

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f(x) = \int \frac{dx}{x^{1/4}(x^{1/4} + 1)}$. If $f(0) = -6$, then $f(2)$ is

- (1) $4 \left[\frac{1}{\sqrt{2}} - 2^{1/4} + \ln|1 + 2^{1/4}| \right] - 6$
- (2) $4 \left[\frac{1}{\sqrt{2}} - 2^{1/4} + \ln|1 + 2^{1/4}| \right] + 6$
- (3) $4 \left[\frac{1}{\sqrt{2}} + 2^{1/3} + \ln|2^{1/4}| \right] - 6$
- (4) $4 \left[3 + 2^{1/3} - \ln 2^{1/4} \right] + 6$

Answer (1)

Sol. $\int \frac{dx}{x^{1/4}(x^{1/4} + 1)}$
 $x^{1/4} = t \Rightarrow dx = 4t^3 dt$
 $\int \frac{4t^3 dt}{t(t+1)} = 4 \int \frac{t^2 dt}{t+1} = 4 \int \left(\frac{t^2 - 1}{t+1} + \frac{1}{t+1} \right) dt$
 $= 4 \int [(t-1)dt + \ln|t+1|] + c$
 $f(x) = 4 \left[\frac{x^{1/2}}{2} - x^{1/4} + \ln|x^{1/4} + 1| \right] + c$
 $f(0) = -6$
 $-6 = 4(0) + c$
 $\Rightarrow c = -6$
 $\Rightarrow f(x) = 4 \left[\frac{\sqrt{x}}{2} - x^{1/4} + \ln|1 + x^{1/4}| \right] - 6$
 $f(2) = 4 \left[\frac{1}{\sqrt{2}} - (2)^{1/4} + \ln|1 + 2^{1/4}| \right] - 6$

2. Evaluate $\sum_{r=1}^{13} \frac{1}{\sin \left[\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right] \sin \left[\frac{\pi}{4} + \frac{r\pi}{6} \right]}$

- (1) $2\sqrt{3} + 2$
- (2) $2\sqrt{3} - 2$
- (3) $3\sqrt{2} + 2$
- (4) $3\sqrt{2} - 4$

Answer (2)

Sol. $\sum_{r=1}^{13} \frac{1}{\sin \left[\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right] \sin \left[\frac{\pi}{4} + \frac{r\pi}{6} \right]}$
 $= \sum_{r=1}^{13} \frac{\sin \left\{ \left(\frac{\pi}{4} + \frac{r\pi}{6} \right) - \left(\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right) \right\}}{\sin \frac{\pi}{6} \cdot \sin \left[\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right] \cdot \sin \left[\frac{\pi}{4} + \frac{r\pi}{6} \right]}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= 2 \cdot \sum_{r=1}^{13} \cot \left(\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right) - \cot \left(\frac{\pi}{4} + \frac{r\pi}{6} \right)$
 $= 2 \cdot \cot \left(\frac{\pi}{4} + 0 \cdot \frac{\pi}{6} \right) - \cot \left(\frac{\pi}{4} + \frac{13\pi}{6} \right)$
 $= 2 \cdot \left\{ \cot \frac{\pi}{4} - \cot \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \right\}$
 $= 2[1 - 2 + \sqrt{3}]$
 $= 2[\sqrt{3} - 1]$

3. Area bounded between the curves $C_1 : x(1 + y^2) - 1 = 0$ and $C_2 : y^2 - 2x = 0$ is (in sq. unit)

- (1) $\frac{\pi}{2} - \frac{1}{3}$
- (2) $\frac{\pi}{4} - \frac{1}{6}$
- (3) $2 \left(\frac{\pi}{2} - \frac{1}{6} \right)$
- (4) $\frac{\pi}{6} + \frac{1}{2}$

Answer (1)

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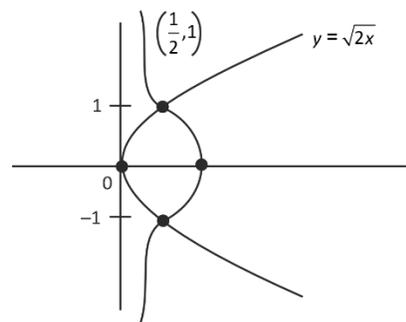
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AIR 15
M Sai Divya Teja Reddy
2 Year Classroom

AIR 19
Rishi Shekher Shukla
2 Year Classroom

Sol.



$$\int_{-1}^1 (x_2 - x_1) dy$$

$$\Rightarrow \int_{-1}^1 \left(\frac{1}{1+y^2} - \frac{y_2}{2} \right) dy$$

$$= \tan^{-1} y - \frac{y_3}{6} \Big|_{-1}^1$$

$$= \left(\tan^{-1} \frac{1}{6} \right) - \left(\tan^{-1}(-1) - \left(\frac{-1}{6} \right) \right)$$

$$= 2 \tan^{-1} 1 - \frac{1}{3} = \frac{\pi}{2} - \frac{1}{3}$$

4. There are three bags such that bag 1 has 4 white, 6 blue, bag 2 has 6 white and 4 blue and bag 3 has 5 white and 5 blue balls. A bag is randomly selected and a ball is randomly picked out of it, it comes out to be white then probability that selected bag was bag 2.

- (1) $\frac{2}{5}$ (2) $\frac{2}{15}$
(3) $\frac{1}{15}$ (4) $\frac{7}{15}$

Answer (1)

Sol. $\boxed{4W+6B}$ $\boxed{6W+4B}$ $\boxed{5W+5B}$
BAG 1 BAG 2 BAG 3

$$P\left(\frac{B_2}{W}\right) = \frac{P(B_2) \cdot P\left(\frac{W}{B_2}\right)}{P(W)}$$

$$P(W) = \sum_{i=1}^3 P(B_i) P\left(\frac{W}{B_i}\right)$$

$$P(B_1) = \frac{1}{3} = P(B_2) = P(B_3)$$

$$P\left(\frac{W}{B_1}\right) = \frac{4}{10}, P\left(\frac{W}{B_2}\right) = \frac{6}{10}, P\left(\frac{W}{B_3}\right) = \frac{5}{10}$$

$$\Rightarrow P\left(\frac{B_2}{W}\right) = \frac{\frac{1}{3} \times \frac{6}{10}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{6}{10} + \frac{1}{3} \times \frac{5}{10}}$$

$$= \frac{\frac{6}{10}}{\frac{4}{10} + \frac{6}{10} + \frac{5}{10}} = \left(\frac{6}{15}\right) = \frac{2}{5}$$

5. If S is a set of words formed by all the letters of word "GARDEN", then find the probability that vowels are not in alphabetical order.

- (1) $\frac{1}{2}$ (2) $\frac{1}{3}$
(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

Answer (1)

Sol. AE GRDN

Only 2 vowels are there.

∴ Only half of the cases will have A before E and vice-versa.

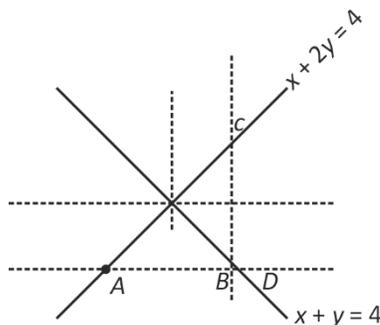
$$\text{Required probability} = \frac{1}{2}$$

6. In isosceles triangle two sides are $x + 2y = 4$, $x + y = 4$ than the sum of all possible value of slope of third side of triangle is

- (1) $\frac{3}{2}$ (2) $\frac{2}{3}$
(3) $-\frac{3}{2}$ (4) $-\frac{2}{3}$

Answer (2)

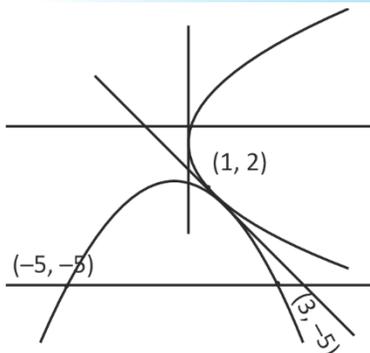
Sol.



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Sol.



To find image of $P(t^2, 2t)$

$$\frac{x-t^2}{1} = \frac{y-2t}{1} = \frac{-2(t^2+2t+1)}{1^2+1^2} = -(t+1)^2$$

$$\Rightarrow x = t^2 - (t+1)^2 = -2t-1$$

$$y = 2t - (t+1)^2 = -t^2 - 1$$

$$t = \left(\frac{-1-x}{2} \right)$$

$$\Rightarrow (y+1) = -\left(\frac{-1-x}{2} \right)^2$$

$$(y+1) = -\frac{(x+1)^2}{4}$$

$$\Rightarrow (x+1)^2 = -4(y+1)$$

$$\Rightarrow X^2 = -4Y, \text{ the vertex is}$$

$$X=0, Y=0$$

$$\Rightarrow \text{Focus} \Rightarrow (-1, -2)$$

Other method would have been to find image of

$$(1, 0) \text{ about } x+y+1=0$$

$$\Rightarrow y = -5 \text{ intersect}$$

$$(-4)(-4) = (x+1)^2$$

$$\Rightarrow (x+1) = \pm 4$$

$$\Rightarrow x = 3, -5$$

$$\Rightarrow a = 8$$

$$d = \frac{1}{2} \times a \times \text{height} = \frac{1}{2} \times 8 \times (-2 - (-5))$$

$$= 4 \cdot 3 = 12 \text{ Sq. unit}$$

$$\Rightarrow \boxed{a+d=20}$$

11. For positive integer n , $4a_n = n^2 + 5n + 6$ and

$$S_n = \sum_{K=1}^n \frac{1}{a_K}. \text{ Then the value of } 507(S_{2025}) \text{ is}$$

- (1) 675 (2) 540
(3) 1350 (4) 725

Answer (1)

$$\text{Sol. } S_n = \sum_{K=1}^n \frac{4}{K^2 + 5K + 6}$$

$$= \sum_{K=1}^n \frac{4}{(K+2)(K+3)} = 4 \sum_{K=1}^n \left(\frac{1}{K+2} - \frac{1}{K+3} \right)$$

$$= 4 \left[\frac{1}{3} - \frac{1}{4} \right]$$

$$= 4 \left[\frac{1}{4} - \frac{1}{5} \right]$$

$$= 4 \left[\frac{1}{n+2} - \frac{1}{n+3} \right]$$

$$S_n = 4 \left[\frac{1}{3} - \frac{1}{n+3} \right]$$

$$S_{2025} = 4 \left[\frac{1}{3} - \frac{1}{2028} \right]$$

$$S_{2025} = 4 \left[\frac{675}{2028} \right]$$

$$507 S_{2025} = 675$$

12. The number of natural numbers between 212 to 999 such that sum of their digit is 15 is equal to

- (1) 63 (2) 61
(3) 62 (4) 65

Answer (2)

$$\text{Sol. } 717 \rightarrow 3$$

$$726 \rightarrow 3!$$

$$735 \rightarrow 3!$$

$$744 \rightarrow 3$$

$$636 \rightarrow 3$$

$$645 \rightarrow 3!$$

$$555 \rightarrow 1$$

$$915 \rightarrow 3!$$

$$924 \rightarrow 3!$$

$$933 \rightarrow 3$$

$$816 \rightarrow 3!$$

$$825 \rightarrow 3!$$

$$834 \rightarrow 3!$$

$$\text{Total : 61}$$

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- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $B = QPQ^T$ and matrix A is defined as $A = Q^T B^{10} Q$ (where $P = \begin{bmatrix} \sqrt{2} & -2 \\ 0 & 1 \end{bmatrix}$), then trace of matrix A is

Answer (33)

Sol. $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
 $B = QPQ^T$
 $A = Q^T B^{10} Q$
 $Q^T Q = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
 $= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
 $\Rightarrow Q^T Q = QQ^T$
 $A = Q^T (QPQ^T)^{10} Q$
 $= IPQ^T B^{10} Q = PQ^T B^{10} Q$
 $\Rightarrow A = PQ^T (QPQ^T)^{10} B^{10} Q$
 $= P^2 Q^T B^{10} Q = P^9 Q^T B^{10} Q$
 $= P^9 Q^T (QPQ^T)^{10} Q = P^{10}$
 Trace $(P^{10}) = (d_1^{10} + d_2^{10})$ where d_1 and d_2 are diagonal elements
 $\Rightarrow \text{Trace}(A) = (\sqrt{2})^{10} + 1^{10}$
 $= 2^{10/2} + 1 = 2^5 + 1 = 33$

22. $f: [0, 3] \rightarrow b, f(x) = 2x^3 - 15x^2 + 36x + 7$ is an onto function
 $g: [0, \infty) \rightarrow d, g(x) = \frac{x^{2025}}{x^{2025} + 1}$ is also an onto function.
 Find the number of elements in the set $S = \{x : x \in Z, x \in b \text{ or } x \in d\}$

Answer (30)

Sol. $f(x) = 2x^3 - 15x^2 + 36x + 7$
 $f'(x) = 6x^2 - 30x + 36 = 0$
 $\Rightarrow x^2 - 5x + 6 = 0$
 $\therefore x = 1, 5$
 $f(0) = 7, f(2) = 35, f(3) = 34$
 $\therefore b = [7, 35]$

$$g(x) = \frac{x^{2025}}{1 + x^{2025}}$$

$d = [0, 1)$
 $\therefore S = [0, 7, 8, 9, \dots, 35]$
 Number of elements = 30

23. The maximum interior angle of a polygon is 171° with n sides such that its angles are in Arithmetic progression with common difference of 6° . Then n is equal to

Answer (10)

Sol. Sum of interior angle
 $\Rightarrow \frac{n}{2}(2a + (n-1)d) = 180^\circ(n-2)$
 $\Rightarrow 171 = a + (n-1)d$
 $\Rightarrow \frac{n}{2}(171 + a) = 180(n-2)$
 $a = 171 - 6(n-1) = 177 - 6n$
 $\Rightarrow \frac{n}{2}(171 + 177 - 6n) = 180(n-2)$
 $\Rightarrow n(174 - 3n) = 180n - 360$
 $\Rightarrow 3n^2 + 6n - 360 = 0$
 $n^2 + 2n - 120 = 0$
 $(n+12)(n-10) = 0$
 $\Rightarrow n = 10$

- 24.
- 25.



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