B throws sum of 8. If A throws first, then the probability that A wins, is

(1) $\frac{8}{19}$

(2) $\frac{9}{19}$

(3) $\frac{8}{17}$

(4) $\frac{9}{17}$

Answer (2)

Sol. For sum 5, (1, 4), (2, 3) (3, 2), (4, 1) $\Rightarrow P(A) = \frac{4}{36}$

For sum 8, (2, 6), (3, 5) (4, 4), (5, 3), (6, 2) $\Rightarrow P(B) = \frac{5}{36}$

$$\Rightarrow P(\bar{A}) = \frac{32}{36}, P(\bar{B}) = \frac{31}{36}$$

$P(A \text{ wins}) =$

$$P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \dots$$

$$= \frac{P(A)}{1 - P(\bar{A})P(\bar{B})} = \frac{\frac{4}{36}}{1 - \frac{32}{36} \cdot \frac{31}{36}} = \frac{9}{19}$$

9. For a distribution of 10 observations, $\sum_{i=1}^{10} x_i = 55$ and

$$\sum_{i=1}^{10} x_i^2 = 328. \text{ If the observations 4 and 5 are replaced by}$$

6 and 8 respectively, then the new variance is

(1) 2.5

(2) 2.7

(3) 3.4

(4) 3.6

Answer (2)

Sol. $x_1 + x_2 + \dots + x_8 + 4 + 5 = 55,$

$$x_1^2 + x_2^2 + \dots + x_8^2 + 16 + 25 = 328$$

$$\mu = \frac{x_1 + x_2 + \dots + x_8 + 6 + 8}{10} = 6,$$

$$x_1^2 + x_2^2 + \dots + x_8^2 + 36 + 64 = 387$$

$$\Rightarrow \sigma^2 = \frac{387}{10} - 36 = \frac{387 - 360}{10} = \frac{27}{10} = 2.7$$

10. If S be the set of 10 distinct primes and let A be the set of product of two or more elements from the set S. If $P = \{(x, y) : x \in S \text{ and } y \in A \text{ and } y \text{ is divided by } x\}$. Then $n(P)$ is equal to

(1) 5110

(2) 5000

(3) 5220

(4) 5420

Answer (1)

Sol. $S = \{P_1, P_2, P_3, \dots, P_{10}\}$

$A = \{\text{product of distinct elements of } S\}$

If $P = \{(x, y) : x \in S, y \in A \text{ and } y \text{ is divided by } x\}$ then $n(P)$ is

\Rightarrow since $x \in S$.

Let $x = P_i \in S$, selected from S in $({}^{10}C_1)$ ways.

Now, $x|y$ iff y contains P_i in product.

$\Rightarrow (P_i)$ is there and at rest one other element as a product

$$\Rightarrow ({}^{10}C_1) \cdot [(1) \cdot (2^9 - 1)] = 10 \cdot (511) = 5110$$

SECTION - B

11. If $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$, $m, n > 0$, then $I(9, 14) + I(10, 13)$ is equal to
- (1) $I(1, 13)$
 - (2) $I(9, 1)$
 - (3) $I(9, 13)$
 - (4) $I(19, 29)$

Answer (3)

Sol. Beta function

$$\beta(p, q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx = \frac{(p-1)!(q-1)!}{(p+q-1)!}, p, q \in I$$

$$\Rightarrow I(9, 14) = \frac{8! \cdot 13!}{22!}$$

$$I(10, 13) = \frac{9! \cdot 12!}{22!}$$

$$\Rightarrow I(9, 14) + I(10, 13) = \frac{1}{22!} (8! \cdot 13! + 9! \cdot 12!)$$

$$\Rightarrow = \frac{1}{22!} \cdot 8! \cdot 12! (13+9) = \frac{8! \cdot 12!}{21!}$$

$$= \frac{(9-1)!(13-1)!}{(9+13-1)!} = I(9, 13)$$

- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The number of 3 digit numbers which is divisible by 2 and 3 but not divisible by 4 and 9?

Answer (125)

Sol. Total number divisible by 6

$$102, 108, \dots, 996$$

$$\text{So total number} = 150$$

Number divisible by 36

$$108, \dots, 972$$

$$\text{Total number} = 25$$

$$\text{Required number} = (\text{divisible by 6}) - (\text{divisible by 36}) = 150 - 25 = 125$$

22. The product of all real roots of equation $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 2$ is

Answer (99)

Sol. $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 2$

$$(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 2$$

$$\text{Let } x^2 - 9x + 11 = t$$

$$t^2 - (t+9) = 2$$

$$t^2 - t - 11 = 0$$

$$t = \frac{1 \pm \sqrt{1+44}}{2} = \frac{1 \pm \sqrt{45}}{2}$$

$$x^2 - 9x + 11 = \frac{1 + \sqrt{45}}{2}$$

$$x^2 - 9x = \frac{\sqrt{45} - 21}{2}$$

$$D_1 : 81 - 4 \left(\frac{\sqrt{45} - 21}{2} \right) > 0$$

Similarly,

$$x^2 - 9x + 11 = \left(\frac{1 - \sqrt{45}}{2} \right)$$

$$\Rightarrow D_2 : 81 - 4 \left(\frac{-21 - \sqrt{45}}{2} \right) > 0$$

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JEE (Main) 2024

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|------------------|----------------|--------|------------------------|------------------|
| Karnataka Topper | 100 PERCENTILE | AIR 1 | Sarvni Jain | 2 Year Classroom |
| Telangana Topper | 100 PERCENTILE | AIR 15 | M Sai Divya Tuja Reddy | 2 Year Classroom |
| Telangana Topper | 100 PERCENTILE | AIR 19 | Rishi Shekher Shukla | 2 Year Classroom |

⇒ All roots are real

⇒ Product of roots = $\left(\frac{\text{Constant term}}{1}\right)$

= $\left(\frac{121 - (20) - 2}{1}\right) = \frac{121 - 22}{1} = \boxed{99}$

23. If $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + n$ terms. The sum of first six terms in A.P. with first term equal to $-p$ and common difference p is $\sqrt{2026 \cdot S_{2025}}$. The absolute value of difference between 20th and 15th terms in A.P. is

Answer (25)

Sol. $S_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)}$
 $= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$
 $= 1 - \frac{1}{n+1} = \frac{n}{n+1} \Rightarrow S_{2025} = \frac{2025}{2026}$

For A.P. $\frac{6}{2}(-2p + (6-1)p) = \sqrt{2026 \times \frac{2025}{2026}} = 45$

⇒ $3p = \frac{45}{3} \Rightarrow p = 5$

Now for A.P. $|a_{20} - a_{15}| = |19 \times 5 - 14 \times 5| = 25$



24. If $f(x)$ satisfies the functional equation $f(x) + 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{7}{2}$, $x \in \mathbb{R} - \{0\}$ and $\lim_{x \rightarrow 0} \left(\frac{1}{\alpha x} + f(x)\right)$ exist finitely and is equal to β , then $(\alpha - 2\beta)$ is

Answer (2)

Sol. $f(x) + 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{7}{2}$

$f\left(\frac{1}{x}\right) + 6f(x) = \frac{35x}{3} - \frac{7}{2}$

$36f(x) + 6f\left(\frac{1}{x}\right) = 70x - 21$

$35f(x) = (70x - 21) - \left[\frac{35}{3x} - \frac{7}{2}\right]$

$35f(x) = 70x - \frac{35}{3x} - \frac{35}{2}$

⇒ $f(x) = 2x - \frac{1}{3x} - \frac{1}{2}$

$\lim_{x \rightarrow 0} \left(\frac{1}{\alpha x} + 2x - \frac{1}{3x} - \frac{1}{2}\right) = \beta$

Limit exist finitely iff

$\alpha = 3 \Rightarrow \beta = -\frac{1}{2}$

$(\alpha - 2\beta) = 3 - 2\left(-\frac{1}{2}\right) = 4$

25.

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