TRUST
Aakash

Stellar Ranks in
JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR **23**

Kushagra Baingaha AIR **7**

Shreyas Lohiya AIR **6**

Harssh A Gupta AIR **15**

Devya Rustagi AIR **28**

Shreyas Lohiya AIR **68**

Rujul Garg AIR **41**

Advay Mayank AIR **36**

Arush Anand AIR **64**

Kotha D Reddy AIR **74**

Top Ranks in
JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students
Aakashians Qualified in **NSEs** 2025-26

777 Classroom Students
Aakashians Qualified in **IOQM** 2025

134 Classroom Students
Aakashians Qualified in **RMO** 2025

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

HOME OF PROBLEM SOLVERS

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $A = \{0, 1, 2, ..., 9\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $x - y$ is a multiple of 3.

   Given below are two statements:

   Statement I: $n(R) = 36$.

   Statement II: R is an equivalence relation.

   In the light of the above statements, choose the correct answer from the options given below

   (1) Both Statement I and Statement II are incorrect

   (2) Statement I is correct but Statement II is incorrect

   (3) Statement I is incorrect but Statement II is correct

   (4) Both Statement I and Statement II are correct

Answer (3)

Sol. $A_1 = \{0, 3, 6, 9\}$, $A_2 = \{1, 4, 7\}$, $A_3 = \{2, 5, 8\}$

$n(R) = 4 \times 4 + 3 \times 3 + 3 \times 3 = 34$

Reflexive: for any $x \in A$, $|x - x| = 0$, Since 0 is multiple of 3, $(x, x) \in R$, True

Symmetric: if $(x, y) \in R$, then $|x - y|$ is multiple of 3.

Since $|y - x| = |x - y|$, then $|y - x|$ is multiple of 3.

$\Rightarrow (y, x) \in R$, True

Transitive: if $(x, y) \in R$, $(y, z) \in R$

$|x - y| = 3k$, $|y - z| = 3m$

$\Rightarrow x - y = \pm 3k$, $y - z = \pm 3m$

$\Rightarrow x - z = 3(\pm k \pm m)$

$\Rightarrow |(x - z)|$ is multiple of 3, True

$\Rightarrow R$ is an equivalence relation

$\Rightarrow$ Option (3) is correct.

2. The sum of all the real solutions of the equation,

   $$\log_{(x^2+3)}\left(6x^2 + 28x + 30\right) = 5 - 2\log_{(6x+10)}\left(x^2 + 6x + 9\right)$$

   is equal to

   (1) 0 (2) 4

   (3) 1 (4) 2

Answer (1)

Sol. $6x^2 + 28x + 30 = 2(x + 3)(3x + 5)$

$x^2 + 6x + 9 = (x + 3)^2$

$\Rightarrow \log_{(x+3)}\left(2(x + 3)(3x + 5)\right)$

$= 5 - 2\log_{2(3x+5)}(x + 3)^2$

$\Rightarrow \log_{(x+3)}(x + 3) + \log_{(x+3)}\left(2(3x + 5)\right)$

$= 5 - 4\log_{2(3x+5)}(x + 3)$

put $\log_{(x+3)}\left(2(3x + 5)\right) = t$

$\Rightarrow 1 + t = 5 - \dfrac{4}{t}$

$\Rightarrow t^2 - 4t + 4 = 0 \Rightarrow t = 2$

$\Rightarrow \log_{(x+3)}\left(2(3x + 5)\right) = 2$

$\Rightarrow 2(3x + 5) = (x + 3)^2$

$\Rightarrow x^2 + 9 + 6x = 6x + 10 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\Rightarrow$ Sum of all real solution $= 1 - 1 = 0$

$\Rightarrow$ option (1) is correct.

3. Let PQ be a chord of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, perpendicular to the x-axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is

(1) $\dfrac{8\sqrt{3}}{5}$

(2) $\dfrac{11}{5}$

(3) $2\sqrt{3}$

(4) $\dfrac{9}{5}$

Answer (1)

Sol. $\because e = \sqrt{3}$

$\Rightarrow b^2 = 4\left(e^2 - 1\right) \Rightarrow b^2 = 4 \times 2 = 8$

Let $m_{OP} = \tan\left(\pm\dfrac{\pi}{6}\right) = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow P$ is intersection point of $y = \pm\dfrac{1}{\sqrt{3}}x$ and the hyperbola

$\Rightarrow \dfrac{x^2}{4} - \dfrac{x^2}{3 \times 8} = 1$

$\Rightarrow 6x^2 - x^2 = 24$

$\Rightarrow x = \pm\sqrt{\dfrac{24}{5}}$

$\Rightarrow P\left(\pm\sqrt{\dfrac{24}{5}}, \pm\sqrt{\dfrac{24}{15}}\right)$

$a^2 = OP^2 = \dfrac{24}{5} + \dfrac{24}{15} = \dfrac{96}{15}$

$\text{Area} = \dfrac{\sqrt{3}}{4} \times \dfrac{96}{15} = \dfrac{8\sqrt{3}}{5}$ square units

$\Rightarrow$ Option (1) is correct.

4. Consider two sets $A = \left\{ x \in \mathbb{Z} : \left|\left(|x-3|-3\right)^2\right| \le 1 \right\}$ and $B = \left\{ x \in \mathbb{R} - \{1,2\} : \dfrac{(x-2)(x-4)}{x-1}\log_e\left(|x-2|\right) = 0 \right\}$.

Then the number of onto functions $f : A \to B$ is equal to

(1) 32　　　　　(2) 62

(3) 81　　　　　(4) 79

Answer (2)

Sol. $\left|\left(|x-3|-3\right)\right| \le 1$

$-1 \le |x-3| - 3 \le 1$

$2 \le |x-3| \le 4 \Rightarrow x \in \left[-1, 1\right] \cup \left[5, 7\right]$

$\Rightarrow A = \left\{-1, 0, 1, 5, 6, 7\right\}$

$\dfrac{(x-2)(x-4)}{(x-1)}\log_e\left(|x-2|\right) = 0$

$\Rightarrow x = 2, 4, 3$

$\Rightarrow B = \{3, 4\}$

$\Rightarrow$ Number of onto function $f : A \to B$

$= 2^6 - 2 = 62$

$\Rightarrow$ Option (2) is correct.

5. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is

(1) $2\left(8 - 3\sqrt{3}\right)$

(2) $4\left(6 + \sqrt{3}\right)$

(3) $4\left(3 - \sqrt{3}\right)$

(4) $2\left(3 + \sqrt{3}\right)$

Answer (3)

Sol.
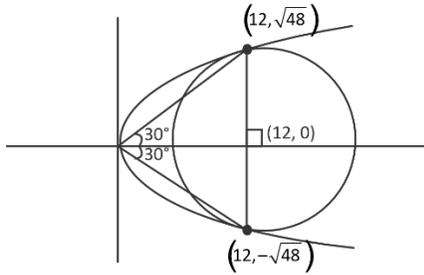
$\Rightarrow$ circle is

$$\left(x-12\right)^2 + \left(y-\sqrt{48}\right)\left(y+\sqrt{48}\right) = 0$$

$\Rightarrow$ $(x-12)^2 + y^2 = 48$ $\Rightarrow$ radius $= 4\sqrt{3}$

Minimum distance is $12 - \sqrt{48}$

$= 4\left(3 - \sqrt{3}\right)$

6. Let $I\left(x\right) = \int \dfrac{3dx}{\left(4x+6\right)\left(\sqrt{4x^2+8x+3}\right)}$ and

$I(0) = \dfrac{\sqrt{3}}{4} + 20$. If $I\left(\dfrac{1}{2}\right) = \dfrac{a\sqrt{2}}{b} + c$, where

$a, b, c \in N, \gcd(a,b) = 1$, then $a + b + c$ is equal to

(1) 29   (2) 30

(3) 28   (4) 31

Answer (4)

Sol. $I\left(x\right) = \int \dfrac{3dx}{\left(4x+6\right)\left(\sqrt{4x^2+8x+3}\right)}$

$= \dfrac{3}{2}\int \dfrac{dx}{\left(2x+3\right)\sqrt{4x^2+8x+3}}$

Let $t = \dfrac{1}{2x+3}$

$dt = \dfrac{-dt}{\left(2x+3\right)^2}$

$x = \dfrac{\dfrac{1}{t} - 3}{2}$

Substitute

$= -\dfrac{3}{2}\int \dfrac{1}{\sqrt{1-2t}}dt$

$= \dfrac{3}{2}\sqrt{1-2t}$

$= \dfrac{3}{4}\sqrt{1 - \dfrac{2}{2x+3}} + C$

$I(0) = \dfrac{\sqrt{3}}{4} + 20 = \dfrac{3}{4}\sqrt{1 - \dfrac{2}{3}} + C$

$\Rightarrow C = 20$

$I(x) = \dfrac{3}{4}\sqrt{1 - \dfrac{2}{2x+3}} + 20$

$I\left(\dfrac{1}{2}\right) = \dfrac{3}{4}\sqrt{1 - \dfrac{2}{4}} + 20$

$= 20 + \dfrac{3}{4\sqrt{2}} = 20 + \dfrac{3\sqrt{2}}{8}$

$a + b + c = 20 + 3 + 8$

$= 31$

7. If $z = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2}, i = \sqrt{-1}$, then $\left(z^{201} - i\right)^8$ is equal to

(1) 0   (2) 1

(3) -1 (4) 256

Answer (4)

Sol. $z = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2} = e^{i\pi/6}$

$(z^{201} - i)^8 = \left(e^{i\frac{201\pi}{6}} - i\right)^8$

$= \left(e^{i\frac{67\pi}{2}} - i\right)^8$

$= \left(-i - i\right)^8$

$= \left(-2i\right)^8$

$= 2^8$

$= 256$

8. Let $A(1,2)$ and $C(-3,-6)$ be two diagonally opposite vertices of a rhombus, whose sides AD and BC are parallel to the line $7x - y = 14$. If $B(\alpha,\beta)$ and $D(\gamma,\delta)$ are the other two vertices, then $|\alpha + \beta + \gamma + \delta|$ is equal to

(1) 6

(2) 9

(3) 3

(4) 1

Answer (1)

Sol.

$A(a, 2)$

$B(\alpha, \beta)$ $D(\gamma, \delta)$

$C(-3, -6)$
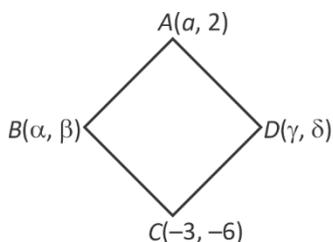
$\because$ AC is parallel to $y = 2x + 14$

$\Rightarrow \dfrac{8}{a+3} = 2 \Rightarrow a = 1$

Now $\dfrac{1-3}{2} = \dfrac{\alpha + \gamma}{2} \Rightarrow \alpha + \gamma = -2$

$\dfrac{2-6}{2} = \dfrac{\beta + \delta}{2} \Rightarrow \beta + \delta = -4$

$|\alpha + \beta + \gamma + \delta| = 6$

9. The system of linear equations

$x + y + z = 6$

$2x + 5y + az = 36$

$x + 2y + 3z = b$

has

(1) infinitely many solutions for $a = 8$ and $b = 16$

(2) unique solution for $a = 8$ and $b = 14$

(3) unique solution for $a = 8$ and $b = 16$

(4) infinitely many solutions for $a = 8$ and $b = 14$

Answer (4)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 8 - a$

for unique solution $a \neq 8$

if $a = 8$

$x + y + z = 6$ ...(i)

$2x + 5y + 8z = 36$ ...(ii)

$x + 2y + 3z = b$ ...(iii)

$3(iii) - (i)$

$2x + 5y + 8z = 3b - 6$ ...(iv)

To have infinite solution eq. (ii) and (iv) must be identical.

$3b - 6 = 36$

$b = 14$

10. The least value of $\left(\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2\right)$ is

(1) $4 + \sqrt{10}$

(2) -1

(3) $4 - \sqrt{10}$

(4) 1

Answer (3)

Sol. $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$

$= 3 - 3\sin 2\theta + 2\sin^2\theta$

$= 3 - 3\sin 2\theta + 1 - \cos 2\theta$

$= 4 - 3\sin 2\theta - \cos 2\theta$

Least value is $4 - \sqrt{10}$

11. The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ is

(1) $\dfrac{2}{3}\left(4\pi - 3\sqrt{3}\right)$

(2) $\dfrac{4}{3}\left(2\pi - \sqrt{3}\right)$

(3) $\dfrac{4}{3}\left(2\pi - 3\sqrt{3}\right)$

(4) $\dfrac{2}{3}\left(2\pi - 3\sqrt{3}\right)$

Answer (1)

Sol.

$$A = 2 \int_0^{\sqrt{3}} \left(\sqrt{4-x^2} - \left(2 - \sqrt{4-x^2}\right) \right) dx$$

$$= 2 \left(\left[x\sqrt{4-x^2} + 4\sin^{-1}\left(\frac{x}{2}\right) \right]_0^{\sqrt{3}} - 2\left(\sqrt{3}\right) \right)$$

$$= 4 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{ sq. units} = \frac{8\pi}{3}$$

12. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$. If $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60°, then $|\vec{a} \cdot \vec{c}|$ is equal to

(1) 0   (2) 1

(3) 4   (4) 2

Answer (2)

Sol. $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$

$\Rightarrow \vec{a} \times (\vec{b} - 2\vec{c}) = 0$

$\Rightarrow \vec{a}$ is parallel to $\vec{b} - 2\vec{c}$

$\therefore \vec{a} = \lambda(\vec{b} - 2\vec{c})$

$\Rightarrow 1 = \lambda^2 \left(16 + 16 - 4 \cdot 4 \cdot 2 \cdot \frac{1}{2} \right)$

$\Rightarrow \lambda = \pm \frac{1}{4}$

$\vec{a} \cdot \vec{c} = \lambda(\vec{b} \cdot \vec{c} - 2|\vec{c}|^2)$

$= \lambda \left(4 \cdot 2 \cdot \frac{1}{2} - 2 \cdot 4 \right)$

$= \lambda(4 - 8)$

$|\vec{a} \cdot \vec{c}| = 1$

13. If $f(x) = \begin{cases} \dfrac{a|x| + x^2 - 2(\sin|x|)(\cos|x|)}{x} &, & x \neq 0 \\ b &, & x = 0 \end{cases}$

is continuous at $x = 0$, then $a + b$ is equal to

(1) 0

(2) 4

(3) 1

(4) 2

Answer (4)

Sol. $\displaystyle\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{a|x| + x^2 - 2(\sin|x|)(\cos|x|)}{x}$

$\text{RHL} = \displaystyle\lim_{x \to 0^+} \frac{ax + x^2 - \sin 2x}{x}$

$\text{RHL} = a - 2$

$\text{LHL} = \displaystyle\lim_{x \to 0^-} \frac{-ax + x^2 - 2\sin(-x) \cdot \cos(-x)}{x}$

$= \displaystyle\lim_{x \to 0^-} \frac{-ax + x^2 + \sin 2x}{x}$

$= -a + 2$

$\text{RHL} = \text{LHL} = f(0)$

$a - 2 = -a + 2 = b$

$\Rightarrow a = 2, b = 0$

$a + b = 2$

14. Let $\dfrac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\dfrac{1}{2\sqrt{2}}$. Then the value of

$\sin\left(\dfrac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\dfrac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$

is equal to

(1) $\dfrac{\sqrt{2}}{\sqrt{3}}$ (2) $-\dfrac{\sqrt{2}}{\sqrt{3}}$

(3) $\dfrac{1-\sqrt{2}}{\sqrt{3}}$ (4) $\dfrac{\sqrt{2}-1}{\sqrt{3}}$

Answer (3)

Sol. $f(\theta) = \sin\left(\dfrac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta)$

$\qquad + \cos\left(\dfrac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$

$= \sin\left(\dfrac{15\theta}{2}\right)\cdot \cos 8\theta - \cos\left(\dfrac{15\theta}{2}\right)\cdot \sin 8\theta$

$\qquad + \sin\left(\dfrac{15\theta}{2}\right)\sin 8\theta + \cos\left(\dfrac{15\theta}{2}\right)\cdot \cos 8\theta$

$= \sin\left(\dfrac{15\theta}{2} - 8\theta\right) + \cos\left(8\theta - \dfrac{15\theta}{2}\right)$

$= \cos\dfrac{\theta}{2} - \sin\dfrac{\theta}{2}$

$\theta \in \left(\dfrac{\pi}{2}, \pi\right)$

$\dfrac{\theta}{2} \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

$\cot\theta = -\dfrac{1}{2\sqrt{2}}$

$\Rightarrow \tan\theta = -2\sqrt{2}$

$\Rightarrow \cos\theta = -\dfrac{1}{3} = 2\cos^2\dfrac{\theta}{2} - 1$

$\Rightarrow 2\cos^2\dfrac{\theta}{2} = \dfrac{2}{3}$

$\Rightarrow \cos\dfrac{\theta}{2} = \dfrac{1}{\sqrt{3}}$

$\sin\dfrac{\theta}{2} = \sqrt{1 - \dfrac{1}{3}} = \dfrac{\sqrt{2}}{\sqrt{3}}$

$\therefore f(\theta) = \dfrac{1}{\sqrt{3}} - \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{1-\sqrt{2}}{\sqrt{3}}$
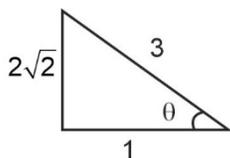
15. If the mean and the variance of the data

Class	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is

(1) 20 (2) 18

(3) 21 (4) 19

Answer (4)

Sol. $\mu = \dfrac{3\times b + \lambda \times 10 + 4\times 14 + 7\times 18}{14\times \lambda} = \dfrac{10\lambda + 200}{14 + \lambda}$

$19 = \sigma^2 = \dfrac{3\times 6^2 + \lambda(10)^2 + 4(14)^2 + 7(18)^2}{14+\lambda} - \left(\dfrac{10\lambda + 200}{14+\lambda}\right)^2$

$\Rightarrow \lambda = 6$

$\mu = 13$

$\lambda + \mu = 19$

16. Let $\displaystyle\sum_{k=1}^{n} a_k = \alpha n^2 + \beta n$. If $a_{10} = 59$ and $a_6 = 7a_1$, then $\alpha + \beta$ is equal to

(1) 5 (2) 3

(3) 12 (4) 7

Answer (1)

Sol. $\because S_n = \alpha n^2 + \beta n$

$\Rightarrow$ Sequence is in AP

$a_{10} = a + 9d = 5a$

$a_6 = a + 5d = 7a \Rightarrow 5d = 6a$

$\Rightarrow a = 5 \qquad d = 6$

Now $S_n = \dfrac{n}{2}[10 + (n-1)6]$

$= n(5 + (n-1)3)$

$= n(3n + 2)$

$\Rightarrow 3n^2 + 2n$

$\Rightarrow \alpha = 3 \quad \beta = 2$

$\boxed{\alpha + \beta = 5}$

17. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$ and $\vec{v} = \vec{a}\times\vec{b}$. If $\vec{v}\cdot\vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p, then $9p^2$ is equal to

(1) 4 (2) 9

(3) 12 (4) 6

Answer (3)

Sol. $\vec{v} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 5\hat{k}$

$\vec{v}.\vec{c} = 11 \Rightarrow (-\hat{i} + 7\hat{j} + 5\hat{k}).(\lambda\hat{i} + \hat{j} + \hat{k}) = 11$

$-\lambda + 7 + 5 = 11$

$\Rightarrow \lambda = 1$

Projection of $\vec{b}$ on $\vec{c} = \dfrac{\vec{b}.\vec{c}}{|\vec{c}|}$

$P = \dfrac{(2\hat{i} + \hat{j} - \hat{k}).(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$

$\Rightarrow \dfrac{2 + 1 - 1}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

$9P^2 = 9 \times \dfrac{4}{3} = 12$

18. If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x^2 + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b), then the value of $ab + 18r^2$ is

(1) 51 (2) 52
(3) 53 (4) 55

Answer (4)

Sol. $x^2 + 2y^2 - 6x - 12y + 23 = 0$ …(1)

$4x^2 + 2y^2 - 20x - 12y + 35 = 0$ …(2)

$(x^2 + 2y^2 - 6x - 12y + 23)$

$+ \lambda(4x^2 + 2y^2 - 20x - 12y + 35) = 0$

$(1 + 4\lambda)x^2 + (2 + 2\lambda)y^2 - 16 + 20\lambda)x$

$-(12 + 12\lambda)y + 23 + 35\lambda = 0$

For this to be circle

$1 + 4\lambda = 2 + 2\lambda$

$\lambda = \dfrac{1}{2}$

Substituting back

$3x^2 + 3y^2 - 16x - 18y + \dfrac{81}{2} = 0$

$x^2 + y^2 - \dfrac{16}{3}x - \dfrac{18}{3}y + \dfrac{81}{6} = 0$

$(a, b) \equiv \left(\dfrac{8}{3}, 3\right)$

$r = \sqrt{\dfrac{64}{9} + 9 - \dfrac{81}{6}}$

$r^2 = \dfrac{47}{18}$

$ab + 18r^2 = 8 + 47 = 55$

19. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is

(1) 455 (2) 384
(3) 403 (4) 429

Answer (1)

Sol. $x_1 + x_2 + x_3 + x_4 = 18$

$x_i \geq 1$ $\qquad X_i = x_i + 1$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 12$

$^{12+4-1}C_{4-1} = {}^{15}C_3 = \dfrac{15!}{3!12!} = \dfrac{13 \times 14 \times 15}{3 \times 2} = 455$

20. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A. Then a ball is randomly drawn from the bag A. If the probability, that the ball drawn is white, is $\dfrac{p}{q}$, $gcd(p,q) = 1$, then $p + q$ is equal to

(1) 21 (2) 23
(3) 24 (4) 22

Answer (2)

Sol.

9W
8B

Bag A

6W
4B
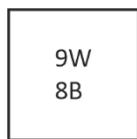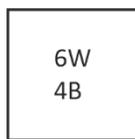
Bag B

A : probability of getting white from bag A

B_w : probability of getting white from bag B

B_b : probability of getting black from bag B

$$P\left(\frac{A}{B_w \cup B_b}\right) = P(B_w)P\left(\frac{A}{B_w}\right) + P(B_b)P\left(\frac{A}{B_b}\right)$$

$$= \frac{6}{10} \times \frac{10}{18} + \frac{4}{10} \times \frac{9}{18}$$

$$= \frac{60+36}{180} = \frac{96}{180} = \frac{8}{15}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the solution curve $y = f(x)$ of the differential equation

$$\left(x^2 - 4\right)y' - 2xy + 2x\left(4 - x^2\right)^2 = 0, x > 2,$$

passes through the point $(3, 15)$, then the local maximum value of f is ____

Answer (16)

Sol. $\left(x^2 - 4\right)\dfrac{dy}{dx} - 2xy + 2x(4 - x^2)^2 = 0, x > 2$

$$\frac{dy}{dx} + \left(\frac{-2x}{x^2 - 4}\right)y = \frac{-2x(4 - x^2)^2}{(x^2 - 4)}$$

$$= (-2x)(x^2 - 4)$$

I.F. $= e^{\int \frac{-2x}{(x^2-4)}dx} = e^{-\ln(x^2-4)} = \frac{1}{x^2-4}$

$$\Rightarrow y \times \frac{1}{(x^2 - 4)} = \int(-2x)dx = -x^2 + C$$

$$\frac{15}{5} = -9 + C \Rightarrow C = 12$$

$$\Rightarrow y = (12 - x^2)(x^2 - 4)$$
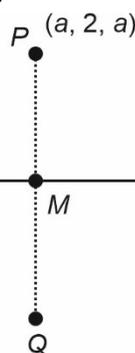
$$= -48 - x^4 + 16x^2$$

$$= 16 - (x^2 - 8)^2$$

$\Rightarrow$ local maximum value is 16 at $x = 2\sqrt{2}$

22. If the image of the point $P(a, 2, a)$ in the line $\dfrac{x}{2} = \dfrac{y+a}{1} = \dfrac{z}{1}$ is Q and the image of Q in the line $\dfrac{x - 2b}{2} = \dfrac{y - a}{1} = \dfrac{z + 2b}{-5}$ is P, then $a + b$ is equal to

Answer (3)

Sol.

$\dfrac{x}{2} = \dfrac{y+a}{1} = \dfrac{z}{1}$

Let M be the mid point of P and Q

$\Rightarrow \overrightarrow{PM} \parallel (2\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} - 5\hat{k})$

$\overrightarrow{PM} \parallel \hat{i} - 2\hat{j}$

Let, $M = (2\lambda, \lambda - a, \lambda)$

$\dfrac{2\lambda - a}{1} = \dfrac{\lambda - a - 2}{-2} = \dfrac{\lambda - a}{0} \Rightarrow \lambda = a$

$M = (2a, 0, a)$

$\overrightarrow{PM} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow 2(2a - a) + (1)(0 - 2) + 1(a - a) = 0$

$\Rightarrow 2a - 2 = 0 \Rightarrow a = 1$

$\Rightarrow P \equiv (1, 2, 1)$

$M \equiv (2, 0, 1)$

$\Rightarrow Q \equiv (3, -2, 1)$

M lie on $\dfrac{x - 2b}{2} = \dfrac{y - a}{1} = \dfrac{z + 2b}{-5}$

$\Rightarrow \dfrac{2 - 2b}{2} = \dfrac{-a}{1} \Rightarrow b = 2$

$\Rightarrow a + b = 3$

23. The number of elements in the set $S = \left\{ x : x \in [0, 100] \text{ and } \displaystyle\int_0^x t^2 \sin(x - t)\, dt = x^2 \right\}$ is _____

Answer (16)

Sol. $\displaystyle\int_0^x t^2 \sin(x - t)\, dt = x^2$

$\Rightarrow \displaystyle\int_0^x (x - t)^2 \sin t\, dt = x^2$

$\Rightarrow \displaystyle\int_0^x (x^2 + t^2 - 2xt) \sin t\, dt = x^2$

$\Rightarrow x^2 \displaystyle\int_0^x \sin t\, dt - 2x \int_0^x t \cdot \sin t\, dt + \int_0^x t^2 \sin t\, dt = x^2$

$\Rightarrow x^2 (-\cos t)\big|_0^x - 2x \left(t(-\cos t)\big|_0^x + \int_0^x \cos t\, dt \right) + $

$\left[t^2(-\cos t)\big|_0^x + 2\int_0^x (t \cos t)\, dt \right] = x^2$

$\Rightarrow x^2(1 - \cos x) - 2x\,(-x \cos x + \sin x) + $

$\left[-x^2 \cos x + 2(t \sin t\big|_0^x - \int_0^x \sin t\, dt) \right] = x^2$

$\Rightarrow x^2(1 - \cos x) + 2x^2\cos x - 2x \sin x - x^2\cos x$

$+ 2x \sin x + 2\cos x - 2 = x^2$

$\Rightarrow x^2 + 2\cos x - 2 = x^2$

$\Rightarrow \cos x = 1$

$x = 2n\pi,\ n \in I$

$\therefore\ n = 0, 1, 2, \ldots, 15$

$\therefore$ Number of elements = 16

24. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that $B(I - A) = I + A$. Then the sum of the diagonal elements of $B^T B$ is equal to _____

Answer (3)

Sol. $B(I - A) = (I + A)$

Since A is skew symmetric matrix

$\Rightarrow A^T = -A$ since, $B^T(I - A^T) = (I + A^T)$

$\Rightarrow B^T(I + A) = (I - A)$

$\Rightarrow BB^T(I + A) = B(I - A) = (I + A)$

$\Rightarrow BB^T(I + A)(I + A)^{-1} = (I + A)(I + A)^{-1}$

$= I_{3\times 3}$

$\Rightarrow \text{Trace }(BB^T) = 3$

25. Let S denote the set of 4-digit numbers $abcd$ such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20. Then $n(S) + n(P)$ is equal to _____

Answer (260)

Sol. $n(S) = {}^{10}C_4 = 210$

Let 5 digit number is

$x_1 x_2 x_3 x_4 x_5 = 20$

One of them has to be 5

$\Rightarrow x_1 x_2 x_3 x_4 = 4$

$5 \times 4 \times 1 \times 1 \times 1$ or $5 \times 2 \times 2 \times 1 \times 1$

$\Rightarrow \dfrac{5!}{3!} + \dfrac{5!}{2!2!} = 50$

$\Rightarrow n(S) + n(P) = 260$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

26. An air bubble of volume 2.9 cm^3 rises from the bottom of a swimming pool of 5 m deep. At the bottom of the pool water temperature is 17°C. The volume of the bubble when it reaches the surface, where the water temperature is 27°C, is _____ cm^3. (g = 10 m/s^2, density of water = 10^3 kg/m^3, and 1 atm pressure is 10^5 Pa)

(1) 3.0

(2) 4.2

(3) 2.0

(4) 4.5

Answer (4)

Sol.

$$\frac{PV}{T} = \text{constant}$$

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

$P_i = P_{atm} + \rho gh$

$T_1 = 273 + 17 = 290k$

$T_f = 273 + 27 = 300k$

$V_i = 2.9$ cm^3

$$V_f = \frac{\left(1 \times 10^5 + 10^3 \times 10 \times 5\right) \times 2.9}{1 \times 10^5} \times \frac{300}{290}$$

$V_f = 4.5$ cm^3

27. A block is sliding down on an inclined plane of slope θ and at an instant $t = 0$ this block is given an upward momentum so that it starts moving up on the inclined surface with velocity u. The distance (S) travelled by the block before its velocity become zero, is _____.

(g = gravitational acceleration)

(1) $\dfrac{u^2}{4g\sin\theta}$

(2) $\dfrac{2u^2}{g\cos\theta}$

(3) $\dfrac{u^2}{\sqrt{2}g\cos\theta}$

(4) $\dfrac{u^2}{2g\cos\theta}$

Answer (BONUS)

Sol. Question should state block was sliding down with uniform speed u before $t = 0$.
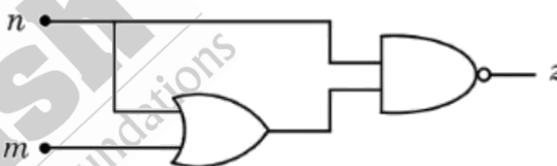
Such that $g\sin\theta = mg\cos\theta$

and when sliding up $s = \dfrac{v^2}{4g\sin\theta}$

(when $a = g\sin\theta + mg\cos\theta = 2g\sin\theta$)   (Option 1)

Since it was not given it should be "BONUS"

28. For the given logic gate circuit, which of the following is the correct truth table?

(1)

n	m	z
0	0	0
0	1	1
1	1	0
1	0	1
n	m	z

(2)

n	m	z
0	0	1
0	1	0
1	1	0
1	0	0
n	m	z

(3)

n	m	z
0	0	1
0	1	1
1	1	0
1	0	0
n	m	z

(4)

n	m	z
0	0	1
0	1	0
1	1	1
1	0	0
n	m	z

Answer (3)

Sol.

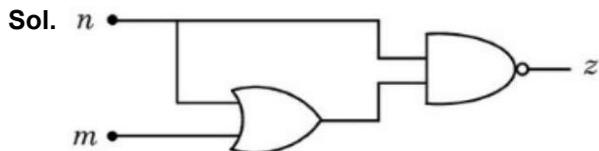$$z = \overline{n(n+m)}$$

$$z = \overline{n}$$

29. A body of mass 14 kg initially at rest explodes and breaks into three fragments of masses in the ratio 2 : 2 : 3. The two pieces of equal masses fly off perpendicular to each other with a speed of 18 m/s each. The velocity of the heavier fragment is _____ m/s.

(1) $24\sqrt{2}$

(2) $10\sqrt{2}$

(3) $12\sqrt{2}$

(4) 12

Answer (3)

Sol. Applying conservation of linear momentum.

$m_1 : m_2 : m_3 = 4 : 4 : 6$

$O = 4 \times 18\hat{i} + 4 \times 18\hat{j} + 6\vec{v}_3$

$\vec{v}_3 = -12(\hat{i} + \hat{j})$

$|\vec{v}_3| = 12\sqrt{2}$ m/s

30. Suppose a long solenoid of 100 cm length, radius 2 cm having 500 turns per unit length, carries a current $I = 10\sin(\omega t)A$, where $\omega = 1000$rad./s. A circular conducting loop (B) of radius 1 cm coaxially slided through the solenoid at a speed $v = 1$ cm/s. The r.m.s. current through the loop when the coil B is inserted 10 cm inside the solenoid is $\alpha / \sqrt{2}$ μA. The value of α is _____.

[Resistance of the loop = 10 Ω]

(1) 197

(2) 100

(3) 280

(4) 80

Answer (1)

Sol. $\Phi = BA$

$$\varepsilon = \frac{dB}{dt}A$$

$$i = \frac{A}{R}\mu_0 n \quad 10\omega\cos\omega t$$

$$i = \frac{500 \times \pi \times 4 \times 10^{-4} \times 4\pi \times 10^{-7}}{4 \times 10} \times 10 \times 1000\cos\omega t$$
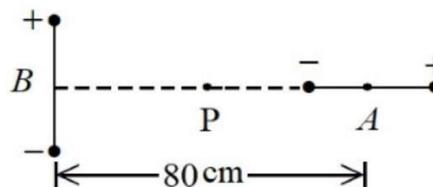
$= 500 \times 16 \times 10^{-7}\cos\omega t$

$= \dfrac{8}{4} \times 10^{-4} <\cos\omega t>_{rms}$

$$<\cos\omega t>_{rms} = \sqrt{\frac{\int\left(\frac{1-\cos 2\omega t}{2}\right)dt}{T}}$$

$$= \frac{1}{\sqrt{2}}$$

$$i = \frac{8}{4} \times 10^{-4}\frac{1}{\sqrt{2}} \approx \frac{200}{\sqrt{2}} \mu A$$

31. Two shorts dipoles (A, B), A having charges ±2μC and length 1 cm and B having charges ±4μC and length 1 cm are placed with their centres 80 cm apart as shown in the figure. The electric field at a point P, equi-distant from the centres of both dipoles is _____ N/C.

(1) $\dfrac{9}{16}\sqrt{2} \times 10^5$

(2) $4.5\sqrt{2} \times 10^4$

(3) $\dfrac{9}{16}\sqrt{2} \times 10^4$

(4) $9\sqrt{2} \times 10^4$

Answer (3)

Sol. $P_A = 2 \times 10^{-6} \times 1 \times 10^{-2} = 2 \times 10^{-8}$ m-c

$P_B = 4 \times 10^{-6} \times 1 \times 10^{-2} = 4 \times 10^{-8}$ m-c

$E_B = \dfrac{kP_B}{r^3}, E_A = \dfrac{2kP_A}{r^3}$

$E_{net} = \dfrac{k}{r^3}\sqrt{P_B^2 + 4P_A^2}$

$E_{net} = \dfrac{9 \times 10^9}{\left(40 \times 10^{-2}\right)^3}\sqrt{\left(4 \times 10^8\right)^2 + 4 \times \left(2 \times 10^{-8}\right)^2}$

$E_{net} = \dfrac{9}{16}\sqrt{2} \times 10^4$ N/C

32. Which of the following pair of nuclei are isobars of the element?

(1) $^{198}_{80}Hg$ and $^{197}_{79}Au$　(2) $^{2}_{1}H$ and $^{3}_{1}H$

(3) $^{3}_{1}H$ and $^{3}_{2}He$　　　(4) $^{236}_{92}U$ and $^{238}_{92}U$

Answer (3)

Sol. Isobars are nuclei with the same mass number (A) but different atomic number (Z)

Both Tritium $\left(^{3}_{1}H\right)$ and Helium $\left(^{3}_{2}He\right)$ are isobars.

33. A small metallic sphere of diameter 2 mm and density 10.5 g$/$cm^3 is dropped in glycerine having viscosity 10 Poise and density 1.5 g/cm³ respectively. The terminal velocity attained by the sphere is _____ cm $/$ s .

($\pi = \dfrac{22}{7}$ and $g = 10$ m$/$s^2)

(1) 3.0　　　　　(2) 1.0

(3) 2.0　　　　　(4) 1.5

Answer (3)

Sol. Terminal velocity is given by equation.

$$V_T = \frac{2g(\rho_s - \rho_l)}{9\eta}r^2$$

Given,

$r = \dfrac{d}{2} = 1\,mm = 1 \times 10^{-3}$ m

$\rho_s = 10.5$ g/cm$^3 = 10.5 \times 10^3$ kg/m^3

$\rho_l = 1.5$ g/cm$^3 = 1.5 \times 10^3$ kg/m^3

$\eta = 10$ poise = 1 Pa.s

$V_T = \dfrac{2 \times 10 \times (10.5 - 1.5) \times 10^3 \times \left(1 \times 10^{-3}\right)^2}{9 \times 1}$

$V_T = 0.02$ m/s = 2 cm/s

34. The internal energy of a monoatomic gas is 3 nRT. One mole of helium is kept in a cylinder having internal cross section area of 17 cm^2 and fitted with a light movable frictionless piston. The gas is heated slowly by suppling 126 J heat. If the temperature rises by 4°C, then the piston will move _____ cm. (atmospheric pressure = 10^5 Pa)

(1) 1.55　　　　　(2) 15.5

(3) 14.5　　　　　(4) 1.45

Answer (2)

Sol. Given,

$\Delta Q = 126$ J

$\Delta T = 4$°C

$A = 17$ cm$^2 = 17 \times 10^{-4}$ m^2

$P = 10^5$ Pa

$\Delta U = nR\Delta T$

$\Delta U = 3 \times 8.314 \times 4 = 99.768$ J

From first law of thermodynamics

$\Delta Q = \Delta U + W$

W = 126 – 99.768 = 26.232 J

Work done by gas = $P\Delta V = PAh$

$h = \dfrac{26.232}{10^5 \times 17 \times 10^{-4}}$

h = 1.543 × 10⁻¹ m

h = 15.43 cm

35. A prism of angle 75° and refractive index $\sqrt{3}$ is coated with thin film of refractive index 1.5 only at the back exit surface. To have total internal reflection at the back exit surface the incident angle must be _____ .($\sin 15° = 0.25$ and $\sin 25° = 0.43$)

(1) > 25°

(2) between 15° and 20°

(3) <15°

(4) 15°

Answer (Bonus)

Sol. $\sin Q_c = \dfrac{1.5}{\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$Q = 60°$

$r_1 + r_2 = A$

$r_1 = A - r_2$

$r_2 > 60°$

$r_2 > A - 60°$

$r_1 < 75° - 60°$

$r_1 < 15°$

$1 \times \sin i = \sqrt{3} \sin r_1$

$i < \sin^{-1}\left(\sqrt{3}\sin(15°)\right)$

$i < 25°$

None of the options is matching.
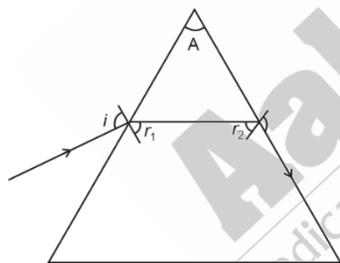
36. One mole of an ideal diatomic gas expands from volume V to $2V$ isothermally at a temperature 27°C and does W joule of work. If the gas undergoes same magnitude of expansion adiabatically from 27°C doing the same amount of work W, then its final temperature will be (close to) ___°C. ($\log_e 2 = 0.693$)

(1) –30   (2) –189

(3) –117 (4) –56

Answer (4)

Sol. Given, n = 1 mole

$v_i = V$ and $v_f = 2V$

$T_i = 300\,\text{K}$

$W_{iso} = |W_{adi}|$

$nRT_i \ln\left(\dfrac{v_2}{v_1}\right) = nC_v(T_1 - T_2)$

$nRT_i \ln(2) = n\dfrac{5}{2}R(T_1 - T_2)$

$300\ln(2) = 2.5(300 - T_2)$

$T_2 = 216.82\,\text{K}$

$T_2 = -56.17\,°\text{C}$

37. Two charges 7 μC and –2 μC are placed at (–9, 0, 0) and (9, 0, 0) respectively in an external field $E = \dfrac{A}{r^2}\hat{r}$, where $A = 9 \times 10^5\,\text{N/C.m}^2$.

Considering the potential at infinity is 0, the electrostatic energy of the configuration is _____ J.

(1) 1.4   (2) 49.3

(3) –90.7 (4) 24.3
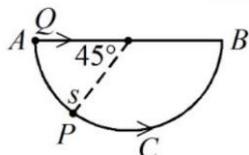
Answer (2)

Sol. $E = \dfrac{A}{r^2}\hat{r}$

$v = \dfrac{A}{r}$

$v_{total} = q_1 v_{ext}(r_1) + q_2 v_{ext}(r_2) + \dfrac{kq_1q_2}{r_{12}}$

$V_{total} = \dfrac{7\times10^{-6}\times9\times10^5}{9\times10^{-2}} + \dfrac{\left(-2\times10^{-6}\right)\times9\times10^5}{9\times10^{-2}}$
$+ \dfrac{9\times10^9\times7\times(-2)\times10^{-12}}{18\times10^{-2}}$

$V_{total} = 50 - 0.7 = 49.3$ J

38. A bead P sliding on a frictionless semi-circular string (ABC) and it is at point S at $t = 0$ and at this instant the horizontal component of its velocity is v. Another bead Q of the same mass as P is ejected from point A at $t = 0$ along the horizontal string AB, with the speed v, friction between the beads and the respective strings may be neglected in both cases. Let t_P and t_Q be the respective times taken by beads P and Q to reach the point B, then the relation between t_P and t_Q is

(1) $t_P > 1.25\, t_Q$

(2) $t_P > t_Q$

(3) $t_P < t_Q$

(4) $t_P = t_Q$

Answer (3)

Sol. The velocity of Q remain the same horizontal as so force it acting on the horizontal dissection but in case P the horizontal velocity first increase reaches has a maximum at lowest point and then decrease this it always remain greater then v.

There force $t_P < t_Q$

39. A paratrooper jumps from an aeroplane and opens a parachute after 2 s of free fall and starts deaccelerating with $3\ \text{m}/\text{s}^2$. At 10 m height from ground, while descending with the help of parachute, the speed of paratrooper is $5\ \text{m}/\text{s}$. The initial height of the airplane is _____ m.

($g = 10$ m/s²)

(1) 20

(2) 82.5

(3) 92.5

(4) 62.5

Answer (3)

Sol. $h_1 = \dfrac{1}{2}\times10\times L^2 = 20$ m

$v_2^2 = v_1^2 + 2ah_2$

$5^2 = 20^2 - 2\times3\times h_2$

$h_2 = 62.5$ m

$h_3 = 10$ m
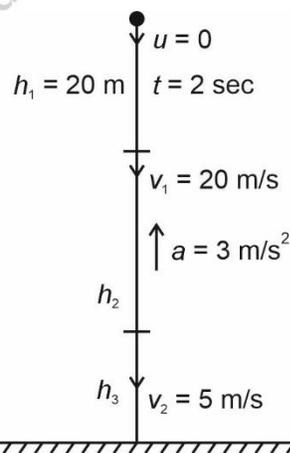
total height of aeroplane from ground is

$= h_1 + h_2 + h_3$

$= 92.5$ m

40. A parallel plate capacitor with plate separation 5 mm is charged by a battery. On introducing a mica sheet of 2 mm and maintaining the connections of the plates with the terminals of the battery, it is found that it draws 25% more charge from the battery. The dielectric constant of mica is _____.

(1) 2.0   (2) 1.5

(3) 1.0   (4) 2.5

Answer (1)

Sol. Given

$$d = 5 \text{ mm}$$

$$t = 2 \text{ mm}$$

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$\frac{1}{C'} = \frac{d-t}{\varepsilon_0 A} + \frac{t}{\varepsilon_0 A k}$$

$$\frac{1}{C'} = \frac{1}{\varepsilon_0 A}\left((d-t) + \frac{t}{k}\right)$$

$$C' = \frac{\varepsilon_0 A k}{(d-t)k + t}$$

$$Q_f = 1.25\, Q_i$$

$$C'v = 1.25\, C_0 V$$

$$\frac{\varepsilon_0 A k}{(d-t)k + t} = 1.25\frac{\varepsilon_0 A}{d}$$

$$\frac{k}{3k+2} = \frac{1.25}{5}$$

$$k = 2$$

41. The current passing through a conducting loop in the form of equilateral triangle of side $4\sqrt{3}$ cm is 2 A. The magnetic field at its centroid is $\alpha \times 10^{-5}$ T. The value of α is _____.

(Given : $\mu_0 = 4\pi \times 10^{-7}$ SI units)

(1) $\dfrac{\sqrt{3}}{2}$ (2) $3\sqrt{3}$

(3) $\sqrt{3}$ (4) $2\sqrt{3}$

Answer (2)

Sol. $B = 3.\dfrac{\mu_0 I}{4\pi d} 2\sin 60°$
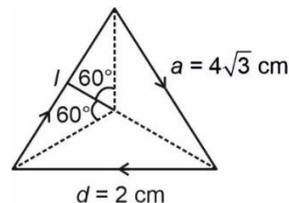
$$B = 3 \times \frac{4\pi \times 10^{-7} \times 2}{4\pi \times 2 \times 10^{-2}} \times 2 \times \frac{\sqrt{3}}{2}$$

$$B = 3\sqrt{3} \times 10^{-5}\ T$$

42. When an unpolarized light falls at a particular angle on a glass plate (placed in air), it is observed that the reflected beam is linearly polarized. The angle of refracted beam with respect to the normal is _____.

$(\tan^{-1}(1.52) = 57.7°$, refractive indices of air and glass are 1.00 and 1.52, respectively.)

(1) 39.6°   (2) 32.3°

(3) 36.3°   (4) 42.6°

Answer (2)

Sol. For Brewster angle, reflected beam is polarized when $\tan i_p = \mu$, $i_p + r = 90°$

$\tan i_p = \mu$

$\tan(90° - r) = 1.52$

$90° - r = \tan^{-1}(1.52)$

$r = 90° - 57.70$

$r = 32.3°$

43. A circular loop of radius 7 cm is placed in uniform magnetic field of 0.2 T directed perpendicular to plane of loop. The loop is converted into a square loop in 0.5 s. The EMF induced in the loop is _____ mV.

(1) 1.32   (2) 13.2

(3) 6.6   (4) 8.25

Answer (1)

Sol. r = 7 cm

$2\pi r = 4l$

$l = \dfrac{2\pi \times 7}{4} = 10.99$ cm

Change in Area ΔA = (153.9 – 120.75) cm^2

ΔA = 33.12 cm^2

e.m.f. = $\dfrac{BA}{t} = \dfrac{0.2 \times 33.12 \times 10^{-4}}{0.5}$

$= 13.248 \times 10^{-4}$ V

e.m.f. = 1.3248 mV

44. To compare EMF of two cells using potentiometer the balancing lengths obtained are 200 cm and 150 cm. The least count of scale is 1 cm. The percentage error in the ratio of EMFs is _____

(1) 1.55　　　　　　(2) 1.75

(3) 1.65　　　　　　(4) 1.45

Answer (BONUS)

Sol. $\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}$

% age error = $\left(\dfrac{\Delta l_1}{l_1} + \dfrac{\Delta l}{l_2}\right) \times 100\%$

$= \left(\dfrac{1}{200} + \dfrac{1}{150}\right) \times 100\%$

$= \dfrac{7}{6}\%$

% age error = 1.16%

None of the options are matching.

45. The ratio of speeds of electromagnetic waves in vacuum and a medium, having dielectric constant k = 3 and permeability of $\mu = 2\mu_0$, is

(μ_0 = permeability of vacuum)

(1) 36 : 1　　　　　　(2) $\sqrt{6}$: 1

(3) 3 : 2　　　　　　(4) 6 : 1

Answer (2)

Sol. $\dfrac{c}{v} = n = \sqrt{\mu_r \varepsilon_r}$

$\Rightarrow \dfrac{c}{v} = \sqrt{6}$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

---

46. The average energy released per fission for the nucleus of $^{235}_{92}U$ is 190 MeV. When all the atoms of 47 g pure $^{235}_{92}U$ undergo fission process, the energy released is $\alpha \times 10^{23}$ MeV. The value of α is _____.

(Avogadro Number = 6 × 10^{23} per mole)

Answer (228)

Sol. 1 nuclei of $U^{235} \longrightarrow$ 190 Mev

Now we need to calculate total number of nucleid in 47 g.

235 g = 6 × 10^{23}

47 g $\equiv \dfrac{6 \times 10^{23}}{235} \times 47 = \dfrac{6}{5} \times 10^{23}$

$\therefore$ Energy released $= 190 \times \dfrac{6}{5} \times 10^{23}$ Mev

$\Rightarrow$ 228 × 10^{23} Mev

47. Suppose there is a uniform circular disc of mass M kg and radius r m shown in figure. The shaded regions are cut out from the disc. The moment of inertia of the remainder about the axis A of the disc is given by $\dfrac{x}{256}Mr^2$. The value of x is _____.
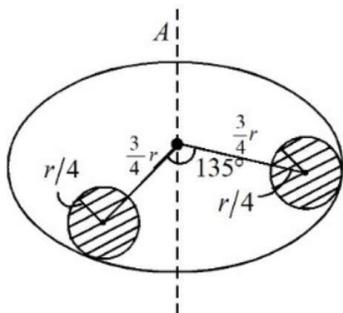
Answer (109)

Sol. $I = \dfrac{Mr^2}{2} - \left(\dfrac{\dfrac{M}{16} \times \dfrac{r^2}{16}}{2} + \dfrac{M}{16} \times \left(\dfrac{3r}{4} \right)^2 \right) \times 2$

$\Rightarrow I = \dfrac{Mr^2}{2} - \left(\dfrac{Mr^2}{2} + 9Mr^2 \right) \dfrac{2}{16 \times 16}$

$\Rightarrow I = \dfrac{Mr^2}{2} - \left(\dfrac{19Mr^2}{16 \times 16} \right)$

$\Rightarrow \dfrac{(128 - 19)Mr^2}{16 \times 16} = \dfrac{109 Mr^2}{256}$

48. A ball of radius r and density ρ dropped through a viscous liquid of density σ and viscosity η attains its terminal velocity at time t, given by $t = A\, \rho^a\, r^b\, \eta^c\, \sigma^d$, where A is a constant and a, b, c and d are integers. The value of $\dfrac{b+c}{a+d}$ is _____.

Answer (1)

Sol. $T^1 = [ML^{-3}]^a [L]^b [ML^{-1}T^{-1}]^c [ML^{-3}]^d$

$a + c + d = 0$

$-3a + b - c - 3d = 0$

$-c = 1$

$\Rightarrow a + d = 1$

$a + d = \dfrac{b - c}{3} = \dfrac{b + 1}{3} = 1$

$b = 2$

$\Rightarrow \dfrac{2 - 1}{1} = 1$

49. The size of the images of an object, formed by a thin lens are equal when the object is placed at two different positions 8 cm and 24 cm from the lens. The focal length of the lens is _____ cm.

Answer (16)

Sol. $m = \dfrac{f}{f + u}$

$\dfrac{f}{f - 24} = \dfrac{-f}{f - 8}$

$f - 8 = -f + 24$

$f = 16$ cm

50. The velocity of sound in air is doubled when the temperature is raised from 0 °C to α °C. The value of α is _____.

Answer (819)

Sol. $v \propto \sqrt{T}$

$\Rightarrow \sqrt{\alpha} = 2\sqrt{273}$

$\alpha(k) = 1092$

$t°c = 1092 - 273$

$= 819$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

51. Which of the following statements are **TRUE** about Haloform reaction?

    A. Sodium hypochlorite reacts with KI to give KOI.

    B. KOI is a reducing agent.

    C. α, β-unsaturated methylketone

    $$(CH_3-CH=CH-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3)$$ will give

    iodoform reaction.
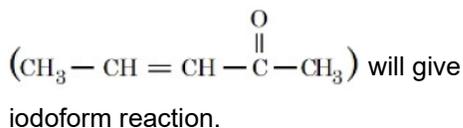
    D. Isopropyl alcohol will not give iodoform test.

    E. Methanoic acid will give positive iodoform test.

    Choose the **correct** answer from the options given below.

    (1) A, C & E Only   (2) B, D & E Only

    (3) A, B & C Only   (4) A & C Only

Answer (4)

Sol. Methanoic acid will not give positive iodoform test.

Isopropyl alcohol will give positive iodoform test.

$$CH_3-CH=CH-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$$ will give iodoform test.
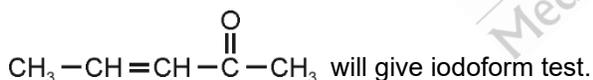
52. In Carius method 0.2425 g of an organic compound gave 0.5253 g silver chloride. The percentage of chlorine in the organic compound is

    (1) 37.57%   (2) 34.79%

    (3) 53.58%   (4) 87.65%

Answer (3)

Sol. % of Cl = $\dfrac{\left(\dfrac{0.5253}{143.5}\right)\times 35.5}{0.2425}\times 100 = 53.58\%$

53. Identify the **CORRECT** set of details from the following:

    A. $\left[Co(NH_3)_6\right]^{3+}$: Inner orbital complex; d^2sp^3 hybridized.

    B. $\left[MnCl_6\right]^{3-}$: Outer orbital complex; sp^3d^2 hybridized.

    C. $\left[CoF_6\right]^{3-}$: Outer orbital complex; d^2sp^3 hybridized.

    D. $\left[FeF_6\right]^{3-}$: Outer orbital complex; sp^3d^2 hybridized.

    E. $\left[Ni(CN)_4\right]^{2-}$: Inner orbital complex; sp^3 hybridized.

    Choose the **correct** answer from the options given below.

    (1) A, B, C, D & E   (2) C & D Only

    (3) A, C & E Only   (4) A, B & D Only

Answer (4)

Sol. (C) is incorrect.

(A) $[Co(NH_3)_6]^{3+} \to d^2sp^3 \to$ Inner orbital

(B) $[MnCl_6]^{3+} \to Mn^{3+} \to 3d^4$, $Cl^- \to WFL$

$\to sp^3d^2$

(D) $FeF_6^{3-} \to Fe^{3+} \to 3d^5$, $F^- \to WFL$, sp^3d^2

(E) is incorrect, $Ni(CN)_4^{2-}$ is dsp^2 hybridised.

54.

Given above is the concentration vs time plot for a dissociation reaction : $A \rightarrow nB$.

Based on the data of the initial phase of the reaction (initial 10 min), the value of n is _____ .

(1) 4

(2) 2

(3) 5

(4) 3

Answer (4)

Sol. $\dfrac{-dA}{dt} = \dfrac{1}{n}\dfrac{dB}{dt}$

So, $\dfrac{0.05 - 0.04}{10} = \dfrac{1}{n}\left[\dfrac{0.03 - 0}{10}\right]$

n = 3

55. The oxidation state of chromium in the final product formed in the reaction between KI and acidified $K_2Cr_2O_7$ solution is:

(1) +2

(2) +3

(3) +4

(4) +6

Answer (2)

Sol. $K_2Cr_2O_7 + 6KI + 7H_2SO_4$

$\rightarrow \overset{+3}{Cr_2}(SO_4)_3 + 3I_2 + 4K_2SO_4 + 7H_2O$

56. Given below are two statements:

Statement I: The second ionisation enthalpy of Na is larger than the corresponding ionisation enthalpy of Mg .

Statement II: The ionic radius of O^{2-} is larger than that of F^- .

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Answer (3)

Sol. $\underset{2,8,1}{Na} \xrightarrow{IE_1} \underset{2,8}{Na^+} \xrightarrow{IE_2} \underset{2,7}{Na^{2+}}$; so IE_2 is high

$Mg(2,8,2)\xrightarrow[IE_1]{} Mg^+(2,8,1) \rightarrow Mg^{2+}(2,8)$

O^{2-} (1.40 Å) is larger than F^- (1.33 – 1.36 Å)

57. Both human DNA and RNA are chiral molecules. The chirality in DNA and RNA arises due to the presence of

(1) D-sugar component

(2) Chiral phosphate unit

(3) L- sugar component

(4) Base unit

Answer (1)

Sol. Chirality is due to D-sugar units.

58. A mixed ether (P), when heated with excess of hot concentrated hydrogen iodide produces two different alkyl iodides which when treated with aq. NaOH give compounds (Q) and (R). Both (Q) and (R) give yellow precipitate with NaOI. Identify the mixed ether (P):

(1)

(2)

(3)

(4)

Answer (4)

Sol.

59. Which statements are **NOT TRUE** about XeO_2F_2?

   A. It has a see-saw shape.

   B. Xe has 5 electron pairs in its valence shell in XeO_2F_2.

   C. The O–Xe–O bond angle is close to 180°.

   D. The F–Xe–F bond angle is close to 180°.

   E. Xe has 16 valence electrons in XeO_2F_2.

   Choose the *correct* answer from the options given below:

   (1) B, D and E Only

   (2) B and D Only

   (3) B, C and E Only

   (4) A and D Only

Answer (3)

Sol.
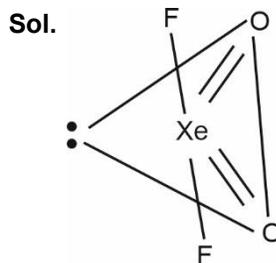
See-saw

Xe has 7 e⁻ pairs.

O–Xe–O bond angle is close to 120°.

60. Given below are two statements:

   Statement I: $(CH_3)_3 \overset{\oplus}{C}$ is more stable than $\overset{\oplus}{C}H_3$ as nine hyperconjugation interactions are possible in $(CH_3)_3 \overset{\oplus}{C}$.

   Statement II: $\overset{\oplus}{C}H_3$ is less stable than $(CH_3)_3 \overset{\oplus}{C}$ as only three hyperconjugation interactions are possible in $\overset{\oplus}{C}H_3$.

   In the light of the above statements, choose the **correct** answer from the options given below

   (1) Both Statement I and Statement II are false

   (2) Statement I is true but Statement II is false

   (3) Statement I is false but Statement II is true

   (4) Both Statement I and Statement II are true

Answer (2)

Sol. No hyperconjugation possible in CH_3^+.

61. Observe the following reactions at T(K).

   I. A → products.

   II. $5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq)$

   $\rightarrow 3Br_2(aq) + 3H_2O(l)$

Both the reactions are started at 10.00 am. The rates of these reactions at 10.10 am are same. The value of $-\dfrac{\Delta[Br^-]}{\Delta t}$ at 10.10 am is 2×10^{-4} mol L^{-1} min^{-1}. The concentration of A at 10.10 am is 10^{-2} mol L^{-1}. What is the first order rate constant (in min^{-1}) of reaction I?

(1) 2×10^{-3} (2) 4×10^{-3}

(3) 10^{-3} (4) 10^{-2}

Answer (2)

Sol. $r = k[A]$ reaction-I

$r = \dfrac{1}{5}\dfrac{\Delta[Br^-]}{\Delta t} = \dfrac{1}{5} \times 2 \times 10^{-4}$

$= 4 \times 10^{-5}$

$4 \times 10^{-5} = k[10^{-2}]$

$k = 4 \times 10^{-3}$ min^{-1}

62. The work functions of two metals (M_A and M_B) are in the 1 : 2 ratio. When these metals are exposed to photons of energy 6 eV, the kinetic energy of liberated electrons of M_A : M_B is in the ratio of 2.642 : 1. The work functions (in eV) of M_A and M_B are respectively.

(1) 1.4, 2.8   (2) 1.5, 3.0

(3) 3.1, 6.2   (4) 2.3, 4.6

Answer (4)

Sol. Let $E°_A$ = Y, then $E°_B$ will be 2Y {ϕ_A = Y, ϕ_B = 2Y}

Let KE_B = x, then KE_A = 2.642x

E(eV) = ω(eV) + KE(eV)

For metal A $\Rightarrow$ 6 = Y + 2.642x   …(I)

For metal B $\Rightarrow$ 6 = 2Y + x   …(II)

From E_v (I) and (II) Y = 1.642x

Now put Y in Eq. (II)

6 = 2(1.642x) + x, on solving x = 1.4, Y = 1.642 × 1.4 = 2.3

So $\underset{(\phi_A)}{\omega_A} = Y = 2.3\,eV$, $\underset{(\phi_B)}{\omega_B} = 2Y = 4.6\,eV$

63. Iodoform test can differentiate between

A. Methanol and Ethanol

B. CH_3COOH and CH_3CH_2COOH

C. Cyclohexene and cyclohexanone

D. Diethyl ether and Pentan-3-one

E. Anisole and acetone

Choose the **correct** answer from the options given below:

(1) B, C & E only

(2) A & D only

(3) A & E only

(4) A, B & E only

Answer (3)

Sol. CH_3CH_2OH and $CH_3 - \overset{\overset{\displaystyle O}{\|}}{C} - CH_3$ gives positive iodoform test.
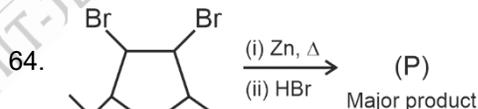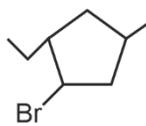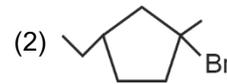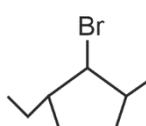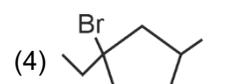
64.

Identify (P)

Answer (2)

Sol.

65.
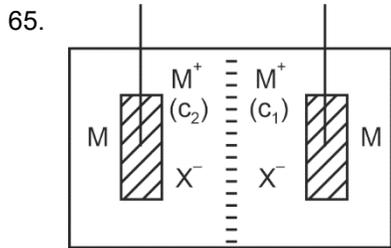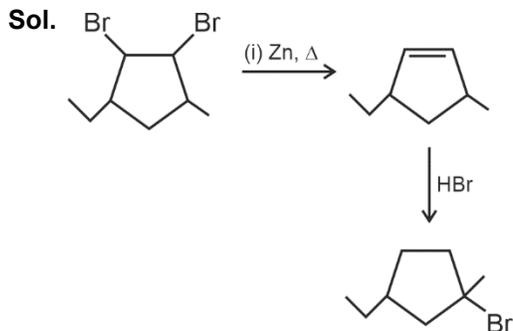
Semi permeable membrane

Consider the above electrochemical cell where a metal electrode (M) is undergoing redox reaction by forming M^+ ($M \rightarrow M^+ + e^-$). The cation M^+ is present in two different concentrations c_1 and c_2 as shown above. Which of the following statement is correct for generating a positive cell potential?

(1) If c_1 is present at anode, then $c_1 > c_2$

(2) If c_1 is present at cathode, then $c_1 > c_2$

(3) If c_1 is present at cathode, then $c_1 < c_2$

(4) If c_1 is present at anode, then $c_1 = c_2$

Answer (2)

Sol. At anode : $\quad M \rightarrow M^+_{(a)} + e^-$

At cathode : $\quad M^+_{(c)} + e^- \rightarrow M$

$$\overline{\qquad\qquad\qquad\qquad\qquad}$$
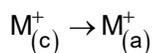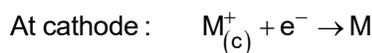
$$M^+_{(c)} \rightarrow M^+_{(a)}$$

$$E_{cell} = -\frac{RT}{F} \ln \frac{M^+_{(a)}}{M^+_{(c)}}$$

In order to have $E_{cell} > 0$

$$\left[M^+_{(c)} \right] > \left[M^+_{(a)} \right]$$

66. Elements X and Y belong to Group 15. The difference between the electronegativity values of 'X' and phosphorus is higher than that of the difference between phosphorus and 'Y'. 'X' & 'Y' respectively

(1) As & Bi   (2) Bi & N

(3) N & As   (4) As & Sb

Answer (3)

Sol. EN of   N = 3.0

            P = 2.1

            As $\Rightarrow$ 2.0

            Sb = 1.9

            Bi = 1.9

ΔEN (N and P) = 0.9

ΔEN (As and P) = 0.1

ΔEN (As and P) = 0.1

ΔEN (Sb and P) = 0.2

ΔEN (Bi and P) = 0.2

67. A student has been given a compound "x" of molecular formula C_6H_7N. 'x' is sparingly soluble in water. However, on addition of dilute mineral acid, 'x' becomes soluble in water. 'x' when treated with $CHCl_3$ and KOH(alc), 'y' is produced. 'y' has a specific unpleasant smell. On treatment with benzensulphonyl chloride, 'x' gives a compound 'z' which is soluble in alkali. The number of different "H" atoms present in 'z' is

(1) 4   (2) 5

(3) 8   (4) 7

Answer (4)

Sol. C_6H_7N has D.U = 4

(x)

(x) + HCl → (water soluble)
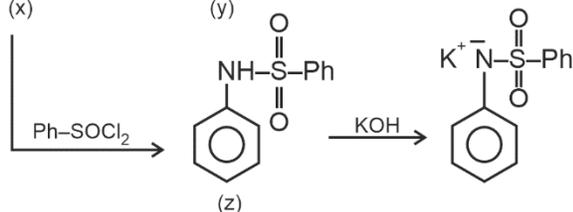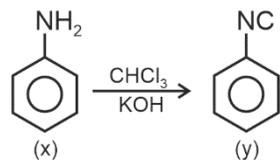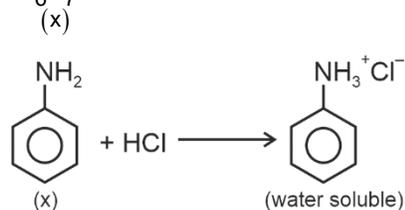
No. of different H-atoms present in (z) = 7

68. Given below are two statements:

Statement I:

can be synthesized from

using simpler reagents in the order (i) Acidic KMnO₄, (ii) Ammonia, (iii) Bromine and alkali

Statement II:

can be converted into

using reagents in the order (i) Bromine H₂O (ii) NaNO₂/HCl (0 – 5°C) (iii) Aq. H₃PO₂.

In the light of above statements, choose the **correct** answer from the options given below.

(1) Both statement I and statement II are false

(2) Statement I is true but statement II is false

(3) Statement I is false but statement II is true

(4) Both statement I and statement II are true

Answer (4)

Sol. (I)

(II)
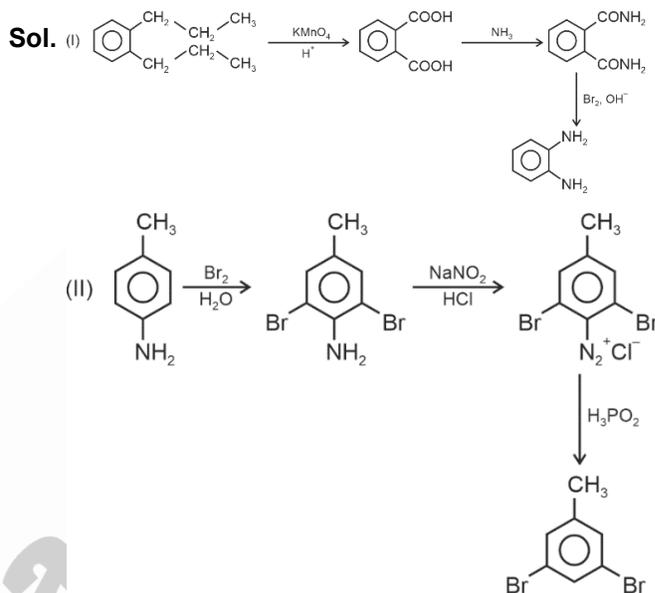
69. It is noticed that Pb^{2+} is more stable than Pb^{4+} but Sn^{2+} is less stable than Sn^{4+}. Observe the following reactions.

$PbO_2 + Pb \rightarrow 2PbO$; $\Delta_rG°(1)$

$SnO_2 + Sn \rightarrow 2SnO$; $\Delta_rG°(2)$
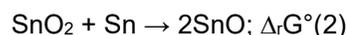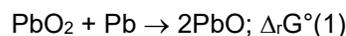
Identify the correct set from the following

(1) $\Delta_rG°(1) < 0$; $\Delta_rG°(2) < 0$

(2) $\Delta_rG°(1) < 0$; $\Delta_rG°(2) > 0$

(3) $\Delta_rG°(1) > 0$; $\Delta_rG°(2) > 0$

(4) $\Delta_rG°(1) > 0$; $\Delta_rG°(2) < 0$

Answer (2)

Sol. As Pb^{2+} is more stable than Pb^{4+}

$(\Delta G°)_1 < 0$

And as Sn^{2+} is less stable than Sn^{4+}

$(\Delta G°)_2 > 0$

70. Identify the INCORRECT statements from the following:

A. Notation $^{24}_{12}Mg$ represents 24 protons and 12 neutrons.

B. Wavelength of a radiation of frequency $4.5 \times 10^{15}\ s^{-1}$ is 6.7×10^{-8} m.

C. One radiation has wavelength = λ_1 (900 nm) and energy = E_1. Other radiation has wavelength = λ_2 (300 nm) and energy = E_2. $E_1 : E_2 = 3 : 1$.

D. Number of photons of light of wavelength 2000 pm that provides 1 J of energy is 1.006×10^{16}.

Choose the **correct** answer from the options given below:

(1) A and B Only
(2) A and C Only
(3) A and D Only
(4) B and C Only

Answer (2)

Sol. (A) $^{24}_{12}Mg$ represents 12 electrons and 24 nucleons (proton + neutron)

(B) $c = \nu\lambda = 4.5 \times 6.7 \times 10^7$

   $= 3.015 \times 10^8$ m/s

(C) $\dfrac{E_1}{E_2} = \dfrac{\dfrac{hc}{\lambda_1}}{\dfrac{hc}{\lambda_2}} = \dfrac{\lambda_2}{\lambda_1} = \dfrac{300}{900} = \dfrac{1}{3}$
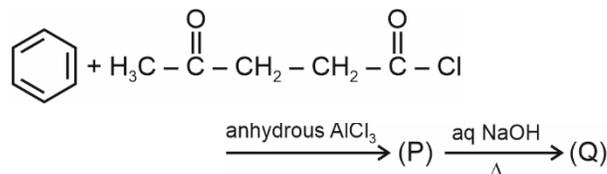
(D) Number of photons

$= \dfrac{E \times \lambda}{hc} = \dfrac{1 \times 2000 \times 10^{-12}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 1.006 \times 10^{16}$
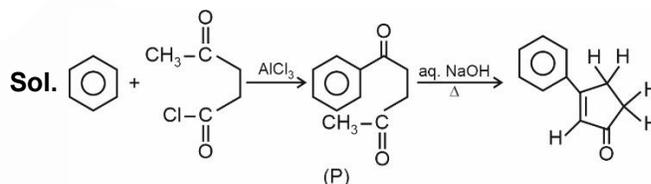

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Consider the following reaction of benzene.

In compound (Q), the percentage of oxygen is _____%. (Nearest integer)

Answer (10)

Sol.

Mass percentage of $O = \dfrac{16 \times 100}{158} = 10.12\%$

72. Two liquids A and B form an ideal solution. At 320 K, the vapour pressure of the solution, containing 3 mol of A and 1 mol of B is 500 mm Hg. At the same temperature, if 1 mol of A is further added to this solution, vapour pressure of the solution increases by 20 mm Hg. Vapour pressure (in mm Hg) of B in pure state is_____. (Nearest integer)

Answer (200)

Sol. $P_S = X_A P_A^o + X_B P_B^o$

$500 = \dfrac{3}{4} \times P_A^o + \dfrac{1}{4} \times P_B^o$
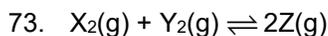
$2000 = 3P_A^o + P_B^o$　　　　…(i)

After adding 1 mole of A,

$$520 = \frac{4}{5}P_A^o + \frac{P_B^o}{5}$$

$$2600 = 4P_A^o + P_B^o \qquad \ldots(ii)$$

$(ii) - (i) \Rightarrow 600 \text{ mm} = P_A^o$

$P_B^o = 2600 - 2400$

$\qquad = 200 \text{ mm Hg}$

73. $X_2(g) + Y_2(g) \rightleftharpoons 2Z(g)$

$X_2(g)$ and $Y_2(g)$ are added to a 1 L flask and it is found that the system attains the above equilibrium at T(K) with the number of moles of $X_2(g)$, $Y_2(g)$ and $Z(g)$ being 3, 3 and 9 mol respectively (equilibrium moles). Under this condition of equilibrium, 10 mol of Z(g) is added to the flask and the temperature is maintained at T(K). Then the number of moles of Z(g) in the flask when the new equilibrium is established is _____. (Nearest integer)

Answer (15)

Sol. At equation $\underset{3\,mol}{X_2\left(g\right)} + \underset{3\,mol}{Y_2\left(g\right)} \rightleftharpoons \underset{9\,mol}{2Z\left(g\right)}$
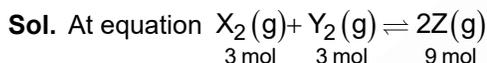
$$K_{eq} = \frac{(9)^2}{3 \times 3} = \frac{9 \times 9}{3 \times 3} = 9$$

when 10 mol of Z is added reaction moves in backward direction

$$\underset{3+x}{X_2\left(g\right)} + \underset{3+x}{Y_2\left(g\right)} \rightleftharpoons \underset{19-2x}{2Z}$$
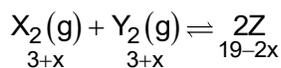
$$9 = \frac{\left(19 - 2x\right)^2}{\left(3 + x\right)^2}$$

$$3 = \frac{19 - 2x}{3 + x}$$
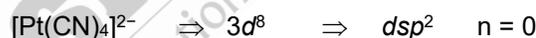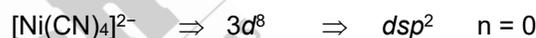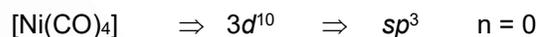
$9 + 3x = 19 - 2x$

$5x = 10$

$$x = \frac{10}{5} = 2$$

Moles of Z at new equilibrium

$= 19 - 2 \times 2 = 15$

74. Total number of unpaired electrons present in the central metal atoms/ions of $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[PtCl_2(NH_3)_2]$, $[Ni(CN)_4]^{2-}$ and $[Pt(CN)_4]^{2-}$ is _____.

Answer (2)

Sol.

$[Ni(CO)_4]$	$\Rightarrow 3d^{10}$	$\Rightarrow sp^3$	$n = 0$
$[NiCl_4]^{2-}$	$\Rightarrow 3d^8$	$\Rightarrow sp^3$	$n = 2$
$[PtCl_2(NH_3)_2]$	$\Rightarrow 3d^8$	$\Rightarrow dsp^2$	$n = 0$
$[Ni(CN)_4]^{2-}$	$\Rightarrow 3d^8$	$\Rightarrow dsp^2$	$n = 0$
$[Pt(CN)_4]^{2-}$	$\Rightarrow 3d^8$	$\Rightarrow dsp^2$	$n = 0$

Total unpaired $e^- = 2$

75. 200 cc of $x \times 10^{-3}$ M potassium dichromate is required to oxidise 750 cc of 0.6 M Mohr's salt solution in acidic medium.

Here x = _____.

Answer (375)

Sol. Number of equivalents $Cr_2O_7^{2-}$ = Number of equivalents of Mohr's salt

$200 \times x \times 10^{-3} \times 6 = 750 \times 0.6 \times 1$

$x = 375$

□ □ □