Aakash
Medical | IIT-JEE | Foundations

Phase-2
2nd April 2026
(Evening Shift)

Answer & Solutions

for

JEE MAIN 2026

TRUST Aakash

Stellar Ranks in JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR 23

Kushagra Baingaha AIR 7

Shreyas Lohiya AIR 6

A Gupta AIR 15

Devya Rustagi AIR 28

Shreyas Lohiya AIR 68

Rujul Garg AIR 41

Advay Mayank AIR 36

Arush Anand AIR 64

Kotha D Reddy AIR 74

Top Ranks in JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students **Aakashians Qualified in NSEs 2025-26**

777 Classroom Students **Aakashians Qualified in IOQM 2025**

134 Classroom Students **Aakashians Qualified in RMO 2025**

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let α, β be the roots of the equation $x^2 - 3x + r = 0$, and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 + 3x + r = 0$.

   If the roots of the equation $x^2 + 6x = m$ are $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$, then m is equal to:

   (1) –135 (2) –567

   (3) 135 (4) 567

Answer (4)

Sol. $\because \alpha, \beta$ are roots of $x^2 - 3x + r = 0$.

$\therefore \alpha + \beta = 3$ and $\alpha \cdot \beta = r$ and $\frac{\alpha}{2}, 2\beta$ are roots of

$x^2 + 3x + r = 0$

$\therefore \frac{\alpha}{2} + 2\beta = -3$ and $\alpha \cdot \beta = r$

$\therefore \alpha = 6, \beta = -3$ and $r = -18$

$\therefore m = -\left(2\alpha + \beta + 2r\right)\left(\alpha - 2\beta - \frac{r}{2}\right)$

$= -(12 - 3 - 36)(6 + 6 + 9) = 567$

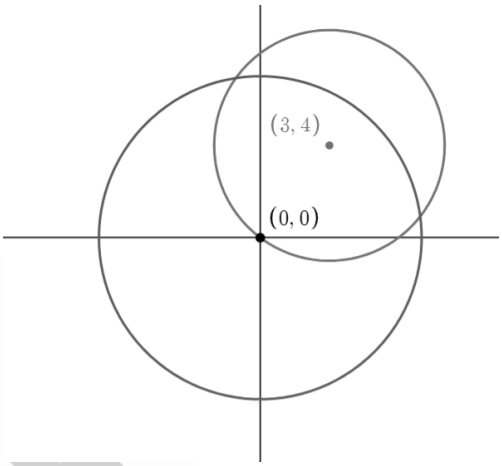
2. Let the circles $C_1 : |z| = r$ and $C_2 : |z - 3 - 4i| = 5, z \in \mathbb{C}$, be such that C_2 lies within C_1. If z_1 moves on C_1, z_2 moves on C_2 and $\min|z_1 - z_2| = 2$, then $\max|z_1 - z_2|$ is equal to

   (1) 12 (2) 17

   (3) 22 (4) 24

Answer (1)

Sol. $C_1 : |z| = r$ and $C_2 : |z - 3 - 4i| = 5$



$\min|z_1 - z_2| = 2$

$\therefore \quad r - 5 = 2$

$\therefore \quad r = 7$

Thus $\max \cdot |z_1 - z_2| = 7 + 5$

= 12 units

3. If the system of equations

   $x + 5y + 6z = 4,$

   $2x + 3y + 4z = 7,$

   $x + 6y + az = b$

   has infinitely many solutions, then the point (a, b) lies on the line

   (1) $y - x = 3$ (2) $x - y = 3$

   (3) $x + y = 11$ (4) $x + y = 12$

Answer (2)

Sol. The system of equations:

$$x+5y+6z=4$$
$$2x+3y+4z=7$$
$$\text{and } x+6y+az=b$$

For infinite many solutions $D=D_x=D_y=D_z=0$

$$\therefore D=\begin{vmatrix}1&5&6\\2&3&4\\1&6&a\end{vmatrix}=0$$

$$\therefore 3a-24-10a+20+54=0$$

$$\therefore a=\frac{50}{7}$$

$$\text{and } D_2=\begin{vmatrix}1&5&4\\2&3&7\\1&6&b\end{vmatrix}=0$$

$$\therefore 3b-42-5(2b-7)+4(12-3)=0$$

$$\therefore b=\frac{29}{7}$$

Here $a-b=3$.

$\therefore$ Point (a, b) lies on the line $x-y=3$

4. Let $a_1, a_2, a_3, \ldots$ be an A.P. and $g_1=a_1, g_2, g_3, \ldots$ be an increasing G.P. If $a_1=a_2+g_2=1$ and $a_3+g_3=4$, then $a_{10}+g_5$ is equal to:

(1) 81 (2) 76

(3) 62 (4) 55

Answer (4)

Sol. Let $a_2-a_1=d$ and $\frac{g_3}{g_2}=r$

$a_1 = 1 = a$, $a_2+g_2=a+d+ar=1$

$d=-br$

$a_3+g_3=(a+2d)+ar^2=4$

$\Rightarrow \quad 1+2d+ar^2=4$

$\Rightarrow \quad ar^2-2ar=3$

$r^2-2r-3=0$

$(r-3)(r+1)=0$

$r=3$

$d=-3$

$a_{10}+g_5=(a+9d)+(ar^4)$

$= 1+9(-3)+(3^4)$

$= 1-27+81$

$= 82-27=55$

5. The sum $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\cdots$ up to 8 terms, is :

(1) 70 (2) 71

(3) 72 (4) 73

Answer (2)

Sol. $T_n=\frac{1^3+2^3+\ldots+n^3}{1+3+5+\ldots+(2n-1)}$

$$T_n=\frac{\left(\frac{n(n+1)}{2}\right)^2}{n^2}$$

$$T_n=\frac{(n+1)^2}{4}=\frac{n^2+2n+1}{4}$$

$$S_8=\sum_{n=1}^{8}\frac{n^2+2n+1}{4}$$

$$=\frac{1}{4}\left[\sum_{n=1}^{8}n^2+2\sum_{n=1}^{n}n+8\right]$$

$$=\frac{1}{4}\left[\frac{8\times9\times17}{6}+2\times\frac{8\times9}{2}+8\right]$$

$$=\frac{1}{4}[204+72+8]$$

$$S_8=71$$

6. If for $3 \leqslant r \leqslant 30, \left({}^{30}C_{30-r}\right) + 3\left({}^{30}C_{31-r}\right) + 3\left({}^{30}C_{32-r}\right) + \left({}^{30}C_{33-r}\right) = {}^{m}C_{r}$, then m equals :

(1) 31 (2) 32

(3) 33 (4) 34

Answer (3)

Sol. $3 \le r \le 30$

$\because \quad {}^{30}C_{30-r} = {}^{30}C_{r}$

${}^{30}C_{31-r} = {}^{30}C_{r-1}$

${}^{30}C_{32-r} = {}^{30}C_{r-2}$

${}^{30}C_{33-r} = {}^{30}C_{r-3}$

$\therefore \quad {}^{30}C_{r} + 3\left({}^{30}C_{r-1}\right) + 3\left({}^{30}C_{r-2}\right) + {}^{30}C_{r-3}$

$= {}^{30}C_{r} + {}^{30}C_{r-1} + 2\left({}^{30}C_{r-1} + {}^{30}C_{r-2}\right)$

${}^{30}C_{r-2} + {}^{30}C_{r-3}$

$= {}^{31}C_{r} + 2\left({}^{31}C_{r-1}\right) + {}^{31}C_{r-2}$

$= {}^{31}C_{r} + {}^{31}C_{r-1} + {}^{31}C_{r-1} + {}^{31}C_{r-2}$

$= {}^{32}C_{r} + {}^{32}C_{r-1}$

$= {}^{33}C_{r}$

on comparing with ${}^{m}C_{r}$

$\Rightarrow m = 33$

7. Let p_n denote the total number of triangles formed by joining the vertices of an n-side regular polygon. If $p_{n+1} - p_n = 66$, then the sum of all distinct prime divisors of n is :

(1) 7 (2) 8

(3) 5 (4) 6

Answer (4)

Sol. $p_n = {}^{n}C_{3}$

$p_{n+1} - p_n = {}^{n+1}C_{3} - {}^{n}C_{3}$

$\Rightarrow \quad {}^{n+1}C_{3} - {}^{n}C_{3} = 66 \qquad \ldots(i)$

$\therefore \quad {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

$\Rightarrow \quad {}^{n+1}C_{3} - {}^{n}C_{3} = {}^{n}C_{2} \qquad \ldots(2)$

From (1) and (2)

${}^{n}C_{2} = 66$

$\frac{n(n-1)}{2} = 66$

$n^2 - n - 132 = 0$

$(n-12)(n+11) = 0$

$\Rightarrow \quad n = 12$ [as n represents the numbers of sides of a polygon]

$12 = 2^2 \times 3$

Distinct prime divisors are 2 and 3

$\therefore$ Sum of prime divisors = 5.

8. A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}, \gcd(m,n) = 1$, then $m+n$ is equal to:

(1) 53

(2) 55

(3) 107

(4) 105

Answer (3)

Sol. Let E_1 represent the event that a head appears and E_1 represent the event that a tail appears.

$\therefore$ $10\,E_1 + 5E_2 = 30$

$\therefore$ $(E_1, E_2) = (0,6), (1,4), (2,2)$ or $(3,0)$

$\because P(E_1) = P(E_2) = \frac{1}{2}$

$\therefore$ Required probability

$$= {}^6C_0\left(\frac{1}{2}\right)^6 + {}^5C_1\left(\frac{1}{2}\right)^5 + {}^4C_2\left(\frac{1}{2}\right)^4 + {}^3C_3\left(\frac{1}{2}\right)^3$$

$$= \frac{1}{64} + \frac{5}{32} + \frac{6}{16} + \frac{1}{8} = \frac{43}{64} = \frac{m}{n}$$

$\therefore$ $m + n = 107$

9. The mean and variance of n observations are 8 and 16, respectively. If the sum of the first $(n - 1)$ observations is 48 and the sum of squares of the first $(n - 1)$ observations is 496, then the value of n is:

(1) 21 (2) 16

(3) 13 (4) 7

Answer (4)

Sol. Let the observations are $x_1, x_2, \ldots, x_n$.

$\therefore$ $\sum_{i=1}^{n} x_i = 8n$ ...(i)

and $\sum_{i=1}^{n}(x_i - 8)^2 = 16n$ ...(ii)

and $\sum_{i=1}^{n-1} x_i = 48$ and $\sum_{i=1}^{n-1} x_i^2 = 496$

$\therefore$ from equation (ii), $\sum_{i=1}^{n} x_i^2 - 16\sum_{i=1}^{n} x_i + 64n = 16n$

$\therefore$ $496 + x_n^2 - 16(48 + x_n) + 48n = 0$

$\therefore$ $x_n^2 - 16n + 48n - 272 = 0$ ...(iii)

From equation (i) : $48 + x_n = 8n$

$\therefore$ $48n = 288 + 6x_n$ ...(iv)

From (iii) and (iv) : $x_n^2 - 10x_n + 16 = 0$

$\therefore$ $x_n = 8$ or 2

$x_n = 2$ is not possible

and for $x_n = 8$, $n = 7$

10. Let a circle pass through the origin and its centre be the point of intersection of two mutually perpendicular lines $x + (k - 1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the point A and B, then $(AB)^2$ is equal to

(1) 10 (2) 27

(3) 18 (4) 34

Answer (3)

Sol. $x + (k-1)y + 3 = 0$

$2x + k^2y - 4 = 0$

$\because$ there 2 lines are perpendicular

$$\frac{1}{1-k} - \left(\frac{2}{k^2}\right) = -1$$

$$2 = k^2(1-k)$$

$$\Rightarrow k^3 - k^2 + 2 = 0$$

$$\Rightarrow k = -1$$

$\therefore$ Lines are $\left.\begin{matrix} x - 2y + 3 = 0 \\ 2x + y - 4 = 0 \end{matrix}\right\} x = 1 \quad y = 2$

$\therefore$ Centre is (1, 2)

Circle will be $x^2 + y^2 - 2x - 4y = 0$

Line $x - y + 2 = 0$ will intersect at $A(-1, 1)$ and $B(2, 4)$

$\therefore$ $(AB)^2 = (2 + 1)^2 + (4 - 1)^2$

$= 9 + 9 = 18$

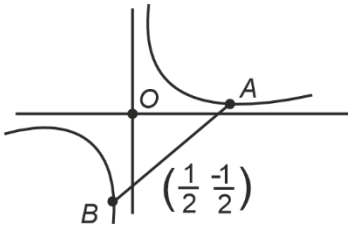
11. Let O be the origin, and P and Q be two points on the rectangular hyperbola $xy = 12$ such that the mid point of the line segment PQ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$. Then the area of the triangle OPQ equals:

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{7}{2}$ (4) $\frac{9}{2}$

Answer (3)

Sol.



$xy = 12$, for chord with mid point

$T = S_1$

$$\Rightarrow \frac{xy_1 + yx_1}{2} - 12 = x_1y_1 - 12$$

$$\Rightarrow \frac{x\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)}{2} = \frac{-1}{4}$$

$\Rightarrow y - x + 1 = 0$

$$xy = 12 \Rightarrow y = x - 1 = \frac{12}{x}$$

$\Rightarrow x^2 - x + 12 = 0$

$(x - 7)(x + 3) = 0$

$x = -3, 4$

$\Rightarrow$ Point A and B are $(-3, -4)$ and $(4, 3)$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -3 & -4 & 1 \\ 4 & 3 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}(-9 + 16) = \frac{7}{2}$$

12. Let the parabola $y = x^2 + px + q$ passing through the point $(1, -1)$ be such that the distance between its vertex and the x-axis is minimum. Then the value of $p^2 + q^2$ is:

(1) 2 (2) 4

(3) 5 (4) 8

Answer (2)

Sol. $-1 = 1 + p + q \Rightarrow p + q = -2$

vertex is $\left(-\frac{p}{2}, -\left(\frac{p^2 - 4q}{4}\right)\right)$

$$= \left(-\frac{p}{2}, \frac{4q - p^2}{4}\right)$$

To minimise:

$$\frac{4q - p^2}{4} = \frac{4q - (-2 - q)^2}{4}$$

$$= \frac{4q - (q^2 + 4q + 4)}{4}$$

$$= \frac{-q^2 - 4}{4}$$

$$\text{distance} = \left|\frac{-q^2 - 4}{4}\right| = \frac{q^2 + 4}{4} \Rightarrow q = 0$$

and $p = -2$

$\Rightarrow p^2 + q^2 = 4$

13. Let $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1\}$ and

$S = \{a \in \mathbf{Z} : 2(\cos^8\theta - \sin^8\theta)\sec 2\theta = a^2, \theta \in P\}$.

Then $n(S)$ is :

(1) 0 (2) 1

(3) 2 (4) 3

Answer (1)

Sol. $a^2 = 2(\cos^8\theta - \sin^8\theta)\sec 2\theta$

$= 2(\cos^4\theta + \sin^4\theta)(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)\sec 2\theta$

$= 2(\cos^4\theta + \sin^4\theta)$

$= 2[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$= 2 - \sin^2 2\theta$

$\because a \in Z \Rightarrow a^2 \in \{0, 1, 4, 9, ...\}$

$\Rightarrow (2 - \sin^2 2\theta) \in \{0, 1, 4, 9...\}$

$\Rightarrow \sin^2 2\theta = 1$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{2}, n \in Z$

$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{4}, n \in Z$

When $\theta = \frac{n\pi}{2} \pm \frac{\pi}{4}, n \in Z$, then $\tan^2\theta = 1$

$\because \tan^2\theta \neq 1$

$\Rightarrow P$ is null set

$\Rightarrow n(P) = 0$

14. Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some $\lambda, \mu \in \mathbf{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$. If $\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is equal to :

(1) 8 (2) 12

(3) 14 (4) 15

Answer (2)

Sol. Let $\vec{x} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\vec{a} \cdot \vec{x} = -3 - 6 + 6 = -3$

$\vec{b} \cdot \vec{x} = 3 - 18 + 2 = -13$

Let $\vec{y} = \hat{i} + \hat{j} + \hat{k}$

$\vec{a} \cdot \vec{x} = 3$

$\vec{b} \cdot \vec{x} = 5$

$\vec{c} \cdot \vec{x} = 10$ ...(given)

$(\lambda a + \mu\vec{b}) \cdot \vec{x} = 10$

$\Rightarrow -3\lambda - 13\mu = 10$ ...(i)

$\vec{c} \cdot \vec{y} = -2$

$\Rightarrow 3\lambda + 5\mu = -2$ ...(ii)

Solving (i) and (ii)

$\lambda = 1, \mu = -1$

$\vec{c} = \vec{a} - \vec{b}$

$= -2\hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{c}|^2 = (\sqrt{12})^2 = 12$

15. Let the point A be the foot of perpendicular drawn from the point $P(a, b, 0)$ on the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3}$. If the midpoint of the line segment PA is $\left(0, \frac{3}{4}, \frac{-1}{4}\right)$, then the value of $a^2 + b^2 + \alpha^2$ is equal to :

(1) 1 (2) 2

(3) 6 (4) 9

Answer (1)

Sol. A: $(2k + 1, 2 + k, \alpha + 3k)$

Mid point M :

$\left(\frac{2k+1+a}{2}, \frac{2+k+b}{2}, \frac{\alpha+3k}{2}\right) \equiv \left(0, \frac{3}{4}, -\frac{1}{4}\right)$

$\therefore$ $a + 1 + 2k = 0$ ...(i)

$6 + k + 2 = \frac{3}{2}$ ...(ii)

$\alpha + 3k = \frac{-1}{2}$ ...(iii)

$\overrightarrow{PA} \perp 2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{PA} = (2k+1)\hat{i} + \left(2 + k - \frac{3}{4}\right)\hat{j} + \left(\alpha + 3k + \frac{1}{4}\right)\hat{k}$

$\therefore \quad 4k + 2 + 2 + k - \frac{3}{4} + 3\alpha + 9k + \frac{3}{4} = 0$

$14k + 4 + 3\alpha = 0$

$14k + 4 + 3\left(-\frac{1}{2} - 3k\right) = 0$

$\Rightarrow \quad 5k + 4 - \frac{3}{2} = 0$

$\Rightarrow \quad 10k + 5 = 0 \Rightarrow k = -\frac{1}{2} \Rightarrow \alpha = 1 \Rightarrow a = 0, b = 0$

$a^2 + b^2 + \alpha^2 = 1$

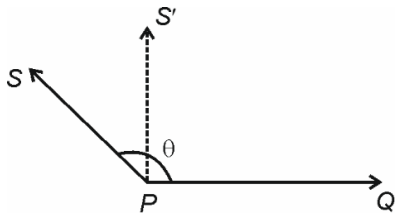
16. Two adjacent sides of a parallelogram $PQRS$ are given by $\overrightarrow{PQ} = \hat{j} + \hat{k}$ and $\overrightarrow{PS} = \hat{i} - \hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$ is equal to:

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{4}$ (4) $\frac{2\sqrt{3}}{5}$

Answer (2)

Sol. $\theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120° = \frac{2\pi}{3}$



$PS' \perp PQ$

$\frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$

$\therefore \quad \alpha = \frac{\pi}{6}$

$\sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$

$= \sin(3\alpha)\cdot\sin(2\alpha)$

$= \sin\left(\frac{\pi}{2}\right)\cdot\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

17. The value of $\int_0^{20\pi} (\sin^4 x + \cos^4 x)dx$ is equal to

(1) $\frac{15\pi}{2}$

(2) 25π

(3) 15π

(4) $\frac{25\pi}{2}$

Answer (3)

Sol. $\int_0^{20\pi} (\sin^4 x + \cos^4 x)dx$

$= 40\int_0^{\pi/2} \left(1 - \frac{1}{2}(\sin 2x)^2\right)dx$

$= 40\int_0^{\pi/2} \left[1 - \frac{1}{2}\left(\frac{1-\cos 4x}{2}\right)\right]dx$

$= 40\int_0^{\pi/2} \left(\frac{3}{4} + \frac{1}{4}\cos 4x\right)dx$

$= 40\left(\frac{3}{4}(\pi/2) + \frac{1}{4}(0)\right) = 15\pi$

18. Let $f(x)$ be a polynomial of degree 5, and have extrema at $x = 1$ and $x = -1$. If $\lim\limits_{x\to 0}\left(\frac{f(x)}{x^3}\right) = -5$, then $f(2) - f(-2)$ is equal to :

(1) 0

(2) 50

(3) 92

(4) 112

Answer (4)

Sol. $f'(x) = (x-1)(x+1)(ax^2+bx+c)$

$$f'(x) = (x^2-1)(ax^2+bx+c)$$

$$f'(x) = ax^4+bx^3+cx^2-ax^2-bx-c$$

$$f'(x) = ax^4+bx^3+(c-a)x^2-bx-c$$

$$f(x) = \frac{ax^5}{5}+\frac{bx^4}{4}+\frac{(c-a)x^3}{3}-\frac{bx^2}{2}-cx+d$$

$\lim\limits_{x\to 0}\frac{f(x)}{x^3} = -5$ Then $b = c = d = 0$

and $\frac{c-a}{3} = -5$

$a = 15$

$f(x) = 3x^5 - 5x^3$

$f(x) - f(-x) = f(x) + f(x)$

$= 2f(x)$

$= 2[3(2)^5 - 5(2)^3]$

$= 112$

19. Let $f(x) = \int\left(\frac{16x+24}{x^2+2x-15}\right)dx$. If $f(4) = 14\log_e(3)$ and $f(7) = \log_e(2^\alpha\cdot 3^\beta)$, $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to

(1) 31 (2) 37

(3) 39 (4) 41

Answer (3)

Sol. $f(x) = \int\left(\frac{16x+24}{x^2+2x-15}\right)dx$

$$= \int\frac{16(x+1)+8}{x^2+2x-15}dx$$

$$= 8\int\frac{2(x+1)}{x^2+2x-15}dx + 8\int\frac{1}{(x+1)^2-4^2}dx$$

$$= 8\ln(|x^2+2x-15|) + 8\times\frac{1}{2\times 4}\ln\left(\left|\frac{x+1-4}{x+1+4}\right|\right)+c$$

$$= 8\ln(|x^2+2x-15|) + \ln\left(\left|\frac{x-3}{x+5}\right|\right)+c$$

$$= 7\ln(|x+5|) + 9\ln(|x-3|)+c$$

$\because\ f(4) = 14\ln 3 \Rightarrow c = 0$

$\Rightarrow\ f(x) = 7\ln(|x+5|) + 9\ln(|x-3|)$

$\Rightarrow\ f(7) = \ln(2^{32}\cdot 3^7) \Rightarrow \alpha+\beta = 39$

Option (3) is correct

20. Let $x = x(y)$ be the solution of the differential equation $2y^2\frac{dx}{dy} - 2xy + x^2 = 0$, $y > 1$, $x(e) = e$. Then $x(e^2)$ is equal to

(1) $\frac{3}{2}e^2$

(2) $\frac{2}{3}e^2$

(3) e^2

(4) $2e^2$

Answer (2)

Sol. $\frac{dx}{dy} = \frac{2xy-x^2}{2y^2}$

Put $x = ty \Rightarrow \frac{dx}{dy} = t + y\frac{dt}{dy}$

$$\Rightarrow\ t + y\frac{dt}{dy} = \frac{2t-t^2}{2} \Rightarrow \frac{dy}{y} = -\frac{2dt}{t^2}$$

$\Rightarrow \ln|y| = \frac{2}{t} + c$

$\Rightarrow \ln|y| = \frac{2y}{x} + c, \because x(e) = e \Rightarrow c = -1$

$\Rightarrow \ln|y| = \frac{2y}{x} - 1 \Rightarrow x(e^2) = \frac{2e^2}{3}$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $A = \{2, 3, 4, 5, 6\}$. Let R be a relation on the set $A \times A$ given by $(x, y)R(z, w)$ if and only if x divides z and $y \le w$. Then the number of elements in R is ____ .

Answer (120)

Sol. The conditions $x \mid z$ and $y \le w$ are independent.

- pairs such that $x \mid z$ in $A = \{2, 3, 4, 5, 6\}$

$x = 2,\ z \in \{2, 4, 6\}$
$x = 3,\ z \in \{3, 6\}$
$x = 4,\ z \in \{4\}$
$x = 5,\ z \in \{5\}$
$x = 6,\ z \in \{6\}$

Total ways = 8

- pars such that $y \le w$

total ways $= \frac{5 \times (5+1)}{2} = 15$

$\Rightarrow$ total elements in $R = 8 \times 15 = 120$

22. Consider the matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$. If matrices P and Q are such that $PA = B$ and $AQ = B$, then the absolute value of the sum of the diagonal elements of $2(P+Q)$ is _____.

Answer (34)

Sol. $PA = B,\ AQ = B$

$\Rightarrow P = BA^{-1}$ and $Q = A^{-1}B$

$\Rightarrow P + Q = BA^{-1} + A^{-1}B,\ A^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ -1 & 1/2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}^{-1} + \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$

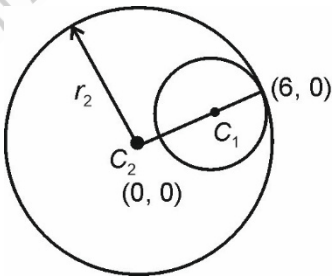
$\begin{bmatrix} -10.5 & 6 \\ -3.5 & 2 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ -2.5 & -7.5 \end{bmatrix}$

$\Rightarrow 2(P+Q) = \left|\text{trace}\left(2(1+Q)\right)\right| = \left|2(-17)\right| = 34$

23. Let A be the point (3, 0) and circles with variable diameter AB touch the circle $x^2 + y^2 = 36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to_____.

Answer (18)

Sol.



Let $B \equiv (h, k)$

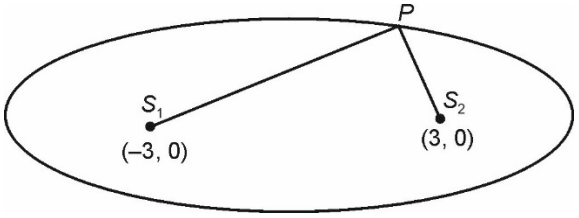
$\Rightarrow C_1 = (x-h)(x-3) + (y-k)(y-0) = 0$

Since the circle touches internally

$r_2 = C_1C_2 + r_1,\ C_1 \equiv \left(\frac{h+3}{2}, \frac{k}{2}\right)$

$\Rightarrow 6 = \sqrt{\left(\frac{h+3}{2}\right)^2 + \left(\frac{k}{2}\right)^2} + \sqrt{\left(\frac{h+3}{2} - 3\right)^2 + \left(\frac{k}{2}\right)^2}$

$\Rightarrow \sqrt{(x+3)^2 + y^2} + \sqrt{(x-3)^2 + y^2} = 12$



$\Rightarrow 2ae = 6$

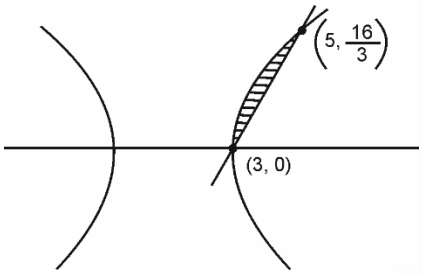
$2a = 12 \Rightarrow e = \frac{1}{2}$

$\Rightarrow 72e^2 = 18$

24. If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A, then $3\left(A + 6\log_e(3)\right)$ is equal to ____.

Answer (24)

Sol.



Solving, $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ together we get $x = 3, 5$

$\Rightarrow A = \int_3^5 \left(\sqrt{\frac{16x^2 - 144}{9}} - \left(\frac{8x-24}{3}\right)\right) dx$

$A = \int_3^5 \left|\frac{8}{3}(x-3) - \frac{4}{3}\sqrt{x^2-9}\right| dx$

$= \frac{4}{3}\left(\frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\ln\left|x + \sqrt{x^2-9}\right|\right) - \frac{4}{3}(x-3)^2\Big|_3^5$

$= A = \left(\frac{36\ln 3 - 24) - (18\ln 3)}{3} = \frac{-18\ln 3 + 24}{3}\right)$

$\Rightarrow 3A + 18\ln 3 = 24$

25. The number of points in the interval [2, 4], at which the function $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, where [·] denotes the greatest integer function, is discontinuous, is

Answer (10)

Sol. $x^2 - x - \frac{1}{2} = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1}{2}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{3}{4}$

$x \in [2, 4]$

$\Rightarrow \left(x - \frac{1}{2}\right) \in \left[\frac{3}{2}, \frac{7}{2}\right]$

$\left(x - \frac{1}{2}\right)^2 \in \left[\frac{9}{4}, \frac{49}{4}\right]$

$\left(\frac{x-1}{2}\right)^2 - \frac{3}{4} \in \left[\frac{6}{4}, \frac{46}{4}\right] \equiv \left[\frac{3}{2}, \frac{23}{2}\right]$

$\Rightarrow \left[x^2 - x - \frac{1}{2}\right]$ will be discontinuous where $x^2 - x - \frac{1}{2}$ becomes integer $\Rightarrow$ 2, 3, … 11

$\Rightarrow$ ((11 – 2) + 1) points = 10

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. Dimensions of universal gravitational constant (G) in terms of Planck's constant (h), distance (L), mass (M) and time (T) are _____.

(1) $\left[hTLM^{-2}\right]$ (2) $\left[hT^{-1}LM^{-2}\right]$

(3) $\left[hTL^{2}M^{-2}\right]$ (4) $\left[h^{-1}\ T^{-1}LM^{-2}\right]$

Answer (2)

Sol. Planck's mass dimensions

$$m \equiv \sqrt{\frac{hc}{G}} \qquad \sqrt{\frac{h}{G}\frac{L}{T}} \equiv M$$

$$G \equiv \frac{hL}{M^2T} = hT^{-1}LM^{-2}$$

27. A 0.5 kg mass is in contact against the inner wall of a cylindrical drum of radius 4 m rotating about its vertical axis. The minimum rotational speed of the drum to enable the mass to remain stuck to the wall (without falling) is 5 rad/s. The coefficient of friction between the drum's inner wall surface and mass is _____. (Take g = 10 m/s^2)

(1) 0.1 (2) 0.5

(3) 0.7 (4) 0.3

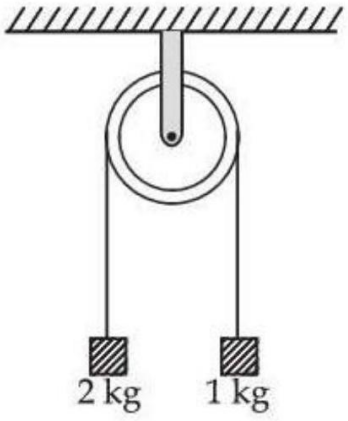
Answer (1)

Sol. $N = m\omega^2 r$, $\mu N = mg$

$\Rightarrow \mu\omega^2 r = g$

$$\mu = \frac{10}{25\times 4} = 0.1$$

28. Two blocks of masses 2 kg and 1 kg respectively, are tied to the ends of a string which passes over a light frictionless pulley as shown in the figure below. The masses are held at rest at the same horizontal level and then released. The distance traversed by the centre of mass in 2 s is _____ m. (Take g = 10 m/s^2)



(1) 3.33 (2) 3.12

(3) 2.22 (4) 1.42

Answer (3)

Sol. $a = g\left(\frac{2-1}{2+1}\right) = \frac{g}{3}$

$$a_{cm} = \frac{2\frac{g}{3} - 1\times\frac{g}{3}}{3} = \frac{g}{9}$$

$$S = \frac{1}{2}\times\frac{10}{9}\times 4 = \frac{20}{9} = 2.22$$

29. A particle having charge 10^{-9} C moving in $x-y$ plane in fields of $0.4\hat{j}$ N/C and $4\times10^{-3}\hat{k}$ T experiences a force of $\left(4\hat{i}+2\hat{j}\right)\times10^{-10}$ N. The velocity of the particle at that instant is _____ m/s.

(1) $50\hat{i}+100\hat{j}$. (2) $100\hat{i}+50\hat{j}$

(3) $-50\hat{i}+100\hat{j}$ (4) $50\hat{i}-100\hat{j}$

Answer (1)

Sol. $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

$= q\{0.4\hat{j} + (v_x\hat{i} + v_y\hat{j}) \times 4 \times 10^{-3}\hat{k}\}$

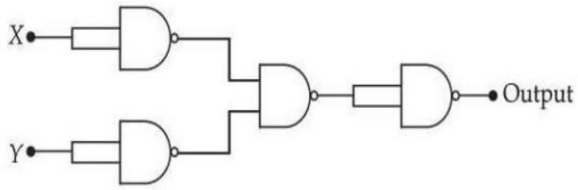
$= q\{0.4\hat{j} + (4 \times 10^{-3} v_x)(-\hat{j}) + (4 \times 10^{-3} v_y\hat{i})\}$

$(4\hat{i} + 2\hat{j}) \times 10^{-10} = 10^{-9}\{(0.4 - 4 \times 10^{-3} v_x)\hat{j} + 4 \times 10^{-3} v_y\hat{i}\}$

$\Rightarrow$ $0.4 = 4 \times 10^{-3} v_y$ $\Rightarrow$ $v_y = 100$

$0.2 = 0.4 - 4 \times 10^{-3} v_x \Rightarrow v_x = 50$

30. If X and Y are the inputs, the given circuit works as _____.



(1) OR gate (2) AND gate

(3) NAND gate (4) NOR gate

Answer (4)

Sol. Output = $\overline{\overline{A} \cdot \overline{B}} = A \cdot B = \overline{A + B}$

31. If a body of mass 1 kg falls on the earth from infinity, it attains velocity (v) and kinetic energy (k) on reaching the surface of earth. The values of v and k respectively are _____. (Take radius of earth to be 6400 km and g = 9.8 m/s^2)

(1) 11.2 km/s; 6.27 × 10^7 J

(2) 11.2 km/s; 12.54 × 10^7 J

(3) 8.8 km/s; 6.27 × 10^7 J

(4) 8.8 km/s; 12.54 × 10^7 J

Answer (1)

Sol. $v = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6400 \times 10^3}$

$= 80 \times 10 \times 14 = 11.2$ km/s

$k = \frac{1}{2} \times 1 \times 11.2 \times 11.2 \times 10^6$

$= 6.27 \times 10^7$

32. In a screw gauge the zero of main scale reference line coincides with the fifth division of the circular scale when two studs are in contact. There are 100 divisions in circular scale and pitch of screw gauge is 0.1 mm. When diameter of a sphere is measured, the reading of main scale is 5 mm and 50th division of circular scale coincides with the reference line of main scale. The diameter of sphere is _____ mm.

(1) 5.045

(2) 5.055

(3) 5.450

(4) 5.550

Answer (1)

Sol. L.C. = $\frac{0.1}{100}$mm = 0.001 mm

ZE = +5 CSD

MSR = 5 mm

Diameter = 5 mm + 50 CSD – 5 CSD

= 5 + 0.045

= 5.045

33. The surface tension of a soap bubble is 0.03 N/m. The work done in increasing the diameter of bubble from 2 cm to 6 cm is $\alpha\pi \times 10^{-4}$ J. The value of α is _____. (Take π = 3.14)

(1) 0.86

(2) 0.64

(3) 1.92

(4) 7.68

Answer (4)

Sol. $\Delta U = \left\{S4\pi\left(R_2^2 - R_1^2\right)\right\}2$

$= 8 \times 0.03 \times 32 \times 10^{-4}\pi$

$= 7.68\pi \times 10^{-4}$

34. A mixture of carbon dioxide and oxygen has volume 8310 cm^3, temperature 300 K, pressure 100 kPa and mass 13.2 g. The number of moles of carbon dioxide and oxygen gases in the mixture respectively are _____.

(Assume both carbon dioxide and oxygen gases behave like ideal gases) [R = 8.31 J / mol.K]

(1) 0.15 and 0.18 (2) 0.25 and 0.08

(3) 0.21 and 0.12 (4) 0.13 and 0.20

Answer (3)

Sol. $\mu = \frac{PV}{RT} = \frac{100\times10^3\times8310\times10^{-6}}{8.31\times300}$

$\mu = \frac{1}{3}$

Let μ_1 = Number of moles of CO_2.

Mass = $\mu_1\times44+\left(\frac{1}{3}-\mu_1\right)32 = 13.2$

$12\mu_1 = 13.20 - 10.67$

$\mu_1 = 0.21$

$\mu_2 = 0.33 - 0.21 = 0.12$

35. If an air bubble of diameter 2 mm rises steadily through a liquid of density 2000 kg/m^3 at a rate of 0.5 cm/s, then the coefficient of viscosity of liquid is _____ Poise. (Take g = 10 m/s^2)

(1) 0.88 (2) 8.8

(3) 88.8 (4) 0.088

Answer (2)

Sol. $6\pi\eta rv = \frac{4}{3}\pi r^3\rho g = (B)$

$\eta = \frac{2}{9}\frac{r^2\rho g}{v}$

$= \frac{2}{9}\times\frac{10^{-6}\times2000\times10}{0.5\times10^{-2}}$

$\eta = 0.88$ deca-poise = 8.8 poise

36. A spherical ball of mass 2 kg falls from a height of 10 m and is brought to rest after penetrating 10 cm into sand. The average force exerted by sand on the ball is _____ N.

(Take g = 10 m/s^2)

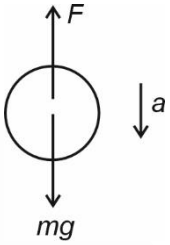
(1) 1980 (2) 2020

(3) 2000 (4) 1000

Answer (2)

Sol. $v = \sqrt{2\times10\times10} = 10\sqrt{2}$ m/s

$a = -\frac{200}{2\times0.1} = -1000$ m/s^2



$mg - F = ma$

F = 2(1010) = 2020 N

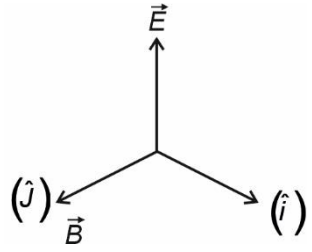
37. An electromagnetic wave travels in free space along the x-direction. At a particular point in space and time, $\vec{B} = 2\times10^{-7}\,\hat{j}$ T is associated with this wave. The value of corresponding electric field $\vec{E}$ at this point is ____ V/m.

(1) 60 $\hat{k}$ (2) –60 $\hat{k}$

(3) 30 $\hat{k}$ (4) –600 $\hat{k}$

Answer (2)

Sol. $E = cB = 3\times10^8\times2\times10^{-7} = 60$



$\vec{E}\times\vec{B} \uparrow\uparrow \hat{i}$

$\Rightarrow \vec{E} \uparrow\uparrow -\hat{k}$

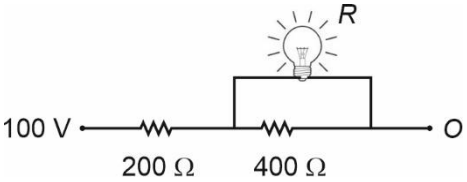
38. Two resistors of 200 Ω and 400 Ω are connected in series with a battery of 100 V. A bulb rated at 200 V, 100 W is connected across the 400 Ω resistance. The potential drop across the bulb is ____ V.

(1) 25 (2) 50

(3) 66.6 (4) 100

Answer (2)

Sol.



$$R_B = \frac{V^2}{P} = 400$$

$$R_{eq} = 200 \oplus 400 \,||\, 400$$

$\Rightarrow \quad V_B = 50$ V.

39. Two metal plates (A, B) are kept horizontally with separation of $\left(\frac{12}{\pi}\right)$ cm, with plate A on the top. An atomizer jet sprays oil (density 1.5g/cm^3) droplets of radius 1 mm horizontally. All oil droplets carry a charge 5 nC. The potentials V_A and V_B are required on plates A and B respectively in order to ensure the droplets do not descend. The values of V_A and V_B are _____.

(Neglect the air resistance to the droplets and take g = 10 m/s^2)

(1) 100 V and 580 V (2) 580 V and 100 V

(3) 60 V and 400 V (4) 0 V and –200 V

Answer (1)

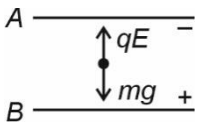
Sol. $qE = mg = \left(\frac{\Delta V}{l} q\right)$

$$5 \times 10^{-9} \frac{\Delta V}{\left(\frac{12}{\pi} \times 10^{-2}\right)} = \frac{4}{3}\pi \times 10^{-9} \times 1.5 \times 10^3 \times 10$$

$$\Delta V = \frac{16 \times 15}{5} \times 10$$

ΔV = 480 V

$V_B > V_A$.



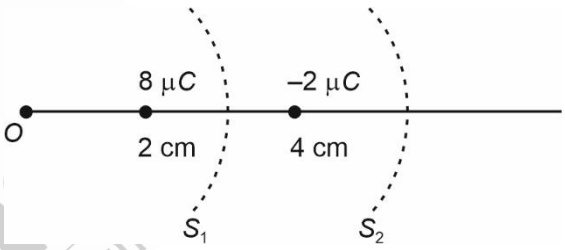
40. Two point charges 8 μC and –2 μC are located at x = 2 cm and x = 4 cm, respectively on the x-axis. The ratio of electric flux due to these charges through two spheres of radii 3 cm and 5 cm with their centers at the origin is _____.

(1) 4 : 1 (2) 3 : 4

(3) 4 : 3 (4) 4 : 5

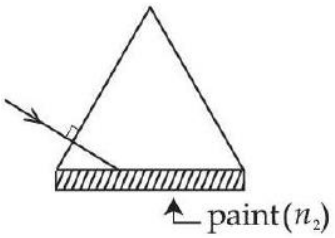
Answer (3)

Sol.



$$\frac{\phi_1}{\phi_2} = \frac{8/\varepsilon_0}{6/\varepsilon_0} = \frac{4}{3}$$

41. One side of an equilateral prism is painted by a transparent material of refractive index n_2. The refractive index of prism is 1.6. The minimum value of n_2 required for total internal reflection from painted face is _____.

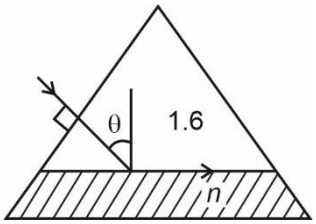


(1) $\frac{3\sqrt{3}}{1.6}$ (2) $\sqrt{3}$

(3) $\frac{3.2}{\sqrt{3}}$ (4) $\frac{4\sqrt{3}}{5}$

Answer (4)

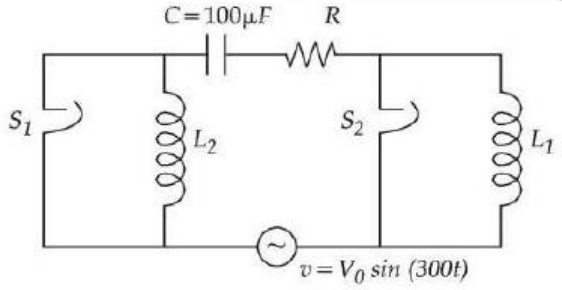
Sol. From geometry $\theta = 60°$



From Snell's law $1.6 \times \frac{\sqrt{3}}{2} = n \sin 90°$

$n = \frac{4\sqrt{3}}{5}$

42. The figure given below shows an *LCR* series circuit with two switches S_1 and S_2. When switch S_1 is closed keeping S_2 open, the phase difference (ϕ) between the current and source voltage is 30° and phase difference is 60° when S_2 is closed keeping S_1 open. The value of $(3L_1 - L_2)$ is _____H.



(1) $\frac{9}{2}$ (2) $\frac{2}{9}$

(3) $\frac{1}{3}$ (4) 3

Answer (2)

Sol. For S_1 closed

$$\tan\theta = \frac{L\omega - \frac{1}{\omega C}}{R}$$

$$R\left(-\frac{1}{\sqrt{3}}\right) = L_1 \times 300 - \frac{1}{300 \times 100 \times 10^{-6}}$$

$$R(-\sqrt{3}) = L_2 \times 300 - \frac{1}{300 \times 100 \times 10^{-6}}$$

$$\Rightarrow \quad 3L_1 \times 300 - \frac{3}{300 \times 100 \times 10^{-6}}$$

$$= 300\, L_2 - \frac{1}{300 \times 100 \times 10^{-6}}$$

$$3L_1 - L_2 = \left(100 - \frac{100}{3}\right)\frac{1}{300}$$

$$\equiv \frac{2}{9}$$

43. A circular current loop of radius *R* is placed inside square loop of side length *L*(*L* >> *R*) such that they are co-planar and their centers coincide. The permeability of free space is μ_0. The mutual inductance between circular loop and square loop is _____.

(1) $2\sqrt{2}\frac{\mu_0 L^2}{R}$

(2) $\sqrt{2}\frac{\mu_0 L^2}{R}$

(3) $\sqrt{2}\frac{\mu_0 R^2}{L}$

(4) $2\sqrt{2}\frac{\mu_0 R^2}{L}$

Answer (4)

Sol. $B = (4)\times\frac{\mu_0 i}{4\pi\frac{L}{2}}\{\sqrt{2}\}$

$$B = \frac{2\sqrt{2}\,\mu_0\, i}{\pi L}$$

$$M = \frac{2\sqrt{2}\,\mu_0\, i\,\pi R^2}{L\,\pi\, i}$$

$$2\sqrt{2}\frac{\mu_0 R^2}{L}$$

44. The binding energy per nucleon of $^{209}_{83}Bi$ is _____ MeV.

[Take $m\left(^{209}_{83}Bi\right) = 208.980388$ u, $m_p = 1.007825$ u, $m_n = 1.008665$ u, 1 u = 931 MeV/c^2]

(1) 7.48 (2) 7.84

(3) 8.79 (4) 6.94

Answer (1)

Sol. $\frac{BE}{N} = \frac{931(83\times1.007825+126\times1.008665-208.980388)}{209}$

= 7.84

45. The equation of motion of a particle is given by $x = a\sin(50t+\pi/3)$cm. The particle will come to rest at time t_1 and it will have zero acceleration at time t_2. The t_1 and t_2 respectively are _____.

(1) $\frac{\pi}{300}$s, $\frac{\pi}{75}$s

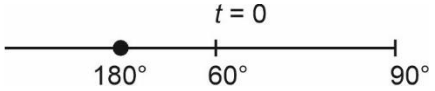
(2) $\frac{\pi}{75}$s, $\frac{\pi}{300}$s

(3) $\frac{\pi}{300}$s, $\frac{\pi}{25}$s

(4) $\frac{\pi}{50}$s, $\frac{\pi}{100}$s

Answer (2)

Sol.



$\omega = 50$

$t = \theta/\omega$

$$t_2 = \frac{\pi}{6\times50} = \frac{\pi}{300}$$

$$t_1 = \frac{2\pi}{3\times50} = \frac{\pi}{75}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. In a Young's double slit experiment, the intensity at some point on the screen is found to be $\frac{3}{4}$ times of the maximum of the interference pattern. The path difference between the interfering waves at this point is $\frac{\lambda}{x}$ where λ is wavelength of the incident light. The value of x is _____.

Answer (6)

Sol. $I = I_0\cos^2\frac{\Delta\phi}{2} = \frac{3}{4}I_0$

$\Delta\phi = 60° \Rightarrow \Delta x = \frac{\lambda}{6}$

47. Using Bohr's model, calculate the ratio of the magnetic fields generated due to the motion of the electrons in the 2nd and 4th orbits of hydrogen atom _____.

Answer (32)

Sol. $B = \frac{\mu_0 q v}{2r \times 2\pi r} \propto \frac{v}{r^2} \propto \frac{1}{n^5}$

$$\frac{B_2}{B_4} = \left(\frac{4}{2}\right)^5 = 32.$$

48. 5 moles of unknown gas is heated at constant volume from 10°C to 20°C. The molar specific heat of this gas at constant pressure $c_p = 8$ cal / mol.°C and $R = 8.36$ J/mol.°C. The change in the internal energy of the gas is _____ calorie.

Answer (300)

Sol. $\Delta U = \mu C_v \Delta T$

$\Delta U = 5 \times (C_P - R)\, \Delta T$

= 5 × (8 – 2) × 10 cal

= 300 cal

49. If sunlight is focused on a paper using convex lens, it starts burning the paper in shortest time when the lens is kept at 30 cm above the paper. If the radius of curvature of the lens is 60 cm then the refractive index of the lens material is $\frac{\alpha}{10}$. The value of α is _____.

Answer (*20)

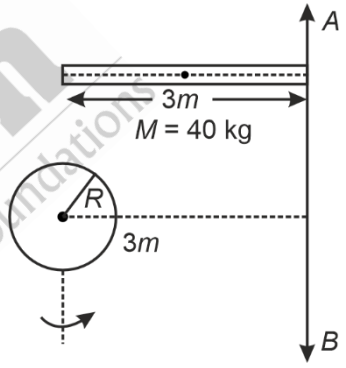
Sol. $\frac{1}{f} = (\mu - 1)\frac{2}{R}$

$$\frac{1}{30} = (\mu - 1)\frac{2}{60}$$

$\mu = 2$

$\Rightarrow$ $\alpha = 20$ (Assuming it is equiconvex)

50. Moment of inertia about an axis *AB* for a rod of mass 40 kg and length 3 m is same as that of a solid sphere of mass of 10 kg and radius *R* about an axis parallel to *AB* axis with separation of 3 m as shown in figure below. The value of *R* is given as $\sqrt{\frac{\alpha}{2}}$. The value of α is _____.



Answer (60)

Sol. $\frac{2}{5}MR^2 = M_r \frac{\ell^2}{3}$

$$\frac{2}{5} \times 10 \times R^2 = 40 \times 3$$

$R = \sqrt{30}$

$\Rightarrow$ $\alpha = 60$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

51. The ratio of mass percentage (w/w) of C : H in a hydrocarbon is 12 : 1. It has two carbon atoms. The weight (in g) of $CO_2(g)$ formed when 3.38 g of this hydrocarbon is completely burnt in oxygen is : (Given : Molar mass in g mol^{-1} C : 12, H : 1, O : 16)

(1) 5.68 (2) 11.44

(3) 22.74 (4) 17.05

Answer (2)

Sol. Atomic ratio:

$C = \frac{12}{12} = 1 \quad \therefore EF = CH$

$H = \frac{1}{1} = 1 \quad \therefore MF = C_2H_2$

$\therefore$ Molar mass $= 26\ g/mol$

$2C_2H_2 + 5O_2 \rightarrow 4CO_2 + 2H_2O$

$n_{C_2H_2} = \frac{3.38}{26} = 0.13\ mol$

$\therefore n_{CO_2} = 2 \times 0.13 = 0.26\ mol$

$\therefore$ Mass of $CO_2 = 44 \times 0.26 = 11.44\ g$

52. The first and second ionization constants of a weak dibasic acid H_2A are 8.1×10^{-8} and 1.0×10^{-13} respectively. 0.1 mol of H_2A was dissolved in 1 L of 0.1 M HCl solution. The concentration of HA^- in the resultant solution is :

(1) 0.1 M (2) 9.53×10^{-6} M

(3) 8.1×10^{-8} M (4) 1.0×10^{-13} M

Answer (3)

Sol. $[H_2A] = 0.1\ M \quad \therefore [H^+] \approx 0.1\ M$

$[HCl] = 0.1\ M$

$Ka_1 = \frac{[H^+][HA^-]}{[H_2A]}$

$\Rightarrow 8.1\times10^{-8} = \frac{0.1\times[HA^-]}{[H_2A]} = \frac{0.1\times[HA^-]}{0.1}$

$\Rightarrow [HA^-] = 8.1\times10^{-8}\ M$

53. SF_4 is isostructural with:

A. $BrF_4^{\ominus}$

B. CH_4

C. $IF_4^{\oplus}$

D. XeF_4

E. XeO_2F_2

Choose the **correct** answer from the options given below :

(1) C Only (2) C and E Only

(3) A and D Only (4) B and E Only

Answer (2)

Sol. C and E only

$SF_4 \Rightarrow$ see-saw shape (sp^3d)

$BrF_4^- \Rightarrow$ square planar

$CH_4 \Rightarrow$ tetrahedral

$IF_4^+ \Rightarrow$ see-saw

$XeF_4 \Rightarrow$ square planar

$XeO_2F_2 \Rightarrow$ see-saw

54. Gas 'A' undergoes change from state 'X' to state 'Y'. In this process, the heat absorbed and work done by the gas is 10 J and 18 J respectively. Now gas is brought back to state 'X' by another process during which 6 J of heat is evolved. In the reverse process of 'Y' to 'X',

(1) 18 J of the work is done by the gas 'A'

(2) 2 J of the work is done by the gas 'A'

(3) 12 J of the work is done on the gas 'A' by the surrounding

(4) 14 J of the work is done on the gas 'A' by the surrounding

Answer (4)

Sol. $\Delta U_{X\to Y} = Q_{X\to Y} + W_{X\to Y} = 10 - 18 = -8 \text{ J}$

$\Delta U_{Y\to X} = Q_{Y\to X} + W_{Y\to X}$

$\therefore \quad W_{Y\to X} = +6 \text{ J} + 8 \text{ J} = +14 \text{ J}$

55. Solution A is prepared by dissolving 1 g of a protein (molar mass $= 50000 \text{ g mol}^{-1}$) in 0.5 L of water at 300 K . Its osmotic pressure is x bar. Solution B is made by dissolving 2 g of same protein in 1 L of water at 300 K . Osmotic pressure of solution B is y bar. Entire solution of A is mixed with entire solution of B at same temperature. The osmotic pressure of resultant solution is z bar, x, y and z respectively are : $\left(R = 0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}\right)$

(1) 9.96×10^{-4}; 9.96×10^{-4}; 9.96×10^{-4}

(2) 9.96×10^{-4}; 9.96×10^{-4}; 19.92×10^{-4}

(3) 4.98×10^{-4}; 4.98×10^{-4}; 9.96×10^{-4}

(4) 4.98×10^{-4}; 4.98×10^{-4}; 4.98×10^{-4}

Answer (1)

Sol. $\pi_A = \dfrac{1}{50000\times0.5}\times0.083\times300$

$= 9.96\times10^{-4} \text{ bar} = x$

$\pi_B = \dfrac{2}{50000\times1}\times0.083\times300$

$= 9.96\times10^{-4} \text{ bar} = y$

$\pi_{total} = \left(\dfrac{n_A + n_B}{v_A + v_B}\right)RT = 9.96\times10^{-4} \text{ bar} = z$

56. At 25ºC, 20.0 mL of 0.2 M weak monoprotic acid HX is titrated against 0.2 M NaOH . The pH of the solution (a) at the start of the titration (when NaOH has not been added) and (b) when 10 mL of NaOH is added respectively, are :

Given: $K_a = 5\times10^{-4}$

$pK_a = 3.3$

$\alpha << 1$

	(a)	(b)
(1)	0.7	2.0
(2)	2.0	3.3
(3)	1.1	2.2
(4)	3.0	2.2

Answer (2)

Sol. pH at the start (pure weak acid)

$[H^+] = \sqrt{K_a \cdot C} = \sqrt{5\times10^{-4}\times0.2} = 10^{-2} \text{ M}$

pH after 10 ml NaOH is added (buffer)

$pH = pK_a + \log\dfrac{\text{salt}}{\text{acid}} = 3.3 + \log\dfrac{2}{2} = 3.3$

57. Consider the reaction $aX \rightarrow bY$, for which the rate constant at 30ºC is 1×10^{-3} mol^{-1} L s^{-1}. Which of the following statements are true?

A. When concentration of ' X′ is increased to four times, the rate of reaction becomes 16 times.

B. The reaction is a second order reaction.

C. The half-life period is independent of the concentration of X.

D. Decomposition of N_2O_5 is an example of the above reaction.

E. $\ln\frac{[R_o]}{[R]}$ vs time → is valid for the above reaction.

Choose the **correct** answer from the options given below :

(1) A and B Only

(2) A, B and C Only

(3) A, B, D and E Only

(4) C and D Only

Answer (1)

Sol. $K = 1 \times 10^{-3}$ mol^{-1} Ls^{-1} (second order reaction)

$\therefore$ $r = K[X]^2$

$X' = 4X \Rightarrow r' = 16r$

$t_{1/2}$ for 2^{nd} order $= \frac{1}{K[A_0]} = \frac{1}{K[X_0]}$

$\therefore$ Decomposition of $N_2O_5 \Rightarrow$ first order reaction

The given plot is for first order reaction.

58. The correct set that contains all kinds (basic, acidic, amphoteric and neutral) of oxides is :

(1) Na_2O, K_2O, Al_2O_3 and As_2O_3

(2) Al_2O_3, As_2O_3, CO and NO

(3) K_2O, Cl_2O_7, As_2O_3 and NO

(4) Na_2O, N_2O, Al_2O_3 and CO

Answer (3)

Sol. K_2O → basic

Cl_2O_7 → acidic

As_2O_3 → amphoteric

NO → neutral

59. Given below are two statements :

Statement I : The second ionization enthalpy of B, Al and Ga is in the order of $B > Al > Ga$.

Statement II : The correct order in terms of first ionization enthalpy is $Si < Ge < Pb < Sn$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (2)

Sol.

$I.E_1$	$\Rightarrow$	Si	> Ge >	Pb	> Sn
kJ / mol		786	761	715	708
$I.E_2$	:	B	> Ga >	Al	
kJ / mol		2427	1979	1816	

S-I $\Rightarrow$ Incorrect

S-II $\Rightarrow$ Incorrect

60. Given below are two statements :

Statement I : Among Zn, Mn, Sc and Cu, the energy required to remove the third valence electron is highest for Zn and lowest for Sc.

Statement II : The correct order of the following complexes in terms of CFSE is $\left[Co(H_2O)_6\right]^{2+} < \left[Co(H_2O)_6\right]^{3+} < \left[Co(en)_3\right]^{3+}$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (1)

Sol. CFSE $\propto$ charge on CMA

$\propto$ = strength of ligand.

CFSE: $[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$

S-II is correct.

$I.E_3$	$\Rightarrow$	Zn	Mn	Sc	Cu
kJ / mol		3837 (highest)	3260	2393 (lowest)	3556

S-I is correct.

61. Which of the following complexes will show coordination isomerism?

A. $\left[Ag(NH_3)_2\right]\left[Ag(CN)_2\right]$

B. $\left[Co(NH_3)_6\right]\left[Cr(CN)_6\right]$

C. $\left[Co(NH_3)_6\right]\left[Co(CN)_6\right]$

D. $\left[Fe(NH_3)_6\right]\left[Co(CN)_6\right]$

E. $\left[Co(NH_3)_6\right]\left[Fe(CN)_6\right]$

Choose the **correct** answer from the options given below :

(1) B, C and D Only

(2) B, D and E Only

(3) A, C and D Only

(4) C, D and E Only

Answer (2)

Sol. A $\Rightarrow$ does not show coordination isomerism.

B, D and E show coordination isomerism.

62. Complete combustion of X g of an organic compound gave 0.25 g of CO_2 and 0.12 g of H_2O. If the % of carbon is 25% and of hydrogen is 4.89%, then X = _____ $\times 10^{-3}$ g. (Nearest integer) (Molar mass of C, H and O are 12,1 and 16 g mol^{-1} respectively.)

(1) 273 (2) 27

(3) 2730 (4) 227

Answer (1)

Sol. Mass of C = $\frac{12}{44} \times 0.25 = 0.06818$ g

$$\% \text{ C} = 25 = \frac{0.06818}{X} \times 100 \approx X = 0.27272 \text{ g}$$

Mass of H = $\frac{2}{18} \times 0.12 = 0.01333$ g

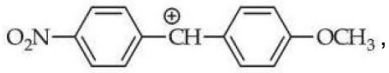
$$\% \text{ H} = \frac{0.01333}{0.2727} \times 100 \Rightarrow 4.89\%$$

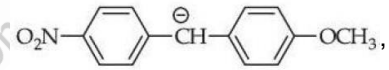
(Matches with question)

$\therefore$ X = 272.7 $\times 10^{-3}$ g

= 273 $\times 10^{-3}$ g

63. Given below are two statements :

Statement I : In  , the carbocation is stabilised by + R effect of $-OCH_3$ group.

Statement II : In  , the carbanion is stabilised by –R effect of $-NO_2$ group.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (1)

Sol.

- Carbocation is stabilised by + M effect of $-OCH_3$ and destabilised by –M of $-NO_2$.
- Carbanion is stabilised by – M effect of $-NO_2$ and destabilised by + M of $-OCH_3$

64. The compound (X) on
(i) on heating in the presence of anhydrous $AlCl_3$ and HCl gas gives 2,4-dimethyl pentane
(ii) aromatization gives toulene and
(iii) cyclisation gives methyl cyclohexane
The correct name of compound (X) is :
(1) Hept-2-ene
(2) Hept-1,3,5-triene
(3) Heptane
(4) Hept-2,4,6-triene

Answer (3)

Sol. $X \xrightarrow[\Delta]{\text{Anhy. } AlCl_3/HCl} 2,4\text{ –dimethyl pentane}$

$\xrightarrow{\text{Aromatization}}$ Toluene

$\xrightarrow{\text{Cyclisation}}$ methyl cyclohexane

X is heptane.

65. Correct statements regarding alkyl halides $(R-X)$ among the following are :
A. Alcohol being less polar solvent as compared to water, alcoholic KOH favours elimination reaction with $R-X$.
B. Order of reactivity towards S_N1 mechanism is $C_6H_5-CH_2-Cl > C_6H_5-CHCl-C_6H_5$.
C. Non substituted aryl halides exhibit properties similar to alkyl halides.
D. Vinyl chloride is an example of haloalkene and allyl chloride is an example of haloalkyne.
E. $R-Cl$ can be prepared by reacting $R-OH$ with $SOCl_2$ but $Ar-Cl$ cannot be prepared by reacting $Ar-OH$ with $SOCl_2$.
Choose the **correct** answer from the options given below :
(1) A, B and C Only (2) B and D Only
(3) A and E Only (4) D and E Only

Answer (3)

Sol.
- S_N1 rate : : $C_6H_5-CHCl-C_6H_5 > C_6H_5-CH_2-Cl$
- $R-OH \xrightarrow{SOCl_2} R-Cl$

$\therefore$ Alcoholic KOH favours elimination.

66. An organic compound "x" where molar ratio of C, O and H are equal, on treatment with 50% KOH under reflux followed by acidification produced "y". The most likely structure of "y" is :
[Molar mass of 'x' is 58 g mol^{-1}]

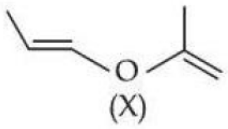
(1) $CH_2=CH-\overset{\overset{O}{\|}}{C}-OH$
(2) $CH_3-CH=CH-CH=O$
(3) $O=\underset{\underset{CH_2-OH}{|}}{C}-OH$
(4) $CH_3-\overset{\overset{O}{\|}}{C}-OH$

Answer (3)

Sol. Molar ratio of C:O:H is same.
EF = $(CHO)_n$
Molar mass = 58 g/mol
MF = $C_2H_2O_2$

$\underset{\underset{CHO}{|}}{CHO} \xrightarrow[\text{(ii) acidification}]{\text{(i) conc. KOH}} \underset{\underset{COOH}{|}}{CH_2OH}$

67. A molecule (X) with following structure under mild acidic condition is hydrolysed to produce (Y) and (Z). Identify the correct statements about (Y) and (Z).



(X)

A. Both (Y) and (Z) have same molar mass.
B. (Y) and (Z) can be distinguished from each other by $NaHCO_3$.

C. (Y) and (Z) react with HCN with same rates.

D. (Y) and (Z) undergo addition reaction with 2,4-DNP.

Choose the **correct** answer from the options given below :

(1) A, B and C Only

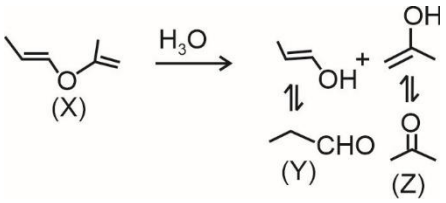
(2) B and C Only

(3) C and D Only

(4) A and D Only

Answer (4)

Sol.



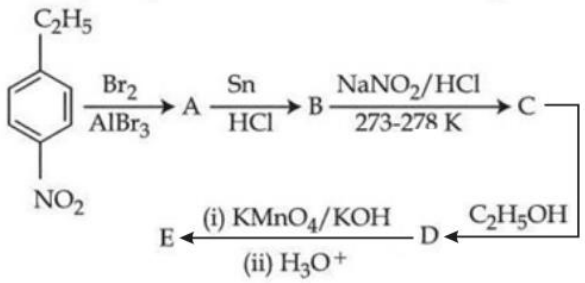
$\therefore$ Same molar mass

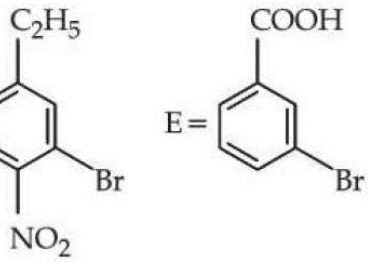
HCN reactivity rate: Y > Z

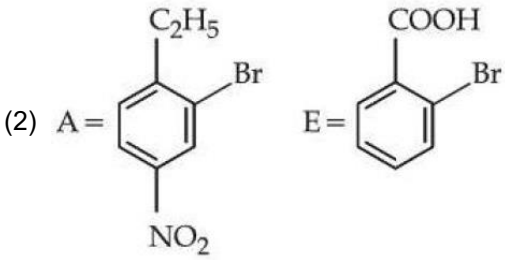
$\therefore$ Y, Z reacts with 2, 4-DNP reagent

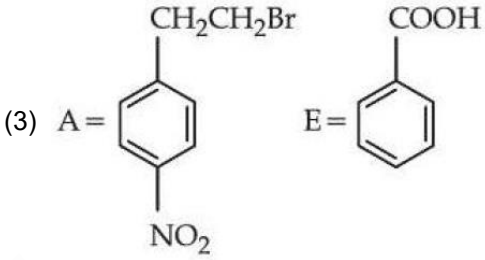
A, D $\rightarrow$ correct

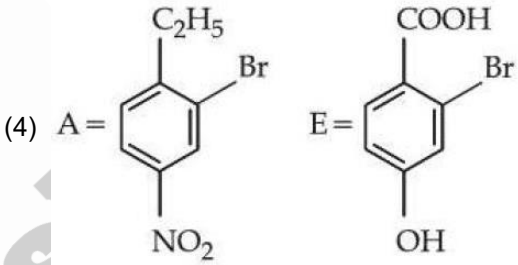
68. Identify compounds A and E in the following reaction sequence.



(1) 

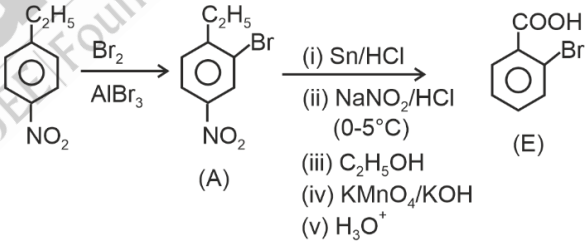
(2) 

(3) 

(4) 

Answer (2)

Sol.



69. Identify the correct pair having amino acid (A) and the hormone (B) that is iodinated derivative of the amino acid (A).

(T and Y represent one letter code for amino acids)

	Amino acid (A)	**Hormone (B)**
(1)	T	Insulin
(2)	T	Thyroxine
(3)	Y	Thyroxine
(4)	Y	Insulin

Answer (3)

Sol. T $\Rightarrow$ Threonine

Y $\Rightarrow$ tyrosine

Thyroxine is iodinated derivative of tyrosine (Y)

70. Among Fe^{2+}, Fe^{3+}, Cr^{2+} and Zn^{2+}, the ion that shows positive borax bead test and with highest ionisation enthalpy is :

(1) Fe^{2+}

(2) Zn^{2+}

(3) Cr^{2+}

(4) Fe^{3+}

Answer (4)

Sol. $Zn^{2+} \Rightarrow$ does not show borax bead test.

Fe^{3+} has highest I.E.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The surface of sodium metal is irradiated with radiation of wavelength x nm. The kinetic energy of ejected electrons is 2.8×10^{-20} J. The work function of sodium is 2.3 eV. The value of x is _____ $\times 10^2$ nm. (Nearest integer)
(Given : h = 6.6 × 10^{-34} J s; 1 eV = 1.6 × 10^{-19} J; c = 3 × 10^8 m s^{-1})

Answer (5)

Sol. $\lambda = x$ nm

$$K.E. = 2.8\times10^{-20}\ J = 0.28\times10^{-19}\ J$$

$$\phi_0 = 2.3\ eV = 3.68\times10^{-19}\ J$$

$$K.E. = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \frac{hc}{\lambda} = (3.68+0.28)\times10^{-19}\ J$$

$$= 3.96\times10^{-19}\ J$$

$$\therefore \lambda = \frac{19.8\times10^{-26}}{3.96\times10^{-19}} = 5\times10^{-7}\ m$$

$$= 500\ nm$$

$$= 5\times10^2\ nm$$

72. Consider the following gas phase reaction being carried out in a closed vessel at 25ºC.

$$2A(g) \rightarrow 4B(g) + C(g)$$

time (min)	total pressure of the system (mm Hg)
30	300
∞	600

The pressure of $C(g)$ at 30 minutes time interval would be _____ mm Hg. (nearest integer)

Answer (20)

Sol.

	$2A(g)$	$\rightarrow 4B(g)$	$+ C(g)$
t = 0	P_0	–	–
t = t	$P_0 - 2x$	4x	x
t = ∞	–	$2P_0$	$0.5P_0$

$\therefore P_0 + 3x = 300 \Rightarrow 3x = 60 \Rightarrow x = 20$ mmHg

$2.5P_0 = 600 \Rightarrow P_0 = 240$ mmHg

$\therefore$ Pressure of C at 30 min

$= x = 20$ mmHg.

73. Consider the following two half-cell reactions along with the standard reduction potential given :

$$CO_2 + 6H^+ + 6e^- \rightarrow CH_3OH + H_2O \quad E^o_{red} = 0.02\ V$$

$$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O \quad E^o_{red} = 1.23\ V$$

A fuel cell was set up using the above two reactions such that the cell operates under the standard condition of 1 bar pressure and 298 K temperature. The fuel cell works with 80% efficiency. If the work derived from the cell using 1 mol of CH_3OH is used to compress an ideal gas isothermally against a constant pressure of 1 kPa , then the change in the volume of the gas, ΔV = _____ m^3. (nearest integer)

Given: $F = 96500\ C\ mol^{-1}$.

Answer (560)

Sol. Anode (Oxidation) :

$$CH_3OH + H_2O \rightarrow CO_2 + 6H^+ + 6e^-, \quad E^\circ_{ox} = -0.02V$$

Cathode (Reduction) :

$$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O, \quad E^\circ_{red} = 1.23V$$

$$E^\circ_{cell} = 1.23 - 0.02 = 1.21V$$

$$\Delta G = -nFE^\circ_{cell}$$

$$= -6 \times 96500 \times 1.21$$

$$= -700590J$$

$$\therefore \quad |W_{max}| = 700590J$$

$$W_{actual} = 0.8 \times 700590 = 560472J$$

$$W_{ext} = -P_{ext}(V_2 - V_1)$$

$$\Delta V = 560.4m^3$$

74. Number of paramagnetic ions among the following d- and f-block metal ions is _____.

Mn^{2+}, Cu^{2+}, Zn^{2+}, Yb^{2+}, Sc^{3+}, La^{3+}, Gd^{3+}, Lu^{3+}, Ti^{4+}, Ce^{4+}

(Atomic number of Mn = 25, Cu = 29, Zn = 30, Yb = 70, Sc = 21, La = 57, Gd = 64, Lu = 71, Ti = 22, Ce = 58)

Answer (3)

Sol. $Mn^{2+} \Rightarrow 3d^5$ (P)

$Cu^{2+} \Rightarrow 3d^9$ (P)

$Zn^{2+} \Rightarrow 3d^{10}$ (D)

$Yb^{2+} \Rightarrow 4f^{14}$ (D)

$Sc^{3+} \Rightarrow 3d^0$ (D)

$La^{3+} \Rightarrow 4f^0$ (D)

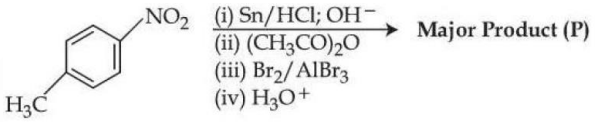
$Gd^{3+} \Rightarrow 4f^7$ (P)

$Lu^{3+} \Rightarrow 4f^{14}$ (D)

$Ti^{4+} \Rightarrow 3d^0$ (D)

$Ce^{4+} \Rightarrow 4f^0$ (D)

75. Consider the following reactions sequence

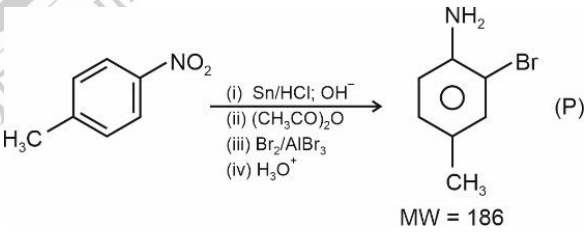


When the product (P) is subjected to Carius analysis using $AgNO_3$, 1.0 g of the product (P) will produce _____ g of the precipitate of AgBr. (Nearest Integer)

(Given : molar mass in g mol^{-1} C : 12, H : 1, O : 16, N : 14, Br : 80, Ag : 108)

Answer (1)

Sol.



$$\therefore \ n_P = \frac{1}{186}\ mol = n_{Br} = n_{AgBr}$$

$$\therefore \ \text{Mass of AgBr ppt.} = \frac{1}{186} \times 188\ g$$

$$= 1.011\ g$$

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