# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. If the letters of the word "KANPUR" are arranged in dictionary, then the 440th word is

(1) PRKAUN

(2) PRKUAN

(3) PRKNAU

(4) PRKUNA

**Answer (1)**

**Sol.** A K N P R U

A --------- 5!

K --------- 5!

N --------- 5!

PA --------- 4!

PK --------- 4!

PN --------- 4!

PRA --------- 3!

438

PRKANU → 439th

PRKAUN → 440th

2. Let $f(x)=\int_0^x t\left(t^2-3t+20\right)dt,\ x\in(1,3)$ and range of $f(x)$ is $(\alpha, \beta)$, then $\alpha+\beta$ is equal to

(1) $\frac{185}{4}$ (2) $\frac{185}{2}$

(3) $\frac{185}{3}$ (4) $\frac{37}{4}$

**Answer (2)**

**Sol.** $f(x)=\frac{x^4}{4}-x^3+10x^2$

$f'(x) = x^3 - 3x^2 + 20x$

$= x(x^2 - 3x + 20)$

in (1, 3) $f'(x)$ is positive

$\therefore$ $f(x)$ is increasing in (1, 3)

$\therefore$ $\alpha = f(1) = \frac{37}{4}$

$\beta = f(3) = 83.25 = \frac{333}{4}$

$\therefore$ $\alpha + \beta = 92.5 = \frac{185}{2}$

3. The value of the limit

$$\lim_{x\to 0}(\operatorname{cosec} x)\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right) \text{ is}$$

(1) 0 (2) 1

(3) $\frac{1}{2\sqrt{5}}$ (4) $-\frac{1}{2\sqrt{5}}$

**Answer (4)**

**Sol.** After rationalization,

$$\lim_{x\to 0}\frac{1}{\sin x}\left(\frac{\left(2\cos^2 x+3\cos x\right)-\left(\cos^2 x+\sin x+4\right)}{\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}}\right)$$

$$\lim_{x\to 0}\frac{\cos^2 x+3\cos x-\sin x-4}{(\sin x)\left(\sqrt{5}+\sqrt{5}\right)}$$

$$=\lim_{x\to 0}\frac{1}{2\sqrt{5}}\frac{\cos^2 x+3\cos x-\sin x-4}{\sin x}$$

L'Hopital,

$$\Rightarrow \lim_{x\to 0}\left(\frac{1}{2\sqrt{5}}\right)\left(\frac{2\cos x(-\sin x)-3\sin x-\cos x}{\cos x}\right)$$

$$=\frac{1}{2\sqrt{5}}\left[\frac{-1}{1}\right]=-\frac{1}{2\sqrt{5}}$$

4. Let the line $L$ be $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{5}$ and foot of perpendicular from (1, –2, –1) to $L$ is ($\alpha$, $\beta$, $\gamma$), then $\alpha + \beta + \gamma$ is

(1) $-\frac{69}{35}$ (2) $\frac{102}{35}$

(3) $\frac{69}{35}$ (4) $-\frac{102}{35}$

**Answer (4)**

**Sol.** $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{5}=\lambda$

Point on line $A(\lambda + 1, 3\lambda + 4, 5\lambda + 7)$

$B(1, -2, -1)$

$\overrightarrow{AB}\cdot\langle 1, 3, 5\rangle = 0$

$\lambda \cdot 1 + 3(3\lambda + 6) + 5(5\lambda + 8) = 0$

$35\lambda + 18 + 40 = 0$

$\lambda = -\frac{58}{35}$

$(\alpha, \beta, \gamma) \equiv \left(-\frac{23}{35}, -\frac{34}{35}, -\frac{9}{7}\right)$

$\alpha + \beta + \gamma = -\frac{102}{35}$

5. If the exhaustive values of $a$ for which the equation $2x^2 + (a - 5)x + 15 = 3a$ has no real roots is ($\alpha$, $\beta$), then $|4(\alpha + \beta)|$ is equal to

(1) 56 (2) 52

(3) 54 (4) 18

**Answer (1)**

**Sol.** No real roots $\Rightarrow$ discriminant is negative

$\Rightarrow (a - 5)^2 - 4(2)(15 - 3a) < 0$

$\Rightarrow a^2 - 10a + 25 - 120 + 24a < 0$

$a^2 + 14a - 95 < 0$

$(a + 19)(a - 5) < 0$

$\Rightarrow a \in (-19, 5)$

$\alpha = -19$

$\alpha = 5$

$|4(\alpha + \beta)|$

$= |4(-19)| = 56$

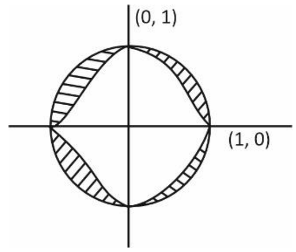
6. Area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is $(\pi - \alpha)$ sq. units, then $9\alpha$ is

(1) 8 (2) 16

(3) 32 (4) 24

**Answer (2)**

**Sol.**




Area $= \pi - 4\int_0^1 (1 - x^2)dx$

$= \pi - 4\left[x - \frac{x^3}{3}\right]_0^1$

$= \pi - 4 \times \frac{2}{3} = \left(\pi - \frac{8}{3}\right)$ sq unit

$= \pi - \alpha$

$\Rightarrow \pi = \frac{8}{3}$

$9\alpha = 16$

7. If $\log y = x \log \frac{2}{5}$, $x \in \mathbb{N} \cup \{0\}$. Then sum of all values of $y$ equals to

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{5}{4}$ (4) $\frac{8}{3}$

**Answer (1)**

**Sol.** $\log y = x \log \frac{2}{5}$

$\log y = \log\left(\frac{2}{5}\right)^x$

$y = \left(\frac{2}{5}\right)^x$

$x \in \mathbb{N} \cup \{0\}$

$\Rightarrow \quad y = 1, \frac{2}{5}, \left(\frac{2}{5}\right)^2$ ... which is in G.P.

Sum of all values of $\sum y = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$

8. There is an arithmetic progression $a_1, a_2, a_3, \ldots a_{2024}$ and $a_1 + (a_5 + a_{10} + a_{15} \ldots a_{2020}) + a_{2024} = 2233$. Find the value of $a_1 + a_2 + a_3 + \ldots a_{2024}$.

(1) 11034 (2) 11132

(3) 10432 (4) 20462

**Answer (2)**

**Sol.** $\because \; a_1, a_2, a_3, \ldots, a_{2024}$ are in A.P.

Then $a_1 + a_{2024} = a_2 + a_{2023} = \ldots = a_r + a_{2024} - r + 1 = l$

$\therefore \; a_1 + (a_5 + a_{10} + \ldots + a_{2020}) + a_{2024} = 2023$

or, $(202l) + l = 2023$

or, $203l = 2233$

$\therefore \; a_1 + a_2 + \ldots + a_{2024} = 1012 \times l$

$= 1012 \times \frac{2233}{203}$

$= 1012 \times 11$

$= 11132$

9. Two points (4, 2) and (0, 2) lie on the circle whose centre lies on $3x + 2y + 2 = 0$, then length of chord whose mid point is (1, 2), is

(1) $\sqrt{3}$ (2) $\sqrt{5}$

(3) $2\sqrt{3}$ (4) $2\sqrt{5}$

**Answer (3)**

**Sol.** Let the centre be $(-2\alpha, 3\alpha - 1)$

$$\sqrt{(-2\alpha-4)^2 + (3\alpha-3)^2} = \sqrt{(-2\alpha-0)^2 + (3\alpha-3)^2}$$

$\Rightarrow \quad (-2\alpha - 4)^2 = (-2\alpha)^2$

$\Rightarrow \quad -2\alpha - 4 = -2\alpha$

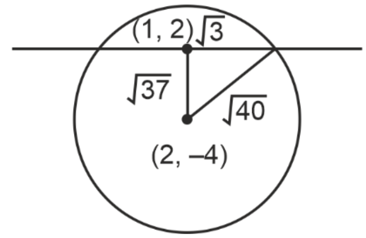
$\Rightarrow$ No solution

$-2\alpha - 4 = -2\alpha$

$\Rightarrow \quad \boxed{\alpha = -1}$

Centre will be (2, –4), radius $\sqrt{4+36} = \sqrt{40}$

$(x - 2)^2 + (y + 4)^2 = 40$




$\Rightarrow$ Length of chord $= 2\sqrt{3}$

10. If $\lim_{t \to 0}\left(\int_0^1 (3x+5)^t \, dx\right)^{\frac{1}{t}} = \frac{\alpha\left(\frac{8}{5}\right)^{\frac{p}{q}}}{4e}$, then $\alpha$ is

(1) 32 (2) 16

(3) 8 (4) 64

**Answer (1)**

**Sol.** Since, $\int_0^1 (3x+5)^t \, dx = \frac{8^{t+1} - 5^{t+1}}{3(t+1)}$

$\Rightarrow L = \lim_{t \to 0}\left(\frac{8^{t+1} - 5^{t+1}}{3(t+1)}\right)^{\frac{1}{t}}$

$\Rightarrow L = \lim_{t \to 0}\left(1 + f(t)\right)^{\frac{1}{f(x)} \cdot \frac{f(t)}{t}}$

Where $f(t) = \frac{8^{t+1} - 5^{t+1}}{3(t+1)} - 1 = \frac{8^{t+1} - 5^{t+1} - 3t - 3}{3(t+1)}$

$\Rightarrow$ Since, $\lim_{t \to 0} f(t) = 0$

$L = \lim_{t \to 0} e^{\frac{f(t)}{t}} = e^{\lim_{t \to 0} \frac{f(t)}{t}} = e^{\lim_{x \to 0} f'(t)}$

$$f'(t) = \frac{8.8^t \ln 8 - 5.5^t \ln 5 - 3}{3(t+1)} - \frac{8^{t-1} - 5^{t+1} - 3t - 3}{3(t+1)^2}$$

$$\lim_{t \to 0} f'(t) = \frac{8\ln 8 - 5\ln 5 - 3}{3} = \frac{1}{3}\ln\left(\frac{8^8}{5^5}\right) - 1$$

$$L = e^{\ln\left(\frac{8^8}{5^5}\right)^{\frac{1}{3}} - 1} = \left(\frac{8^8}{5^5}\right)^{\frac{1}{3}} . e^{-1} = \frac{\left(\frac{8^8}{5^5}\right)^{\frac{1}{3}}}{e}$$

$$= \frac{1}{e}\left(\frac{8}{5}\right)^{\frac{5}{3}} . 8^{\frac{1}{3}} = \frac{32\left(\frac{8}{5}\right)^{\frac{5}{3}}}{4e}$$

$\Rightarrow \alpha = 32$

11. The value of $\int_0^{\frac{\pi}{4}}\left(\sin\left|\left(4x - \frac{\pi}{2}\right)\right| + \sin[2x]\right)dx$ is

(where [·] denotes the greatest integer function)

(1) $\frac{1}{2} + \left(\frac{\pi - 2}{4}\right)\sin 1$ (2) $\frac{1}{4} + \left(\frac{\pi - 2}{2}\right)\sin 1$

(3) $\frac{1}{2} - \left(\frac{\pi - 2}{4}\right)\sin 1$ (4) $\frac{1}{4} - \left(\frac{\pi - 2}{2}\right)\sin 1$

**Answer (1)**

**Sol.** $\int_0^{\frac{\pi}{4}}\left(\sin\left|4x - \frac{\pi}{2}\right| + \sin[2x]\right)dx$

$$= \int_0^{\frac{\pi}{4}} \sin\left|4x - \frac{\pi}{2}\right| dx + \int_0^{\frac{\pi}{4}} \sin[2x]dx$$

$$= \int_0^{\frac{\pi}{8}} \sin\left|\frac{\pi}{2} - 4x\right| dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin\left[4x - \frac{\pi}{2}\right]dx + \int_0^{\frac{1}{2}} 0dx + \int_{\frac{1}{2}}^{\frac{\pi}{4}} \sin(1)dx$$

$$= \int_0^{\frac{\pi}{8}} \cos 4x dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos 4x dx + \sin 1 \cdot \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \left[\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{8}} - \left[\frac{\sin 4x}{4}\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}} + \frac{(x - 2)\sin 1}{4}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{(\pi - 2)\sin 1}{4}$$

$$= \frac{(\pi - 2)\sin(1) + 2}{4} = \frac{1}{2} + \left(\frac{\pi - 2}{4}\right)\sin 1$$

12.

13.

14.

15.

16.

17.

18.

19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $a_{ij} = \left(\sqrt{2}\right)^{i+j}, A = \left[a_{ij}\right]_{3\times 1}$. If sum of third row of $A^2$ is $\alpha + \beta\sqrt{2}$, then $\alpha + \beta$ is

**Answer (224)**

**Sol.** $$\begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}\begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix} = \begin{bmatrix} 28 & 28\sqrt{2} & 56 \\ 28\sqrt{2} & 56 & 56\sqrt{2} \\ 56 & 56\sqrt{2} & 112 \end{bmatrix}$$

$56 + 112 + 56\sqrt{2}$

$168 + 56\sqrt{2}$

$\alpha + \beta\sqrt{2}$

$\alpha + \beta = 224$

22. If $3^{107}$ is divided by 23, then remainder is

**Answer (06)**

**Sol.** Notice that, $3^4 \equiv (12)$ (mod23)

$\Rightarrow$ $3^8 \equiv 144 \equiv 6$(mod 23)

$3^{11} \equiv 1$(mod 23)

$(3^{11})^9 \equiv 1$(mod 23)

$3^{99} \equiv 1$(mod 23)

$3^8 \cdot 3^{99} \equiv 1$(mod 23)

$\Rightarrow$ $3^{107} \equiv 6$(mod 23)

23. If $\alpha$, $\beta$ are the values of $m$ where

$x + y + 2z = 1$

$x + 2y + 4z = m$

$x + 4y + 8 = m^2$ have infinitely many solutions.

Then $\sum_{n=1}^{10}\left(n^{\alpha} + n^{\beta}\right)$ is equal to

**Answer (440)**

**Sol.** For infinite solution

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 4 & 8 \end{vmatrix} = 0$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 2 \\ m & 2 & 4 \\ m^2 & 4 & 8 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & m & 4 \\ 1 & m^2 & 8 \end{vmatrix} = 0 \Rightarrow m^2 + 3m - 2$$

$$\Rightarrow m^2 - 3m + 2 = 0$$

$m = 2, 1$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & m \\ 1 & 4 & m^2 \end{vmatrix} = 0 \Rightarrow m^2 - 3m + 2 = 0$$

$\Rightarrow m = 2, 1 \Rightarrow \alpha = 1, \beta = 2$

$$\sum_{n=1}^{10}(n)^1 + (n)^2 = \frac{10 \times 11}{2} + \frac{10 \times 11 \times 21}{6}$$

$= 440$

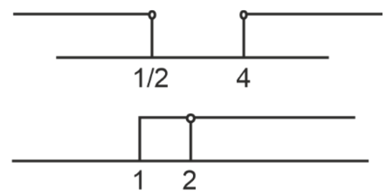
24. If the domain of $\log_{x-1}\left(\frac{2x^2 - 9x + 4}{x^2 - 4x + 5}\right)$ is $(\alpha, \infty)$ and $\log_5(18x - x^2 - 77)$ is $(\beta, \gamma)$, then the value of $\alpha^2 + \beta^2 + \gamma^2$ is

**Answer (186)**

**Sol.** $\frac{2x^2 - 9x + 4}{x^2 - 4x + 5} > 0$ ...(i)

$x - 1 > 0$, $x - 1 \neq 1$

$\Rightarrow (2x - 1)(x - 4) > 0$




$\therefore x \in (4, \infty)$

$\therefore \alpha = 4$

$\log_5(18x - x^2 - 77)$

$\Rightarrow 18x - x^2 - 77 > 0$

$\Rightarrow x^2 - 18x + 77 < 0$

$\Rightarrow (x - 7)(x - 11) < 0$

$x \in (7, 11)$

$\therefore \beta = 7, \gamma = 11$

$\therefore \alpha^2 + \beta^2 + \gamma^2$

$= 16 + 49 + 121$

$= 186$

25. The equation $\alpha x + \beta y = 109$ is chord of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ having midpoint $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is

**Answer (58)**

**Sol.** Chord with given middle point

$T = S_1$

$$\frac{5}{18}x + \frac{y}{8} = \frac{25}{36} + \frac{1}{16} = \frac{109}{144}$$

$40x + 18y = 109$

$\equiv \alpha x + \beta y = 109$

$\Rightarrow \alpha = 40 \quad \beta = 18$

$\alpha + \beta = 58$

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