

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $7 = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \dots \infty$ terms, then α is equal to

- (1) 6 (2) $\frac{6}{7}$
 (3) $\frac{1}{7}$ (4) 1

Answer (1)

$$S = a + (a + d)r + (a + 2d)r^2 + \dots \infty$$

$$\text{Then } S = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$7 = \frac{S}{1 - \frac{1}{7}} + \frac{\alpha \cdot \frac{1}{7}}{\left(1 - \frac{1}{7}\right)^2}$$

$$\Rightarrow \alpha = 6$$

2. If A and B are binomial coefficients of 30th and 12th term of binomial expansion $(1 + x)^{2n-1}$. If $2A = 5B$, then the value of n is

- (1) 20 (2) 21
 (3) 14 (4) 20

Answer (2)

Sol. $T_{r+1} = {}^{2n-1}C_r x^r$

$$\text{Coefficient of } T_{30} = {}^{2n-1}C_{29} = A$$

$$\text{Coefficient of } T_{12} = {}^{2n-1}C_{11} = B$$

$$\Rightarrow 2({}^{2n-1}C_{29}) = 5({}^{2n-1}C_{11})$$

$$\Rightarrow \text{Solving we get } n = 21$$

3. The equation of chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with (3, 1) as mid-point is

- (1) $48x + 25y - 169 = 0$ (2) $25x + 5y - 125 = 0$
 (3) $65x + 2y - 12 = 0$ (4) $45x + 4y - 135 = 0$

Answer (1)

Sol. Chord with given mid-point

$$\Rightarrow T = S_1$$

$$\Rightarrow \left(\frac{xx_1}{25} + \frac{yy_1}{16} - 1 \right) = \frac{x_1^2}{25} + \frac{y_1^2}{16} - 1$$

$$\Rightarrow \frac{3x}{25} + \frac{y}{16} - 1 = \frac{9}{25} + \frac{1}{16} - 1$$

$$\Rightarrow 48x + 25y - 400 = 144 + 25 - 400$$

$$\Rightarrow 48x + 25y = 169$$

4. If system of equations

$$x + 2y - 3z = 2$$

$$2x + \lambda y + 5z = 5$$

$4x + 3y + \mu z = 33$ has infinite solutions, then $\lambda + \mu$ is equal to

- (1) $\frac{1334}{5}$ (2) $\frac{1269}{5}$
 (3) $\frac{261}{5}$ (4) $\frac{1063}{5}$

Answer (1)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 4 & 3 & \mu \end{vmatrix} = 0$

$$\Rightarrow 12\lambda + \lambda\mu - 4\mu + 7 = 0$$

$$\Delta z = 25\lambda - 95 = 0$$

$$\Rightarrow \lambda = \frac{95}{25} = \frac{19}{5}$$

$$\Delta y = 0$$



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$$\Rightarrow \frac{263 - \mu}{5} = 0$$

$$\Rightarrow \mu = 263$$

$$\therefore \lambda + \mu = \frac{1334}{5}$$

5. Let S_n denotes the sum of the first n terms of an arithmetic progression. If $S_{40} = 1030$ and $S_{12} = 57$, then the value of $S_{30} - S_{10}$ is

- (1) 505 (2) 510
(3) 515 (4) 520

Answer (3)

Sol. $S_{40} = 1030 \Rightarrow \frac{40}{2}[2a + 39d] = 1030 \dots(i)$

$$S_{12} = 57 \Rightarrow \frac{12}{2}[2a + 11d] = 57 \dots(ii)$$

From (i) & (ii) $a = \frac{-7}{2}, d = \frac{3}{2}$

$\Rightarrow$

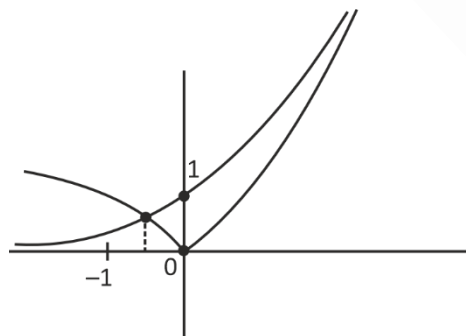
$$S_{30} - S_{10} = \frac{30}{2}[2a + 29d] - \frac{10}{2}[2a + 9d] = 20a + 390d = 515$$

6. The area of region enclosed by the curves $y = e^x, y = |e^x - 1|$ and y -axis is (in sq. units)

- (1) 1 (2) $1 - \ln 2$
(3) $1 + \ln 2$ (4) $\ln 2$

Answer (2)

Sol.



$$e^x = 1 - e^x \Rightarrow 2e^x = 1$$

$$\Rightarrow e^x = \frac{1}{2}$$

$$\Rightarrow x = \ln \frac{1}{2}$$

$$\Rightarrow \int_{\ln \frac{1}{2}}^0 [e^x - (1 - e^x)] dx$$

$$= \int_{\ln 2}^0 (2e^x - 1) dx = 2e^x - x \Big|_{-\ln 2}^0$$

$$= 2 - (1 + \ln 2)$$

$$= 1 - \ln 2$$

7. Consider an event E such that a matrix of order 2×2 is invertible with entries 0 or 1. Then, $P(E)$ is (where $P(X)$ denotes the probability of event X)

- (1) $\frac{5}{8}$ (2) $\frac{3}{8}$
(3) $\frac{1}{8}$ (4) $\frac{7}{8}$

Answer (2)

Sol. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$$\Rightarrow ad - bc = 0$$

$$\Rightarrow ad = bc$$

Case-I $ad = bc = 1$

$$a = b = c = d = 1$$

Case-II $ad = bc = 0$

$$a = 0, d = 0$$

$$b = 0, c = 0$$

$$a = 0, d = 1$$

$$b = 0, c = 1$$

$$a = 1, d = 0$$

$$b = 1, c = 0$$

$$3 \times 3 = 9 \text{ cases}$$

$$\therefore \frac{2^4 - 10}{2^4} = \frac{6}{16} = \frac{3}{8}$$

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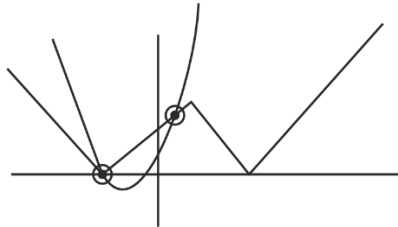
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8. The number of real roots of the equation

$$x^2 + 3x + 2 = \min(|x+2|, |x-3|)$$

- (1) 0
(2) 1
(3) 2
(4) 3

Answer (3)**Sol.**

9. A function
- $f: R \rightarrow (-1, 1)$
- such that
- $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$
- . The

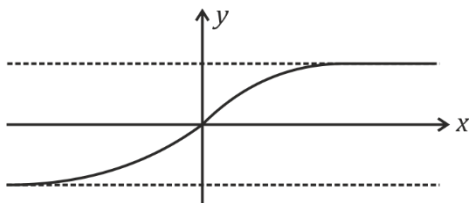
function f is

- (1) Both one-one and onto
(2) only one-one
(3) only onto
(4) Both many-one and onto

Answer (1)

Sol. $f(x) = \frac{2^{2x} - 1}{2^{2x} + 1} = 1 - \frac{2}{2^{2x} + 1}$

$$f'(x) = \frac{2}{(2^{2x} + 1)^2} \cdot 2^{2x} \cdot (\ln 2) \cdot 2 > 0$$

 $\therefore f(x)$ is increasing $\therefore$ one-oneas $x \rightarrow -\infty, f(x) \rightarrow -1$ as $x \rightarrow \infty, f(x) \rightarrow 1$ $\therefore$ range is $(-1, 1) \therefore$ onto

10. Let
- $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$
- ,
- $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{j})$
- and
- $\vec{c} = \vec{b} \times \hat{k}$
- , then projection of
- $\vec{c} - 2\hat{j}$
- on
- $\vec{a}$
- is equal to

- (1) $2\sqrt{14}$
(2) $3\sqrt{7}$
(3) $2\sqrt{7}$
(4) $\frac{3\sqrt{14}}{14}$

Answer (4)

Sol. $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 1 & -2 & 0 \end{vmatrix} = -2\hat{i} - \hat{j} - 8\hat{k}$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & -8 \\ 0 & 0 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j}$$

$$\Rightarrow \vec{c} - 2\hat{j} = -\hat{i}$$

$$\Rightarrow \text{Projection of } \vec{c} - 2\hat{j} \text{ on } \vec{a} = \frac{(\vec{c} - 2\hat{j}) \cdot \vec{a}}{|\vec{a}|}$$

$$= \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14}$$

11. If
- $\alpha > \beta > \gamma > 0$
- then find

$$\cot^{-1}\left(\frac{1+\alpha\beta}{\alpha-\beta}\right) + \cot^{-1}\left(\frac{1+\beta\gamma}{\beta-\gamma}\right) + \cot^{-1}\left(\frac{1+\gamma\alpha}{\gamma-\alpha}\right)$$

- (1) π (2) 0
(3) $\frac{\pi}{2} - (\alpha + \beta + \gamma)$ (4) 3π

Answer (1)

Sol. $\cot^{-1}\left(\frac{1+\alpha\beta}{\alpha-\beta}\right) + \cot^{-1}\left(\frac{1+\beta\gamma}{\beta-\gamma}\right) + \cot^{-1}\left(\frac{1+\gamma\alpha}{\gamma-\alpha}\right)$

$$= \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right) + \tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right) + \pi + \tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$$

$$\because \gamma - \alpha < 0$$

$$= \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right) + \tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right) + \tan^{-1}\left(\frac{\gamma-\alpha}{1+4\alpha}\right) + \pi$$



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$$\begin{aligned} &= \tan^{-1}\alpha - \tan^{-1}\beta + \tan^{-1}\beta - \tan^{-1}\gamma + \tan^{-1}\gamma - \tan^{-1}\alpha + \pi \\ &= 0 + \pi \\ &= \pi \end{aligned}$$

12.
13.
14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $\lim_{x \rightarrow 0} \begin{vmatrix} a + \frac{\sin x}{x} & 1 & 1 \\ a & 1 + \frac{\sin x}{x} & 1 \\ a & 1 & a + \frac{\sin x}{x} \end{vmatrix}$

$$= \mu a^2 + \lambda a + \alpha, \text{ then } \mu + \lambda + \alpha \text{ is}$$

Answer (4)

Sol. $\begin{vmatrix} a+1 & 1 & 1 \\ a & 2 & 1 \\ a & 1 & a+1 \end{vmatrix} = a^2 + 2a + 1$

$$\therefore \mu = 1$$

$$\lambda = 2$$

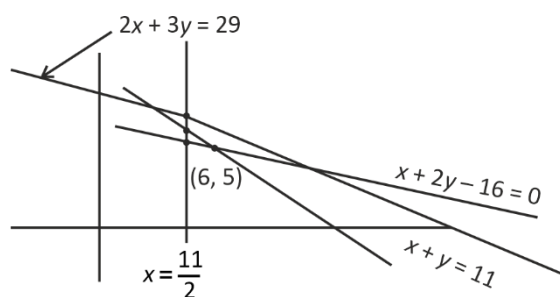
$$\alpha = 1$$

$$\therefore \mu + \lambda + \alpha = 4$$

22. The point $P\left(\frac{11}{2}, \alpha\right)$ lies on or inside the triangle formed by the lines $x + y = 11$, $x + 2y = 16$ and $2x + 3y = 29$, then minimum value of 10α is equal to

Answer (55)

Sol.



Clearly, $x = \frac{11}{2}$ intersect

$$x + y - 11 = 0 \text{ at } \left(\frac{11}{2}, \frac{11}{2}\right) \text{ and}$$

$$2x + 3y - 29 = 0 \text{ at } \left(\frac{11}{2}, 6\right).$$

$$\Rightarrow \alpha \in \left[\frac{11}{2}, 6\right]$$

$$\Rightarrow \text{minimum value of } 10\alpha = 55$$

23. If $\int \frac{2x^2 + 5x + 9}{x^2 + x + 1} dx = x\sqrt{x^2 + x + 1} + \alpha\sqrt{x^2 + x + 1} + \beta \ln\left(x + \frac{1}{2} + \sqrt{x^2 + x + 1}\right) + C$, then $\alpha + 2\beta$ equals to

Answer (16)

Sol. $I = \int \frac{2x^2 + 5x + 9}{x^2 + x + 1} dx.$

$$\text{Let } \frac{2x^2 + 5x + 9}{x^2 + x + 1} = \frac{A(x^2 + x + 1) + B(2x + 1) + C}{x^2 + x + 1}$$

$$\text{Then, } A = 2, B = \frac{3}{2} \text{ and } C = \frac{11}{2}$$

$$\therefore I = \int \frac{2(x^2 + x + 1) + \frac{3}{2}(2x + 1) + \frac{11}{2}}{x^2 + x + 1} dx$$

$$= 2 \int \sqrt{x^2 + x + 1} dx + \frac{3}{2} \cdot 2 \int \sqrt{x^2 + x + 1} dx + \frac{11}{2} \int \frac{1}{\sqrt{x^2 + x + 1}} dx$$

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$$\begin{aligned}
 &= 2 \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx + 3\sqrt{x^2 + x + 1} \\
 &\quad + \frac{11}{2} \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
 &= 2 \left\{ \frac{x + \frac{1}{2}}{2} \sqrt{x^2 + x + 1} + \frac{3}{8} \ln \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1} \right| \right\} \\
 &\quad + 3\sqrt{x^2 + x + 1} + \frac{11}{2} \ln \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C \\
 &= \left(\frac{2x+1}{2} \right) \sqrt{x^2 + x + 1} + \frac{3}{4} \ln \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| \\
 &\quad + 3\sqrt{x^2 + x + 1} + \frac{22}{7} \ln \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C \\
 &= \frac{2x+7}{2} \sqrt{x^2 + x + 1} + \frac{25}{4} \ln \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C
 \end{aligned}$$

$$\begin{aligned}
 &= x\sqrt{x^2 + x + 1} + \frac{7}{2} \sqrt{x^2 + x + 1} \\
 &\quad + \frac{25}{4} \ln \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C
 \end{aligned}$$

$$\therefore \alpha = \frac{7}{2}, \beta = \frac{25}{4}$$

$$\text{Then } \alpha + 2\beta = \frac{7}{2} + \frac{25}{2} = 16$$

24. In group A, there are 7 boys and 3 girls and in group B, there are 4 boys and 5 girls. For a picnic trip 4 boys and 4 girls are required such that 5 are selected from group A and 3 are selected from group B. Then the total number of ways to select the team for picnic trip is

Answer (5880)

Sol. Group A (7B + 3G), Group B (4B + 5G)

$$\begin{aligned}
 \text{Number of required ways} &= ({}^7C_4 \cdot {}^3C_1) \cdot ({}^4C_0 \cdot {}^5C_3) \\
 &\quad + ({}^7C_3 \cdot {}^3C_2) \cdot ({}^4C_1 \cdot {}^5C_2) \\
 &\quad + ({}^7C_2 \cdot {}^3C_3) \cdot ({}^4C_2 \cdot {}^5C_1)
 \end{aligned}$$

$$= 5880$$

25.



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