Aakash
Medical | IIT-JEE | Foundations

Phase-1
21st January 2026
(Morning Shift)

Answer & Solutions

for

JEE MAIN 2026

TRUST Aakash

Stellar Ranks in JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR 23

Kushagra Baingaha AIR 7

Shreyas Lohiya AIR 6

Harssh A Gupta AIR 15

Devya Rustagi AIR 28

Shreyas Lohiya AIR 68

Rujul Garg AIR 41

Advay Mayank AIR 36

Arush Anand AIR 64

Kotha D Reddy AIR 74

Top Ranks in JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students Aakashians Qualified in NSEs 2025-26

777 Classroom Students Aakashians Qualified in IOQM 2025

134 Classroom Students Aakashians Qualified in RMO 2025

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

HOME OF PROBLEM SOLVERS

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The value of $cosec10^\circ - \sqrt{3}sec10^\circ$ is equal to:

(1) 2 (2) 6

(3) 8 (4) 4

Answer (4)

Sol. $\frac{1}{\sin 10} - \frac{\sqrt{3}}{\cos 10}$

$= \frac{1}{2}\left[\frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\frac{1}{2}\sin 10^\circ \cos 10^\circ}\right]$

$= 2\times 2\frac{[\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ]}{\sin 20^\circ} = 4$

2. Let (α, β, γ) be the co-ordinates of the foot of the perpendicular drawn from the point (5, 4, 2) on the line $\vec{r} = \left(-\hat{i} + 3\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$.

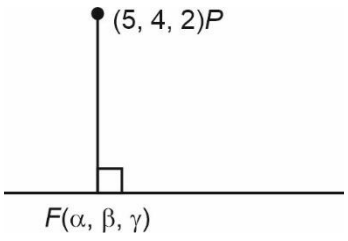
Then the length of the projection of the vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ on the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is:

(1) 4 (2) $\frac{15}{7}$

(3) $\frac{18}{7}$ (4) 3

Answer (3)

Sol.



$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$

$(2\lambda - 1,\ 3\lambda + 3,\ 1 - \lambda)$

$\overrightarrow{PF} = (2\lambda - 6)\hat{i} + (3\lambda - 1)\hat{j} + (-\lambda - 1)\hat{k}$

$\overrightarrow{PF} \cdot \left(2\hat{i} + 3\hat{j} - \hat{k}\right) = 0$

$4\lambda - 12 + 9\lambda - 3 + \lambda + 1 = 0 \Rightarrow \lambda = 1$

$\Rightarrow F \equiv (1,\ 6,\ 0)$

$\Rightarrow$ length of projection of $\hat{i} + 6\hat{j}$ on $6\hat{i} + 2\hat{j} + 3\hat{k}$

$= \left|\frac{\left(\hat{i} + 6\hat{j}\right)\cdot\left(6\hat{i} + 2\hat{j} + 3\hat{k}\right)}{\left|6\hat{i} + 2\hat{j} + 3\hat{k}\right|}\right|$

$= \frac{1}{7}(6 + 12) = \frac{18}{7}$

Option (3) is correct.

3. Let $\vec{c}$ and $\vec{d}$ be vectors such that $\left|\vec{c} + \vec{d}\right| = \sqrt{29}$ and $\vec{c} \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \vec{d}$. If λ_1, λ_2 $(\lambda_1 > \lambda_2)$ are the possible values of $\left(\vec{c} + \vec{d}\right)\cdot\left(-7\hat{i} + 2\hat{j} + 3\hat{k}\right)$, then the equation

$K^2x^2 + \left(K^2 - 5K + \lambda_1\right)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$ represents a circle, for K equal to:

(1) 2 (2) –1

(3) 4 (4) 1

Answer (4)

Sol. $\left|\vec{c} + \vec{d}\right| = \sqrt{29}$

$\vec{c} \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \vec{d} = 0$

$(\vec{c}+\vec{d})\times(2\hat{i}+3\hat{j}+4\hat{k})=0$

$\Rightarrow \vec{c}+\vec{d}=\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

$\Rightarrow \sqrt{29}=|\lambda|\sqrt{29}$

$\Rightarrow \lambda=\pm1 \Rightarrow \vec{c}+\vec{d}=\pm(2\hat{i}+3\hat{j}+4\hat{k})$

$\Rightarrow (\vec{c}+\vec{d})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})=\pm(-14+6+12)=\pm4$

$\Rightarrow \lambda_1=4,\lambda_2=-4$

Given equation represents a circle

$\Rightarrow K^2=3K+\frac{\lambda_2}{2}$ and $K^2-5K+\lambda_1=0$

$\Rightarrow K^2=3K-2$ and $K^2-5K+4=0$

$\Rightarrow K = 1, 2$ and $K = 4, 1$

$\Rightarrow K = 1$

$\Rightarrow$ option (4) is correct.

4. Let PQ and MN be two straight lines touching the circle $x^2+y^2-4x-6y-3=0$ at the points A and B respectively. Let O be the centre of the circle and $\angle AOB=\pi/3$. Then the locus of the point of intersection of the lines PQ and MN is:

(1) $x^2+y^2-18x-12y-25=0$

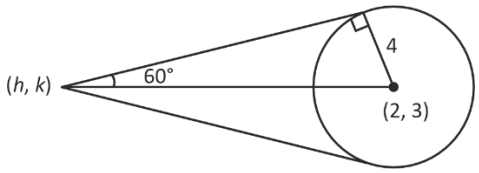
(2) $3(x^2+y^2)-12x-18y-25=0$

(3) $x^2+y^2-12x-18y-25=0$

(4) $3(x^2+y^2)-18x-12y+25=0$

Answer (2)

Sol.



$x^2+y^2-14x-6y-3=0$

$\tan60^\circ=\frac{4}{\sqrt{h^2+k^2-4h-6k-3}}$

squaring both side

$3(h^2+k^2-4h-6k-3)=16$

To get locus replace h, k by x and y;

$3x^2+3y^2-12x-18y-9-16=0$

$3x^2+3y^2-12x-18y-25=0$

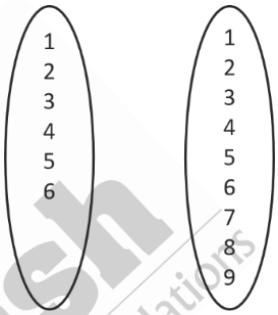
5. The number of strictly increasing functions f from the set $\{1,2,3,4,5,6\}$ to the set $\{1, 2, 3,\ldots., 9\}$ such that $f(i)\neq i$ for $1\le i\le 6$, is equal to :

(1) 27 (2) 22

(3) 21 (4) 8

Answer (4)

Sol.



Case (i) If $f(1) = 2$ then ${}^7C_5$ functions

Case (ii) $f(1) = 3$ then ${}^6C_5$

Case (iii) $f(1) = 4$ then ${}^5C_5$

$\Rightarrow 21 + 6 + 1 = 28$

6. Let the mean and variance of 7 observations $2,4,10,x,12,14,y, x>y$, be 8 and 16 respectively. Two numbers are chosen from $\{1, 2, 3, x-4, y, 5\}$ one after another without replacement, then the probability, that the smaller number among the two chosen numbers is less than 4, is :

(1) $\frac{3}{5}$ (2) $\frac{4}{5}$

(3) $\frac{2}{5}$ (4) $\frac{1}{3}$

Answer (2)

Sol. Mean = 8

$$\frac{2+4+10+x+y+12+14}{7}=8$$

$\Rightarrow x + y = 14$

variance = 16

$$16=\frac{2^2+4^2+10^2+x^2+y^2+12^2+14^2+y^2}{7}-8^2$$

$\Rightarrow \quad x^2 + y^2 = 100$

$\therefore \quad x = 8, y = 6$

New set : {1, 2, 3, 4, 5, 6}

Required probability $= 1 - P$(both number are ≥ 4)

$$=1-\frac{{}^3C_2}{{}^6C_2}$$

$$=1-\frac{3}{15}$$

$$=\frac{4}{5}$$

7. Let $\vec{a}=-\hat{i}+2\hat{j}+2\hat{k}, \vec{b}=8\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}=\vec{b}$.

If $\vec{c}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=4$, then $|\vec{a}+\vec{c}|^2$ is equal to :

(1) 35 (2) 33

(3) 27 (4) 30

Answer (3)

Sol. Let $\vec{c}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\vec{a}\times\vec{c}=\vec{b}\begin{vmatrix}\hat{i} & \hat{j} & k\\ -1 & 2 & 2\\ x & y & z\end{vmatrix}$$

$$=(2z-2y)\hat{i}+(z+2x)\hat{j}+(-y-2x)\hat{k}$$

$$\equiv 8\hat{i}+7\hat{j}-3\hat{k}$$

$\Rightarrow \quad 2z - 2y = 8$

$$\left.\begin{aligned}z+2x&=7\\ -y-2x&=3\end{aligned}\right\}y=3-2x,\ z=7-2x$$

$$\vec{c}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=4$$

$x + y + z = 4$

Now $x + 3 - 2x + 7 - 2x = 4$

$\Rightarrow \quad x = 2$

$y = -1$

$z = 3$

$\Rightarrow \quad \vec{c}=2\hat{i}-\hat{j}+3\hat{k}$

$$|\vec{a}+\vec{c}|^2=\left|\hat{i}+\hat{j}+5\hat{k}\right|^2$$

$= 27$

8. Let $y = y(x)$ be the solution curve of the differential equation $\left(1+x^2\right)dy+\left(y-\tan^{-1}x\right)dx=0, y(0)=1$.

Then the value of $y(1)$ is :

(1) $\frac{4}{e^{\pi/4}}-\frac{\pi}{2}-1$ (2) $\frac{2}{e^{\pi/4}}-\frac{\pi}{4}-1$

(3) $\frac{2}{e^{\pi/4}}+\frac{\pi}{4}-1$ (4) $\frac{4}{e^{\pi/4}}+\frac{\pi}{2}-1$

Answer (3)

Sol. $\left(1+x^2\right)dy+\left(y-\tan^{-1}x\right)dx=0$

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{\tan^{-1}x}{1+x^2}$$

$$\text{IF}=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$$

$$ye^{\tan^{-1}x}=\int e^{\tan^{-1}x}\frac{\tan^{-1}x}{1+x^2}dx$$

Let $\tan^{-1}x = t$

$$\frac{1}{1+x^2}dx=dt$$

$$ye^{\tan^{-1}x} = \int e^t t \cdot dt$$

$$ye^{\tan^{-1}x} = e^t t - e^t + c$$

$$ye^{\tan^{-1}x} = e^{\tan^{-1}x}\left(\tan^{-1}x - 1\right) + c$$

Now $y(0) = 1$

$\Rightarrow \quad 2 = c$

$\therefore \quad y = \left(\tan^{-1}x - 1\right) + 2e^{-\tan^{-1}x}$

$$y(1) = \left(\frac{\pi}{4} - 1\right) + 2e^{-\pi/4}$$

$$= \frac{2}{e^{\frac{\pi}{4}}} + \frac{\pi}{4} - 1$$

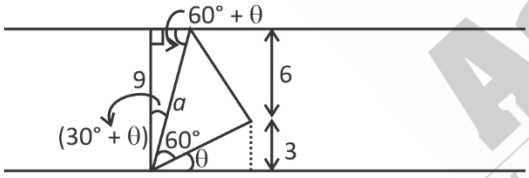
9. Let a point A lie between the parallel lines L_1 and L_2 such that its distances from L_1 and L_2 are 6 and 3 units, respectively. Then the area (in sq. units) of the equilateral triangle ABC, where the points B and C lie on the lines L_1 and L_2, respectively, is :

(1) $21\sqrt{3}$ (2) $12\sqrt{2}$

(3) $15\sqrt{6}$ (4) 27

Answer (1)

Sol. Let a be the side of ΔABC



$$\sin\theta = \frac{3}{a}$$

$$\sin\left(60° + \theta\right) = \frac{9}{a}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} \times \sqrt{1 - \frac{9}{a^2}} + \frac{3}{a} \times \frac{1}{2} = \frac{9}{a}$$

$$\Rightarrow \quad \sqrt{3}\left(\sqrt{a^2 - 9}\right) + 3 = 18$$

$\Rightarrow \quad 3(a^2 - 9) = 15^2 \qquad \Rightarrow a^2 - 9 = 15 \times 5$

$\Rightarrow a^2 = 84$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = 21\sqrt{3}$$

10. Let $f : \mathbf{R} \to (0, \infty)$ be a twice differentiable function such that $f(3) = 18, f'(3) = 0$ and $f''(3) = 4$. Then $\lim_{x \to 1}\left(\log_e\left(\frac{f(2+x)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)$ is equal to :

(1) 2 (2) 9

(3) 1 (4) 18

Answer (1)

Sol. $\ln\left(\lim\limits_{x \to 1}\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)(1^\infty)$

$\ln\left(e^{\lim\limits_{x \to 1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)}{f(3)} - 1\right)}\right)$

$\ln\left(e^{\lim\limits_{x \to 1}\frac{f'(x+2)}{2(x-1)}}\right)$

$\ln\left(e^{\lim\limits_{x \to 1}\frac{f''(x+2)}{2}}\right)$

$\lim\limits_{x \to 1}\frac{f''(3)}{2} = \frac{4}{2} = 2$

11. The number of relations, defined on the set $\{a, b, c, d\}$, which are both reflexive and symmetric, is equal to:

(1) 1024 (2) 64

(3) 256 (4) 16

Answer (2)

Sol. Number of such relations:

From $A \to A$ such that

$n(A) = N$ is

$\Rightarrow 2^{\left(\frac{N^2-N}{2}\right)}$

for $N = 4 \Rightarrow 2^6 = 64$

Alter :

$$\begin{bmatrix} O & - & - & - \\ \Theta & O & - & - \\ \Theta & \Theta & O & - \\ \Theta & \Theta & \Theta & O \end{bmatrix} \Rightarrow \text{each } \Theta \text{ has two choices}$$

$\Rightarrow 2^6 = 64$

12. Let $a_1, a_2, a_3, \ldots$ be G.P. of increasing positive terms such that $a_2 \cdot a_3 \cdot a_4 = 64$ and $a_1 + a_3 + a_5 = \frac{813}{7}$. Then $a_3 + a_5 + a_7$ is equal to:

(1) 3252 (2) 3248

(3) 3244 (4) 3256

Answer (1)

Sol. $a_2 \cdot a_3 \cdot a_4 = 64$

$\frac{a}{r} \cdot a \cdot ar = 64$

$\Rightarrow \quad a = 4$

$a_1 + a_3 + a_5 = \frac{813}{7}$

$= \frac{4}{r^2} + 4 + 4r^2 = \frac{813}{7}$

Let $r^2 = t$

$28t^2 - 785t + 28 = 0$

$(28t - 1)(t - 28) = 0$

$\Rightarrow \quad t = 28, \frac{1}{28}$

$r^2 = 28$ ($\because$ G.P. is increasing)

$a_3 + a_5 + a_7$

$= a + ar^2 + ar^4 = r^2\left(a + \frac{a}{r^2} + r^2\right)$

$= 28 \times \frac{813}{7}$

$= 3252$

13. Let the foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If the eccentricity of the hyperbola is 5, then the length of its latus rectum is:

(1) $\frac{96}{\sqrt{5}}$ (2) $24\sqrt{5}$

(3) 12 (4) 16

Answer (1)

Sol. $E : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

$H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

For con-focal E and H

$\because \quad A^2 - B^2 = a^2 + b^2$

(Given that $A^2 = 36$ and $B^2 = 16$)

$36 - 16 = a^2 + b^2$

$a^2 + b^2 = 20 \qquad \ldots(1)$

$e_H = 5$

$\Rightarrow \quad 25 = 1 + \frac{b^2}{a^2}$

$\Rightarrow \quad 25a^2 = 20 \Rightarrow a^2 = \frac{4}{5} \Rightarrow a = \frac{2}{\sqrt{5}}$

Also $a^2 + b^2 = 20$

$b^2 = 20 - \frac{4}{5} \Rightarrow \frac{96}{5}$

L(LR) = $= 2 \times \frac{b^2}{a} = \frac{2 \times \frac{96}{5}}{\frac{2}{\sqrt{5}}} = \frac{96}{\sqrt{5}}$

14. The value of $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left(\frac{\pi + 4x^{11}}{1 - \sin\left(|x| + \frac{\pi}{6}\right)}\right) dx$ is equal to:

(1) 8π (2) 4π

(3) 6π (4) 2π

Answer (2)

Sol. $\int_0^{\frac{\pi}{6}} \frac{\pi+4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}+\frac{\pi-4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}dx$

$=2\pi\int_0^{\frac{\pi}{6}} \frac{1}{1-\sin\left(x+\frac{\pi}{6}\right)}dx$

$=2\pi\int_0^{\frac{\pi}{6}} \frac{1+\sin\left(x+\frac{\pi}{6}\right)}{\cos^2\left(x+\frac{\pi}{6}\right)}dx$

$=2\pi\int_0^{\frac{\pi}{6}} \sec^2\left(x+\frac{\pi}{6}\right)+\tan\left(x+\frac{\pi}{6}\right)\sec\left(x+\frac{\pi}{6}\right)dx$

$\Rightarrow 2\pi\left[\tan\left(x+\frac{\pi}{6}\right)+\sec\left(x+\frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$

$\Rightarrow 4\pi$

15. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2x-5}{11-3x}\right)+\sin^{-1}\left(2x^2-3x+1\right)$ is the interval $[\alpha, \beta]$, then $\alpha + 2\beta$ is equal to :

(1) 3

(2) 2

(3) 1

(4) 5

Answer (1)

Sol. $\underbrace{-1\le\frac{2x-5}{11-3x}\le 1}_{(2)\quad(1)}$ and $\underbrace{-1\le 2x^2-3x+1\le 1}_{(4)\quad(3)}$

(1) $\frac{2x-5}{11-3x}-1\le 0$

$\Rightarrow \frac{2x-5-11+3x}{11-3x}\le 0$

$\Rightarrow \frac{5x-16}{11-3x}\le 0$

$\Rightarrow x\in\left(-\infty,\frac{16}{5}\right]\cup\left(\frac{11}{3},\infty\right)$ ...(i)

(2) $\frac{2x-5}{11-3x}\ge -1$

$\Rightarrow \frac{2x-5}{11-3x}+1\ge 0$

$\Rightarrow \frac{6-x}{11-3x}\ge 0$

$\Rightarrow \frac{x-6}{3x-11}\ge 0$

$\Rightarrow x\in\left(-\infty,\frac{11}{3}\right)\cup[6,\infty)$ ...(ii)

(3) $2x^2-3x+1\le 1$

$\Rightarrow x(2x-3)\le 0$

$x\in\left[0,\frac{3}{2}\right]$

(4) $2x^2-3x+1\ge -1$

$\Rightarrow 2x^2-3x+2\ge 0$

$x\in R$ ...(iv)

taking intersection of (i), (ii), (iii), (iv)

$x\in\left[0,\frac{3}{2}\right]$

$\therefore\ \alpha+2\beta$

$=3$

16. If $x^2+x+1=0$, then the value of $\left(x+\frac{1}{x}\right)^4+\left(x^2+\frac{1}{x^2}\right)^4+\left(x^3+\frac{1}{x^3}\right)^4+\ldots+\left(x^{25}+\frac{1}{x^{25}}\right)^4$ is :

(1) 175

(2) 162

(3) 128

(4) 145

Answer (4)

Sol. $x^2+x+1=0 \begin{matrix} w \\ w^2 \end{matrix}$

$$\left(w+\frac{1}{w}\right)^4+\left(w^2+\frac{1}{w^2}\right)^4+\left(w^3+\frac{1}{w^3}\right)^4+\ldots.$$

$$\left(w^{25}+\frac{1}{w^{25}}\right)^4$$

$$\sum\left(w^k+\frac{1}{w^k}\right)^4$$

$$K=3x \Rightarrow w^{3x}+\frac{1}{w^{3x}}=2$$

$$k\neq 3x \Rightarrow w^k+\frac{1}{w^k}=-1$$

$$\sum_{k=1}^{25}\left(w^k+\frac{1}{w^k}\right)^4 \Rightarrow 8\left(1+1+2^4\right)+1$$

$= 145$

17. The area of the region, inside the ellipse $x^2+4y^2=4$ and outside the region bounded by the curves $y=|x|-1$ and $y=1-|x|$, is :

(1) $3(\pi-1)$

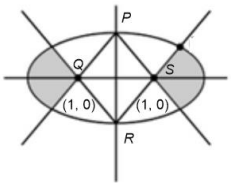
(2) $2\pi-\frac{1}{2}$

(3) $2\pi-1$

(4) $2(\pi-1)$

Answer (4)

Sol.



$E: \frac{x^2}{4}+\frac{y^2}{1}=1$

Area inside $E = 2\pi$

Area $PQRS = \left(\sqrt{2}\right)^2$

$= 2$

Required area $= 2\pi - 2$

$= 2(\pi - 1)$

18. The sum of all the roots of the equation $(x-1)^2-5|x-1|+6=0$, is :

(1) 3 (2) 4

(3) 1 (4) 5

Answer (2)

Sol. $|x-1|2-5|x-1|+6=0$

Let $|x-1| = t$

$t^2 - 5 + 6 = 0$

$(t-3)(t-2) = 0$

$t = 2, 3$

$|x-1| = 2$ or $|x-1| = 3$

$x-1 = \pm 2$ or $x-1 = \pm 3$

$\Rightarrow x = 3, -1, 4, -2$

$\Rightarrow$ Sum $= 3 - 1 + 4 - 2$

$= 4$

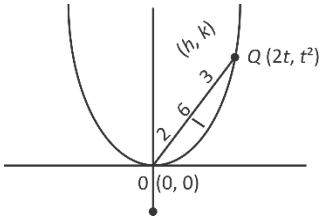
19 Let O be the vertex of the parabola $x^2=4y$ and Q be any point on it. Let the locus of the point P, which divides the line segment OQ internally in the ratio $2:3$ be the conic C. Then the equation of the chord of C, which is bisected at the point (1, 2), is

(1) $5x-y-3=0$ (2) $4x-5y+6=0$

(3) $5x-4y+3=0$ (4) $x-2y+3=0$

Answer (3)

Sol.



$h = \frac{4t}{5}$, $k = \frac{2t^2}{5}$

$\Rightarrow \frac{2t^2}{5} = t$, $\frac{2t^2}{5} = t^2$

$\frac{5k}{2} = \left(\frac{5h}{4}\right)^2$

Replace (h, k) with (x, y)

$\frac{5y}{2} = \frac{25x^2}{16} =$

$C : \boxed{8y = 5x^2}$

chord with given middle point

$T = S_1$

$5xx_1 - 4(y + y_1) = 5x_1^2 - 8y_1$

$5x - 4(y + 2) = 5 - 16$

$5x - 4y - 8 = -11$

$5x - 4y + 3 = 0$

20 If the coefficient of x in the expansion of $(ax^2 + bx + c)(1 - 2x)^{26}$ is –56 and the coefficients of x^2 and x^3 are both zero, then $a + b + c$ is equal to

(1) 1500 (2) 1403

(3) 1300 (4) 1483

Answer (2)

Sol. $(ax^2 + bx + c)(1 - 2x)^{26}$

$ax^2(1 - 2x)^{26} + bx(1 - 2x)^{26} + c(1 - 2x)^{26}$

Coefficient of

$x \Rightarrow \left[0 + b \cdot {}^{26}C_0 + c \cdot {}^{26}C_1(-2)\right]x$

$= b - 52c$

Coefficient of

$x^2 = (a({}^{26}C_0) + b \cdot {}^{26}C_1(-2) + c\,{}^{26}C_2(-2)^2)x^2$

$= a - 52b + {}^{26}C_2 \times 4 = a - 52b + 1300c$

Coefficient of

$x^3 = [a \cdot {}^{26}C_1(-2) + b \cdot {}^{26}C_2(26)^2 + c\,{}^{26}C_3(-2)^3]$

$= -52a + 1300b - 20800c$

$\Rightarrow b - 52c = -56$

$\Rightarrow b - 52c = -56$

$a - 52b + 1300c = 0 \quad \Rightarrow a = 1300$

$-a + 25b - 400c = 0 \quad b = 100$

$c = 3$

$\Rightarrow a + b + c = 1403$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be a twice differentiable function such that the quadratic equation $f(x)m^2 - 2f'(x)m + f''(x) = 0$ in m, has two equal roots for every $x \in \mathbf{R}$. If $f(0) = 1, f'(0) = 2$, and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to _____ .

Answer (1)

Sol. For equal roots, discriminant is 0.

$4(f'(x))^2 - 4(f(x))(f''(x)) = 0$

Let $y = f(x)$

$(y')^2 = yy''$

$\Rightarrow \frac{y'}{y} = \frac{y''}{y'}$

$\Rightarrow \ln|y| = \ln|y'| + \ln K$

$\ln(1) = \ln|2| + \ln(K) \Rightarrow$

$\ln K = -\ln 2$

$\Rightarrow \ln|y| = \ln|y'| - \ln 2$

$\Rightarrow \ln\left|\frac{y}{y'}\right| = -\ln 2 = \ln\left(\frac{1}{2}\right)$

$\Rightarrow 2y = y'$

$\Rightarrow 2 = \left|\frac{y'}{y}\right| \Rightarrow 2x = \ln|y| + \ln\lambda$

$0 = \ln|1| + \ln\lambda$

$\Rightarrow \ln\lambda = 0$

$\Rightarrow \ln|y| = 2x$

$y = e^{2x}$

$f^{(\ln x - x)} = e^{2(\ln x - x)} = e^{2\left(\ln x^2 - 2x\right)}$

$= \frac{x^2}{e^{2x}}$

The function $\frac{x^2}{e^{2x}}$ is increasing in (0, 1)

$\Rightarrow \alpha + \beta = 1$

22. $6\int_0^{\pi}\left|(\sin 3x + \sin 2x + \sin x)\right|dx$ is equal to ____ .

Answer (17)

Sol. $I = 6\int_0^{\pi}|\sin x + \sin 3x + \sin 2x|dx$

$= 6\int_0^{\pi}|\sin 2x||1 + 2\cos x|dx$

$= 6\int_0^{\pi}\left|(2\sin x)(2\cos^2 x + \cos x)\right|dx$

Since $\sin x > 0 \forall x \in (0, \pi)$

$\Rightarrow I = 6\int_0^{\pi}(2\sin x)\cos x(2\cos x + 1)dx$

$\Rightarrow I = 6\int_0^{\pi/2}(2\sin x)(2\cos^2 x + \cos x)dx$

$-6\int_{\pi/2}^{2\pi/3}(2\sin x)(2\cos^2 x + \cos x)dx$

$+6\int_{2\pi/3}^{\pi}(2\sin x)(2\cos^2 x + \cos x)dx$

$6\left(\frac{7}{3} + \frac{1}{12} + \frac{5}{12}\right) = 17$

23. Let $S = \{(m, n) : m, n \in \{1, 2, 3, \ldots\ldots, 50\}\}$. If the number of elements (m, n) in S such that $6^m + 9^n$ is a multiple of 5 is p and the number of elements (m, n) in S such that $m + n$ is a square of a prime number is q, then $p + q$ is equal to _____.

Answer (1333)

Sol. $5 \mid 6^m + 9^n$

$\Rightarrow 5 \mid 1^m + (-1)^n$

$\Rightarrow m$ and n has to be opposite parity.

${}^2C_1 \times {}^{25}C_1 \cdot {}^{25}C_1 = 625 \times 2 = 1250$

For, $m + n = K^2$ for some prime K.

$m + n \in \{2, 3, \ldots, 100\}$

$m + n = 4, 9, 25, 49$

$m + n = 4 \Rightarrow$ 3 pairs

$m + n = 9 \Rightarrow$ 8 pairs

$m + n = 25 \Rightarrow$ 24 pairs

$m + n = 49 \Rightarrow$ 48 pairs

$\Rightarrow$ 83 pairs

$\Rightarrow$ 1333

24. For some $\alpha, \beta \in \mathbf{R}$, let $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ be such that $A^2 - 4A + 2I = B^2 - 3B + I = O$. Then $\left(\det\left(\text{adj}\left(A^3 - B^3\right)\right)\right)^2$ is equal to _____.

Answer (225)

Sol. Using characteristic equation,

$$\begin{vmatrix} \alpha - \lambda & 2 \\ 1 & 2 - \lambda \end{vmatrix} = 0$$

$(\lambda - 2)(\lambda - \alpha) - 2 = 0$

$\lambda^2 + \lambda(-2 - \alpha) + 2\alpha - 2 = 0$

$\Rightarrow A^2 + A(-2 - \alpha) + (2\alpha - 2)I = 0$

$\Rightarrow -2 - \alpha = -4 \Rightarrow \alpha = 2$

Similarly,

$$\begin{vmatrix} 1 - K & 2 \\ 1 & \beta - K \end{vmatrix} = 0$$

$\Rightarrow (K - \beta)(K - 1) - 1 = 0$

$K^2 + K(-1 - \beta) + \beta - 1 = 0$

$\Rightarrow -1 - \beta = -3 \Rightarrow \beta = 2$

$\Rightarrow A = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Let $A^3 - B^3 = \begin{bmatrix} 15 & 20 \\ 6 & 7 \end{bmatrix} = C$, $\det(C) = -15$

$|\text{adj } C|^2 = |C|^2 = 225$

25. Let $a_1 = 1$ and for $n \geq 1, a_{n+1} = \frac{1}{2}a_n + \frac{n^2 - 2n - 1}{n^2(n+1)^2}$. Then $\left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right|$ is equal to _____ .

Answer (2)

Sol. $a_{n+1} = \frac{1}{2}a_n + \frac{1}{(n+1)^2} - \frac{\left((n+1)^2 - n^2\right)}{n^2(n+1)^2}$

$a_{n+1} = \frac{a_n}{2} + \frac{2}{(n+1)^2} - \frac{1}{n^2}$

$a_{n+1} - \frac{2}{(n+1)^2} = \frac{1}{2}\left(a_n - \frac{2}{n^2}\right)$

Let $b_n = a_n \frac{-2}{n^2}$

then $\langle b \rangle$ is geometric progression with ratio $= \frac{1}{2}$

$b_1 = a_1 - \frac{2}{1} = 1 - 2 = (-1)$

$\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right) = \sum_{n=1}^{\infty} b_n = \frac{(-1)}{1 - \left(\frac{1}{2}\right)} = \frac{-1}{1/2} = -2$

$\Rightarrow \left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right| = 2$

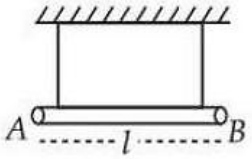
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. A uniform rod of mass m and length l suspended by means of two identical inextensible light strings as shown in figure. Tension in one string immediately after the other string is cut, is _____. (g acceleration due to gravity)



(1) $mg/2$

(2) $mg/4$

(3) mg

(4) $mg/3$

Answer (2)

Sol. $\tau_{net} = mg\frac{l}{2} = I\alpha$

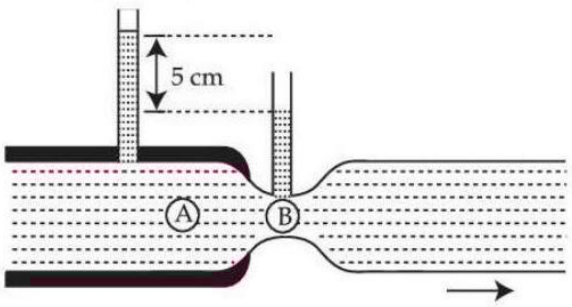
$\Rightarrow \quad mg\frac{l}{2} = \frac{ml^2}{3}\alpha$

$\Rightarrow \quad \alpha = \frac{3g}{2l}$

$r_{cm} = \frac{\alpha l}{2} = \frac{3g}{4}$

$\therefore \quad T = mg - \frac{3mg}{4} = \frac{mg}{4}$

27. Water flows through a horizontal tube as shown in the figure. The difference in height between the water columns in vertical tubes is 5 cm and the area of cross-sections at A and B are 6 cm^2 and 3 cm^2 respectively. The rate of flow will be _____. cm^3/s. (take g = 10 m/s^2)



(1) $200\sqrt{3}$

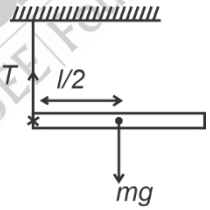
(2) $100\sqrt{3}$

(3) $200\sqrt{6}$

(4) $\frac{200}{\sqrt{3}}$

Answer (1)

Sol.



$P_A - P_B = \rho gh = 10000 \times \frac{5}{100} = 500$ Pa

and $P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$

$\Rightarrow \quad P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$

$\Rightarrow \quad 1000 = 1000(V_B^2 - V_A^2)$

and $V_A A_A = V_B A_B$

$\Rightarrow V_A \times 6 = V_B \times 3$

$\therefore V_A = \frac{V_B}{2}$

Hence, $4V_A^2 - V_A^2 = 1$

$\Rightarrow V_A = \frac{1}{\sqrt{3}}$ m/s

Rate of flow = $V_A A_A = \frac{1}{\sqrt{3}} \times 6 \times 10^{-4}$ m/s

$\Rightarrow 200\sqrt{3}$

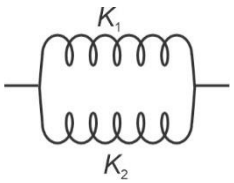
28. In an experiment the values of two spring constants were measured as $k_1 = (10 \pm 0.2)$ N/m and $k_2 = (20 \pm 0.3)$ N/m. If these springs are connected in parallel, then the percentage error in equivalent spring constant is:

(1) 1.33% (2) 2.67%

(3) 2.33% (4) 1.67%

Answer (4)

Sol.



$K_1 = 10 \pm 0.2$ N/m

$K_2 = 20 \pm 0.3$ N/m

$\because K_{ev} = K_1 + K_2 = 30$ N/m

$\Delta K_{eq} = 0.5$ N/m

$\%\Delta K_{eq} = \frac{\Delta K_{eq}}{K_{eq}} \times 100 = \frac{50}{30} = \frac{5}{3} = 1.67\%$

29. Two strings (A, B) having linear densities $\mu_A = 2 \times 10^{-4}$ kg/m and, $\mu_B = 4 \times 10^{-4}$ kg/m and lengths L_A = 2.5 m and L_B = 1.5 m respectively are joined. Free ends of A and B are tied to two rigid supports C and D, respectively creating a tension of 500 N in the wire. Two identical pulses, sent from C and D ends, take time t_1 and t_2, respectively, to reach the joint. The ratio t_1 / t_2 is:

(1) 1.67

(2) 1.08

(3) 1.90

(4) 1.18

Answer (4)

Sol. Since strings are joined in series, Tension in both will be same.

For string A

$t_1 = \frac{L_A}{V_A} = L_A\sqrt{\frac{M_A}{T}}$

And For string B

$t_2 = \frac{L_B}{V_B} = L_B\sqrt{\frac{M_B}{T}}$

$\Rightarrow \frac{t_1}{t_2} = \frac{L_A}{L_B}\sqrt{\frac{M_A}{M_B}}$

$\Rightarrow \frac{t_1}{t_2} = \frac{2.5}{1.5}\sqrt{\frac{2 \times 10^{-4}}{4 \times 10^{-4}}}$

$\Rightarrow \frac{t_1}{t_2} = 1.178 \approx 1.18$

30. A point charge of $10^{-8}C$ is placed at origin. The work done in moving a point charge $2\mu C$ from point $A(4, 4, 2)$ m to $B(2, 2, 1)$ m is _____ J.

$\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ in SI units}\right)$

(1) 15×10^{-6} (2) 30×10^{-6}

(3) 45×10^{-6} (4) 0

Answer (2)

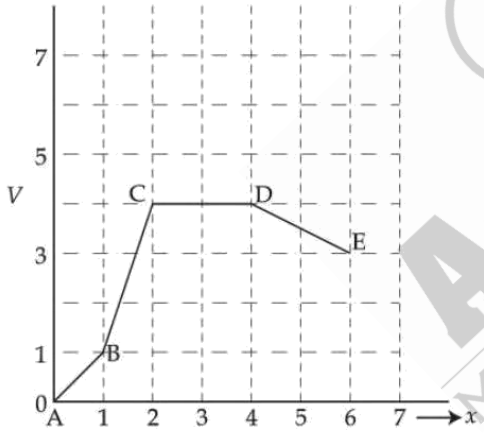
Sol. $W = \frac{Kq_1q_2}{r_2} - \frac{Kq_1q_2}{r_1}$

$\Rightarrow W = 9\times10^9 \times10^{-8} \times2\times10^{-6}$

$$\left(\frac{1}{\sqrt{2^2+2^2+1^2}} - \frac{1}{\sqrt{4^2+4^2+2^2}}\right)$$

$\Rightarrow W = \frac{180\times10^{-6}}{6} = 30\times10^{-6}$ J

31. Potential energy (V) versus distance (x) is given by the graph. Rank various regions as per the magnitudes of the force (F) acting on a particle from high to low.



(1) $F_{BC} > F_{AB} > F_{DE} > F_{CD}$

(2) $F_{CD} > F_{DE} > F_{AB} > F_{BC}$

(3) $F_{BC} > F_{CD} > F_{DE} > F_{AB}$

(4) $F_{CD} > F_{AB} > F_{BC} > F_{DE}$

Answer (1)

Sol. In Potential energy *v/s* distance (x) graph, magnitude of Force is the slope of the graph.

$\therefore$ greater the slope, more the force.

$\therefore$ $F_{BC} > F_{AB} > F_{DE} > F_{CD}$

32. A parallel plate capacitor has capacitance C, when there is vacuum within the parallel plates. A sheet having thickness $\left(\frac{1}{3}\right)^{rd}$ of the separation between the plates and relative permittivity K is introduced between the plates. The new capacitance of the system is

(1) $\frac{4KC}{3K-1}$

(2) $\frac{CK}{2+K}$

(3) $\frac{3KC}{2K+1}$

(4) $\frac{3CK^2}{(2K+1)^2}$

Answer (3)

Sol. $C_{new} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

$\Rightarrow \frac{\varepsilon_0 A}{d\left(1 - \frac{t}{d} + \frac{t}{dK}\right)}$

$\Rightarrow \frac{C}{1 - \frac{1}{3} + \frac{1}{3K}}$

$C_{new} = \frac{3CK}{2K+1}$

33. An aluminium and steel rods having same lengths and cross-sections are joined to make total length of 120 cm at 30°C. The coefficient of linear expansion of aluminium and steel are 24×10^{-6}/°C and 1.2×10^{-5}/°C, respectively. The length of this composite rod when its temperature is raised to 100°C, is _____ cm.

(1) 120.20

(2) 120.15

(3) 120.03

(4) 120.06

Answer (2)

Sol. Since the rods are joined in series,

$\Delta l_{eq} = \Delta l_1 + \Delta l_2$

$\Rightarrow \quad l\alpha_1\Delta T + l\alpha_2\Delta T$

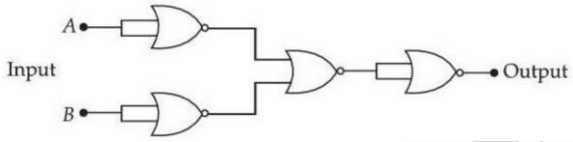
$\Rightarrow \quad l\Delta T(\alpha_1 + \alpha_2)$

$\Rightarrow \quad 60 \times (100 - 30)\left(24 \times 10^{-6} + 1.2 \times 10^{-5}\right)$

$\Rightarrow \quad 0.1512$ cm

$\therefore$ New length = 120 + 0.1512 = 120.1512 cm

34. The given circuit works as:



(1) NOR gate

(2) AND gate

(3) OR gate

(4) NAND gate

Answer (4)

Sol. O/P = $\bar{A} + \bar{B} = \overline{A \cdot B}$

$\therefore$ It is Nand gate

35. Consider a modified Bernoulli equation.

$$\left(P + \frac{A}{Bt^2}\right) + \rho g(h + Bt) + \frac{1}{2}\rho V^2 = \text{constant}$$

If t has the dimension of time then the dimensions of A and B are _____, _____ respectively.

(1) $\left[ML^0T^{-1}\right]$ and $\left[M^0LT\right]$

(2) $\left[ML^0T^{-2}\right]$ and $\left[M^0LT^{-2}\right]$

(3) $\left[ML^0T^{-1}\right]$ and $\left[M^0LT^{-1}\right]$

(4) $\left[ML^0T^{-2}\right]$ and $\left[M^0LT^{-1}\right]$

Answer (3)

Sol. $P + \frac{A}{Bt^2} + \rho g(h + Bt) + \frac{1}{2}\rho V^2 = \text{constant}$

dimensions of quantities being added should be equal.

$\therefore \quad [h] = [Bt]$

$\therefore \quad [B] = LT^{-1}$

And $[P] = \left[\frac{A}{Bt^2}\right]$

$\therefore \quad [A] = [P][B][t^2]$

$[A] = ML^{-1}T^{-2}\ LT^{-1}\ T^2$

$[A] = M^1L^0T^{-1}$

36. The electric field in a plane electromagnetic wave is given by :

$$E_y = 69\sin\left[0.6 \times 10^3 x - 1.8 \times 10^{11} t\right] \text{V/m}$$

The expression for magnetic field associated with this electromagnetic wave is _____ T.

(1) $B_y = 69\sin\left[0.6 \times 10^3 x + 1.8 \times 10^{11} t\right]$

(2) $B_z = 2.3 \times 10^{-7}\sin\left[0.6 \times 10^3 x + 1.8 \times 10^{11} t\right]$

(3) $B_z = 2.3 \times 10^{-7}\sin\left[0.6 \times 10^3 x - 1.8 \times 10^{11} t\right]$

(4) $B_y = 2.3 \times 10^{-7}\sin\left[0.6 \times 10^3 x - 1.8 \times 10^{11} t\right]$

Answer (3)

Sol. Magnitudes of $\vec{E}$ and $\vec{B}$ vary synchronously.

and $\frac{E_0}{C} = B_0$

$\therefore \quad B_0 = \frac{69}{3\times10^8} = 23\times10^{-8}$ or 2.3×10^{-7} SI

$\therefore \quad \beta = 2.3\times10^{-7}\sin\left(0.6\times10^3 x - 1.8\times10^{11}t\right)$

37. A conducting circular loop of area 1.0 m^2 is placed perpendicular to a magnetic field which varies as $B = \sin(100t)$ Tesla. If the resistance of the loop is 100 Ω, then the average thermal energy dissipated in the loop in one period is _____ J.

(1) 2π

(2) $\frac{\pi}{2}$

(3) π^2

(4) π

Answer (4)

Sol. According to Faradys law,

$|\varepsilon| = \frac{d\phi}{dt}$

$\Rightarrow \quad |\varepsilon| = A\frac{dB}{dt} = 100\cos(100t)$

and Power $= \frac{\varepsilon^2}{R} = \frac{100\times100\cos^2(100t)}{100}$

$P = 100\cos^2(100t) = \frac{dQ}{dt}$

$Q = \int_0^T 100\cos^2(100t)dt$

$Q = 50\times\frac{\pi}{50} = \pi$ J

38. A light wave described by

$E = 60\left[\sin\left(3\times10^{15}\right)t + \sin\left(12\times10^{15}\right)t\right]$ (in SI units)

falls on a metal surface of work function 2.8 eV. The maximum kinetic energy of ejected photoelectron is (approximately) _____ eV.

(h = 6.6 × 10^{-34} J.s. and e = 1.6 × 10^{-19} C)

(1) 3.8

(2) 5.1

(3) 7.8

(4) 6.0

Answer (2)

Sol. $E = 60\left[\sin\left(3\times10^{15}t\right) + \sin\left(12\times10^{15}t\right)\right]$

highest frear $= \frac{12\times10^{15}}{2\pi} = \frac{6}{\pi}\times10^{15}$

$\therefore \quad KE_{max} = h\nu - \phi_0$

$\Rightarrow \quad \frac{6.6\times10^{-34}\times6\times10^{15}}{1.6\times10^{-19}\pi} - 2.8$

$\Rightarrow \quad 7.87 - 2.8 \approx 5.1$ eV

39. A current carrying solenoid is placed vertically and a particle of mass *m* with charge *Q* is released from rest. The particle moves along the axis of solenoid. If *g* is acceleration due to gravity then the acceleration (a) of the charged particle will satisfy

(1) $a = 0$

(2) $a > g$

(3) $0 < a < g$

(4) $a = g$

Answer (4)

Sol. Since charge moves along magnetic field, magnetic force on it will be zero.

$\therefore \quad a = g$

40. If an alpha particle with energy 7.7 MeV is bombarded on a thin gold foil, the closest distance from nucleus it can reach is _____ m.

(Atomic number of gold = 79 and $\frac{1}{4\pi \in_0} = 9\times10^9$ in SI units)

(1) 3.85×10^{-16}

(2) 2.95×10^{-14}

(3) 3.85×10^{-14}

(4) 2.95×10^{-16}

Answer (2)

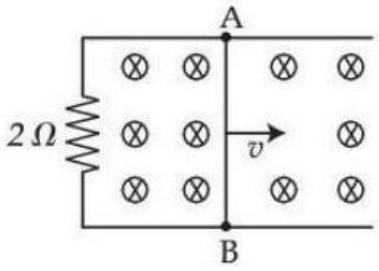
Sol. Kinetic energy would convert into electrostatic P.E.

$$\text{K.E.} = \frac{Kze\cdot 2e}{r}$$

$$\Rightarrow 7.7\times10^6\times1.6\times10^{-19} = \frac{9\times10^9\times79\times\left(1.6\times10^{-19}\right)^2\times2}{r}$$

$r = 2.95 \times 10^{-14}$ m

41. A 1 m long metal rod AB completes the circuit as shown in figure. The area of circuit is perpendicular to the magnetic field of 0.10 T. If the resistance of the total circuit is 2 Ω then the force needed to move the rod towards right with constant speed (v) of 1.5 m/s is _____N.



(1) 7.5×10^{-3} (2) 5.7×10^{-3}

(3) 5.7×10^{-2} (4) 7.5×10^{-2}

Answer (1)

Sol. Force on rod directly will be :

$$F = \frac{B^2v l^2}{r} = \frac{(0.1)^2\times1.5\times1^2}{2}$$

$\Rightarrow F = 7.5 \times 10^{-3}$ N

42. A gas based geyser heats water flowing at the rate of 5.0 litres per minute from 27 °C to 87 °C. The rate of consumption of the gas is _____ g/s.

(Take heat of combustion of gas = 5.0×10^4 J/g) specific heat capacity of water = 4200 J/kg. °C

(1) 4.2

(2) 2.1

(3) 0.42

(4) 0.21

Answer (3)

Sol. Heat given by gas is equal to heat taken by water

$$\Rightarrow m\times5\times10^4 = \frac{5}{60}\times4200\times60$$

$\Rightarrow m = 0.42\ g$

43. In a double slit experiment the distance between the slits is 0.1 cm and the screen is placed at 50 cm from the slits plane. When one slit is covered with a transparent sheet having thickness t and refractive index n(= 1.5), the central fringe shifts by 0.2 cm. The value of t is _____ cm.

(1) 5.0×10^{-3} (2) 5.6×10^{-4}

(3) 6.0×10^{-3} (4) 8×10^{-4}

Answer (4)

Sol. $d = 0.1$ cm $= 0.1 \times 10^{-2}$ m

$D = 0.5$ mm

$n = 1.5$

Thickness

$$t = \frac{xd}{D(m-1)}$$

Or 8×10^{-4} cm

44. Initially a satellite of 100 kg is in a circular orbit of radius $1.5R_E$. This satellite can be moved to a circular orbit of radius $3R_E$ by supplying $\alpha \times 10^6$ J of energy. The value of α is _____.

(Take Radius of Earth $R_E = 6 \times 10^6$ m and g = 10 m/s^2)

(1) 150

(2) 100

(3) 500

(4) 1000

Answer (4)

Sol. To send satellite from r_1 radius orbit to r_2 radius orbit, energy needed to be supplied will be

$$E = -\frac{GM_m}{2r_2} + \frac{GM_m}{2r_1}$$

$$\therefore \quad E = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow \quad E = \frac{gR^2m}{2}\left(\frac{2}{3R} - \frac{1}{3R}\right)$$

$E = 10^9 = 1000 \times 10^6$ J

45. A 4 kg mass moves under the influence of a force $\vec{F} = \left(4t^3\hat{i} - 3t\hat{j}\right)N$ where t is the time in second. If mass starts from origin at t = 0, the velocity and position after t = 2 s will be :

(1) $\vec{v} = 4\hat{i} + \frac{5}{2}\hat{j}$ $\vec{r} = \frac{8}{5}\hat{i} + 2\hat{j}$

(2) $\vec{v} = 4\hat{i} + \frac{3}{2}\hat{j}$ $\vec{r} = \frac{6}{5}\hat{i} - \hat{j}$

(3) $\vec{v} = 3\hat{i} + \frac{3}{2}\hat{j}$ $\vec{r} = \frac{6}{5}\hat{i} + \hat{j}$

(4) $\vec{v} = 4\hat{i} - \frac{3}{2}\hat{j}$ $\vec{r} = \frac{8}{5}\hat{i} - \hat{j}$

Answer (4)

Sol. $\vec{F} = m\vec{a}$

$$\vec{a} = \frac{1}{4}(4t^3\hat{i} - 3t\hat{j})$$

$$\frac{d\vec{v}}{dt} = \frac{1}{4}(4t^3\hat{i} - 3t\hat{j})$$

$$\vec{v} = \frac{1}{4}(t^4\hat{i} - \frac{3}{2}t^2\hat{j}) = \frac{t^4}{4}\hat{i} - \frac{3}{8}t^2\hat{j}$$

At t = 2

$$\vec{v} = 4\hat{i} - \frac{3}{2}\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^4}{4}\hat{i} - \frac{3}{8}t^2\hat{j}$$

$$\vec{r} = \frac{t^5}{20}\hat{i} - \frac{t^3}{8}\hat{j}$$

At t = 2

$$\vec{r} = \frac{8}{5}\hat{i} - \hat{j}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. In a microscope the objective is having focal length f_0 = 2 cm and eye-piece is having focal length f_e = 4 cm. The tube length is 32 cm. The magnification produced by this microscope for normal adjustment is _____.

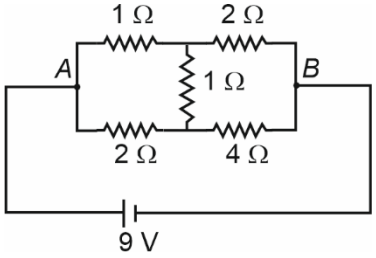
Answer (100)

Sol. Magnification(M) = $\frac{L}{f_0} \cdot \frac{D}{f_e}$

$$= \frac{32}{2} \cdot \frac{25}{4}$$

$$= 100$$

47. The heat generated in 1 minute between points *A* and *B* in given circuit, when a battery of 9 V with internal resistance of 1 Ω is connected across these points is _____*J*.



Answer (1080)

Sol. It is a balanced wheatstone bridge

$$R = \frac{3\times 6}{3+6} = 2\Omega$$

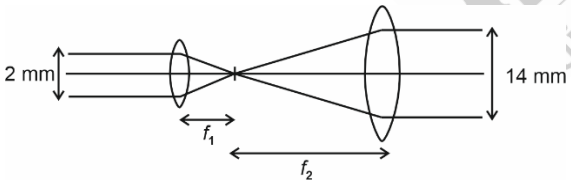
$$P = \left(\frac{9}{2+1}\right)^2 \times 2 = 18$$

$H = 18 \times 60 = 1080$ J

48. A collimated beam of light of diameter 2 mm is propagating along *x*-axis. The beam is required to be expanded in a collimated beam of diameter 14 mm using a system of two convex lenses. If first lens has focal length 40 mm, then the focal length of second lens is ___ mm.

Answer (280)

Sol.



$$\frac{f_1}{1} = \frac{f_2}{7}$$

$f_2 = 7 \times 40$

$f_2 = 280$ mm

49. 10 mole of oxygen is heated at constant volume from 30°C to 40°C. The change in the internal energy of the gas is ___ cal. (The molecular specific heat of oxygen at constant pressure, C_p = 7 cal/mol. °C and R = 2 cal./mol. °C.)

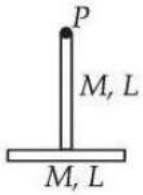
Answer (500)

Sol. $\Delta U = nC_V\Delta T$

$$\Delta U = 10\times\frac{5}{2}\times 2\times 10$$

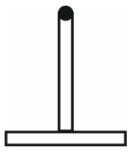
= 500 cal

50. Two identical thin rods of mass *M* kg and length *L* m are connected as shown in figure. Moment of inertia of the combined rod system about an axis passing through point *P* and perpendicular to the plane of the rods is $\frac{x}{12}ML^2$ kg m^2. The value of *x* is _____.



Answer (17)

Sol.



$$I = \frac{ml^2}{3} + \frac{ml^2}{12} + ml^2$$

$$I = \frac{17}{12}ml^2$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. Given below are two statements :

Statement I :

Among $\left[Cu(NH_3)_4\right]^{2+}$, $[Ni(en)_3)]^{2+}$, $\left[Ni(NH_3)_6\right]^{2+}$ and $\left[Mn(H_2O)_6\right]^{2+}$, $\left[Mn(H_2O)_6\right]^{2+}$ has the maximum number of unpaired electrons.

Statement II :

The number of pairs among $\left\{\left[NiCl_4\right]^{2-},\left[Ni(CO)_4\right]\right\}$, $\left\{\left[NiCl_4\right]^{2-},\left[Ni(CN)_4\right]^{2-}\right\}$ and $\left\{\left[Ni(CO)_4\right],\left[Ni(CN)_4\right]^{2-}\right\}$ that contain only diamagnetic species is two.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Answer (1)

Sol. $[Cu(NH_3)_4]^{2+} \Rightarrow Cu^{2+} \Rightarrow 3d^9$ (n = 1)

$[Ni(en)_3)]^{2+} \Rightarrow Ni^{2+} \Rightarrow 3d^8$ (n = 2)

$[Ni(NH_3)_6]^{2+} \Rightarrow Ni^{2+} \Rightarrow 3d^8$ (n = 2)

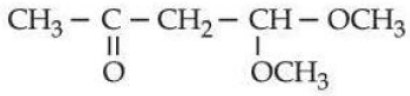
$[Mn(H_2O)_6]^{2+} \Rightarrow Mn^{2+} \Rightarrow 3d^5$ (n = 5)

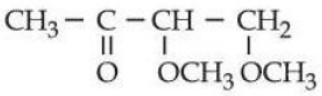
$[NiCl_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow sp^3$ (n = 2) $\Rightarrow$ paramagnetic

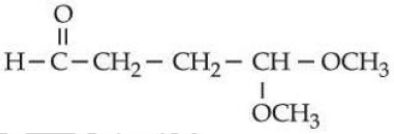
$[Ni(CO)_4] \Rightarrow sp^3 \Rightarrow$ diamagnetic

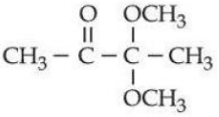
$[Ni(CN)_4]^{2-} \Rightarrow dsp^2 \Rightarrow$ diamagnetic

52. An organic compound "P" of molecular formula $C_6H_{12}O_3$ gives positive Iodoform test but negative Tollen's test. When "P" is treated with dilute acid, it produces "Q". "Q" gives positive Tollen's test and also iodoform test. The structure of "P" is :

(1) 

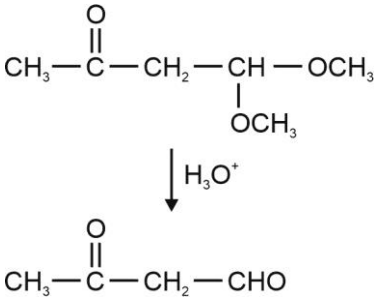
(2) 

(3) 

(4) 

Answer (1)

Sol.  gives positive iodoform test



(gives positive Tollen's test and iodoform test)

53. 14.0 g of calcium metal is allowed to react with excess HCl at 1.0 atm pressure and 273 K.

Which of the following statements is incorrect?

[Given : Molar mass in $gmol^{-1}$ of Ca – 40, Cl –35.5, H – 1]

(1) 0.35 mol of H_2 gas is evolved.

(2) The limiting reagent is calcium metal.

(3) 7.84 L of H_2 gas is evolved.

(4) 33.3 g of $CaCl_2$ is produced.

Answer (4)

Sol. $\underset{(LR)}{Ca} + \underset{(excess)}{2HCl} \longrightarrow CaCl_2 + H_2 \uparrow$

$$n_{Ca} = \frac{14}{40} = 0.35$$

$$n_{Ca} = n_{H_2} = 0.35 = n_{CaCl_2}$$

$$V_{H_2} = 0.35 \times 22.4 \text{ L} = 7.84 \text{ L}$$

Mass of $CaCl_2$ = 0.35 × 111 g

= 38.85 g

54. Identify correct statements from the following:

A. Propanal and propanone are functional isomers.

B. Ethoxyethane and methoxypropane are metamers.

C. But-2-ene shows optical isomerism.

D. But-1-ene and but-2-ene are functional isomers.

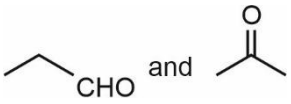
E. Pentane and 2, 2-dimethyl propane are chain isomers.

Choose the correct answer from the options given below:

(1) A, B and C only (2) C, D and E only

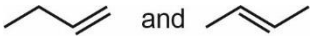
(3) B, C and D only (4) A, B and E only

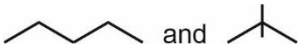
Answer (4)

Sol.  ⇒ functional isomer

 ⇒ Metamers

$CH_3 - CH = CH - CH_3$ ⇒ doesn't show optical isomer

 ⇒ position isomer

 ⇒ chain isomer

A, B and E are correct

55. Given below are two statements:

Statement I: The number of pairs among $[SiO_2, CO_2]$, $[SnO, SnO_2]$, $[PbO, PbO_2]$ and $[GeO, GeO_2]$, which contain oxides that are both amphoteric is 2 .

Statement II: BF_3 is an electron deficient molecule, can act as a Lewis acid, forms adduct with NH_3 and has a trigonal planar geometry.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Answer (2)

Sol. GeO_2, SiO_2, CO_2 ⇒ acidic oxides

SnO, SnO_2, PbO, PbO_2 ⇒ amphoteric oxide

GeO ⇒ distinctly acidic

BF_3 ⇒ Lewis acid

$$\underset{\substack{(sp^2)\\ \text{trigonal}\\ \text{planar}}}{BF_3} + NH_3 \longrightarrow \underset{\substack{\text{Both B and N are}\\ sp^3 \text{ hybridised}}}{BF_3^- - NH_3^+}$$

56. Given below are two statements :

Statement I: When an electric discharge is passed through gaseous hydrogen, the hydrogen molecules dissociate and the energetically excited hydrogen atoms produce electromagnetic radiation of discrete frequencies.

Statement II: The frequency of second line of Balmer series obtained from $He^+$is equal to that of first line of Lyman series obtained from hydrogen atom.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Answer (2)

Sol. Frequency $\propto \frac{Z^2}{n^3}$

$\therefore$ Frequency of second line of Balmer series of

$He+ \propto \frac{2^2}{2^3}$ (Z = 2, n = 2)

$\therefore$ Frequency of first line of Lyman series of H $\propto \frac{1^2}{1^3}$

(Z = 1, n = 1)

57. Consider the following reactions.

$PbCl_2 + K_2CrO_4 \rightarrow A + 2KCl$

(Hot solution)

$A + NaOH \rightleftharpoons B + Na_2CrO_4$

$PbSO_4 + 4CH_3COONH_4 \rightarrow (NH_4)_2SO_4 + X$

In the above reactions, A, B and X are respectively.

(1) $Na_2[Pb(OH)_2]$, $PbCrO_4$ and $[Pb(NH_3)_4]SO_4$

(2) $PbCrO_4$, $Na_2[Pb(OH)_4]$ and $[Pb(NH_3)_4]SO_4$

(3) $Na_2[Pb(OH)_2]$, $PbCrO_4$ and $(NH_4)_2[Pb(CH_3COO)_4]$

(4) $PbCrO_4$, $Na_2[Pb(OH)_4]$ and $(NH_4)_2[Pb(CH_3COO)_4]$

Answer (4)

Sol. $\underset{\text{(Hot solution)}}{PbCl_2} + K_2CrO_4 \rightarrow \underset{\text{(A)}}{PbCrO_4} + 2KCl$

$PbCrO_4 + NaOH \rightleftharpoons \underset{\text{(B)}}{Na_2[Pb(OH)_4]} + Na_2CrO_4$

$PbSO_4 + 4CH_3COONH_4 \rightarrow (NH_4)_2SO_4 + \underset{\text{(X)}}{(NH_4)_2[Pb(CH_3COO)_4]}$

58. Identify the **correct** statements.

A. Arginine and Tryptophan are essential amino acids.

B. Histidine does not contain heterocyclic ring in its structure.

C. Proline is a six membered cyclic ring amino acid.

D. Glycine does not have chiral centre.

E. Cysteine has characteristic feature of side chain as MeS – CH_2 – CH_2 –.

Choose the **correct** answer from the options given below.

(1) B and E Only

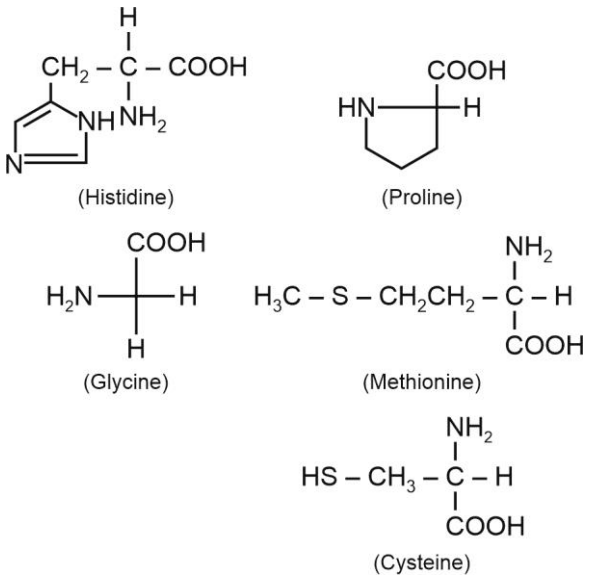
(2) C and D Only

(3) A and D Only

(4) C and E Only

Answer (3)

Sol. Arginine and Tryptophan are essential amino acids.



59. An organic compound (P) on treatment with aqueous ammonia under hot condition forms compound (Q) which on heating with Br_2 and KOH forms compound (R) having molecular formula C_6H_7N. Names of P, Q and R respectively are

(1) Benzoic acid, 4-methylbenzamide, 4-methylaniline

(2) Benzoic acid, benzamide, aniline

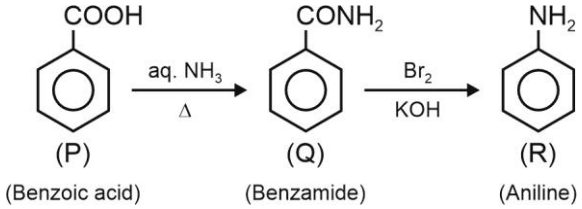
(3) Toluic acid, methylbenzamide, 2-methylaniline

(4) Phenylethanoic acid, phenylethanamide, benzamine

Answer (2)

Sol. $C_6H_7N \Rightarrow DU = 4$

C_6H_7N is aniline.



60. Which of the following represents the correct trend for the mentioned property?

A. F > P > S > B – First Ionisation Energy

B. Cl > F > S > P – Electron Affinity

C. K > Al > Mg > B – Metallic character

D. $KO_2 > Na_2O > MgO > Al_2O_3$ – Basic character

Choose the **correct** answer from the options given below.

(1) A and B only (2) A, B and D only

(3) A, B, C and D (4) B and C only

Answer (2)

Sol. Statements A, B and D are correct.

Metallic character order : K > Mg > Al > B

Statement C is incorrect.

	F	P	S	B
I.E. (kJ/mol)	1680	1012	1000	800

61. Elements P and Q form two types of non-volatile, non-ionizable compounds PQ and PQ_2. When 1 g of PQ is dissolved in 50 g of solvent 'A', ΔT_b was 1.176 K while when 1 g of PQ_2 is dissolved in 50 g of solvent 'A', ΔT_b was 0.689 K. (K_b of 'A' = 5 K kg mol^{-1}). The molar masses of elements P and Q (in g mol^{-1}) respectively, are

(1) 70, 110 (2) 25, 60

(3) 65, 145 (4) 60, 25

Answer (2)

Sol. $1.176 = (5)\left(\frac{1\times1000}{(MW)(50)}\right)$

$MW_{PQ} = 85.03$

P + Q = 85

$$0.689 = (5)\left(\frac{1000}{(MW)50}\right)$$

$MW_{PQ_2} = 145.1$

P + 2Q = 145

Q = 60

P = 25

62. 80 mL of a hydrocarbon on mixing with 264 mL of oxygen in a closed U-tube undergoes complete combustion. The residual gases after cooling to 273 K occupy 224 mL . When the system is treated with KOH solution, the volume decreases to 64 mL . The formula of the hydrocarbon is

(1) C_2H_2 (2) C_4H_{10}

(3) C_2H_4 (4) C_2H_6

Answer (1)

Sol. $V_{CO_2} = 224 - 64 = 160$ mL

V_{O_2} remaining = 64 mL

C_xH_y : Hydrocarbon

$$x = \frac{160}{80} = 2$$

V_{O_2} used = 264 – 64 = 200

$$x + \frac{y}{4} = \frac{200}{80} = 2.5$$

$$2 + \frac{y}{4} = 2.5$$

$$\frac{y}{4} = 0.5$$

$y = 2 \Rightarrow C_2H_2$

63. MnO_4^{2-}, in acidic medium, disproportionates to :

(1) Mn_2O_7 and MnO_2 (2) Mn_2O_7 and MnO

(3) MnO_4^- and MnO_2 (4) MnO_4^- and MnO

Answer (3)

Sol. $4H^+ + 3MnO_4^{2-} \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

64. In Carius method, 0.75 g of an organic compound gave 1.2 g of barium sulphate, find percentage of sulphur (molar mass 32 g mol^{-1}). Molar mass of barium sulphate is 233 g mol^{-1}.

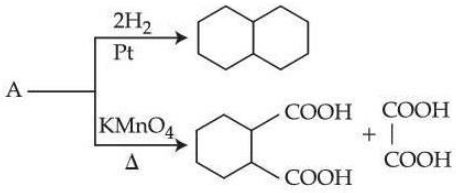
(1) 21.97% (2) 10.30%

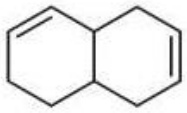
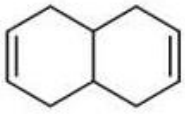
(3) 4.55% (4) 16.48%

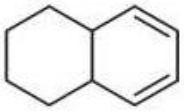
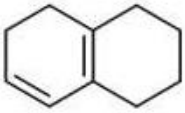
Answer (1)

Sol. $\%\ \text{of S} = \frac{1.2 \times 32}{233 \times (0.75)} \times 100 = 21.97$

65. Identify A in the following reaction.

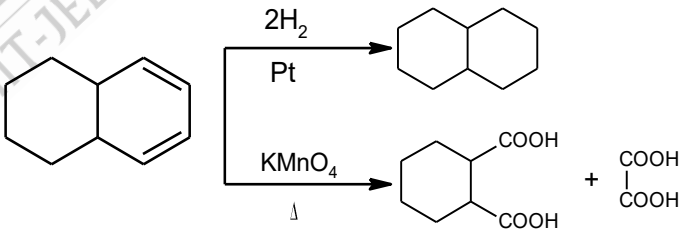


(1)  (2) 

(3)  (4) 

Answer (3)

Sol.



66. Given below are two statements :

Statement I :

The number of species among $SF_4, NH_4^+, [NiCl_4]^{2-}$, $XeF_4, [PtCl_4]^{2-}, SeF_4$ and $[Ni(CN)_4]^{2-}$, that have tetrahedral geometry is 3 .

Statement II :

In the set $[NO_2, BeH_2, BF_3, AlCl_3]$, all the molecules have incomplete octet around central atom.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is true but **Statement II** is false

(3) **Statement I** is false but **Statement II** is true

(4) Both **Statement I** and **Statement II** are false

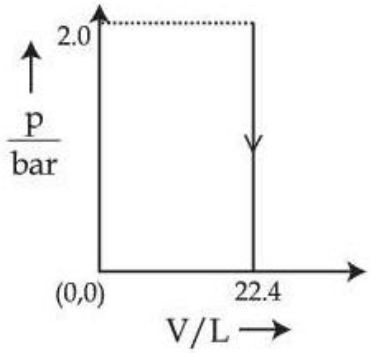
Answer (3)

Sol. Statement I is incorrect as only NH_4^+ and $[NiCl_4]^{2-}$ are tetrahedral

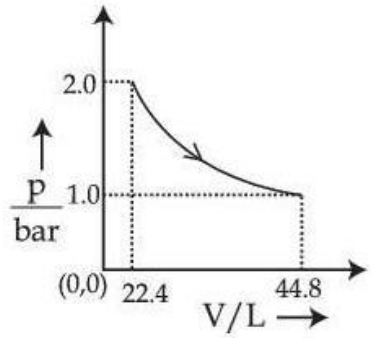
Statement II is correct

67. Which of the following graphs between pressure 'p' versus volume 'V' represents the maximum work done?

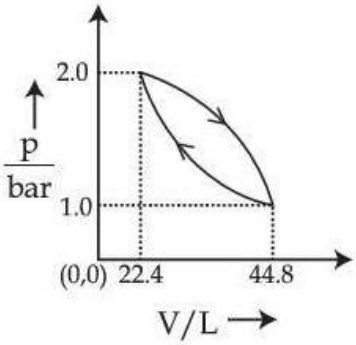
(1)



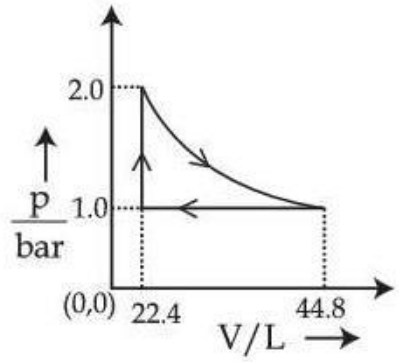
(2)



(3)



(4)



Answer (2)

Sol. $W_1 = 0$

$W_2 = (44.8) \times \ln 2 = 31.04$

$W_3 < 31.04$

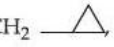
$W_4 < 31.04$

W_2 is max

68. A hydrocarbon 'P' (C_4H_8) on reaction with HCl gives an optically active compound 'Q' (C_4H_9Cl) which on reaction with one mole of ammonia gives compound 'R'($C_4H_{11}N$). 'R' on diazotization followed by hydrolysis gives 'S'. Identify P, Q, R and S .

(1) $P = CH_3-CH=CH-CH_3$, $Q = CH_3-CH_2-\underset{\underset{Cl}{|}}{CH}-CH_3$,

$R = CH_3-CH_2-\underset{\underset{NH_2}{|}}{CH}-CH_3$, $S = CH_3-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3$

(2) P = CH_3  , Q = Cl – CH_2

R = H_2N-CH_2  , S = 

(3) P = $CH_3-CH=CH-CH_3$, Q = $CH_3-CH_2-CH_2-CH_2-Cl$,

R = $CH_3-CH_2-CH_2-\underset{\underset{NH_2}{|}}{CH_2}$, S = $CH_3-CH_2-\underset{\underset{OH}{|}}{CH_2}$

(4) P = $CH_3-CH_2-CH=CH_2$, Q = $CH_3-CH_2-CH_2-\underset{\underset{Cl}{|}}{CH_2}$,

R = $CH_3-CH_2-CH_2-\underset{\underset{CH_3}{|}}{NH}$, S = $CH_3-CH_2-\underset{\underset{CH_3}{|}}{\overset{\overset{OH}{|}}{CH}}$

Answer (1)

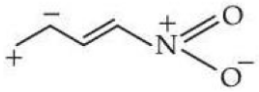
Sol. P : $CH_3—CH = CH — CH_3$

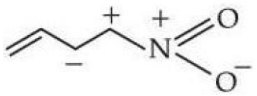
Q : $CH_3—CH_2—\underset{\underset{Cl}{|}}{CH}—CH_3$

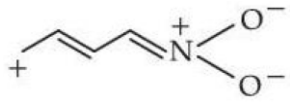
R : $CH_3—CH_2—\underset{\underset{NH_2}{|}}{CH}—CH_3$

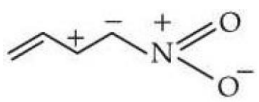
S : $CH_3—CH_2—\underset{\underset{OH}{|}}{CH}—CH_3$

69. From the following, the least stable structure is:

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. Structure in option (2) is least stable as two +ve charges are present on adjacent atoms.

70. For the reaction, $N_2O_4 \rightleftharpoons 2NO_2$, graph is plotted as shown below. Identify correct statements.

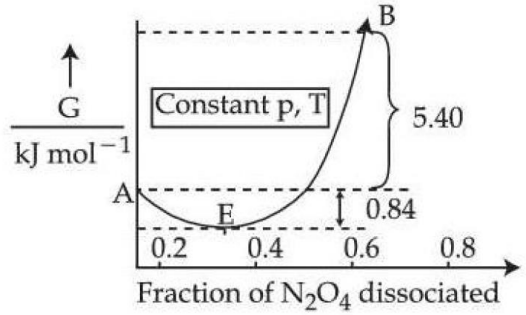
A. Standard free energy change for the reaction is –5.40 kJ mol^{-1}.

B. As $\Delta G^{\ominus}$ in graph is positive, N_2O_4 will not dissociate into NO_2 at all.

C. Reverse reaction will go to completion.

D. When 1 mole of N_2O_4 changes into equilibrium mixture, value of $\Delta G^{\ominus} = -0.84$ kJ mol^{-1}

E. When 2 mole of NO_2 changes into equilibrium mixture, $\Delta G^{\ominus}$ for equilibrium mixture is. –6.24 kJ mol^{-1}.



Choose the **correct** answer from the options given below :

(1) C and E only (2) D and E only

(3) B and C only (4) A and D only

Answer (2)

Sol. D is correct.

E is correct.

$\Delta G^{\circ}_{reaction} = G_B - G_A = 5.40$ kJ mol^{-1}

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The pH and conductance of a weak acid (HX) was found to be 5 and 4×10^{-5} S, respectively. The conductance was measured under standard condition using a cell where the electrode plates having a surface area of 1 cm^2 were at a distance of 15 cm apart. The value of the limiting molar conductivity is _____ S m^2 mol^{-1}. (nearest integer)

(Given : degree of dissociation of the weak acid $(\alpha) \ll 1$)

Answer (6)

Sol. Resistance = $\frac{10^5}{4}$ Ohm

$$R = \frac{\rho l}{A}$$

$$\frac{10^5}{4} = (\rho)\left(\frac{15\ cm}{1\ cm^2}\right)$$

$$\rho = \frac{10^5}{4 \times 15}\ \text{Ohm.cm}$$

Conductivity = 60×10^{-5} S cm^{-1}

$= 6 \times 10^{-4}$ S cm^{-1}

$$\alpha = \frac{\lambda_m}{\lambda_m^\circ}$$

$C\alpha = 10^{-5}$

$$\lambda_m = \frac{k}{H^+_{total}}$$

In m^3 (1000 L)

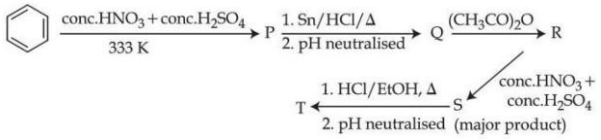
$H^+ = 0.01$ mol m^{-3}

$$\lambda_m = \frac{0.06}{0.01} = 6\ Sm^2\ mol^{-1}$$

As $\lambda_m \approx \lambda_m^\circ$

$\lambda_m^\circ = 6\ Sm^2\ mol^{-1}$

72. Consider the following reaction sequence

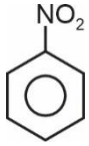
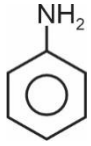
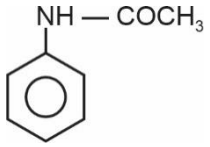
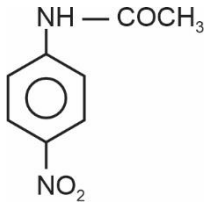


The percentage of nitrogen in product 'T' formed is _____ %. (Nearest integer)

(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16)

Answer (20)

Sol.

A :	
B :	
R :	
S :	

T :	

$T = C_6H_4(NH_2)(NO_2)$

$\%\ of\ N = \frac{2\times14}{138}\times100$

$= 20.28\%$

≈ 20

73. Consider the following reactions :

$NaCl + K_2Cr_2O_7 + H_2SO_4 \rightarrow$

$A + KHSO_4 + NaHSO_4 + H_2O$

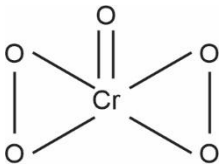
$A + NaOH \rightarrow B + NaCl + H_2O$

$B + H_2SO_4 + H_2O_2 \rightarrow C + Na_2SO_4 + H_2O$

In the product 'C', 'X' is the number of O_2^{2-} units, 'Y' is the total number oxygen atoms present and 'Z' is the oxidation state of Cr. The value of X + Y + Z is _____ .

Answer (13)

Sol. C is CrO_5



X = 2

Y = 5

Z = +6

X + Y + Z = 13

74. Use the following data :

Substance	$\frac{\Delta_f H^{\ominus}(500\ K)}{kJmol^{-1}}$	$\frac{S^{\ominus}(500\ K)}{JK^{-1}\ mol^{-1}}$
AB(g)	32	222
A_2(g)	6	146
B_2(g)	x	280

One mole each of A_2(g) and B_2(g) are taken in a 1L closed flask and allowed to establish the equilibrium at 500 K.

$A_2(g) + B_2(g) \rightleftharpoons 2AB(g)$

The value of x (in kJ mol^{-1}) is _____ . (Nearest integer)

(Given : log K = 2.2 R = 8.3 J K^{-1} mol–1)

Answer (70)

Sol. $\Delta G^\circ = -2.303 \times R \times T \times \log K$

$= -2.303 \times 8.3 \times 500 \times 2.2$

$= -21.026\ \frac{kJ}{mole}$

$A_2 + B_2 \rightleftharpoons 2AB$

$\Delta H^\circ = 64 - 6 - x$

$= (58 - x)\ kJ\ mol^{-1}$

$\Delta S^\circ = 444 - 146 - 280$

$= 18$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$-21.026 = (58 - x) - (0.5)(18)$

$= (58 - x) - 9 = 49 - x$

$-21.026 = 49 - x$

$x = 49 + 21.026$

$= 70.026$

≈ 70

75. Pre-exponential factors of two different reactions of same order are identical. Let activation energy of first reaction exceeds the activation energy of second reaction by 20 kJ mol^{-1}. If k_1 and k_2 are the rate constants of first and second reaction respectively at 300 K , then $\ln\frac{k_2}{k_1}$ will be _____.

(nearest integer) [R = 8.3 J K^{-1} mol^{-1}]

Answer (8)

Sol. $\ln\frac{k_2}{k_1} = \frac{\Delta E}{RT}$

$= \frac{20\times1000}{8.3\times300} = 8.03$

≈ 8

❑ ❑ ❑