Aakash
Medical | IIT-JEE | Foundations

Phase-1
24th January 2026
(Evening Shift)

Answer & Solutions

for

JEE MAIN 2026

TRUST Aakash

Stellar Ranks in JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR 23

Kushagra Baingaha AIR 7

Shreyas Lohiya AIR 6

Harssh A Gupta AIR 15

Devya Rustagi AIR 28

Shreyas Lohiya AIR 68

Rujul Garg AIR 41

Advay Mayank AIR 36

Arush Anand AIR 64

Kotha D Reddy AIR 74

Top Ranks in JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students Aakashians Qualified in NSEs 2025-26

777 Classroom Students Aakashians Qualified in IOQM 2025

134 Classroom Students Aakashians Qualified in RMO 2025

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

HOME OF PROBLEM SOLVERS

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f(x)=\int \frac{7x^{10}+9x^8}{\left(1+x^2+2x^9\right)^2}dx, x>0$, $\lim\limits_{x\to 0} f(x)=0$ and $f(1)=\frac{1}{4}$.

If $A=\begin{bmatrix}0 & 0 & 1\\ \frac{1}{4} & f'(1) & 1\\ \alpha^2 & 4 & 1\end{bmatrix}$ and $B=\text{adj}(\text{adj}A)$ be such that $|B|=81$, then α^2 is equal to

(1) 3 (2) 2

(3) 1 (4) 4

Answer (4)

Sol. $f(x)=\int \frac{7x^{10}+9x^8}{(1+x^2+2x^9)^2}dx,\ x>0$

$$=\int \frac{\frac{7}{x^8}+\frac{9}{x^{10}}}{\left(\frac{1}{x^9}+\frac{1}{x^7}+2\right)^2}dx$$

Let $\frac{1}{x^9}+\frac{1}{x^7}+2=t$

$\therefore \frac{-9}{x^{10}}-\frac{7}{x^8}=\frac{dt}{dx}$

$\therefore \left(\frac{7}{x^8}+\frac{9}{x^{10}}\right)dx=-dt$

$f(x)=\int\frac{-dt}{t^2}=\frac{1}{t}+C$

$f(x)=\frac{x^9}{1+x^2+2x^9}+C$

$\lim\limits_{x\to 0} f(x)=0+C=0\Rightarrow C=0$

$f(1)=\frac{1}{4}$ and $f'(1)=\frac{16}{16}=1$

$\therefore A=\begin{bmatrix}0 & 0 & 1\\ \frac{1}{4} & 1 & 1\\ \alpha^2 & 4 & 1\end{bmatrix}$ then $1-\alpha^2=|A|$

and $|B|=|\text{adj}(\text{adj}A)|=|A|^4=(1-\alpha^2)^4$

$\therefore (1-\alpha^2)^4=81$

or, $|1-\alpha^2|=3$

or, $\alpha^2=-2$ or 4.

$\therefore \alpha^2=4$

2. The smallest positive integral value of a, for which all the roots of $x^4-ax^2+9=0$ are real and distinct, is equal to

(1) 3 (2) 4

(3) 9 (4) 7

Answer (4)

Sol. $x^4-ax^2+9=0$

$a=x^2+\frac{9}{x^2}$

$\because$ A.M $\geq$ G.M

$\Rightarrow \frac{x^2+\frac{9}{x^2}}{2}\geq\sqrt{x^2\cdot\frac{9}{x^2}}$

$\Rightarrow a \geq 6$

If $a = 6$, then $x^2 + \frac{9}{x^2} \Rightarrow x^4 = 9 \Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$ (two distinct roots)

$\because$ all roots are distinct and real

$\Rightarrow a > 6$

$\Rightarrow$ minimum integral value of a is 7

3. If the domain of the function

$f(x) = \sin^{-1}\left(\frac{1}{x^2 - 2x - 2}\right)$, is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$,

then $\alpha + \beta + \gamma + \delta$ is equal to

(1) 2 (2) 3

(3) 5 (4) 4

Answer (4)

Sol. $f(x) = \sin^{-1}\left(\frac{1}{x^2 - 2x - 2}\right)$

$$-1 \leq \frac{1}{x^2 - 2x - 2} \leq 1$$

$$\frac{1}{x^2 - 2x - 2} \geq -1$$

$$\Rightarrow \frac{1 + x^2 - 2x - 2}{x^2 - 2x - 2} \geq 0$$

$$\Rightarrow \frac{x^2 - 2x - 1}{x^2 - 2x - 2} \geq 0$$

Roots of numerator,

$x^2 - 2x - 1 = 0$

$x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

Roots of denominator,

$x^2 - 2x - 2 = 0$

$x = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$

Solution is

$x \in \left(-\infty, 1-\sqrt{3}\right) \cup \left[1-\sqrt{2}, 1+\sqrt{2}\right] \cup \left(1+\sqrt{3}, \infty\right)$ ...(i)

Now, $\frac{1}{x^2 - 2x - 2} \leq 1$

$$\Rightarrow \frac{1 - x^2 - 2x - 2}{x^2 - 2x - 2} \leq 0$$

$$\Rightarrow \frac{1 - x^2 + 2x + 2}{x^2 - 2x - 2} \leq 0$$

$$\Rightarrow \frac{-x^2 + 2x + 3}{x^2 - 2x - 2} \leq 0$$

$$\Rightarrow \frac{x^2 - 2x - 3}{x^2 - 2x - 2} \geq 0$$

$$\Rightarrow \frac{(x-3)(x+1)}{x^2 - 2x - 2} \geq 0$$

Solution is, $x \in (-\infty, -1] \cup \left(1-\sqrt{3}, 1+\sqrt{3}\right) \cup [3, \infty)$ (ii)

from (i) & (ii)

$x \in \left(-\infty, -1\right] \cup \left[1-\sqrt{2}, 1+\sqrt{2}\right] \cup \left[3, \infty\right)$

$\therefore \alpha = -1,\ \beta = 1-\sqrt{2},\ \gamma = 1+\sqrt{2},\ \delta = 3$

$\therefore \alpha + \beta + \gamma + \delta = -1 + 1 - \sqrt{2} + 1 + \sqrt{2} + 3 = 4$

4. Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to

(1) $\frac{\sqrt{21}}{2}$ (2) 7

(3) 13 (4) $\frac{\sqrt{35}}{2}$

Answer (4) Bonus

Sol. Any vector in the plane of $\vec{a}$ and $\vec{b}$ can be written as

$\vec{v} = x\vec{a} + y\vec{b}$

Projection of $\vec{v}$ on vector $\vec{c} = \frac{1}{\sqrt{14}}$

$\Rightarrow \frac{\vec{c}\cdot\vec{v}}{|\vec{c}|} = \frac{1}{\sqrt{14}}$

$= \frac{2(2x+y)+1(-x+3y)+3(-x)}{\sqrt{14}} = \frac{1}{\sqrt{14}}$

$\Rightarrow 2y = 1 \Rightarrow y = \frac{1}{2}$

$\Rightarrow \vec{v} = x\vec{a} + \frac{\vec{b}}{2}$

Now, since $\vec{v}$ was independent on $\vec{a}$ for project, we can assume $x = 1$, but it should be bonus.

$\Rightarrow |\vec{v}| = \left|\vec{a} + \frac{\vec{b}}{2}\right| = \sqrt{\frac{35}{4}} = \frac{\sqrt{35}}{2}$

5. Let the angles made with the positive x-axis by two straight lines drawn from the point $P(2, 3)$ and meeting the line $x + y = 6$ at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2. Then the value of $(\theta_1 + \theta_2)$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{12}$ (4) $\frac{\pi}{3}$

Answer (2)

Sol. Perpendicular distance from $P(2, 3)$ to the line $x + y - 6 = 0$ is

$d = \frac{|2+3-6|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

The two lines drawn from P meet the line $x + y = 6$ at distⁿ $r = \frac{\sqrt{2}}{3}$ from P.

Let α be the angle between the lines from P & $\perp^r$ from P to the line.

$\sin\alpha = \frac{d}{r} = \frac{\sqrt{3}}{2}$

$\therefore \alpha = \frac{\pi}{3}$

Slope of line $x + y = 6$ is, $m = -1$

Let the angle made by this line with +ve x-axis is ϕ.

$\therefore \tan\phi = -1 \Rightarrow \phi = \frac{3\pi}{4}$

Slope of perpendicular from P to line $x + y = 6$ is 1.

$\therefore \theta_p = \frac{\pi}{4}$

$\theta_1 = \theta_P - \alpha = \frac{\pi}{4} - \frac{\pi}{3} = \frac{-\pi}{12}$

$\theta_2 = \theta_P + \alpha = \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$

$\therefore \theta_1 + \theta_2 = \frac{\pi}{2}$

6. Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by $x = 0, x = 1, y^2 = x$ and $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$. Then $\left(f(0) + f(1)\right)$ is equal to

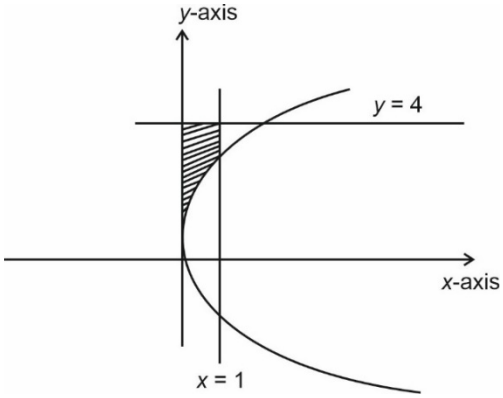
(1) 9 (2) 12

(3) 7 (4) 14

Answer (3)

Sol. For $f(0)$, $\alpha = 0$

$x = 0$, $x = 1$, $y^2 = x$, $y = 4$



$$f(0)=\int_0^1\left(4-\left(\sqrt{x}\right)\right)dx$$

$$=4-\frac{x^{3/2}}{3/2}\Big|_0^1$$

$$=4-\frac{2}{3}$$

$$=\frac{10}{3}$$

For $f(1)$, $\alpha = 1$

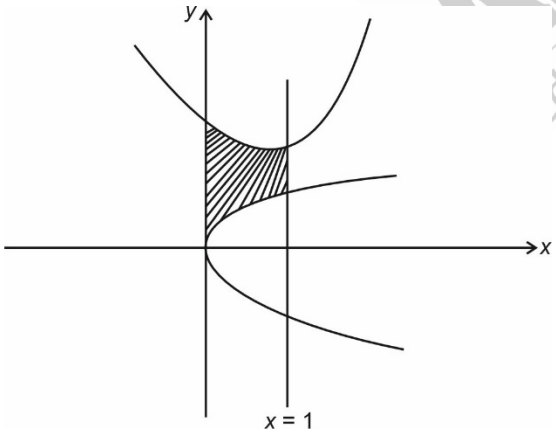
$x = 0$, $x = 1$, $y^2 = x$, $y = |x-5| - |x-1| + x^2$

For $0 < x < 1$

$y = 5 - x + x - 1 + x^2$

$y = x^2 + 4$

$$f(1)=\int_0^1\left[\left(x^2+4\right)-\left(\sqrt{x}\right)\right]dx$$



$$=\frac{1}{3}+4-\frac{2}{3}$$

$$=4-\frac{1}{3}=\frac{11}{3}$$

$$f(0)+f(1)=\frac{21}{3}=7$$

7. Let $X=\{x\in N:1\le x\le 19\}$ and for some $a,b\in\mathbb{R}, Y=\{ax+b:x\in X\}$. If the mean and variance of the elements of Y are 30 and 750, respectively, then the sum of all possible values of b is

(1) 80

(2) 60

(3) 100

(4) 20

Answer (2)

Sol. $x = \{1, 2, 3, \ldots\ldots, 19\}$

$$M_x=\frac{1+2+3+\ldots+19}{19}=\frac{19\times 20}{2\times 19}=10$$

$$\sigma_x^2=\frac{\sum x_i^2}{19}-(10)^2=30$$

$y = ax + b$

$M_y = aM_x + b$

$\sigma_y^2 = a^2\sigma_{x^2}$

$750 = a^2 \times 30$

$a = \pm 5$

If $a = 5$

$30 = 5(10) + b \Rightarrow b = -20$

If $a = -5$

$30 = -5(10) + b \Rightarrow b = 80$

Sum of all possible values of $b = -20 + 80 = 60$

8. Let f be a function such that $3f(x)+2f\left(\frac{m}{19x}\right)=5x, x\neq 0$, where $m=\sum_{i=1}^{9}(i)^2$.

Then $f(5)-f(2)$ is equal to

(1) 18 (2) –9

(3) 36 (4) 9

Answer (1)

Sol. $m=\sum_{i=1}^{9}i^2=\frac{9\times10\times19}{6}=15\times19$

$\therefore 3f(x)+2f\left(\frac{15}{x}\right)=5x$ ...(i)

Replace x by $\frac{15}{x}$ we get:

$3f\left(\frac{15}{x}\right)+2f(x)=\frac{5\times15}{x}$ ...(ii)

$\therefore 5f(x)=15x-\frac{150}{x}$

$\therefore f(x)=3x-\frac{30}{x}$

$\therefore f(5)-f(2)=(15-6)-(6-15)=18$

9. The largest value of n, for which 40n divides 60!, is

(1) 11 (2) 14

(3) 13 (4) 12

Answer (2)

Sol. $40 = 2^3 \times 5$

Let $60! = 2^a \cdot 3^b \cdot 5^c \ldots$

$a=\left[\frac{60}{2}\right]+\left[\frac{60}{4}\right]+\left[\frac{60}{8}\right]+\left[\frac{60}{16}\right]+\left[\frac{60}{32}\right]+\left[\frac{60}{64}\right]$

$= 30 + 15 + 7 + 3 + 1$

$= 56$

$\therefore 2^a = (8)^{a/3}$

$\Rightarrow$ max power of $8 = \left[\frac{56}{3}\right] = 18$

$c=\left[\frac{60}{5}\right]+\left[\frac{60}{25}\right]+\left[\frac{60}{125}\right]$

$= 12 + 2 = 14$

$\min\left(c,\frac{a}{3}\right)=c=14$

$\therefore$ 14

10. Let a_1,a_2,a_3,a_4 be an A.P. of four terms such that each term of the A.P. and its common difference l are integers. If $a_1+a_2+a_3+a_4=48$ and $a_1a_2a_3a_4+l^4=361$, then the largest term of the A.P. is equal to

(1) 27

(2) 21

(3) 23

(4) 24

Answer (1)

Sol. Let the four terms of A.P. be $a - 3d$, $a - d$, $a + d$, $a + 3d$

where $\ell = 2d$

$4a = 48$

$\Rightarrow a = 12$

$a_1 \cdot a_2 \cdot a_3 \cdot a_4 + \ell^4 = 361$

$(a-3d)(a-d)(a+d)(a+3d)+\ell^4=361$

$(a^2-9d^2)(a^2-d^2)+\ell^4=361$

$(144-9d^2)(144-d^2)+\ell^4=361$

$(144-9d^2)(144-d^2)=(19-\ell^2)(19+\ell^2)$

$(144-9d^2)(144-d^2)=(19-4d^2)(19+4d^2)$

$19 + 4d^2 = 144 - d^2$

$\Rightarrow 5d^2 = 125$

$d^2 = 25$

$d = 5$

$\ell = 10$

largest term = $a + 3d = 12 + 15 = 27$

11. The sum of all values of α, for which the shortest distance between the lines $\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha}$ and $\frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha}$ is $\sqrt{2}$, is

(1) 6 (2) –8

(3) –6 (4) 8

Answer (3)

Sol. $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{j} - 3\hat{k}$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}$$

$= 0\hat{i} - 3\alpha^2\hat{j} + 3\alpha\hat{k}$

$|\vec{n}_1 \times \vec{n}_2| = \sqrt{9\alpha^4 + 9\alpha^2} = 3\sqrt{\alpha^4 + \alpha^2}$

$$d = \sqrt{2} = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|}\right|$$

$$\Rightarrow \sqrt{2} = \left|\frac{3\alpha^2 - 9\alpha}{3\sqrt{\alpha^4 + \alpha^2}}\right|$$

$$\Rightarrow \sqrt{2} = \frac{\alpha^2 - 3\alpha}{\sqrt{\alpha^4 + \alpha^2}}$$

$\Rightarrow 2(\alpha^4 + \alpha^2) = \alpha^4 + 9\alpha^2 - 6\alpha^3$

$= \alpha^4 + 6\alpha^3 - 7\alpha^2 = 0$

$= \alpha^2 + 6\alpha - 7 = 0 \qquad (\alpha \neq 0)$

Sum = –6 + 0 = –6

12. Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be 30. If its eccentricity is the maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$, then $(a^2 + b^2)$ is equal to

(1) 516 (2) 256

(3) 496 (4) 276

Answer (3)

Sol. $f(t) = -\frac{3}{4} + 2t - t^2$

$= -\frac{3}{4} - (t-1)^2 + 1$

$f(t) = \frac{1}{4} - (t-1)^2$

Maximum value of $f(t) = \frac{1}{4} = e$

$\frac{2b^2}{a} = 30$

$\Rightarrow \frac{b^2}{a} = 15$

$\Rightarrow b^2 = 15a$

$e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{1}{16} = 1 - \frac{15a}{a^2}$

$\Rightarrow \frac{15}{a} = \frac{15}{16}$

$\Rightarrow a = 16$

$\therefore b^2 = 15 \times 16 = 240$

$a^2 + b^2 = 240 + 256 = 496$

13. Let the image of parabola $x^2 = 4y$, in the line $x - y = 1$ be $(y + a)^2 = b(x - c)$, $a, b, c \in N$. Then $a + b + c$ is equal to

(1) 8 (2) 12

(3) 6 (4) 4

Answer (3)

Sol. Any point $P : (2t, t^2)$

Image of point P in the line $x - y = 1$ is

$$\frac{x-2t}{1} = \frac{y-t^2}{-1} = \frac{-2(2t-t^2-1)}{2}$$

$$\Rightarrow x - 2t = \frac{y-t^2}{-1} = t^2 - 2t + 1$$

$$\Rightarrow x = t^2 + 1$$

$$y = 2t - 1$$

$$x = \left(\frac{y+1}{2}\right)^2 + 1 \Rightarrow 4(x-1) = (y+1)^2$$

14. Let $P = [p_{ij}]$ and $Q = [q_{ij}]$ be two square matrices of order 3 such that $q_{ij} = 2^{(i+j-1)} p_{ij}$ and $\det(Q) = 2^{10}$. Then the value of $\det(\text{adj}(\text{adj } P))$ is:

(1) 81 (2) 32

(3) 16 (4) 124

Answer (3)

Sol. $q_{ij} = 2^{i+j-1}p^{ij}$, $|Q| = 2^{10}$

$$\Rightarrow Q = \begin{bmatrix} 2p_{11} & 2^2 p_{12} & 2^3 p_{13} \\ 2^2 p_{21} & 2^3 p_{22} & 2^4 p_{24} \\ 2^3 p_{31} & 2^4 p_{32} & 2^5 p_{33} \end{bmatrix}$$

$$\Rightarrow Q = 2 \cdot 2^2 \cdot 2^3 \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{22} & 2p_{22} & 2p_{24} \\ 2^2 p_{31} & 2^2 p_{32} & 2^2 p_{33} \end{bmatrix}$$

$= 2^{1+2+3+1+2}|P|$

$|Q| = 2^9|P|$

$\Rightarrow |P| = 2$

$\Rightarrow |\text{adj}(\text{adj}(P))| = |\text{adj}(P)|^2 = |P|^4 = 2^4 = 16$

15. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is

(1) 1579 (2) 1580

(3) 1581 (4) 1578

Answer (2)

Sol. UDAYPUR

A, D, P, R, U, U, Y

words starting with A = $\frac{6!}{2!} = 360$

words starting with D = $\frac{6!}{2!} = 360$

words starting with P = $\frac{6!}{2!} = 360$

words starting with R = $\frac{6!}{2!} = 360$

words starting with UA = 5! = 120

words starting with UDAP = 3! = 6

words starting with UDAR = 3! = 6

words starting with UDAU = 3! = 6

Then next word is UDAYPRU

and next word is UDAYPUR

$\Rightarrow$ Rank of UDAYPUR is (360) × 4 + 120 + (6) × 3 + 1 + 1

= 1580

16. $\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\frac{1}{3^2}+\frac{1}{3}\times\frac{4}{7}+\frac{4^2}{7^2}\right)+\left(\frac{1}{3^3}+\frac{1}{3^2}\times\frac{4}{7}+\frac{1}{3}\times\frac{4^2}{7^2}+\frac{4^3}{7^3}\right)+\ldots$ upto infinite terms, is equal to

(1) $\frac{7}{4}$ (2) $\frac{4}{3}$

(3) $\frac{5}{2}$ (4) $\frac{6}{5}$

Answer (3)

Sol. Let $\frac{1}{3}=a, \frac{4}{7}=b$

then

$s=(a+b)+(a^2+b^2+ab)+(a^3+a^2b+ab^2+b^3)+\ldots$

multiplying by $(a-b)$

$s(a+b)=(a^2-b^2)+(a^3-b^3)+(a^4-b^4)+\ldots$

$\Rightarrow s=\frac{1}{(a-b)}\left((a^2+a^3+a^4+\ldots-(b^2+b^3+b^4+\ldots)\right)$

$\Rightarrow s=\frac{1}{(a-b)}\left[\frac{a^2}{1-a}-\frac{b^2}{1-b}\right]$

$=\frac{1}{(a-b)}\left[\frac{a^2-a^2b-b^2+ab^2}{(1-a)(1-b)}\right]$

$=\frac{1}{(a-b)}\left[\frac{a^2-b^2}{(1-a)(1-b)}+\frac{ab(-a+b)}{(1-a)(1-b)}\right]$

$=\frac{a+b}{(1-a)(1-b)}-\frac{ab}{(1-a)(1-b)}$

$=\frac{\frac{19}{21}}{\frac{2}{3}\times\frac{3}{7}}-\frac{\frac{4}{21}}{\frac{6}{21}}=\frac{19}{6}-\frac{4}{6}=\frac{15}{6}=\frac{5}{2}$

17. Let $y = y(x)$ be a differentiable function in the interval $(0,\infty)$ such that $y(1) = 2$, and $\lim\limits_{t\to x}\left(\frac{t^2y(x)-x^2y(t)}{x-t}\right)=3$ for each $x > 0$. Then $2y(2)$ is equal to

(1) 18 (2) 12

(3) 27 (4) 23

Answer (4)

Sol. $\lim\limits_{t\to x}\frac{t^2y(x)-x^2y(t)}{x-t}=3$

$\lim\limits_{t\to x}-\left(2ty(x)-x^2y^1(t)\right)=3$

$-\left(2xy(x)-x^2y^1(x)\right)=3$

Now

$x^2\frac{dy}{dx}-2xy=3$

$\frac{dy}{dx}-\frac{2}{x}y=\frac{3}{x^2}$

IF $=\ell^{-\int\frac{2}{x}dx}=\frac{1}{x^2}$

Solution $\frac{y}{x^2}=\int\frac{3}{x^4}$

$\frac{y}{x^2}=-\frac{1}{x^3}+c$

$y(x)=\frac{-1}{x}+cx^2$

$\because y(1)=2$

$2=-1+c\Rightarrow c=3$

$y(x)=\frac{-1}{x}+3x^2$

$2y(2)=2\left(-\frac{1}{2}+12\right)=-1+24=23$

18. Let $[t]$ denote the greatest integer less than or equal to t. If the function

$$f(x)=\begin{cases} b^2\sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x+\sin x)\cos x\right]\right), & x<0 \\ \frac{\sin x-\frac{1}{2}\sin 2x}{x^3}, & x>0 \\ a, & x=0 \end{cases}$$

is continuous at $x = 0$, then a^2+b^2 is equal to

(1) $\frac{5}{8}$ (2) $\frac{3}{4}$

(3) $\frac{1}{2}$ (4) $\frac{9}{16}$

Answer (2)

Sol.

$$f(x)=\begin{cases} b^2\sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x+\sin x)\cos x\right]\right), & x<0 \\ \dfrac{\sin x-\frac{1}{2}\sin 2x}{x^3}, & x>0 \\ a, & x=0 \end{cases}$$

$$\text{RHL}=\lim_{x\to 0^+}\frac{\sin x-\frac{1}{2}\sin 2x}{x^3}=a$$

$$\lim_{x\to a^+}\frac{\sin x-\sin x\cos x}{x^3}=a=\frac{1}{2}$$

$$\text{LHL}=\lim_{x\to 0} b^2\sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\sin x+\cos x)\cos x\right]\right)$$

When $x\to 0^-$, $\sin x\cos x\to 0^-$ and $\cos^2 x\to 1^-$

$$\Rightarrow \frac{\pi}{2}(\sin x+\cos x)\cos x\to \frac{\pi^-}{2}$$

$$\Rightarrow \left[\frac{\pi}{2}(\sin x+\cos x)\cos x\right]=1$$

$$\Rightarrow \text{LHL}=b^2\sin\left(\frac{\pi}{2}\right)=b^2=a$$

$$\therefore a^2+b^2=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$$

19. Consider the following three statements for the function $f:(0,\infty)\to\mathbb{R}$ defined by $f(x)=|\log_e x|-|x-1|$:

(I) f is differentiable at all $x>0$.

(II) f is increasing in (0, 1).

(III) f is decreasing in $(1,\infty)$.

Then.

(1) Only (I) is TRUE

(2) Only (I) and (III) are TRUE

(3) Only (II) and (III) are TRUE

(4) All (I), (II) and (III) are TRUE

Answer (2)

Sol. $f(x)=|\ln x|-|x-1|$

$$f'(x)=\begin{cases}\ln x-x+1 & x>1\\ x-1-\ln x & 0<x\le 1\end{cases}$$

$$f'(x)=\begin{cases}\frac{1}{x}-1 & x>1\\ 1-\frac{1}{x} & 0<x\le 1\end{cases}$$

$\Rightarrow f(x)$ is differentiable $\forall\ x\in R$

$f(x)$ is always singular

$\Rightarrow$ Statement (I) and (III) is correct

20. Let $\vec{a}=2\hat{i}-5\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+3\hat{k}$. If $\vec{c}$ is a vector such that $2(\vec{a}\times\vec{c})+3(\vec{b}\times\vec{c})=\vec{0}$ and $(\vec{a}-\vec{b})\cdot\vec{c}=-97$, then $|\vec{c}\times\hat{k}|^2$ is equal to

(1) 205 (2) 233

(3) 193 (4) 218

Answer (4)

Sol. $2(\vec{a}\times\vec{c})+3(\vec{b}\times\vec{c})=\vec{0}$

taking dot product with $\vec{a}$ and $\vec{b}$ respectively

$3[\vec{a}\,\vec{b}\,\vec{c}]=0$

$\Rightarrow \vec{c}$ vector is in the plane of vector $\vec{a}$ and $\vec{b}$.

$\Rightarrow \vec{c}=\vec{a}x+y\vec{b}$ for some $x, y\in R$.

$2(\vec{a}\times(\vec{a}x+y\vec{b}))+3(\vec{b}\times(\vec{a}x+y\vec{b}))=0$

$2y(\vec{a}\times\vec{b})+3x(\vec{b}\times\vec{a})=0$

Since $\vec{a}\times\vec{b}\neq 0$

$\Rightarrow 2y=3x$

Also,

$(\vec{a}-\vec{b})\cdot(\vec{a}x+y\vec{b})=-97$

$54x-11y-22x+22y=-97$

$32x+11y=-97 \Rightarrow x=-2, y=\frac{3(-2)}{2}=-3$

$\Rightarrow \vec{c}=-2\vec{a}-3\vec{b}$

$\Rightarrow (\vec{c}\times\hat{k})=(-2\vec{a}\times\hat{k}-3\vec{b}\times\hat{k})$

$=(-2)(-2\hat{j}-5\hat{i})-3(-\hat{j}-\hat{i})$

$\Rightarrow \left|\vec{c}\times\hat{k}\right|^2=218.$

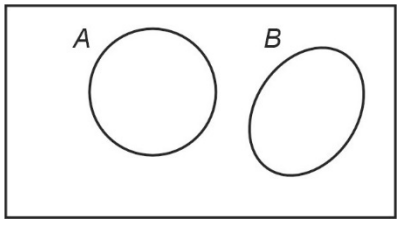
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let S be a set of 5 elements and $P(S)$ denote the power set of S. Let E be an event of choosing an ordered pair (A, B) from the set $P(S)\times P(S)$ such that $A\cap B=\varnothing$. If the probability of the event E is $\frac{3^p}{2^q}$, where $p,q\in N$, then $p + q$ is equal to ____

Answer (15)

Sol.



Every element has 3 choice to be either in A, B or $A^C\cap B^C$.

Number of favourable ways = 3^5

Total ways to choose (A, B) pairs from power set

$\Rightarrow 2^5\cdot 2^5$

$\Rightarrow P(A\cap B=\phi)=\frac{3^5}{2^{10}} \Rightarrow p + q = 15$

22. The number of elements in the set

$\left\{x\in\left[0,180^\circ\right]:\tan\left(x+100^\circ\right)=\tan\left(x+50^\circ\right)\tan x\tan\left(x-50^\circ\right)\right\}$

is _____.

Answer (4)

Sol. $\frac{\tan(x+100)}{\tan x}=\tan(x+50)\tan(x-50)$

$\frac{\sin(x+100)\cos x}{\cos(x+100)\sin x}=\frac{\sin(x+50)\sin(x-50)}{\cos(x+50)\cos(x-50)}$

Apply C & D

$\frac{\sin(2x+100)}{\sin100}=\frac{\cos100}{-\cos2x}$

$2\sin(2x + 100)\cos2x + 2\sin100\cos100 = 0$

$\sin(4x + 100) + \sin100 + \sin200 = 0$

$\sin(4x + 100) = -2\sin150^\circ\sin50^\circ$

$\sin(4x + 100) = -\sin50^\circ$

$4x + 100 = n\pi + (-1)^n(-50^\circ)$

$x=\frac{n\pi+(-1)^{n+1}(50^\circ)-100^\circ}{4}$

$x=\frac{130^\circ}{4},\frac{210^\circ}{4},\frac{490^\circ}{4},\frac{570^\circ}{4}$

$\therefore$ 4 solutions are possible

23. Let (h, k) lie on the circle $C: x^2 + y^2 = 4$ and the point $(2h + 1, 3k + 2)$ lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to ____ .

Answer (9)

Sol. $\because$ (h, k) lies on $x^2 + y^2 = 4$

$\therefore$ $h = 2\cos\theta$, $k = 2\sin\theta$

then $(2h + 1, 3k + 2) \equiv (4\cos\theta + 1$ and $6\sin\theta + 2)$

Here let $4\cos\theta + 1 = x$ and $6\sin\theta + 2 = y$

$\therefore$ Required ellipse is:

$$\frac{(x-1)^2}{4^2} + \frac{(y-2)^2}{6^2} = 1$$

$$\therefore \text{ eccentricity } = e^2 = 1 - \frac{4^2}{6^2} = \frac{20}{36} = \frac{5}{9}$$

$$\therefore \frac{5}{e^2} = 9$$

24. Let $z = (1 + i)(1 + 2i)(1 + 3i)\ldots(1 + ni)$, where $i = \sqrt{-1}$. If $|z|^2 = 44200$, then n is equal to ____

Answer (5)

Sol. $z = (1 + i)(1 + 2i)(1 + 3i).....(1 + ni)$

$z\bar{z} = (1 + i)(1 - i)(1 + 2i)(1 - 2i).....(1 + ni)(1 - ni)$

$|z|^2 = (2)(5)(1 + 9).....(1 + n^2)$

$\Rightarrow 2 \times 5 \times 10 \times.....(1 + n^2) = 44200$

$$\prod_{r=1}^{n}\left(1+r^2\right) = 44200$$

Since $210 < \sqrt{44200} < 211$

$\Rightarrow n$ will be small

$\Rightarrow$ Since $13|44200 \Rightarrow (5^2 + 1)|44200$

$\Rightarrow$ checking small values of n, $\prod_{r=1}^{5}\left(1+r^2\right) = 44200$

$\Rightarrow n = 5$

25. If $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 (y + xe^x) f(y)\,dy$, then $e + f(0)$ is equal to ____ .

Answer (2)

Sol. $f(x) = e^x + \int_0^1 yf(y)dy + xe^x \int_0^1 f(y)dx$

Let $A = \int_0^1 yf(y)dy$, $B = \int_0^1 f(y)dx$

$\Rightarrow f(x) = e^x + A + xe^x B$

$B = \int_0^1 \left(e^y + A + ye^y B\right)dy$

$= e^y + Ay + B\left(e^y y - e^y\right)\Big|_0^1$

$= e + A + B(0) - [1 + B(-1)]$

$= e + A - 1 + B \Rightarrow A = 1 - e$

$\Rightarrow f(0) = 1 + A$

$\Rightarrow e + f(0) = 1 + A + e$

$= 1 + (1 - e) + e = 2$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. The binding energy for the following nuclear reactions are expressed in MeV.

$_2He^3 + {}_0n^1 \rightarrow {}_2He^4 + 20MeV$

$_2He^4 + {}_0n^1 \rightarrow {}_2He^5 - 0.9MeV$

If X_3, X_4, X_5 denote the stability of $_2He^3$, $_2He^4$ and $_2He^5$, respectively, then the correct order is :

(1) $X_4 < X_5 < X_3$ (2) $X_4 > X_5 < X_3$

(3) $X_4 > X_5 > X_3$ (4) $X_4 = X_5 = X_3$

Answer (3)

Sol. More energy released means more stable product and if energy is absorbed reactant is more stable.

Thus $X_4 > X_5 > X_3$

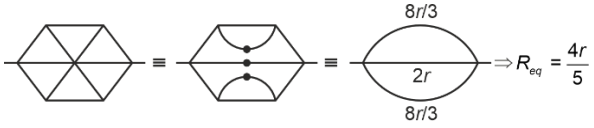
27. A regular hexagon is formed by six wires each of resistance $r\Omega$ and the corners are joined to the centre by wires of same resistance. If the current enters at one corner and leaves at the opposite corner, the equivalent resistance of the hexagon between the two opposite corners will be

(1) $\frac{3}{5}r$ (2) $\frac{4}{5}r$

(3) $\frac{3}{4}r$ (4) $\frac{5}{8}r$

Answer (2)

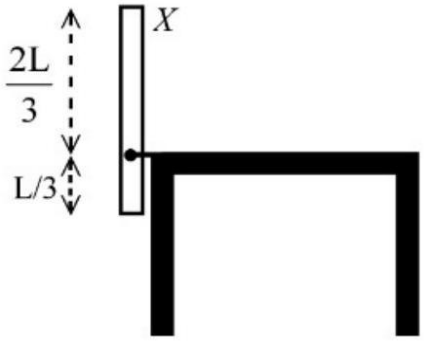
Sol.



$\Rightarrow R_{eq} = \frac{4r}{5}$

28. A thin uniform rod (X) of mass M and length L is pivoted at a height $\left(\frac{L}{3}\right)$ as shown in the figure. The rod is allowed to fall from a vertical position and lie horizontally on the table. The angular velocity of this rod when it hits the table top, is _____.

(g = gravitational acceleration)



(1) $\sqrt{\frac{3}{2}\frac{g}{L}}$ (2) $\frac{1}{\sqrt{2}}\sqrt{\frac{g}{L}}$

(3) $\sqrt{\frac{3g}{L}}$ (4) $\frac{3}{\sqrt{2}}\sqrt{\frac{g}{L}}$

Answer (3)

Sol. Let table surface be at zero gravitational potential then

loss in PE = gain in KE

$$mg\frac{l}{6} = \frac{1}{2}\left(\frac{ml^2}{12} + m\left(\frac{l}{6}\right)^2\right)\omega^2$$

$$mg\frac{l}{6} = \frac{1}{2}\times\frac{1}{36g}4ml^2\omega^2$$

$$\sqrt{\frac{3g}{L}} = \omega$$

29. In a vernier callipers, 50 vernier scale divisions are equal to 48 main scale divisions. If one main scale division = 0.05 mm, then the least count of the vernier callipers is _____ mm .

(1) 0.05 (2) 0.005

(3) 0.002 (4) 0.02

Answer (3)

Sol. $\text{VSD} = \frac{48}{50}\text{MSD} = \frac{48}{50}\times 0.5 = 48\,\mu\text{m} = 0.048$

LC = MSD – VSD = 0.05 – 0.048 = 0.002

30. A cubical block of density $\rho_b = 600\ \text{kg/m}^3$ floats in a liquid of density $\rho_e = 900\text{kg/m}^3$. If the height of block is $H = 8.0$ cm then height of the submerged part is _____ cm .

(1) 7.3 (2) 4.3

(3) 5.3 (4) 6.3

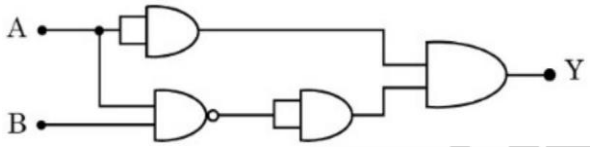
Answer (3)

Sol. From condition of floatation

$AH\rho_b g = Ah_i\rho_e g$

$\Rightarrow \quad h_i = H\frac{\rho_b}{\rho_e} = 8\times\frac{6}{9} = \frac{16}{3} = 5.33$

31. Identify the correct truth table of the given logic circuit.



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(3)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Answer (2)

Sol. $y = AA^*\overline{AB}\cdot\overline{AB}$

$= A^*\overline{AB}$

$= A^*\left(\bar{A}+\bar{B}\right)$

$= A\bar{A} + A\bar{B}$

$= A\bar{B}$

32. When a light of a given wavelength falls on a metallic surface the stopping potential for photoelectrons is 3.2 V. If a second light having wavelength twice of first light is used, the stopping potential drops to 0.7 V. The wavelength of first light is _____ m.

$\left(h = 6.63\times10^{-34}\ \text{J.s}, e = 1.6\times10^{-19}\text{C}, c = 3\times10^{8}\ \text{m/s}\right)$

(1) 2.9×10^{-8}

(2) 3.1×10^{-7}

(3) 2.5×10^{-7}

(4) 2.2×10^{-8}

Answer (3)

Sol. $\frac{hc}{\lambda} - \phi_0 = 3.2e$ ...(i)

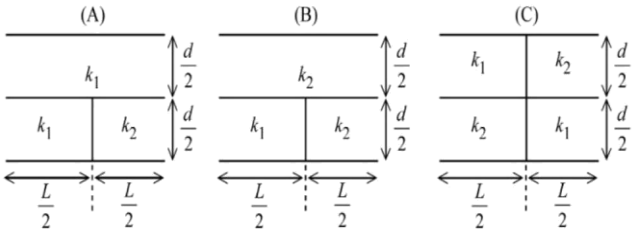
$\frac{hc}{2\lambda} - \phi_0 = 0.7e$ ...(ii)

Substituting (i) and (ii)

$\frac{hc}{2\lambda} = 2.5e \Rightarrow \lambda = \frac{hc}{5e}$

$\lambda = 2.486\times10^{-7}\ \text{m}$

33. Three parallel plate capacitors each with area A and separation d are filled with two dielectric (k_1 and k_2) in the following fashion. Which of the following is true? $(k_1 > k_2)$



(1) $C_A > C_C > C_B$ (2) $C_B > C_C > C_A$

(3) $C_C > C_B > C_A$ (4) $C_C > C_A > C_B$

Answer (1)

Sol. Let $\frac{\varepsilon_0 A/2}{d/2} = C$

$$C_{eq_A} = \frac{K_1 C}{2} + \frac{K_1 K_2}{K_1 + K_2} C$$

$$C_{eq_B} = \frac{K_1 K_2}{K_1 + K_2} C + \frac{K_2}{2} C$$

$$C_{eq_C} = \frac{K_1 K_2 C}{K_1 + K_2} + \frac{K_1 K_2}{K_1 + K_2} C$$

let $k_1 = 2$ or $k_2 = 1$ we get

$$C_{eq_A} = \frac{5}{3}C,\ C_{eq_B} = \frac{7}{6}C,\ C_{eq_C} = \frac{4}{3}C$$

Clearly, $C_A > C_C > C_B$

34. Five persons P_1, P_2, P_3, P_4 and P_5 recorded object distance (u) and image distance (v) using same convex lens having power +5D as $(25, 96)$, $(30, 62)$, $(35, 37)$, $(45, 35)$ and $(50, 32)$ respectively. Identify correct statement

(1) Readings recorded by all persons are correct

(2) Readings recorded by P_3 and P_2 persons are incorrect

(3) Readings recorded by P_3 person are incorrect

(4) Readings recorded by P_4 and P_5 persons are incorrect

Answer (3)

Sol. $+5D \equiv 20$ cm of focal length

$$\frac{1}{v} - \frac{1}{(-u)} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$P_1 \equiv \frac{1}{25} + \frac{1}{96} \approx \frac{1}{20}$$

$$P_2 \equiv \frac{1}{30} + \frac{1}{60} \approx \frac{1}{20}$$

$$P_3 \equiv \frac{1}{35} + \frac{1}{37} \approx \frac{1}{18}$$

$$P_4 \equiv \frac{1}{45} + \frac{1}{35} \approx \frac{1}{20}$$

Hence, P_3 is incorrect

35. The fifth harmonic of a closed organ pipe is found to be in unison with the first harmonic of an open pipe. The ratio of lengths of closed pipe to that of the open pipe is $5/x$. The value of x is ____ .

(1) 1 (2) 3

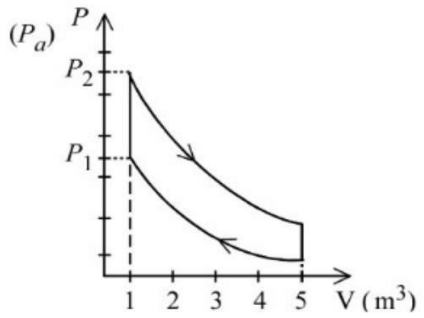
(3) 2 (4) 4

Answer (3)

Sol. For 5th harmonic = 1st harmonic

$$\frac{5c}{4l_1} = \frac{c}{2l_2} \Rightarrow \frac{l_1}{l_2} = \frac{5}{2}$$

36. 10 mole of an ideal gas is undergoing the process shown in the figure. The heat involved in the process from P_1 to P_2 is α Joule ($P_1 = 21.7$ Pa and $P_2 = 30$ Pa, $C_v = 21$ J/K.mol, $R = 8.3$ J/mol.K). The value of α is _____ .



(1) 21 (2) 24

(3) 15 (4) 28

Answer (1)

Sol. For isochoric process $\Delta Q = nC_V\Delta T$

OR $\Delta Q = \frac{\gamma C_V}{R}(P_2 - P_1)$

$= \frac{21}{8.3}(8.3) = 21$ J

37. A flexible chain of mass m hangs between two fixed points at the same level. The inclination of the chain with the horizontal at the two points of support is 30°. Considering the equilibrium of each half of the chain, the tension of the chain at the lowest point is _____.

(1) $\sqrt{3}mg$ (2) $\frac{1}{2}mg$

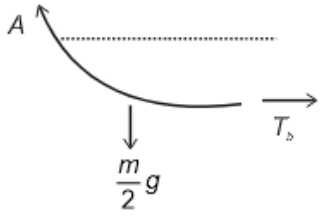
(3) mg (4) $\frac{\sqrt{3}}{2}mg$

Answer (4)

Sol. Analysing FBD of full chain

$2T\sin 30^\circ = mg \Rightarrow T = mg$

Now, FBD of half chain



$T\cos 30^\circ = T_b$

$mg\frac{\sqrt{3}}{2} = T_b$

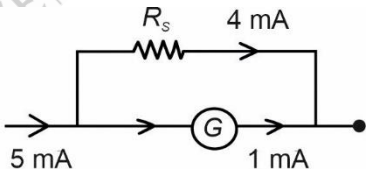
38. A moving coil galvanometer of resistance 100 Ω shows a full-scale deflection for a current of 1 mA. The value of resistance required to convert this galvanometer into an ammeter, showing full scale deflection for a current of 5 mA, is _____ Ω

(1) 10 (2) 25

(3) 2.5 (4) 0.5

Answer (2)

Sol.



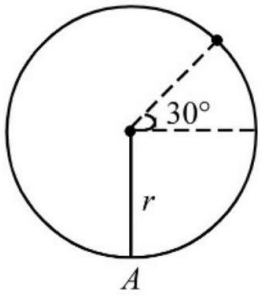
Voltage drop across G is 0.1 V

$\Rightarrow 0.1 = 4 \times 10^{-3} R_s$

$\Rightarrow R_s = 25\ \Omega$

39. In case of vertical circular motion of a particle by a thread of length r if the tension in the thread is zero at an angle 30° shown in figure, the velocity at the bottom point (A) of the circular path is

(g = gravitational acceleration)

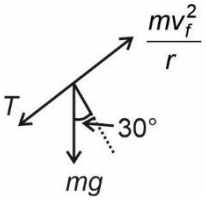


(1) $\sqrt{\frac{7}{2}gr}$ (2) $\sqrt{4gr}$

(3) $\sqrt{\frac{5}{2}gr}$ (4) $\sqrt{5gr}$

Answer (1)

Sol. Analysing *FBD* at final position



$mg\sin30° + T = \frac{mv_f^2}{r}$

if $T = 0$

$\frac{1}{2}mv_f^2 = \frac{mgr}{4}$

Applying conservation of energy

$mg\left(r+\frac{r}{2}\right)+\frac{1}{2}mv_f^2 = \frac{1}{2}mv_b^2$

$\Rightarrow \quad 3gr+\frac{gr}{2} = v_6^2$

$\Rightarrow \quad v_b = \sqrt{\frac{7}{2}gr}$

40. In the Young's double slit experiment the intensity produced by each one of the individual slits is I_0. The distance between two slits is 2 mm. The distance of screen from slits is 10 m. The wavelength of light is 6000 Å. The intensity of light on the screen in front of one of the slits is _____.

(1) $2I_0$ (2) $4I_0$

(3) $\frac{I_0}{2}$ (4) I_0

Answer (4)

Sol. $I_R = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$

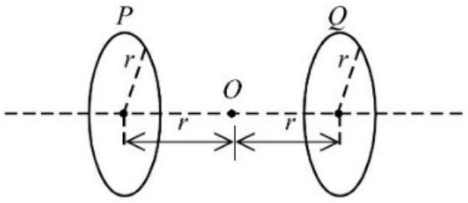
$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \quad \& \quad \Delta x = \frac{yd}{D}$

$\Rightarrow \quad \Delta x = \frac{(1\,mm)(2\,mm)}{10} = 2\times10^{-7}\ m$

$\Rightarrow \quad \Delta\phi = \frac{2\pi}{6\times10^{-7}}\times2\times10^{-7} = \frac{2\pi}{3}$

$\Rightarrow \quad \frac{\Delta\phi}{2} = \frac{\pi}{3}$

$\Rightarrow \quad I_R = I_0$

41. 

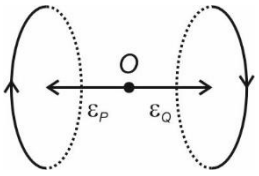
Two identical circular loops *P* and *Q* each of radius *r* are lying in parallel planes such that they have common axis. The current through *P* and *Q* are *I* and 4*I* respectively in clockwise direction as seen from *O*. The net magnetic field at *O* is:

(1) $\frac{3\mu_0 I}{4\sqrt{2}r}$ towards *Q* (2) $\frac{3\mu_0 I}{4\sqrt{2}r}$ towards *P*

(3) $\frac{\mu_0 I}{4\sqrt{2}r}$ towards *Q* (4) $\frac{\mu_0 I}{4\sqrt{2}r}$ towards *P*

Answer (1)

Sol. $\frac{\mu_0 r^2}{2(\sqrt{2}r)^3}(4I - I) = \frac{3\mu_0 I}{4\sqrt{2}r}$ towards Q



42. A point source is kept at the center of a spherically enclosed detector. If the volume of the detector increased by 8 times, the intensity will

(1) decrease by 4 times (2) increase by 8 times

(3) increase by 64 times (4) decrease by 8 times

Answer (1)

Sol. $I \propto \frac{1}{r^2} \Rightarrow (\text{for } v_f = 8v_i, r_f = 2r_i)$

$\Rightarrow I_f = \frac{1}{4} I_i$

43. Distance between an object and three times magnified real image is 40 cm. The focal length of the mirror used is _____ cm.

(1) –15 (2) –10

(3) –20 (4) $-\frac{15}{2}$

Answer (1)

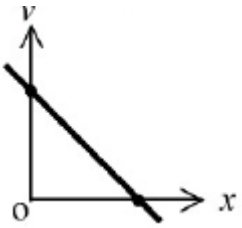
Sol. As per sign convention

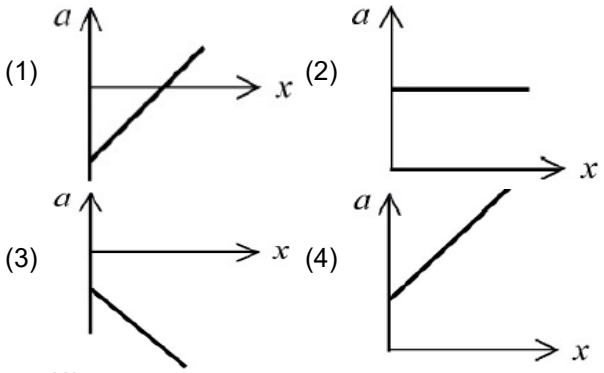
$u - v = 40$ & $\frac{v}{u} = 3$

$\Rightarrow v = -60$ cm $u = -20$ cm

$f = \frac{vu}{v+u} = -\frac{120}{8} = -15$ cm

44. The velocity (v) – Distance (x) graph is shown in figure. Which graph represents acceleration(a) versus distance (x) variation of this system?





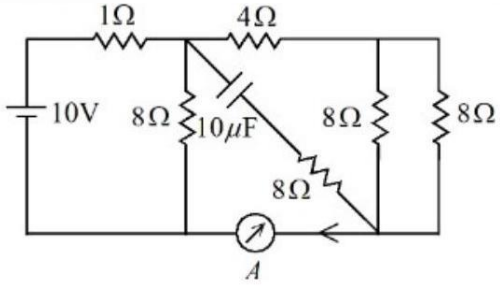
Answer (1)

Sol. $a = +v\frac{dv}{dx}$

$\frac{dv}{dx}$ = slope

$\Rightarrow$ a = (linear function of v) × (negative slope as constant)

45. The reading of the ammeter (A) in steady state in the following circuit (assuming negligible internal resistance of the ammeter) is _____A.

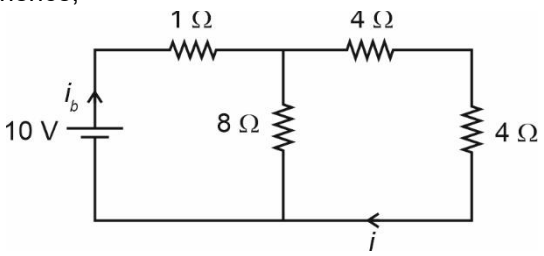


(1) $\frac{1}{2}$ (2) 0

(3) 2 (4) 1

Answer (4)

Sol. In steady state capacitor branch cannot conduct hence,



$i_b = \frac{10}{5} = 2$ A & $i = \frac{i_b}{2} = 1$ A

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A soap bubble of surface tension 0.04 N/m is blown to a diameter of 7 cm. If (15000 – x) μJ of work is done in blowing it further to make its diameter 14 cm, then the value of x is _____.
(x = 22/7)

Answer (200)

Sol. $\Delta U = \Delta W = T\left(8\pi r_f^2 - 8\pi r_i^2\right)$

ΔW = 3696 μJ

= (15000 – 11304) μJ

$\Rightarrow$ x = 11304

47. In a meter bridge experiment to determine the value of unknown resistance, first the resistances 2 Ω and 3 Ω are connected in the left and right gaps of the bridge and the null point is obtained at a distance l cm from the left. Now when an unknown resistance x Ω is connected in parallel to 3 Ω resistance, the null point is shifted by 10 cm to the right of wire. The value of unknown resistance x is _____ Ω.

Answer (6)

Sol. Given $\frac{2}{3} = \frac{l}{100-l}$

$\Rightarrow$ l = 40 cm

$\Rightarrow$ $\frac{2}{3x}(3+x) = \frac{l+10}{90-l} = \frac{50}{50} = 1$

x = 6 Ω

48. When 300 J of heat given to an ideal gas with $C_P = \frac{7}{2}R$ its temperature raises from 20 °C to 50 °C keeping its volume constant. The mass of the gas is (approximately) _____ g. (R = 8.314 J/mol.K)

Answer (BONUS)

Sol. $C_V = C_P - R = \frac{5}{2}R$

$\Delta Q_V = nCv\Delta T$

$\Rightarrow$ $n = \frac{300\times 2}{30\times 5R} = \frac{4}{R} \approx 0.481$

Mass = nM_0

Since M_0 is not given

49. A uniform solid cylinder of length L and radius R has moment of inertia about its axis equal to I_1. A small co-centric cylinder of length L/2 and radius R/3 carved from this cylinder has moment of inertia about its axis equals to I_2. The ratio I_1/I_2 is _____.

Answer (162)

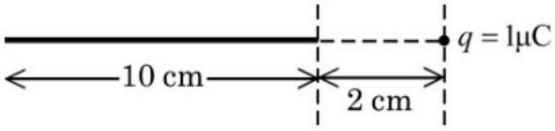
Sol. $I_1 = \frac{MR^2}{2}$

$I_2 = \frac{Mr^2}{2}$

$\frac{I_1}{I_2} = \frac{M}{m}\left(\frac{R}{r}\right)^2 = \frac{M}{m}\times 9 = \frac{V_1}{V_2}9$

$V_2 = \pi\left(\frac{R}{3}\right)^2\left(\frac{L}{2}\right) = \frac{V_1}{18}$

50. A point charge q = 1 μC is located at a distance 2 cm from one end of a thin insulating wire of length 10 cm having a charge Q = 24 μC, distributed uniformly along its length, as shown in figure. Force between q and wire is _____ N.



Answer (90)

Sol. $F = \frac{KQq}{\left(2\times 10^{-2}\right)\left[(2+10)\times 10^{-2}\right]}$

$F = \frac{9\times 10^9 \times 24\times 10^{-12+4}}{24} = 90$ N

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

51. The heat of atomisation of methane and ethane are 'x' kJ mol^{-1} and 'y' kJ mol^{-1} respectively. The longest wavelength (λ) of light capable of breaking the C – C bond can be expressed in SI unit as

(1) $\frac{hc}{1000}\left(\frac{y-6x}{4}\right)^{-1}$

(2) $\frac{N_A hc}{250(4y-6x)}$

(3) $N_A hc\left(y-\frac{6x}{4}\right)^{-1}$

(4) $\frac{N_A hc}{250(y-6x)}$

Answer (2)

Sol. $CH_4 \rightarrow C + 4H$ x kJ/mol

$C_2H_6 \rightarrow 2C + 6H$ y kJ/mol

$y = E_{C-C} + 6\times\frac{x}{4}$

$E_{C-C} = y - \frac{6x}{4}$ kJ/mol

So, $\frac{hc}{\lambda} = \frac{1}{N_A}\left(y-\frac{6x}{4}\right)\times 10^3$

$\lambda = \frac{4hc\,N_A}{4y-6x}\times\frac{1}{1000}$

$\lambda = \frac{hc\,N_A}{250(4y-6x)}$

52. Find out the statements which are **not** true.

A. Resonating structures with more number of covalent bonds and lesser charge separation are more stable.

B. In electromeric effect, an unsaturated system shows +E effect with nucleophile and –E effect with electrophile.

C. Inductive effect is responsible for high melting point, boiling point and dipole moment of polar compounds.

D. The greater the number of alkyl groups attached to the doubly bonded carbon atoms, higher is the heat of hydrogenation.

E. Stability of carbanion increases with the increase in s – character of the carbon carrying the negative charge.

Choose the **correct** answer from the options given below:

(1) B & D only

(2) A, D & E only

(3) B, D & E only

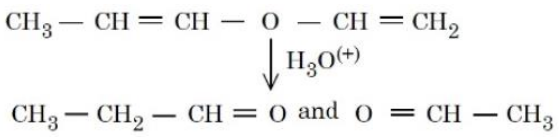
(4) A, C & D only

Answer (1)

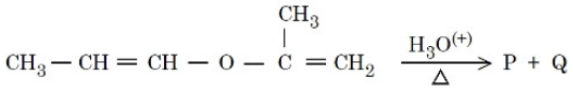
Sol. B. In electromeric effect, an unsaturated system shows –E effect with nucleophile and +E effect with electrophile.

D. The greater the number of alkyl groups attached to the doubly bonded carbon atoms, lower is the heat of hydrogenation.

53. The unsaturated ether on acidic hydrolysis produces carbonyl compounds as shown below:



Based on this, predict the solution/reagent that will help to distinguish "P" and "Q" obtained in the following reaction:



(1) Fehling solution

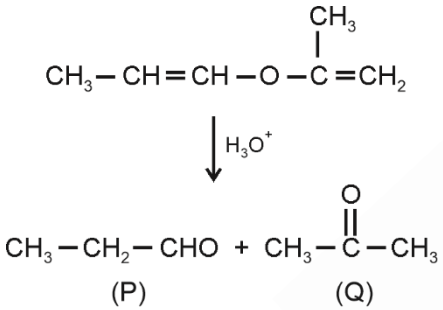
(2) Lucas reagent

(3) Saturated $NaHSO_3$ solution

(4) 2, 4 - DNP reagent

Answer (1)

Sol.



54. Given below are two statements:

Statement I: The dipole moment of R-CN is greater than R-NC and R-NC can undergo hydrolysis under acidic medium to produce $R-\overset{O}{\overset{\|}{C}}-OH$.

Statement II: R-CN hydrolyses under acidic medium to produce a compound which on treatment with $SOCl_2$, followed by the addition of NH_3 gives another compound (x). This compound (x) on treatment with $NaOCl/NaOH$ gives a product, that on treatment with $CHCl_3/KOH/\Delta$ produces R-NC.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

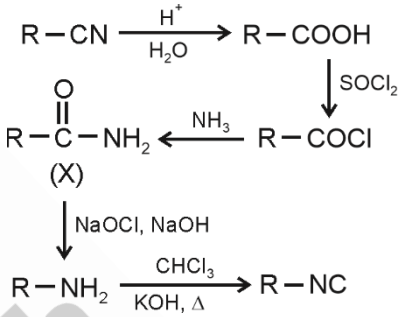
(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Answer (3)

Sol. (I) $R-NC \xrightarrow[H_2O]{H^+} R-NH_2 + HCOOH$

(II)



55. Given below are two statements:

Statement I: Cross aldol condensation between two different aldehydes will always produce four different products.

Statement II: When semicarbazide reacts with a mixture of benzaldehyde and acetophenone under optimum pH , it forms a condensation product with acetophenone only.

In the light of the above statements, choose the **correct** answer from the options given below

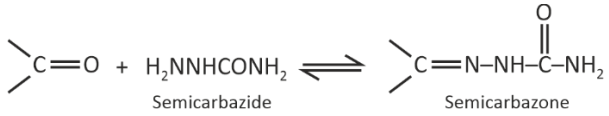
(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

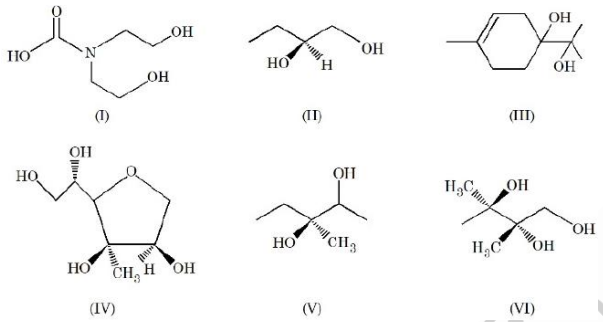
Answer (4)

Sol. 

$\therefore$ Both PhCHO and $PhCOCH_3$ reacts with semicarbazide.

If one aldehyde has no α-H and other with α-H are taken for aldol condensation number of products will not be four in that case this statement-I will be false

56. From the following, how many compounds contain at least one secondary alcohol?



Choose the **correct** answer from the options given below:

(1) Three

(2) Two

(3) Four

(4) Five

Answer (1)

Sol. Compounds having atleast one secondary alcohol.

II, IV, V

57. The wavelength of spectral line obtained in the spectrum of Li^{2+} ion when the transition takes place between two levels whose sum is 4 and difference is 2, is

(1) 2.28×10^{-6} cm (2) 1.14×10^{-6} cm

(3) 2.28×10^{-7} cm (4) 1.14×10^{-7} cm

Answer (2)

Sol. $n_1 + n_2 = 4$; $n_2 - n_1 = 2$

Hence, $n_2 = 3$, $n_1 = 1$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times 3^2 \times \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = 9.873 \times 10^7 \times \frac{8}{9}$$

$\lambda = 0.114 \times 10^{-7}$ m

$\lambda = 1.14 \times 10^{-8}$ m

$= 1.14 \times 10^{-6}$ cm

58. One mole of $Cl_2(g)$ was passed into 2 L of cold 2M KOH solution. After the reaction, the concentrations of Cl^-, ClO^- and OH^- are respectively (assume volume remains constant)

(1) 0.75M, 0.75M, 1M

(2) 0.5M, 0.5M, 1M

(3) 0.5M, 0.5M, 0.5M

(4) 1M, 1M, 1M

Answer (2)

Sol. $\underset{1\,mol}{Cl_2} + \underset{4\,mol}{2KOH} \rightarrow KCl + KClO + H_2O$

Here, Cl_2 is limiting reagent. Hence, 1 mole of KCl and KClO forms.

$$[KCl] = \frac{1}{2} = 0.5\,M$$

$$[KClO] = \frac{1}{2} = 0.5\,M$$

$$[KOH] = \frac{4-2}{2} = 1M$$

59. Pair of species among the following having same bond order as well as paramagnetic character will be

(1) O_2^+, N_2^-

(2) O_2^-, N_2^-

(3) O_2^-, N_2^+

(4) O_2^+, N_2^{2-}

Answer (1)

Sol. $O_2^+(15e) \rightarrow$ Paramagnetic.

$$\text{Bond order} = \frac{\text{Bonding e} - \text{Antibonding e}}{2}$$

$$= \frac{(10-5)}{2} = 2.5$$

$N_2^-(15e) \rightarrow$ Paramagnetic.

$$\text{Bond order} = \frac{(10-5)}{2} = 2.5$$

$N_2^+(13e) \rightarrow$ Paramagnetic.

$$\text{Bond order} = \frac{(9-4)}{2} = 2.5$$

$O_2^-(17e) \rightarrow$ Paramagnetic.

$$\text{Bond order} = \frac{(10-7)}{2} = 1.5$$

$N_2^{2-}(16e) \rightarrow$ Paramagnetic.

$$\text{Bond order} = \frac{(10-6)}{2} = 2$$

60. Consider the following gaseous equilibrium in a closed container of volume ' V ' at $T(K)$.

$$P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$$

2 moles each of $P_2(g), Q_2(g)$ and $PQ(g)$ are present at equilibrium. Now one mole each of 'P_2' and 'Q_2' are added to the equilibrium keeping the temperature at $T(K)$. The number of moles of P_2, Q_2 and PQ at the new equilibrium, respectively, are

(1) 1.66, 1.66, 1.66

(2) 2.67, 2.67, 2.67

(3) 2.56, 1.62, 2.24

(4) 1.21, 2.24, 1.56

Answer (2)

Sol.

	P_2	+	Q_2	$\rightleftharpoons$	2PQ
$t = t_{eq}$	2 mol		2 mol		2 mol
$t = t'_{eq}$	$3-x$		$3-x$		$2+2x$

$$\therefore \quad K_{eq} = \frac{(2)^2}{2\times 2} = 1$$

$$\therefore \quad 1 = \frac{(2+2x)^2}{(3-x)^2} \Rightarrow \frac{2+2x}{3-x} = 1$$

$$\Rightarrow \quad 2+2x = 3-x$$

$$\Rightarrow \quad 3x = 1$$

$$\Rightarrow \quad x = \frac{1}{3}$$

$$\therefore \quad n_{P_2} = 3-x = 3-\frac{1}{3} = \frac{8}{3} = 2.67$$

$$\therefore \quad n_{Q_2} = 3-x = \frac{8}{3} = 2.67$$

$$\therefore \quad n_{PQ} = 2+2X = 2+\frac{2}{3} = \frac{8}{3} = 2.67$$

61. In the Group analysis of cations, Ba^{2+} & Ca^{2+} are precipitated respectively as

(1) sulphide & sulphide

(2) carbonate & carbonate

(3) chromate & sulphide

(4) hydroxide & carbonate

Answer (2)

Sol. Group V cations precipitate as carbonates

(Reagent – $NH_4Cl + (NH_4)_2CO_3$)

62. Choose the **INCORRECT** statement

(1) Carbon cannot exceed its covalency more than four

(2) CO_2 is the most acidic oxide among the dioxides of group of 14 elements.

(3) Among the isotopes of carbon, ^{13}C is a radioactive isotope.

(4) Carbon exhibits negative oxidation states along with +4 and +2.

Answer (3)

Sol. C-13 isotope is not a radioactive isotope.

63. At 298 K, the mole percentage of $N_2(g)$ in air is 80%. Water is in equilibrium with air at a pressure of 10 atm. What is the mole fraction of $N_2(g)$ in water at 298 K? (K_H for N_2 is 6.5×10^7 mm Hg)

(1) 1.23×10^{-7} (2) 9.35×10^{5}

(3) 1.17×10^{-4} (4) 9.35×10^{-5}

Answer (4)

Sol. $Y_{N_2} = 0.8 \qquad Y_{H_2O} = 0.2$

P_{Total} = 10 atm

$$P_{N_2} = K_H \cdot \chi_{N_2} = P_{Total} \cdot Y_{N_2}$$

$$\frac{6.5\times10^7}{760} \times \chi_{N_2} = 10 \times 0.8$$

$$\chi_{N_2} = 9.35 \times 10^{-5}$$

64. The wavelength of light absorbed for the following complexes are in the order

$\underset{(I)}{\left[Co(NH_3)_6\right]^{3+}}$; $\underset{(II)}{\left[Co(H_2O)_6\right]^{3+}}$; $\underset{(III)}{\left[Co(CN)_6\right]^{3-}}$;

$\underset{(IV)}{\left[Co(NH_3)_5(H_2O)\right]^{3+}}$; $\underset{(V)}{\left[CoF_6\right]^{3-}}$

(1) III < I < IV < II < V (2) III < IV < I < II < V

(3) III < I < IV < V < II (4) III < I < II < IV < V

Answer (1)

Sol. As all the complexes have Co^{3+} ion hence Δ_o will depend on crystal field of the ligand.

Crystal field strength of

$CN^- > NH_3 > H_2O > F^-$

$\because \Delta_o \uparrow \Rightarrow \lambda_{max} \downarrow$

So, order is : III < I < IV < II < V

65. The number of possible tripeptides formed involving alanine (ala), glycine (gly) and valine (val), where no amino acids has been used more than once is:

(1) 4 (2) 6

(3) 8 (4) 3

Answer (2)

Sol. Gly – Ala – Val

Gly – Val – Ala

Ala – Gly – Val

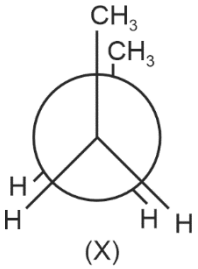
Ala – Val – Gly

Val – Gly – Ala

Val – Ala – Gly

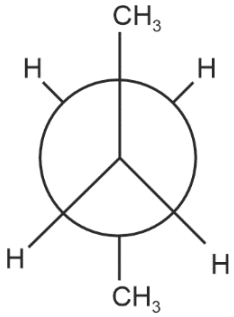
66. Given below are two statements:

Statement I: There are several conformers for n-butane. Out of those conformers,



(X)

is the least stable and most stable conformer is



(Y)

Statement II: As the dihedral angle increases, torsional strain decreases from (X) to (Y).

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is true but statement II is false

(2) Both statement I and statement II are true

(3) Statement I is false but statement II is true

(4) Both statement I and statement II are false

Answer (2)

Sol. Anti form is more stable than fully eclipsed form.

Torsional strain deceases with increase in dihedral angle from (X) to (Y)

67. A student has planned to prepare acetanilide from aniline using acetic anhydride.

The student has started from 9.3 g of aniline. However, the student has managed to obtain 11 g of dry acetanilide.

The % yield of this reaction is

(1) 97.5% (2) 59.5%

(3) 81.5% (4) 72.5%

Answer (3)

Sol. Moles of aniline = $\frac{9.3}{93} = 0.1$

Moles of 'P' formed = 0.1 (if 100% yield)

Actual moles of 'P' formed = $\frac{11}{135} = 0.0814$

% yield = $\frac{0.0814}{0.1} \times 100 = 81.48\% = 81.5\%$

68. "X" is an oxoanion of the lightest element of group 7 (in the periodic table). The metal is in +6 oxidation state in "X". The color of the potassium salt of X is

(1) Green (2) Yellow

(3) Orange (4) Purple

Answer (1)

Sol. The lightest element of group 7 = Mn

The oxoanion is : K_2MnO_4 which is green.

69. The correct order of C, N, O and F in terms of second ionisation potential is

(1) $F < N < C < O$ (2) $C < F < N < O$

(3) $C < O < N < F$ (4) $C < N < F < O$

Answer (4)

Sol. $O \xrightarrow{-e^-} O^-(2p^3)$

$C \xrightarrow{-e^-} C^+(2p^1)$

$N \xrightarrow{-e^-} N^+(2p^2)$

$F \xrightarrow{-e^-} F^+(2p^4)$

$IE = C^+ < N^+ < F^+ < O^+$

$IE_2 = C < N < F < O$

70. Two liquids A and B form an ideal solution at temperature T K. At T K, the vapour pressures of pure A and B are 55 and 15 k Nm^{-2} respectively. What is the mole fraction of A in solution of A and B in equilibrium with a vapour in which the mole fraction of A is 0.8?

(1) 0.480

(2) 0.663

(3) 0.5217

(4) 0.340

Answer (3)

Sol. $P_A^o = 55\ kN/m^2$

$P_B^o = 15\ kN/m^2$

$Y_A = 0.8$

$\chi_A = ?$

$P_{Total} = 55 \cdot \chi_A + 15 \times (1 - \chi_A)$

$= 40\ \chi_A + 15$

$P_A = P_A^o \cdot \chi_A = P_{Total} \times Y_A$

$\Rightarrow 55\ \chi_A = (40\ \chi_A + 15)0.8$

$\Rightarrow 55\ \chi_A = 32\ \chi_A + 12$

$23\ \chi_A = 12$

$\chi_A = 0.5217$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The half-life of ^{65}Zn is 245 days. After x days, 75% of original activity remained. The value of x in days is _____. (Nearest integer)

(Given: $\log 3 = 0.4771$ and $\log 2 = 0.3010$)

Answer (102)

Sol. $t_{25\%} = \frac{1}{k}\ln\frac{100}{75} = \frac{245}{\ln 2} \times (\ln 4 - \ln 3)$

$= \frac{245 \times (2 \times \log 2 - \log 3)}{\log 2}$

$t_{25\%} = 101.66$ days

72. Grignard reagent RMgBr(P) reacts with water and forms a gas (Q). One gram of Q occupies 1.4 dm^3 at STP. (P) on reaction with dry ice in dry ether followed by H_3O^+ forms a compound (Z). 0.1 mole of (Z) will weigh _____ g. (Nearest integer)

Answer (6)

Sol. $\underset{(P)}{RMgBr} \xrightarrow{H_2O} \underset{(Q)}{R-H}$

1 g of (Q) will occupy 1.4 L at STP

So, $\frac{1}{M_Q} \times 22.4 = 1.4$

$M_Q = 16$

The RMgBr is CH_3MgBr

$CH_3MgBr \xrightarrow[(ii)\ H^+]{(i)\ CO_2} \underset{(Z)}{CH_3 - COOH}$

Molar mass of (Z) = 60

0.1 mole of (Z) = 6 g

73. A chromium complex with a formula $CrCl_3 \cdot 6H_2O$ has a spin only magnetic moment value of 3.87 BM and its solution conductivity corresponds to 1 : 2 electrolyte. 2.75 g of the complex solution was initially passed through a cation exchanger. The solution obtained after the process was reacted with excess of $AgNO_3$. The amount of AgCl formed in the above process is _____ g. (Nearest integer)
[Given: Molar mass in g mol^{-1} Cr : 52; Cl : 35.5, Ag : 108, O : 16, H : 1]

Answer (3)

Sol. μ = 3.87 BM

$\Rightarrow$ Number of unpaired e^- = 3

and it corresponds to 1 : 2 electrolyte means the complex is : $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$

Moles of complex $= \frac{2.75}{266.5} = 0.01$

Mass of AgCl formed = 0.01 × 2 × 143.5 = 2.87 g

74. Molar conductivity of a weak acid HQ of concentration 0.18 M was found to be 1/30 of the molar conductivity of another weak acid HZ with concentration of 0.02 M. If $\lambda^\circ_{Q^-}$ happened to be equal with $\lambda^\circ_{Z^-}$, then the difference of the pK_a values of the two weak acids (pK_a(HQ) – pK_a(HZ)) is _____ (Nearest integer).
[Given: degree of dissociation (α) << 1 for both weak acids, λ° : limiting molar conductivity of ions]

Answer (2)

Sol. $\lambda^\infty_m Q^\ominus = \lambda^\infty_m Z^\ominus$, so $A^\infty_m HQ = \Lambda^\infty_m HZ$ (Let it Y)

For 0.02 M Hz, let $\Lambda_m = x$

Then for 0.18 M HQ, $\Lambda_m = \frac{x}{30}$

For HQ

$\Rightarrow Ka_1 = C \times \alpha^2 = 0.18 \times \left(\frac{x}{30Y}\right)^2$ ...(I) {α <<1}

For HZ

$\Rightarrow Ka_2 = C \times \alpha^2 = 0.02 \times \left(\frac{x}{Y}\right)^2$ ...(II) {α <<1}

$pKa_1 - pKa_2 = -\log\left(\frac{1}{100}\right) = 2$

So, pKa(HQ) – pKa(Hz) = 2

75. 0.25 g of an organic compound "A" containing carbon, hydrogen and oxygen was analysed using the combustion method. There was an increase in mass of $CaCl_2$ tube and potash tube at the end of the experiment. The amount was found to be 0.15 g and 0.1837 g, respectively. The percentage of oxygen in compound A is _____%. (Nearest integer)
(Given: molar mass in g mol^{-1} H : 1, C : 12, O : 16)

Answer (73)

Sol. Mass of H_2O = Mass gain of $CaCl_2$
= 0.15 g

Moles of $H_2O = \frac{0.15}{18}$ moles

Mass of CO_2 = Mass gain by potash = 0.1837 g

Moles of $CO_2 = \frac{0.1837}{44}$ moles

Mass of C in compound $= n_{CO_2} \times 12 = 0.0501$ g

Mass of H in compound $= n_{H_2O} \times 2 = 0.0167$ g

Mass of O in compound = 0.25 × 0.0501 – 0.0167
= 0.183 g

Mass % of $O = \frac{0.183}{0.25} \times 100 = 73.3\%$

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