MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2 \cos^2 x}{1+e^x} dx = \pi(\alpha\pi^2 + \beta)$, then $(\alpha + \beta)^2$ is

(1) 100 (2) 144

(3) 169 (4) 196

Answer (1)

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2 \cos^2 x}{1+e^x} dx$

$$= \int_0^{\frac{\pi}{2}} 96x^2 \cos^2 x \, dx$$

$$= 48\int_0^{\frac{\pi}{2}} x^2(1+\cos 2x)dx$$

$$= 48\left(\frac{x^3}{3}\right)\Bigg|_0^{\frac{\pi}{2}} + 48\int_0^{\frac{\pi}{2}} x^2 \cos 2x dx$$

$$= 16\left[\frac{\pi^2}{8}\right] + 48\left[\frac{x^2 \sin 2x}{2}\Bigg|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}(2x)\left(\frac{\sin 2x}{2}\right)dx\right]$$

$$= 2\pi^2 + 48(0-0) - 48\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

$$= 2\pi^3 + 48\left[\frac{-x\cos x}{2}\Bigg|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}(1)\left(\frac{-\cos 2x}{2}\right)dx\right]$$

$$= 2\pi^3 - 48\left[\left(\frac{\pi}{2}\right)\times\frac{1}{2}\right] + 48\int_0^{\frac{\pi}{2}}\frac{\cos 2x}{2}dx$$

$$= 2\pi^2 - 12\pi + 24\left(\frac{\sin 2x}{2}\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$= 2\pi^2 - 12\pi + (0-0) = \pi\left[2\pi^2 - 12\right]$$

$$= \pi\left[\alpha\pi^2 + \beta\right]$$

$\Rightarrow$ $\alpha = 2, \beta = -12$

$\Rightarrow$ $(\alpha + \beta)^2 = (2 - 12)^2 = 100$

2. Number of ways to form 5 digit numbers greater than 50000 with the use of digits 0, 1, 2, 3, 4, 5, 6, 7 such that sum of first and last digit is not more than 8, is equal to

(1) 5119 (2) 5120

(3) 4607 (4) 4608

Answer (3)

Sol. $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}\ \underline{e}$

$a \geq 5$

$a + e \leq 8$

not all b, c, d, e are zero if $a = 5$.

$\Rightarrow$ (i) $a = 5$

5 _ _ _ $\underline{e}$ $\Rightarrow e \in \{3, 2, 1, 0\}$

$8^3 \cdot 4 - 1$ ways

$\Rightarrow$ (ii) $a = 6$

6 _ _ _ $\underline{e}$ $\Rightarrow e \in \{2, 1, 0\}$

$8^3 \cdot 3$ ways

$\Rightarrow$ (iii) $a = 7$

7 _ _ _ $\underline{e}$ $\Rightarrow e \in \{1, 0\}$

$\Rightarrow 8^3 \cdot 2$ ways

$\Rightarrow$ Total ways

$\Rightarrow 8^3(2 + 3 + 4) - 1 = 4608 - 1 = 4607$

3. If the image of the point $P(4,\ 4,\ 3)$ in the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{1}$ is $Q(\alpha, \beta, \gamma)$. Then $(\alpha + \beta + \gamma)$ is equal to

(1) 7

(2) $\frac{31}{3}$

(3) $\frac{11}{3}$

(4) 8

Answer (2)

Sol. $P(4, 4, 3)$

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{1}=\lambda$$

Any pint of line $R(2\lambda + 1, \lambda + 2, \lambda + 1)$

$$\overrightarrow{PR}:(2\lambda-3)\hat{i}+(\lambda-2)\hat{j}+(\lambda-2)\hat{k}$$

$$\overrightarrow{PR} < 2, 1,\ 1 > = 0$$

$2(2\lambda - 3) + (\lambda - 2) + 2(\lambda - 2) = 0$

$6\lambda = 10$

$$\lambda=\frac{5}{3}$$

$$\therefore\ R\left(\frac{13}{3},\frac{11}{3},\frac{8}{3}\right)$$

Now, $Q(\alpha, \beta, \gamma)$

$$\frac{\alpha+4}{2}=\frac{13}{3},\ \frac{\beta+4}{2}=\frac{11}{3},\ \frac{\gamma+3}{2}=\frac{8}{3}$$

$$\alpha=\frac{14}{3},\ \beta=\frac{10}{3},\ \gamma=\frac{7}{3}$$

$$\alpha+\beta+\gamma=\frac{14+10+7}{3}=\frac{31}{3}$$

4. If $\int_0^x tf(t)dt = x^2 f(x)$ and $f(2) = 3$, then $f(6)$ equals to

(1) 1 (2) 6

(3) 3 (4) 2

Answer (1)

Sol. $\int_0^x tf(t)dt = x^2 f(x)$

Differentiating both sides w.r.t 'x'

$xf(x) = x^2 f'(x) + 2xf(x)$

$$\frac{x^2 dy}{dx}+xy=0$$

$$\frac{dy}{y}=\frac{-dx}{x}$$

$\ln y + \ln x = \ln c$

$yx = c$

As $f(2) = 3$

$6 = c$

$\therefore\ yx = 6$

$\therefore$ Put $x = 6$

$y(6) = 6$

$y = 1$

Option (1) is correct

5. Let R be a relation such that $R = \{(x,\ y) : x,\ y \in Z$ and $(x + y)$ is even$\}$, then the relation R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive and transitive but not symmetric

(3) Transitive only

(4) Equivalence relation

Answer (4)

Sol. **for reflexive**

If $(x, x) \in Z$

R : $x + x + 2x$ $\Rightarrow$ R is relexive

For symmetric

If $(x, y) \in R$ $\Rightarrow$ $x + y$ = even

$\Rightarrow$ $y + x$ = even $6(y, x) \in R$

$\Rightarrow$ R is symmetric

If $(x, y) \in R$ $\Rightarrow$ $x + y$ = even

$(y, z) \in R$ $\Rightarrow$ $y + z$ = even

$\Rightarrow \quad x + 2y + Z \in$ even $\quad \Rightarrow \quad x + Z =$ even $- 2y$ even

$\Rightarrow \quad \underline{x + z \in \text{even}}$

$\Rightarrow \quad (x, z) \in R$

$\Rightarrow \quad R$ is equivalence relation.

6. Evaluate

$$\cos\left(\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{33}{65}\right)\right)$$

(1) 0

(2) 1

(3) $\cos\frac{5}{13}$

(4) 2

Answer (1)

Sol. $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{33}{65}\right)$

$$=\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\left[\frac{5}{13}\sqrt{\frac{1-33^2}{65^2}}+\frac{33}{65}\sqrt{1-\frac{5^2}{13^2}}\right]\right)$$

$$=\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$$

$$=\cos\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{3}{5}\right)$$

$$=\cos\left(\frac{\pi}{2}\right)=0$$

7. The sum of squares of real roots of the equation: $x^2 + |2x - 3| - 4 = 0$, is

(1) $6\left(2-\sqrt{2}\right)$ (2) $3\left(2-\sqrt{2}\right)$

(3) $3\left(2+\sqrt{2}\right)$ (4) $6\left(2+\sqrt{2}\right)$

Answer (1)

Sol. $x^2 + |2x - 3| - 4 = 0$

(i) $2x - 3 \geq 0 \quad \Rightarrow \quad x \geq \frac{3}{2}$

$\Rightarrow \quad x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0 \quad \Rightarrow \quad (x + 1)^2 = 8$

$\Rightarrow \quad x = \pm 2\sqrt{2} - 1$

$\Rightarrow \quad x = \left(2\sqrt{2} - 1\right)$

as $-2\sqrt{2} - 1 < \frac{3}{2}$

(ii) $x \leq \frac{3}{2} \quad \Rightarrow \quad x^2 - (2x - 3) - 4 = 0$

$\Rightarrow \quad x^2 - 2x - 1 - 0 = 0 \quad \Rightarrow \quad (x - 1)^2 = 2$

$\Rightarrow \quad x = \pm 2\sqrt{2} + 1 \quad \Rightarrow \quad x = -2\sqrt{2} + 1$

as $\sqrt{2} + 1 > \frac{3}{2}$

$\Rightarrow$ two roots are $x = -\sqrt{2} + 1, 2\sqrt{2} - 1$

$\Rightarrow$ Sum of squares $= 12 - 6\sqrt{2} = 6\left(2 - \sqrt{2}\right)$

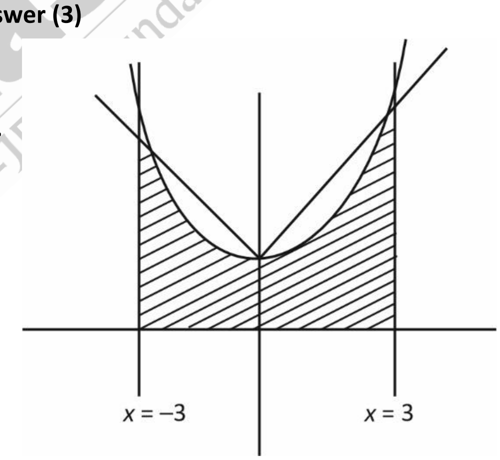
8. Area enclosed by $\left\{(x, y): 0 \leq y \leq 2|x| + 1, 0 \leq y \leq x^2 + 1, |x| \leq 3\right\}$ is equals to

(1) $\frac{17}{3}$ (2) $\frac{32}{3}$

(3) $\frac{64}{3}$ (4) $\frac{80}{3}$

Answer (3)

Sol.



$$\text{Area} = 2\left[\int_0^2 (x^2 + 1)dn + \frac{1}{2}[5 + 7] \times 1\right]$$

$$= \frac{64}{3}$$

9. There are 2 bad oranges mixed with 7 good oranges and 2 oranges are drawn at random. Let X be the number of bad oranges. The variance of X is

(1) $\frac{51}{268}$ (2) $\frac{49}{162}$

(3) $\frac{63}{108}$ (4) $\frac{91}{206}$

Answer (2)

Sol.

X	0	1	2
$P(X)$	$\frac{^7C_2}{^9C_2}$	$\frac{^7C_1 \cdot {}^2C_1}{^9C_2}$	$\frac{^2C_2}{^9C_2}$

$$\text{Variance} = 0^2 \cdot \frac{^7C_2}{^9C_2} + 1^2 \cdot \frac{^7C_1 \cdot {}^2C_1}{^9C_2} + 2^2 \cdot \frac{^2C_2}{^9C_2}$$

$$-\left(\frac{0 \cdot {}^7C_2}{^9C_2} + \frac{1 \cdot {}^7C_1 \cdot {}^2C_1}{^9C_2} + \frac{2 \cdot {}^2C_2}{^9C_2}\right)^2$$

$$= \frac{7}{18} + \frac{4}{36} - \left(\frac{7}{18} + \frac{2}{36}\right)^2$$

$$= \frac{49}{162}$$

10. Let $f(x) = \frac{2^x}{2^x + \sqrt{2}}$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to

(1) $\frac{81}{2}$ (2) 41

(3) $41\sqrt{2}$ (4) 81

Answer (1)

Sol. $f(x) = \frac{2^x}{2^x + 2^{1/2}} = \frac{2^x}{2^x + \sqrt{2}}$

$$f(1-x) = \frac{2^{1-x}}{2^{1-x} + 2^{1/2}} = \frac{\frac{2}{2^x}}{\frac{2}{2^x} + 2^{1/2}} = \frac{2}{2 + \sqrt{2}\, 2^x}$$

$$= \frac{\sqrt{2}}{2^x + \sqrt{2}}$$

$$\Rightarrow f(x) + f(1-x) = \frac{\sqrt{2} + 2^x}{\sqrt{2} + 2^x} = 1$$

$$\Rightarrow \sum_{k=1}^{81} f\left(\frac{k}{82}\right) + f\left(\frac{2}{82}\right) + \left(f\left(\frac{3}{82}\right)\right) + \ldots.$$

$$\ldots + f\left(\frac{40}{82}\right) + f\left(\frac{41}{82}\right) + f\left(\frac{42}{82}\right)$$

$$+ \ldots + f\left(\frac{79}{82}\right) + f\left(\frac{80}{82}\right) + f\left(\frac{81}{82}\right)$$

$$= \left[f\left(\frac{1}{82}\right) + f\left(\frac{81}{82}\right)\right] + \left[f\left(\frac{2}{82}\right) + f\left(\frac{80}{82}\right)\right] + \ldots$$

$$+ \left[f\left(\frac{40}{82}\right) + f\left(\frac{42}{82}\right) + f\left(\frac{41}{82}\right)\right]$$

$$= \underset{40 \text{ times}}{\longleftarrow 1 + 1 + \ldots. + 1 \longrightarrow} + f\left(\frac{1}{2}\right)$$

$$= 40 + \frac{\sqrt{2}}{\sqrt{2} + \sqrt{2}} = 40 + \frac{1}{2} = \frac{81}{2}$$

11. If $2a_{n+2} = 5a_{n+1} - 3a_n$, where $n = 0, 1, 2, \ldots$. If $a_0 = 3$ and $a_1 = 4$, then the value of $\sum_{k=1}^{100} a_k$ is equal to

(1) $3a_{100} - 91$ (2) $3a_{99} - 91$

(3) $3a_{100} + 91$ (4) $3a_{99} + 91$

Answer (1)

Sol. $2a_{n+2} = 5a_{n+1} - 3a_n = 0$

$\Rightarrow 2t^2 - 5t + 3 = 0$

$\Rightarrow t = 1, \frac{3}{2}$

$\therefore \quad a_n = A \cdot (1)^n + B \cdot \left(\frac{3}{2}\right)^2$

$a_0 = 3, a_1 = 4$

$\therefore \quad A = 1, B = 2$

$a_n = 1 + 2 \cdot \left(\frac{3}{2}\right)^n$

$S_{100} = 100 - 6\left(1 - \left(\frac{3}{2}\right)^{100}\right)$

$= 3a_{100} - 99$

Using $\Rightarrow \left(\frac{3}{2}\right)^{100} = \left(\frac{a_{100}-1}{2}\right)$

$$a_{100} = 2\cdot\left(\frac{3}{2}\right)^{100} + 1$$

12. Let k_1 and k_2 be two randomly selected natural numbers.

The probability that $(i)^{k_1} + (i)^{k_2}$ is non-zero is (where $i = \sqrt{-1}$)

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{1}{4}$ (4) $\frac{1}{6}$

Answer (2)

Sol. $(i)^{k_1} + (i)^{k_2}$ is non zero

$\Rightarrow$ $k_1 : 4\lambda_1 + r_1$, $r_1 \in \{0, 1, 2, 3\}$

$K_2 : 4\lambda_2 + r_2$, $r_2 \in \{0, 1, 2, 3\}$

The pairs to get zero will be

$(1, -1)$, $(i, -i)$

$\Rightarrow$ (i) $(1, -1)$ pair

$\Rightarrow (r_1, r_2) \in \{(2, 0), (0, 2)\}$

(ii) $(i, -i)$ pair

$\Rightarrow (r_1, r_2) \in \{(1, 3), (3, 1)\}$

$\Rightarrow$ probablity $\left(i^{k_1} + i^{k_2} \neq 0\right)$

$= 1 -$ probablity $\left(i^{k_1} + i^{k_2} \neq 0\right)$

$$= 1 - \frac{4}{16} = \frac{12}{16} = \frac{3}{4}$$

13. In ΔABC, $A(4\sin\theta, 4\cos\theta)$, $B(-2\cos\theta, 0)$ and $C(2, 2\sin\theta)$. If locus of centroid is $(3x-2)^2 + (3y)^2 = \alpha$, then α is

(1) 20

(2) 4

(3) 16

(4) 12

Answer (1)

Sol. $h = \frac{4\sin\theta - 2\cos\theta + 2}{3} \Rightarrow 3h - 2 = 4\sin\theta - 2\cos\theta$ ...(1)

$k = \frac{4\cos\theta - 2\sin\theta}{3} \Rightarrow 3k = 4\cos\theta + 2\sin\theta$ ...(2)

$(1)^2 + (2)^2$

$$(3h-2)^2 + (3k)^2 = (4\sin\theta - 2\cos\theta)^2 + (4\cos\theta + 2\sin\theta)^2$$

$$= 16\sin^2\theta + 4\cos^2\theta - 8\sin\theta\cos\theta +$$
$$16\cos^2\theta + 4\sin^2\theta - 8\sin\theta\cos\theta$$

$(3h-2)^2 + (3k)^2 = 20$

$(3x-2)^2 + (3y)^2 = 20$

14. Let $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ be an ellipse and a series of ellipse are drawn that E_{i+1} has same centre, eccentricity as E_i and E_{i+1}'s major axis is minor axis of E_i. If S_i be the area of E_i, then $\left(\frac{5}{\pi}\sum_{i=1}^{\infty} S_i\right)$ is equal to

(1) 63 (2) 54

(3) 45 (4) 72

Answer (2)

Sol. Let b_i be minor axis of E_i

a_i be major axis of E_i

$$\Rightarrow e_i = 1 - \frac{b_i^2}{a_i^2}$$

Now, b_{i+1} minor axis of E_{i+1}

A_{i+1} major axis of E_{i+1}

$\Rightarrow$ $e_{i+1} = 1 - \frac{b_{i+1}^2}{a_{i+1}^2}$, also $a_{i+1} = b_i$ and $e_i = e_{i+1}$

$$= \frac{b_i^2}{a_i^2} = \frac{b_{i+1}^2}{(b_i)^2} \Rightarrow b_{i+1} = \frac{b_i^2}{a_i}$$

$\Rightarrow$ Area of $E_i = S_i = p a_i b_i$

$\Rightarrow$ $S_{i+1} = \pi a_{i+1} b_{i+1}$

$$= \pi(b_i)\left(\frac{b_i^2}{a_i}\right)$$

$$= \pi(b_i a_i)\underbrace{\left(\frac{b_i}{a_i}\right)^2}$$

$$S_{i+1} = S_i(1-e_i^2)$$

$$\Rightarrow \; S_{i+1} = S_i\left(1-\left(1-\frac{4}{9}\right)\right) = S_i\cdot\frac{4}{9}$$

$$\Rightarrow \; S_i = 6\pi,\, S_2 = 6\pi\cdot\frac{4}{9}\, S_3 = 6\pi\cdot\left(\frac{4}{9}\right)^2$$

$$\Rightarrow \; \sum_{k=1}^{\infty} S_i = \left(\frac{6\pi}{1-\frac{4}{9}}\right) = \frac{54\pi}{5}$$

$$\Rightarrow \; \frac{5}{\pi}\sum_{k=1}^{\infty} S_i = \frac{5}{\pi}\cdot\frac{54\pi}{5} = 54$$

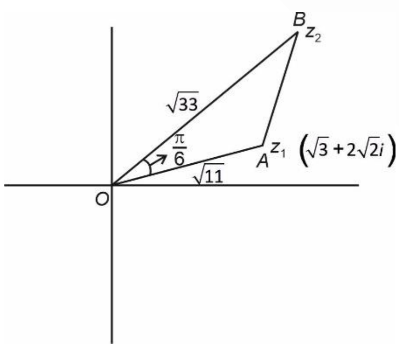
15. Let $z_1 = \sqrt{3}+2\sqrt{2}i$ and $\sqrt{3}|z_1| = |z_2|$ and $\arg(z_2) = \arg(z_1)+\frac{\pi}{6}$, then the area of triangle with vertices z_1, z_2 and origin is (in sq. units)

(1) $\frac{11\sqrt{3}}{4}$ (2) $\frac{11\sqrt{2}}{3}$

(3) $\frac{11}{4}$ (4) $\frac{2\sqrt{2}}{3}$

Answer (1)

Sol.



Area of ΔOAB = $\frac{1}{2}\times\sqrt{11}\times\sqrt{33}\sin\frac{\pi}{6}$

= $\frac{11\sqrt{3}}{4}$ square units

16. Let $f(x) = \begin{cases} 2x, & x<0 \\ \min(1+x+[x], 1+2[x]), & 0\le x<2 \\ 5, & x\ge 2 \end{cases}$

If α is the number of points of discontinuity and β is the number of points of non-differentiability, then $(\alpha+\beta)$ is equal to (where [.] denote greatest integer function)

(1) 6 (2) 5

(3) 4 (4) 8

Answer (1)

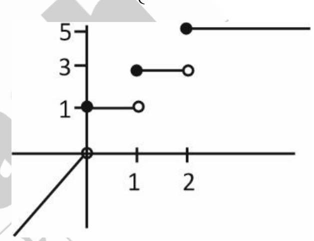
Sol. $f(x) = \begin{cases} 2x, & x<0 \\ \min(1+x+[x], 1+2[x]), & 0\le x<2 \\ 5 & x\ge 2 \end{cases}$

$1 + x + [x] = 1 + \{x\} + 2[x]$

Since $\{x\}\ge 0\ \forall x\in R$

$\Rightarrow \; 1+x+[x]\ge 1+2[x]$

$\Rightarrow \; f(x) = \begin{cases} 2x & , x<0 \\ 1+2[x], & 0\le x<0 \\ 5 & x\ge 2 \end{cases}$



Number of discontinuity = 3 $\Rightarrow \alpha = 3$

Number of point of non-differentiability

= 3 $\Rightarrow \beta = 3$

17. If $\alpha = 1+\sum_{n=1}^{6}(-3)^{n-1}\cdot{}^{12}C_{2n-1}$, then distance of point $(12, \sqrt{3})$ from the line $\alpha x-\sqrt{3}y+100=0$ is,

(1) $\frac{109}{2}$

(2) 55

(3) 54

(4) 109

Answer (1)

Sol. $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1} \cdot {}^{12}C_{2r-1}$

$= 1 + \sum_{r=1}^{6} \left[\left(\sqrt{3}i\right)^2\right]^{r-1} \cdot {}^{12}C_{2r-1}$

$= 1 + \frac{1}{\sqrt{3}i} \sum_{r=1}^{6} \left(\sqrt{3}i\right)^{2r-1} \cdot {}^{12}C_{2r-1}$

Let $\sqrt{3}i = x$

$\Rightarrow \quad \alpha = 1 + \frac{1}{\sqrt{3}i} \sum_{r=1}^{6} {}^{12}C_{2r-1} \cdot x^{2r-1}$

$$\alpha = 1 + \frac{1}{\sqrt{3}i}\left[{}^{12}C_1 \cdot x^1 + {}^{12}C_3 \cdot x^3 + {}^{12}C_5 \cdot x^5 + \ldots + {}^{12}C_{11} \cdot x^{11}\right]$$

Let $(1+x)^{12} = {}^{12}C_0 \cdot x^0 + {}^{12}C_1 \cdot x^1 + {}^{12}C_2 \cdot x^2 + \ldots + {}^{12}C_{12} \cdot x^{12}$

$$(1-x)^{12} = {}^{12}C_0 \cdot x^0 - {}^{12}C_1 \cdot x^1 + {}^{12}C_2 \cdot x^2 - {}^{12}C_3 \cdot x^3 \ldots + {}^{12}C_{12} \cdot x^{12}$$

$$(1+x)^{12} - (1-x)^{12} = 2\left({}^{12}C_1 \cdot x^1 + {}^{12}C_3 \cdot x^3 + \ldots + {}^{12}C_{11} \cdot x^{11}\right)$$

$$\Rightarrow \quad \alpha = 1 + \frac{1}{\sqrt{3}i}\left[\frac{(1+x)^{12} - (1-x)^{12}}{2}\right]$$

$$\alpha = 1 + \frac{1}{\sqrt{3}i}\left[\frac{\left(1+\sqrt{3}i\right)^{12} - \left(1-\sqrt{3}i\right)^{12}}{2}\right]$$

Since, $\omega = \frac{-1}{2} + \frac{\sqrt{3}i}{2} \qquad \Rightarrow \quad -2\omega = 1 - \sqrt{3}i$

$\omega^2 = \frac{-1}{2} - \frac{\sqrt{3}i}{2} \qquad \Rightarrow \quad -2\omega^2 = 1 + \sqrt{3}i$

$$\Rightarrow \quad \alpha = 1 + \frac{1}{\sqrt{3}i}\left(\frac{\left(-2\omega^2\right)^{12} - (2\omega)^{12}}{2}\right)$$

$$= 1 + \frac{2^{11}}{\sqrt{3}i}\left(\omega^{24} - \omega^{12}\right)$$

$$= 1 + \frac{2^{11}}{\sqrt{3}i}(1-1) = 1$$

$\alpha = 1, \ \Rightarrow$ perpendicular distance from $\left(12, \sqrt{3}\right)$ is

$$\frac{\left|12 - \sqrt{3}\left(\sqrt{3}\right) + 100\right|}{\sqrt{1^2 + \left(\sqrt{3}\right)^2}} = \frac{109}{2}$$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In an AP, $T_m = \frac{1}{25}, T_{25} = \frac{1}{20}$ and $20\sum_{r=1}^{25} T_r = 13$, then $5m\sum_{r=m}^{2m} T_r$ equals

Answer (126)

Sol. $T_m = a + (m-1)d \cdot \frac{1}{25} \qquad \ldots(1)$

$T_{25} = a + 24d = \frac{1}{20}$

$20 \times \frac{2r}{2}\left[a + \frac{1}{20}\right] = 13 \Rightarrow a = \frac{1}{20 \times 25}$

$20\sum_{r=1}^{25} T = 20 \times \frac{25}{2}[2a + 24d] = 13$

$d = \frac{1}{20 \times 25}$

Substitute a and d in (i)

$\Rightarrow m = 20$

Now $5m\sum_{r=m}^{2m} T_r = 5 \times 20\left[\sum_{r=20}^{40} T_r\right]$

$$= 100\left[\frac{40}{2}[2a + 39d]\right] - \frac{19}{2}[2a + 18d]$$

$$= 100\left[\frac{40}{2} \times 41d - \frac{19}{2} \cdot 20d\right]$$

$$= 100\left[\frac{40}{2} \times \frac{41}{20 \times 25} - \frac{19.20}{2 \times 20 \times 25}\right]$$

= 126

22.

23.

24.

25.

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