Aakash
Medical | IIT-JEE | Foundations

Phase-2
2nd April 2026
(Morning Shift)

Answer & Solutions

for

JEE MAIN 2026

TRUST Aakash

Stellar Ranks in JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR 23

Kushagra Baingaha AIR 7

Shreyas Lohiya AIR 6

A Gupta AIR 15

Devya Rustagi AIR 28

Shreyas Lohiya AIR 68

Rujul Garg AIR 41

Advay Mayank AIR 36

Arush Anand AIR 64

Kotha D Reddy AIR 74

Top Ranks in JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students **Aakashians Qualified in NSEs 2025-26**

777 Classroom Students **Aakashians Qualified in IOQM 2025**

134 Classroom Students **Aakashians Qualified in RMO 2025**

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let α, $\alpha + 2$, $\alpha \in Z$, be the roots of the quadratic equation

$x(x + 2) + (x + 1)(x + 3) + (x + 2)(x + 4) + \cdots + (x + n - 1)(x + n + 1) = 4n$ for some $n \in N$.

Then $n + \alpha$ is equal to:

(1) 0 (2) 1

(3) 2 (4) 3

Answer (3)

Sol. $\sum_{k=0}^{n-1}(x+k)(x+k+2)=4n$

k^{th} term = $(x + k)(x + k + 2)$

$= (x + k + 1 - 1)(x + k + 1 + 1)$

$= (x + k + 1)^2 - 1$

$\therefore \quad \sum_{k=0}^{n-1}\left[(x+k+1)^2-1\right]=4n$

$\sum_{k=0}^{n-1}(x+k+1)^2-\sum_{k=0}^{n-1}1=4n$

$\sum_{k=1}^{n}(x+k)^2-n=4n$

$\sum_{k=1}^{n}x^2+k^2+2kx=5n$

$nx^2+\frac{n(n+1)(2n+1)}{6}+2x\frac{(n)(n+1)}{2}=5n$

$\Rightarrow \quad x^2+(n+1)x+\frac{(n+1)(2n+1)}{6}-5=0$

Now, difference of roots = $\frac{\sqrt{D}}{|a|}$

$\left|(\alpha+2)-\alpha\right|=\frac{\sqrt{b^2-4ac}}{1}$

$\Rightarrow b^2-4ac=4$

$\Rightarrow (n+1)^2-4\left[\frac{(n+1)(2n+1)}{6}-5\right]=4$

$\Rightarrow (n+1)^2-\frac{2(n+1)(2n+1)}{3}+20=4$

$\Rightarrow 3(n+1)^2-2(n+1)(2n+1)+60=12$

$\Rightarrow 3n^2+6n+3-4n^2-6n-2+48=0$

$\Rightarrow n^2=49 \Rightarrow n=7 \{\because n \in N\}$

Now, sum of roots $=\alpha+\alpha+2=-(n+1)$

$2\alpha+2=-8$

$\therefore n+\alpha=7-5=2$

2. Let x and y be real numbers such that $50\left(\frac{2x}{1+3i}-\frac{y}{1-2i}\right)=31+17i$, $i=\sqrt{-1}$. Then the value of $10(x - 3y)$ is:

(1) 20 (2) 31

(3) 35 (4) 75

Answer (4)

Sol. $50\left(\frac{2x}{1+3i}-\frac{y}{1-2i}\right)=31+17i$

$\Rightarrow 50\left(\frac{2x(1-3i)}{10}-\frac{y(1+2i)}{5}\right)=31+17i$

$\Rightarrow 10x-(30x)i-10y-20yi=31+17i$

$10(x-y)=31$ ...(i)

$-30x-20y=17$ ...(ii)

From (i) and (ii)

$x=\frac{9}{10}, y=\frac{-11}{5}$

$\therefore\ 10(x-3y)=10\left(\frac{9}{10}+\frac{33}{5}\right)=75$

3. Let $\alpha,\ \beta\in\mathbb{R}$ be such that the system of linear equations

$x+2y+z=5$

$2x+y+\alpha z=5$

$8x+4y+\beta z=18$

has no solution. Then $\frac{\beta}{\alpha}$ is equal to:

(1) –4 (2) 4

(3) 8 (4) –8

Answer (2)

Sol. $x+2y+z=5$

$2x+y+\alpha z=5$

$8x+4y+\beta z=18$

$\Delta=0$

$\begin{vmatrix}1&2&1\\2&1&\alpha\\8&4&\beta\end{vmatrix}=0$

$\Rightarrow 1(\beta-4\alpha)-2(2\beta-8\alpha)+1(8-8)=0$

$\Rightarrow \beta-4\alpha-4\beta+16\alpha=0$

$\Rightarrow 12\alpha-3\beta=0$

$\Rightarrow \frac{\beta}{\alpha}=4$

4. Let $A=\begin{bmatrix}1&2\\1&\alpha\end{bmatrix}$ and $B=\begin{bmatrix}3&3\\\beta&2\end{bmatrix}$ If $A^2-4A+I=O$ and $B^2-5B-6I=O$, then among the two statements:

(S1): $[(B-A)(B+A)]^T=\begin{bmatrix}13&15\\7&10\end{bmatrix}$

and (S2): $\det(\text{adj}(A+B))=-5$,

(1) Only (S1) is correct

(2) Only (S2) is correct

(3) Both (S1) and (S2) are correct

(4) Both (S1) and (S2) are wrong

Answer (2)

Sol. $A=\begin{bmatrix}1&2\\1&\alpha\end{bmatrix}$ and $A^2-4A+I=0$

$\text{tr}(A)=4$

$\Rightarrow 1+\alpha=4\Rightarrow A=\begin{bmatrix}1&2\\1&3\end{bmatrix}$

$B=\begin{bmatrix}3&3\\\beta&2\end{bmatrix}$ and $B^2-5B-6I=0$

$\det(B)=-6$

$6-3\beta=-6$

$3\beta=12\Rightarrow B=\begin{bmatrix}3&3\\4&2\end{bmatrix}$

For statement 1: $B-A=\begin{bmatrix}2&1\\3&-1\end{bmatrix}$

and $B+A=\begin{bmatrix}4&5\\5&5\end{bmatrix}$

$(B-A)(B+A)=\begin{bmatrix}2&1\\3&-1\end{bmatrix}\begin{bmatrix}4&5\\5&5\end{bmatrix}=\begin{bmatrix}13&15\\7&10\end{bmatrix}$

$[(B-A)(B+A)]^T=\begin{bmatrix}13&7\\15&10\end{bmatrix}$

For Statement 2:

$\det(\text{adj}(A+B))=\det(A+B)$

$=\begin{vmatrix}4&5\\5&5\end{vmatrix}=-5$

5. Let A be the set of first 101 terms of an A.P., whose first term is 1 and the common difference is 5 and let B be the set of first 71 terms of an A.P., whose first term is 9 and the common difference is 7. Then the number of elements in $A \cap B$, which are divisible by 3, is :

(1) 4 (2) 5

(3) 6 (4) 7

Answer (2)

Sol. $A = \{1,6,11,16,\ldots,501\}$

$B = \{9,16,23,30,\ldots,499\}$

For $A \cap B$

common difference, $D = \text{LCM}(5,7) = 35$

and first common term is 16

Now $16 + (k-1)35 \le \min(501,499)$

$16 + (k-1)35 \le 499$

$k - 1 \le 13.8$

$k \le 14.8$

$k = 14$

Now $16 + (k-1)35$ to be a multiple of 3

$16 + (k-1)35 \equiv 0 (\text{mod } 3)$

$1 + 2(k-1) \equiv 0 (\text{mod } 3)$

$\Rightarrow 2k - 1 \equiv 0 (\text{mod } 3)$

$\Rightarrow k \equiv 2 (\text{mod } 3)$

$k = 2,5,8,11,14$

$\Rightarrow$ we have 5 such elements

Number of elements in $A \cap B$ which are divisible by 3 is 5.

6. The number of seven-digit numbers, that can be formed by using the digits 1, 2, 3, 5 and 7 such that each digit is used at least once, is :

(1) 15400 (2) 17800

(3) 16800 (4) 29400

Answer (3)

Sol. **Case 1:** One digit repeats 3 times and the other used once (x,x,x,a,b,c,d)

Total number $= {}^5C_1 \times \frac{7!}{3!} = 5 \times 840 = 4200$

Case 2: Two digit are repeating twice each and other repeated once (x,x,a,a,b,c,c)

Total number $= {}^5C_2 \times \frac{7!}{2!2!} = 10 \times 1260 = 12600$

Total number $= 12600 + 4200 = 16800$

7. The number of elements in the set $S = \left\{(r,k) : k \in \mathbf{Z} \text{ and } {}^{36}C_{r+1} = \frac{6\left({}^{35}C_r\right)}{\left(k^2-3\right)}\right\}$, is :

(1) 2 (2) 4

(3) 8 (4) 16

Answer (2)

Sol. ${}^{36}C_{r+1} = \frac{6\left({}^{35}C_r\right)}{k^2-3}$

$\left(\frac{36}{r+1}\right){}^{35}C_r = \frac{6\left({}^{35}C_r\right)}{k^2-3} \left[\because {}^nC_r = \frac{n}{r}\left({}^{n-1}C_{r-1}\right)\right.$

$\frac{36}{r+1} = \frac{6}{k^2-3}$

$\frac{6}{r+1} = \frac{1}{k^2-3}$

$\Rightarrow k^2 - 3 = \frac{r+1}{6}$

$\Rightarrow k^2 = \frac{r+19}{6}$

$0 \le r \le 35$

For $r = 5$ $\quad k^2 = 4 \Rightarrow k = \pm 2$

For $r = 35$ $\quad k^2 = 9 \Rightarrow k = \pm 3$

The valid orders pairs (r,k) are

$(5,2),(5,-2),(35,3),(35,-3)$

8. If the mean of the data

Class	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	2	k	28	54	$k+1$	5

is 21, then k is one of the roots of the equation:

(1) $2x^2-23x-10=0$

(2) $4x^2-35x+24=0$

(3) $2x^2-19x-10=0$

(4) $2x^2-35x+98=0$

Answer (3)

Sol. Mean of the data

$$=\frac{\frac{15}{2}\times 2+\frac{25}{2}\times k+\frac{35}{2}\times 28+\frac{45}{2}\times 54+\frac{55}{2}\times(k+1)+\frac{65}{2}\times 5}{90+2k}=21$$

$$\therefore 40k+1910=21\cdot(90+2k)$$

$$\therefore 2k=20\Rightarrow k=10$$

which the roots of equation $2x^2-19x-10=0$

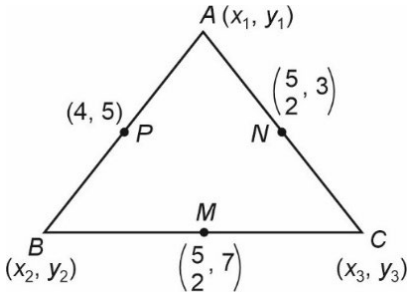
9. Let the mid points of the sides of a triangle ABC be $\left(\frac{5}{2},7\right),\left(\frac{5}{2},3\right)$ and (4, 5). If its incentre is (h, k), then $3h + k$ is equal to:

(1) 11 (2) 12

(3) 13 (4) 14

Answer (3)

Sol.



$\therefore$ $x_1 + x_2 = 8$, $x_2 + x_3 = 5$ and $x_3 + x_1 = 5$

and $y_1 + y_2 = 10$, $y_2 + y_3 = 14$ and $y_3 + y_1 = 6$

$\therefore$ $x_1 = 4$, $x_2 = 4$, $x_3 = 1$

and $y_1 = 1$, $y_2 = 9$, $y_3 = 5$

$\therefore$ $A = (4, 1)$, $B = (4, 9)$ and $C = (1, 5)$

$\therefore$ $AB = 8$, $BC = 5$ and $CA = 5$

$\therefore$ incentre of $\Delta ABC = (h, K)$

$$=\left(\frac{20+20+8}{18},\frac{5+45+40}{18}\right)$$

$$=\left(\frac{8}{3},5\right)$$

$\therefore$ $3h + k = 13$

10. Let an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a<b$, pass through the point (4, 3) and have eccentricity $\frac{\sqrt{5}}{3}$. Then the length of its latus rectum is:

(1) $\frac{4\sqrt{5}}{3}$ (2) $2\sqrt{5}$

(3) $\frac{7\sqrt{5}}{3}$ (4) $\frac{8\sqrt{5}}{3}$

Answer (4)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{16}{a^2}+\frac{9}{b^2}=1$ ($\because$ (4, 3) lies on ellipse) ...(1)

Also, $e^2=1-\frac{a^2}{b^2}$

$\frac{5}{9}=1-\frac{a^2}{b^2}$

$\frac{a^2}{b^2}=\frac{4}{9}$ ...(2)

Solving (1) and (2)

$a^2 = 20$

$b^2 = 45$

$E : \frac{x^2}{20} + \frac{y^2}{45} = 1$

$L(LR) = \frac{2a^2}{b} = \frac{2\times 20}{\sqrt{45}} = \frac{40}{3\sqrt{5}} = \frac{8}{3}\sqrt{5}$ units

11. If $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right) = K$, then the value of $\sin\left(\frac{10K\pi}{3}\right)$ is:

(1) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (2) $\frac{\sqrt{3}-1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$

$= \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{\pi}{3}-\frac{\pi}{18}\right)\sin\left(\frac{\pi}{3}+\frac{\pi}{18}\right)$

$= \frac{1}{4}\sin\left(3\cdot\frac{\pi}{18}\right) = \frac{1}{4}\sin\left(\frac{\pi}{6}\right)$

$= \frac{1}{8} = K$

Now $\sin\left(\frac{10K\pi}{3}\right) = \sin\left(\frac{10\pi}{8\times 3}\right)$

$= \sin\left(\frac{5\pi}{12}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

12. Let $S = \{x\in[-\pi, \pi]: \sin x(\sin x + \cos x) = a, a\in Z\}$. Then $n(S)$ is equal to:

(1) 3

(2) 6

(3) 7

(4) 9

Answer (4)

Sol. $x\in[-\pi,\pi]$

Let $E = \sin^2 x + \sin x \cdot \cos x$

$\Rightarrow \frac{1}{2}(2\sin^2 x + 2\sin x\cdot\cos x) = E$

$\Rightarrow \frac{1}{2}(1-\cos 2x + \sin 2x) = E$

$\Rightarrow \frac{1}{2}+\frac{1}{2}[\sin 2x - \cos 2x] = E$

$\sin 2x - \cos 2x \in [-\sqrt{2}, \sqrt{2}]$

$\frac{1}{2}+\frac{1}{2}(\sin 2x - \cos 2x) \in \left[\frac{1}{2}-\frac{1}{\sqrt{2}}, \frac{1}{2}+\frac{1}{\sqrt{2}}\right]$

$a = 0$ and 1

For $a = 0$

$\sin x(\sin x + \cos x) = 0 \Rightarrow \sin x = 0, \tan x = -1$

$\Rightarrow x = -\pi, -\frac{\pi}{4}, 0, \frac{3\pi}{4}, \pi$

For $a = 1$

$\sin x\cdot\cos x = \cos^2 x$

$\Rightarrow \cos x(\sin x - \cos x) = 0$

$\Rightarrow \cos x = 0, \tan x = 1$

$x = -\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{2}$

$\therefore$ Number of elements = 9

13. If the point of intersection of the lines $\frac{x+1}{3}=\frac{y+a}{5}=\frac{z+b+1}{7}$ and $\frac{x-2}{1}=\frac{y-b}{4}=\frac{z-2a}{7}$ lies on xy-plane, then the value of $a + b$ is:

(1) 2 (2) 5

(3) 7 (4) 9

Answer (3)

Sol. $L_1:\frac{x+1}{3}=\frac{y+a}{5}=\frac{z+b+1}{7}=\lambda$

$L_2:\frac{x-2}{1}=\frac{y-b}{4}=\frac{z-2a}{7}=\mu$

any point on L_1 : $(3\lambda-1,5\lambda-a,7\lambda-b-1)$

any point on L_2 : $(\mu+2, 4\mu+b, 7\mu+2a)$

$\because$ L_1 and L_2 intersect in x-y plane.

$\Rightarrow 7\lambda = b+1$ ...(i)

and $7\mu = -2a$ ...(ii)

also $3\lambda-1=\mu+2$ ...(iii)

and $5\lambda-a=4\mu+b$ ...(iv)

Solving (i), (ii), (iii) and (iv)

$a = 3$, $b = 4$, $\lambda=\frac{5}{7}$, $\mu=\frac{-6}{7}$

$\Rightarrow a+b=7$

14. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=2$ and $|\vec{b}|=3$, then the maximum value of $3\left|\left(3\vec{a}+2\vec{b}\right)\right|+4\left|\left(3\vec{a}-2\vec{b}\right)\right|$ is :

(1) 30 (2) 36

(3) 60 (4) 72

Answer (3)

Sol. $3\left(\sqrt{9a^2+4b^2+12\vec{a}\cdot\vec{b}}\right)+4\left(\sqrt{9a^2+4b^2-12\vec{a}\cdot\vec{b}}\right)$

$=3\left(36+36+72\cos\theta\right)^{\frac{1}{2}}+4\left(36+36-72\cos\theta\right)^{\frac{1}{2}}$

$=3\left(6\sqrt{2}\right)\left(1+\cos\theta\right)^{\frac{1}{2}}+4\cdot\left(6\sqrt{2}\right)\left(1-\cos\theta\right)^{\frac{1}{2}}$

$=18\sqrt{2}\left|\left(\sqrt{2}\cos\left(\frac{\theta}{2}\right)\right)\right|+24\sqrt{2}\left|\left(\sqrt{2}\sin\left(\frac{\theta}{2}\right)\right)\right|$

$=36\left|\cos\frac{\theta}{2}\right|+48\left|\sin\frac{\theta}{2}\right|$

max value = $\sqrt{36^2+48^2}$

= 60

15. Let a line L passing through the point (1, 1, 1) be perpendicular to both the vectors $2\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. If $P(a, b, c)$ is the foot of perpendicular from the origin on the line L, then the value of $34(a + b + c)$ is :

(1) 50

(2) 80

(3) 100

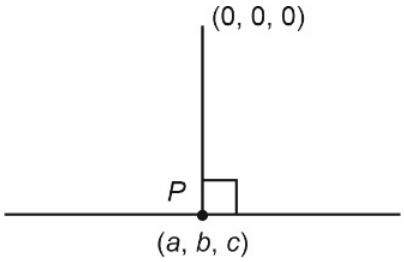
(4) 120

Answer (3)

Sol. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&2&1\\1&2&2\end{vmatrix}$

$=2\hat{i}-3\hat{j}+2\hat{k}$

$\therefore L:\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-1}{2}=\lambda$



$P(2\lambda+1, -3\lambda+1, 2\lambda+1)$

$\overrightarrow{OP}\cdot(\vec{a}\times\vec{b})=0$

$\Rightarrow (2\lambda+1)(2)+(-3\lambda+1)(-3)+(2\lambda+1)(2)=0$

$\Rightarrow 17\lambda=-1$

$\Rightarrow \lambda=\frac{-1}{17}$

$\therefore a+b+c=\lambda+3=\frac{50}{17}$

16. If $\lim\limits_{x\to 2}\frac{\sin\left(x^3-5x^2+ax+b\right)}{\left(\sqrt{x-1}-1\right)\log_e(x-1)}=m$, then $a+b+m$ is equal to :

(1) 5 (2) 6

(3) 8 (4) 10

Answer (2)

Sol. $\lim\limits_{x\to 2}\frac{\sin(x^3-5x^2+ax+b)}{(\sqrt{x-1}-1)\ln(x-1)}=m \quad \left(\frac{0}{0}\right)$

$\therefore 8-20+2a+b=0$

$= 2a+b=12$

$\lim\limits_{x\to 2}\frac{\left(x^3-5x^2+ax+b\right)\left(\sqrt{x-1}+1\right)}{(x-2)\frac{\ln(1+(x-2)}{(x-2)}(x-2)}$

$2\lim\limits_{x\to 2}\frac{x^3-5x^2+ax+b}{(x-2)^2}$

$=2\lim\limits_{x\to 2}\frac{3x^2-10x+a}{2(x-2)} \quad \left(\frac{0}{0}\right)$

$12-20+a=0\Rightarrow a=8\Rightarrow b=-4$

$=2\lim\limits_{x\to 2}\frac{6x-10}{2}=m$

$\Rightarrow 2(1)=m$

$\therefore 8-4+2=6$

17. If the curve $y=f(x)$ passes through the point $(1, e)$ and satisfies the differential equation $dy = y(2+\log_e x)dx$, $x>0$, then $f(e)$ is equal to

(1) e^e

(2) e^{e^2}

(3) e^{2e}

(4) e^{2^e}

Answer (3)

Sol. $\int\frac{dy}{y}=\int(2+\ln x)\,dx$

$\Rightarrow \ln|y|=2x+(x\ln x-x)+c\Big|_{(1,e)}$

$\Rightarrow 1=2+(-1)+c\Rightarrow c=0$

$x=e$

$\ln|y|=2e+(e-e)\Rightarrow y=e^{2e}$

18. The number of critical points of the function $f(x)=\begin{cases}\left|\frac{\sin x}{x}\right|, & x\neq 0\\ 1, & x=0\end{cases}$ in the interval $(-2\pi, 2\pi)$ is equal to :

(1) 1

(2) 3

(3) 5

(4) 7

Answer (3)

Sol. Critical points: Point at which derivative is zero or function is not differentiable

$f(x)=\begin{cases}\left|\frac{\sin x}{x}\right| & , \quad x\neq 0\\ 1 & , \quad x=0\end{cases}$

$f'(0^+)=\lim\limits_{h\to 0}\left|\frac{\frac{\sin h}{h}-1}{h}\right|=\lim\limits_{h\to 0}\frac{\sin h-h}{h^2}=\lim\limits_{h\to 0}\frac{\cos h-1}{2h}=0$

Similarly, $f'(0^-) = 0$

$f'(0) = 0$

$\Rightarrow$ $x = 0$ is critical point

$$f'(\pi^+) = \lim_{h\to 0} \frac{\left|\frac{\sin(\pi+h)}{\pi+h}\right| - 0}{h} = \lim_{h\to 0} \frac{-\sin(\pi+h)}{(\pi+h)h}$$

$$= \lim_{h\to 0} \frac{-\cos(\pi+h)}{2h+\pi} = \frac{1}{\pi}$$

Similarly, $f'(\pi^-) = -\frac{1}{\pi}$

$\Rightarrow f(x)$ is not differentiable at $x = \pi$ and $-\pi$

($\because$ $f(x)$ is even function)

And $f'(x) = \pm\left(\frac{x\cos x - \sin x}{x^2}\right) \Rightarrow \tan x = x$

$\Rightarrow$ $x \simeq 4.49$

$\Rightarrow$ $\tan x = x$ has 2 solutions in $x \in (-2\pi, 2\pi)$

$\therefore$ 2 critical points

$\Rightarrow$ Total critical points = 5

19. Let [·] denote the greatest integer function. Then the value of $\int_0^3 \left(\frac{e^x + e^{-x}}{[x]!}\right) dx$ is

(1) $e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}$

(2) $\frac{1}{2}\left(e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}\right)$

(3) $e^2 + e^3 - \frac{1}{2e^2} - \frac{1}{2e^3}$

(4) $\frac{1}{2}\left(e^2 + e^3\right) - \frac{1}{e^2} - \frac{1}{e^3}$

Answer (2)

Sol. $I = \int_0^3 \frac{e^x + e^{-x}}{[x]!} dx$

$$= \int_0^1 \frac{e^x + e^{-x}}{1} dx + \int_1^2 \frac{e^x + e^{-x}}{1} dx + \int_2^3 \frac{e^x + e^{-x}}{2} dx$$

$$= \int_0^2 (e^x + e^{-x})\, dx + \frac{1}{2}\int_2^3 (e^x + e^{-x})\, dx$$

$$= \frac{1}{2}\left(e^2 + e^3 - \frac{1}{e^2} - \frac{1}{e^3}\right)$$

20. Let $y = y(x)$ be the solution curve of the differential equation $(1+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$, $y(0) = 0$. If the curve $y = y(x)$ passes through the point $\left(\alpha, \frac{-1}{2}\right)$, then a value of α is

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{2}$

Answer (4)

Sol. $(1+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$

$(1 + \sin x)\, dy + (y + 1)\cos x\, dx = 0$

$d((1 + \sin x)(y + 1)) = 0$

$\Rightarrow$ $(1 + \sin x)(y + 1) = c$

$\because$ $y(0) = 0 \Rightarrow c = 1 \Rightarrow (1 + \sin x)(1 + y) = 1$

$\Rightarrow$ $(1+\sin\alpha)\left(1 - \frac{1}{2}\right) = 1$

$\Rightarrow$ $\sin\alpha = 1 \Rightarrow \alpha = \frac{\pi}{2}$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the domain of the function

$$f(x)=\sqrt{\log_{(0.6)}\left(\left|\frac{2x-5}{x^2-4}\right|\right)}$$ is $(-\infty, a] \cup \{b\} \cup [c, d) \cup (e, \infty)$, then the value of $a + b + c + d + e$ is ____.

Answer (4)

Sol. $f(x)=\sqrt{\log_{(0.6)}\left(\left|\frac{2x-5}{x^2-4}\right|\right)}$

$\Rightarrow \log_{0.6}\left(\left|\frac{2x-5}{x^2-4}\right|\right)\geq 0$ ... (i)

$\left|\frac{2x-5}{x^2-4}\right|>0$ ... (ii)

$|x^2-4|\neq 0$ ... (iii)

from (i), $\left|\frac{2x-5}{x^2-4}\right|\leq 1\Rightarrow (2x-5)^2\leq (x^2-4)^2$

$\Rightarrow [(2x-5)-(x^2-4)][(2x-5)+(x^2-4)]\leq 0$

$\Rightarrow (x-1)^2((x+1)^2-10)\geq 0$

$\Rightarrow x\in(-\infty,-1-\sqrt{10}]\cup\{1\}\cup[-1+\sqrt{10},\infty)$

from (2) and (3), $x\neq \pm 2, \frac{5}{2}$

$\Rightarrow$ Domain = $(-\infty,-1-\sqrt{10}]\cup\{1\}\cup\left[-1+\sqrt{10},\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

$\Rightarrow a+b+c+d+e=4$

22. If $\sum_{k=1}^{n} a_k = 6n^3$, then $\sum_{k=1}^{6}\left(\frac{a_{k+1}-a_k}{36}\right)^2$ is equal to _____.

Answer (91)

Sol. $\sum_{k=1}^{n} a_k = 6n^3$

$\Rightarrow a_1 + a_2 + \ldots + a_n = 6n^3$

$a_1 + \ldots + a_n + a_{n+1} = 6(n+1)^3$

$\Rightarrow a_{n+1} = 6(n+1)^3 - 6n^3$

$a_n = 6n^3 - 6(n-1)^3$

$\Rightarrow a_{n+1} - a_n = 6\left[(n+1)^3 + (n-1)^3 - 2n^3\right]$

$= 6\left[n^3 + 3n^2 + 3n + 1 + n^3 - 3n^2 + 3n - 1 - 2n^3\right]$

$\Rightarrow a_{n+1} - a_n = 36n$

$\Rightarrow \sum_{k=1}^{6}\left(\frac{a_{k+1}-a_k}{36}\right)^2 = \sum_{k=1}^{6}\left(\frac{36k}{36}\right)^2$

$= 1^2 + 2^2 + \ldots + 6^2$

$= \frac{6\times 7\times 13}{6} = 91$

23. Let $a, b, c \in \{1, 2, 3, 4\}$. If the probability, that $ax^2+2\sqrt{2}bx+c>0$ for all $x\in R$ is $\frac{m}{n}$, $gcd(m, n) = 1$, then $m + n$ is equal to _____.

Answer (81)

Sol. $\therefore \quad ax^2+2\sqrt{2}bx+c>0$ and $a > 0$, $\forall x\in R$

$\Rightarrow D < 0$ for no roots

$\Rightarrow \left(2\sqrt{2}b\right)^2 - 4ac < 0$

$8b^2 - 4ac < 0$

$\Rightarrow 2b^2 < ac$

Total choices = $4 \times 4 \times 4 = 64$

Favourable cases :

(i) $b = 1 \Rightarrow ac > 2 \Rightarrow$ All except pairs (a, c)

(1, 1), (2, 1), (1, 2)

$\Rightarrow 16 - 3 = \boxed{13}$

(ii) $b = 2 \Rightarrow ac > 8$

Pairs can be (3, 3), (4, 3), (4, 3), (4, 4)

$\boxed{4}$

(iii) $b = 3 \Rightarrow ac > 18 \Rightarrow$ no such pair

(iv) $b = 4 \Rightarrow ac > 32 \Rightarrow$ no such pair

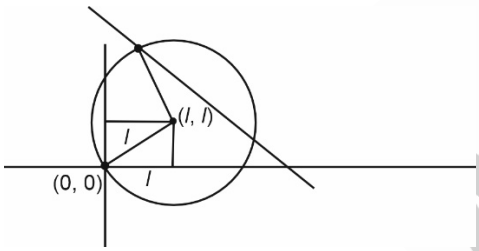
$\Rightarrow$ total favourable $(a, b, c) = 17$

$$\Rightarrow P = \frac{17}{64} = \frac{m}{n} \Rightarrow m + n = \boxed{81}$$

24. Let a circle C have its centre in the first quadrant, intersect the coordinate axes at exactly three points and cut off equal intercepts from the coordinate axes. If the length of the chord of C on the line $x + y = 1$ is $\sqrt{14}$, then the square of the radius of C is _____.

Answer (8)

Sol.



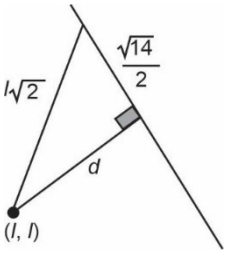
$$R = \sqrt{l^2 + l^2} = l\sqrt{2}$$

Equation of circle

$$= (x - l)^2 + (y - l)^2 = 2l^2$$

$$\Rightarrow x^2 + y^2 - 2xl - 2yl = 0$$

$$d = \left|\frac{l + l - 1}{\sqrt{2}}\right|$$



$$\Rightarrow \left(\frac{2l-1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{14}}{2}\right)^2 = \left(l\sqrt{2}\right)^2$$

$$\Rightarrow \frac{(2l-1)^2}{2} + \frac{14}{4} = 2l^2$$

$$\Rightarrow 2(2l-1)^2 + 14 = 8l^2$$

$$\Rightarrow 8l^2 - 8l + 2 + 14 = 8l^2$$

$$\Rightarrow l = 2$$

$$\Rightarrow R^2 = 2l^2 = 8$$

25. If $\alpha = \int_0^{2\sqrt{3}} \log_2(x^2 + 4)\,dx + \int_2^4 \sqrt{2^x - 4}\,dx$, then α^2 is equal to _____.

Answer (192)

Sol. $\alpha = \int_0^{2\sqrt{3}} \log_2(x^2 + 4)\,dx + \int_2^4 \sqrt{2^x - 4}\,dx$

Notice that

$f(x) = \log_2(x^2 + 4)$

$$\Rightarrow 2^{f(x)} = x^2 + 4 \Rightarrow x = \sqrt{2^{f(x)} - 4}$$

$\Rightarrow$ $\log_2(x^2 + 4)$ and $\sqrt{2^x - 4}$ are inverse of each other

Since, $\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx = bf(b) - af(a)$

$$\Rightarrow \alpha = 2\sqrt{3}\log_2\left((2\sqrt{3})^2 + 4\right) = 2\sqrt{3}\log_2 16 = 8\sqrt{3}$$

$$\Rightarrow \alpha^2 = 64 \times 3 = 192$$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. The dimensional formula of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of vacuum and E = electric field) is $M^aL^bT^c$. The value of $2a - b + c$ = _____.

(1) 0 (2) 1

(3) –1 (4) 2

Answer (2)

Sol. $\frac{1}{2}\varepsilon_0 E^2 \equiv \frac{\varepsilon}{V} \equiv \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

$\Rightarrow$ $2 \times 1 + 1 - 2 = 1$

27. The diameter of a wire measured by a screw gauge of least count 0.001 cm is 0.08 cm. The length measured by a scale of least count 0.1 cm is 150 cm. When a weight of 100 N is applied to the wire, the extension in length is 0.5 cm, measured by a micrometer of least count 0.001 cm. The error in the measured Young's modulus is $\alpha \times 10^9$ N/m^2. The value of α is _____.

(Ignore the contribution of the load to Young's modulus error calculation)

(1) 1.3 (2) 1.65

(3) 0.13 (4) 0.25

Answer (2)

Sol. $Y = \frac{Fl}{A\Delta l} = \frac{Fl}{\pi r^2 \Delta l} = \frac{100 \times 150 \times 10^{-2}}{\pi \times (0.04)^2 \times 10^{-4} \times 0.5 \times 10^{-2}}$

$\frac{\Delta Y}{Y} = \frac{0.1}{150} + 2\frac{0.001}{0.080} + \frac{0.001}{0.500}$

$= \frac{1}{1500} + \frac{2}{80} + \frac{1}{500}$

$= \frac{4}{1500} + \frac{2}{80}$

$\frac{\Delta Y}{Y} = \frac{32 + 300}{12000} = \frac{0.332}{12}$

$Y = \frac{150}{\pi \times 16 \times 0.5 \times 10^{-10}} = 5.97 \times 10^{10}$

$\Delta Y = \frac{0.332}{12} \times 59.7 \times 10^9 \Rightarrow \alpha = 1.65$

28. The velocity of a particle is given as $\vec{v} = -x\hat{i} + 2y\hat{j} - z\hat{k}$ m/s. The magnitude of acceleration at point (1, 2, 4) is _____ m/s^2.

(1) $\sqrt{6}$

(2) 9

(3) $\sqrt{33}$

(4) 0

Answer (2)

Sol. $\vec{v} = -x\hat{i} + 2y\hat{j} - z\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$\vec{a} = -v_x\hat{i} + 2v_y\hat{j} - v_z\hat{k}$

$= -\hat{i} + 2 \times 4\hat{j} - (-4)\hat{k}$

$\vec{a} = -\hat{i} + 8\hat{j} + 4\hat{k}$

a = 9 m/s^2

29. The position of a object having mass 0.1 kg as a function of time t is given as $\vec{r} = \left(10t^2\hat{i} + 5t^3\hat{j}\right)$ m.

At t = 1 s, which of the following statements are **correct**?

A. The linear momentum $\vec{p} = \left(2\hat{i} + 1.5\hat{j}\right)$ kg·m/s.

B. The force acting on the object $\vec{F} = \left(2\hat{i} + 3\hat{j}\right)$ N.

C. The angular momentum of the object about its origin $\vec{L} = 15\hat{k}$ J s.

D. The torque acting on the object about its origin $\vec{\tau} = 20\hat{k}$ Nm.

Choose the **correct** answer from the options given below:

(1) A, B and C only

(2) B, C and D only

(3) A, C and D only

(4) A, B and D only

Answer (4)

Sol. $\vec{r} = 10t^2\hat{i} + 5t^3\hat{j} = 10\hat{i} + 5\hat{j}$

$\vec{v} = 20t\hat{i} + 15t^2\hat{j} = 20\hat{i} + 15\hat{j}$

$\Rightarrow \vec{p} = (2\hat{i} + 1.5\hat{j})$

$\vec{a} = 20\hat{i} + 30t\hat{j}$

$\Rightarrow \vec{F} = 2\hat{i} + 3\hat{j}$

$\Rightarrow \vec{L} = (10\hat{i} + 5\hat{j}) \times (2\hat{i} + 1.5\hat{j})$

$= 15\hat{k} - 10\hat{k} = 5\hat{k}$

$\Rightarrow \vec{\tau} = (10\hat{i} + 5\hat{j}) \times (2\hat{i} + 3\hat{j})$

$= 30\hat{k} - 10\hat{k} = 20\hat{k}$

30. A planet (P_1) is moving around the star of mass $2M$ in the orbit of radius R. Another planet (P_2) is moving around another star of mass $4M$ in a orbit of radius $2R$. Ratio of time periods of revolution of P_2 and P_1 is _____.

(1) $\frac{1}{2}$

(2) 2

(3) 4

(4) $\frac{1}{4}$

Answer (2)

Sol. $T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$

$\Rightarrow$ Ratio $= \frac{T_2}{T_1} = \frac{2^{3/2}}{\sqrt{2}} = 2$

31. A particle is rotating in a circular path and at any instant its motion can be described as $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$. The angular acceleration of the particle after 10 seconds is ____ rad / s^2.

(1) 150 (2) 120

(3) 130 (4) 170

Answer (3)

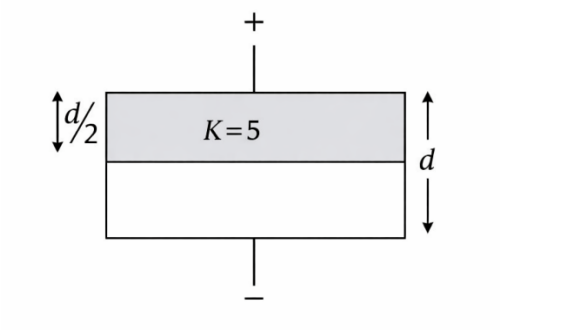
Sol. $\theta = \frac{5t^4}{40} - \frac{t^3}{3}$, $\alpha = \frac{d^2\theta}{dt^2}$

$\Rightarrow \alpha = \frac{5 \times 4 \times 3t^2}{40} - \frac{3 \times 2t}{3}$

$= \frac{3}{2} \times 100 - 20$

= 130 rad/s^2

32. A parallel plate air capacitor has a capacitance C. When it is half filled as show in figure with a dielectric constant $K = 5$, the percentage increase in the capacitance is _____.



(1) 33.34 (2) 66.67

(3) 200 (4) 400

Answer (2)

Sol. $C = \dfrac{\varepsilon_0 A}{\left(\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}\right)}$

$$C_{new} = \frac{\varepsilon_0 A}{\frac{d}{10} + \frac{d}{2}} = \frac{10}{6}\frac{\varepsilon_0 A}{d} = \frac{5}{3}C$$

$$\% \text{ increase} = \frac{\frac{5}{3}C - C}{C} \times 100 = 66.67\%$$

33. Heat is supplied to a diatomic gas at constant pressure. Then the ratio of $\Delta Q : \Delta U : \Delta W$ is ____.

(1) 2 : 3 : 5 (2) 5 : 3 : 2

(3) 2 : 5 : 7 (4) 7 : 5 : 2

Answer (4)

Sol. At constant pressure for diatomic gas $\Delta Q : \Delta U : \Delta W = \mu C_p \Delta T : \mu C_v \Delta T : \mu R \Delta T$.

$$= \frac{7}{2}R : \frac{5}{2}R : R$$

$\Rightarrow$ 7 : 5 : 2

34. Two charged conducting spheres S_1 and S_2 of radii 8 cm and 18 cm are connected to each other by a wire. After equilibrium is established, the ratio of electric fields on S_1 and S_2 spheres are E_{S1} and E_{S2} respectively. The value of $\dfrac{E_{S1}}{E_{S2}}$ is ____.

(1) $\frac{3}{2}$

(2) $\frac{2}{3}$

(3) $\frac{4}{9}$

(4) $\frac{9}{4}$

Answer (4)

Sol.

$$V_1 = V_2 \Rightarrow \frac{Q_1}{R_1} = \frac{Q_2}{R_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$$

$$\frac{E_1}{E_2} = \frac{Q_1 R_2^2}{R_1^2 Q_2^2} = \frac{R_2}{R_1} = \frac{18}{8} = \frac{9}{4}$$

35. The equation of a plane progressive wave is given by $y = 5\cos\pi\left(200t - \dfrac{x}{150}\right)$ where x and y are in cm and t is in second. The velocity of the wave is ___ m/s.

(1) 120 (2) 150

(3) 200 (4) 300

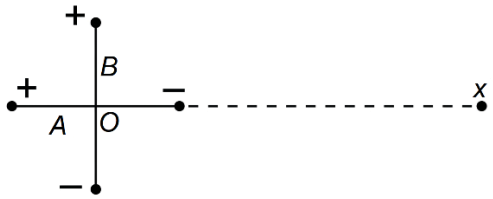
Answer (4)

Sol. $y = 5\cos\left(200\pi t - \dfrac{x\pi}{150}\right) = A\sin(\omega t - kx)$

$$k = \frac{\omega}{v}$$

$$v = \frac{200\pi}{\pi/150} = 150 \times 200 \text{ cm/s} = 300 \text{ m/s}$$

36. Two short electric dipoles *A* and *B* having dipole moment p_1 and p_2 respectively are placed with their axis mutually perpendicular as shown in the figure. The resultant electric field at a point *x* is making an angle of 60° with the line joining points *O* and *x*. The ratio of the dipole moments p_2 / p_1 is _____.

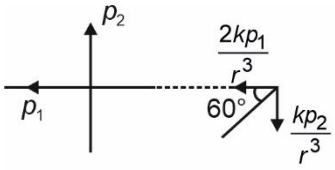


(1) $\frac{\sqrt{3}}{2}$ (2) $2\sqrt{3}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\sqrt{3}$

Answer (2)

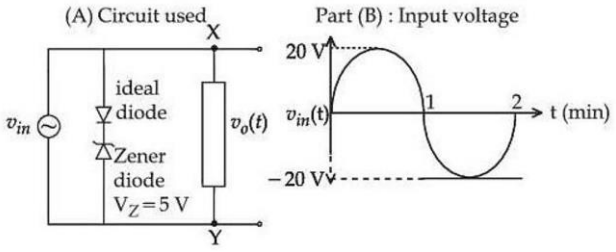
Sol.



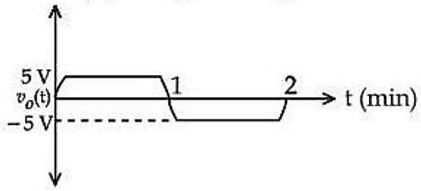
$$\Rightarrow \tan 60° = \frac{p_2}{2p_1}$$

$$\Rightarrow \frac{p_2}{p_1} = 2\sqrt{3}$$

37. For the given circuit (shown in part (A)) the time dependent input voltage $v_{in}(t)$ and corresponding output $v_o(t)$ are shown in part (B) and part (C), respectively. Identify the components that are used in the circuit between points *X* and *Y*.



Part (C) : Output voltage



(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. For positive cycle, left branch will give out put as right branch is reverse biased.

For negative cycle, right branch must be similar but in opposite sense to that of left.

38. When a coil is placed in a time dependent magnetic field the power dissipated in it is *P*. The number of turns, area of the coil and radius of the coil wire are *N*, *A* and *r* respectively. For a second coils number of turns, area of the coil and radius of the coil wire are 2*N*, 2*A* and 3*r* respectively. When the first coil is replaced with second coil the power dissipated in it is $\sqrt{2}\alpha P$. The value of α is _____.

(1) 36

(2) $128\sqrt{2}$

(3) 16

(4) 64

Answer (1)

Sol. $\varepsilon = NA\frac{dB}{dt}$ and Power $= \frac{\varepsilon^2}{R}$

$$R = \frac{\rho l}{A} = \frac{\rho 2\pi R_C N}{\pi r^2} = \frac{CN\sqrt{A}}{r^2}$$

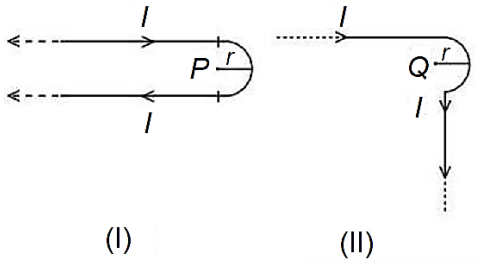
$\Rightarrow$ Power $= \frac{N^2A^2}{CN\sqrt{A}}\left(\frac{dB}{dT}\right)^2 r^2$

Power $\propto NA^{3/2}\, r^2$

$\frac{P}{P'} = \frac{1}{2\times 2^{3/2}\times 9}$

$\frac{P'}{P} = 36\sqrt{2} \Rightarrow r = 36$

39. Two identical long current carrying wires are bent into the shapes shown in the following figures. If the magnitude of magnetic fields at the centres P and Q of a semicircular arc are B_1 and B_2 respectively, then the ratio $\frac{B_1}{B_2}$ is _____.



(I) (II)

(1) $\frac{2+\pi}{1+\pi}$ (2) $\frac{1+\pi}{1-\pi}$

(3) $\frac{2+\pi}{1-\pi}$ (4) $\frac{1+\pi}{2-\pi}$

Answer (1)

Sol. $B_1 = \frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{4r}$

$B_2 = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r}$

$\frac{B_1}{B_2} = \frac{\frac{1}{2\pi}+\frac{1}{4}}{\frac{1}{4\pi}+\frac{1}{4}} = \frac{2+\pi}{1+\pi}$

40. For a thin symmetric prism made of glass (refractive index 1.5), the ratio of incident angle and minimum deviation will be _____.

(1) 3 : 4 (2) 3 : 2

(3) 2 : 1 (4) 1 : 2

Answer (2)

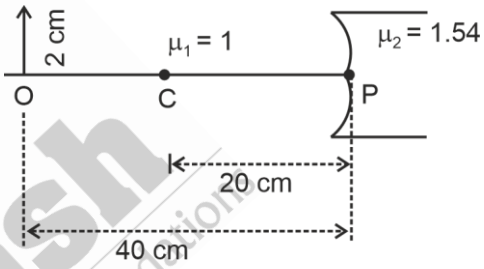
Sol. $\delta = (\mu - 1)A$

$\delta = 0.5$ A

$\Rightarrow \frac{A}{\delta} = 2,\ i + e - A = \delta,\ i = \frac{\delta + A}{2}$

$= i = \frac{3\delta}{2} \Rightarrow \frac{i}{\delta} = \frac{3}{2}$

41. Refer the figure given below. μ_1 and μ_2 are refractive indices of air and lens material. The height of image will be _____ cm.



(1) 1 (2) 0.5

(3) 1.2 (4) 0.25

Answer (1)

Sol. |R| = 20, |4| = 40

$\frac{1.54}{v} = \frac{1}{(-40)} = \frac{1.54-1}{(-20)}$

$\frac{1.54}{v} = -\frac{1}{40} - \frac{0.54}{20}$

$\frac{1.54}{v} = \frac{-2.08}{40}$

$m = \frac{v/\mu_2}{u/\mu_1} = \frac{v}{1.54(-40)} = \frac{1}{2.08} = \frac{h_i}{2}$

$\Rightarrow h_i \approx 1$

42. For a certain metal, when monochromatic light of wavelength λ is incident, the stopping potential for photoelectrons is $3V_O$. When the same metal is illuminated by light of wavelength 2λ, then the stopping potential becomes V_O,. The threshold wavelength for photoelectric emission for the given metal is $\alpha\lambda$. The value of α is _____.

(1) 1 (2) 4

(3) 2 (4) 4

Answer (2)

Sol. $\frac{hc}{\lambda} = \phi + e3V_O$ ...(i)

$\frac{hc}{2\lambda} = \phi + eV_O$ ...(ii)

3(ii) (i) gives

$\frac{hc}{2\lambda} = 2\phi$

$\phi = \frac{hc}{4\lambda} = \frac{hc}{\lambda_O} \Rightarrow \propto = 4$

43. An electromagnetic wave travelling in x-direction is described by field equation $E_y = 300 \sin\omega\left(t - \frac{x}{c}\right)$.

If the electron is restricted to move in y-direction only with speed of 1.5×10^6 m/s then ratio of maximum electric and magnetic forces acting on the electron is ____.

(1) 200 (2) 150

(3) 400 (4) 300

Answer (1)

Sol. $E = CB$

$\frac{F_E}{F_B} = \frac{qE}{qvB} = \frac{CB}{VB} = \frac{C}{V} = \frac{3\times10^8}{1.5\times10^6}$

$= 200$

44. Angular momentum of an electron in a hydrogen atom is $\frac{3h}{\pi}$, then the energy of the electron is _____ eV.

(1) –1.51 (2) –0.85

(3) –0.38 (4) –0.28

Answer (3)

Sol. $\frac{nh}{2\pi} = \frac{3h}{\pi} \Rightarrow n = 6$

$E = \frac{-13.6}{6^2} = -0.38$

45. A liquid drop of diameter 2 mm breaks into 512 droplets. The change in surface energy is $\alpha \times 10^{-6}$ J. The value of α is _____. (Take surface tension of liquid = 0.08 N/m)

(1) 10 (2) 7

(3) 8 (4) 11

Answer (2)

Sol. $U_i = S4\pi R^2$

$R^3 = 512r^3$

$\Rightarrow r = \frac{R}{8}$

$U_f = \left\{S\,4\pi\left(\frac{R}{8}\right)^2\right\}512 = 8\left(S\,4\pi R^2\right)$

$\Delta U = 28\ S\pi R^2$

$= 28 \times 0.08 \times \frac{22}{7} \times 10^{-6}$

$= 7 \times 10^{-6}$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. In single slit diffraction pattern, the wavelength of light used is 628 nm and slit width is 0.2 mm, the angular width of central maximum is $\alpha \times 10^{-2}$ degrees. The value of α is _____.

Answer (36)

Sol. $\theta = \frac{\lambda}{a}$

$$\varphi = 2\theta = \frac{2\times628\times10^{-9}}{0.2\times10^{-3}}\times\frac{180}{3.14} \text{ degrees}$$

$$= 10\times200\times180\times10^{-6}$$

$$\varphi = 36\times10^{-2}$$

47. A vessel contains 0.15 m^3 of a gas at pressure 8 bar and temperature 140 °C with $c_p = 3R$ and $c_v = 2R$. It is expanded adiabatically till pressure falls to 1 bar. The work done during this process is _____ kJ.

(R is gas constant)

Answer (120)

Sol. $\gamma = 1.5$

PV^{γ} = constant for adiabatic forces

$8 \times (0.15)^{1.5} = 1\ V^{1.5}$

$$2^{\frac{3}{1.5}}\times0.15 = V$$

$V = 0.6\ m^3$

$$w = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$= \frac{(1\times0.6 - 8\times0.15)}{-0.5}\times10^5$$

$$= \frac{6}{5}\times10^5 = 120 \text{ kJ}$$

48. 1 μC charge moving with velocity $\vec{v} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ m/s in the region of magnetic field $\vec{B} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$ T. The magnitude of force acting on it is $\sqrt{\alpha}\times10^{-6}$ N. The value of α is _____.

Answer (171)

Sol. $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

$$= 10^{-6}\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)\times\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

$$= 10^{-6}\left(3\hat{k} + 5\hat{j} + 4\hat{k} + 10\hat{i} + 6\hat{j} - 9\hat{i}\right)$$

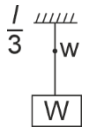
$$= 10^{-6}\left(\hat{i} + 11\hat{j} + 7\hat{k}\right)$$

$$= \sqrt{171}\times10^{-6}$$

49. A uniform wire of length l of weight w is suspended from the roof with a weight of W at the other end. The stress in the wire at $\frac{l}{3}$ distance from the top is $\left(\frac{W}{A} + \frac{2}{\gamma}\frac{w}{A}\right)$, where, A is the cross sectional area of the wire. The value of γ is _____.

Answer (3)

Sol. $\sigma = \frac{\frac{2w}{3} + W}{A}$



50. A tub is filled with water and a wooden cube 10 cm × 10 cm × 10 cm is placed in the water. The wooden cube is found to float on the water with a part of it submerged in water. When a metal coin is placed on the wooden cube, the submerged part is increased by 3.87 cm. The mass of the metal coin is _____ gram.

(Take water density as 1 g/cm^3 and density of wood as 0.4 g/cm^3)

Answer (387)

Sol. $\Delta mg = \Delta v\rho g = a^2\Delta x\rho g$

$\Delta m = 10\times10\times3.87\times1$

= 387 g

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. The mass of iron converted into Fe_3O_4 by the action of 18 g of steam is

(Given: Molar mass of H, O and Fe are 1, 16 and 56 g mol^{-1} respectively)

Assume iron is present in excess:

(1) 2.1 g (2) 4.2 g

(3) 21 g (4) 42 g

Answer (4)

Sol. $3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2$

$$n_{steam} = \frac{18}{18} = 1$$

4 mole steam reacts with 3 moles Fe

1 moles steam reacts with $\frac{3}{4}$ moles Fe

$$m_{Fe} = \frac{3}{4} \times 56 = 42 \text{ g}$$

52. What is the energy (in J $atom^{-1}$) required for the following process?

$$Li^{2+}(g) \rightarrow Li^{3+}(g) + e^-$$

(Take the ionization energy for the H atom in the ground state as 2.18×10^{-18} J $atom^{-1}$)

(1) 8.72×10^{-18}

(2) 1.962×10^{-18}

(3) 1.962×10^{-17}

(4) 6.54×10^{-17}

Answer (3)

Sol. $Li^{2+} \rightarrow Li^{3+} + e^-$

1^{st} IE of Li^{2+} = $K \times Z^2$ ($K \times 1^2 = 2.18 \times 10^{-18}$)

$= 2.18 \times 10^{-18} \times 9$

$= 19.62 \times 10^{-18}$

$= 1.962 \times 10^{-17}$ J/atom

53. Given below are two statements :

Statement (I) : The correct sequence of bond lengths in the following species is:

$O_2^+ < O_2 < O_2^- < O_2^{2-}$

Statement (II) : The correct sequence of number of unpaired electrons in the following species is:

$O_2 > O_2^+ > O_2^- > O_2^{2-}$

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Answer (3)

Sol. O_2 $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2$ $\pi^* 2p_x^1 \pi^* 2p_y^1$

	B.O (Bond order)	Unpaired e^-
O_2^+	2.5	1
O_2	2.0	2
O_2^-	1.5	1
O_2^{2-}	1.0	0

$$BO \propto \frac{1}{BL}$$

54. Consider the following data :

(i) $2Al(s)+6HCl(aq)\rightarrow Al_2Cl_6(aq)+3H_2(g)+1200\ kJ/mol$

(ii) $H_2(g)+Cl_2(g)\rightarrow 2HCl(g)+164\ kJ/mol$

(iii) $HCl(g)+aq\rightarrow HCl(aq)+83\ kJ/mol$

(iv) $Al_2Cl_6(s)+aq\rightarrow Al_2Cl_6(aq)+663\ kJ/mol$

The enthalpy of formation of anhydrous solid Al_2Cl_6 is

(1) –648 kJ mol^{-1} (2) –1350 kJ mol^{-1}

(3) –2002 kJ mol^{-1} (4) –1527 kJ mol^{-1}

Answer (4)

Sol. Required reaction :

$2Al(s)+3Cl_2(g)\rightarrow Al_2Cl_6(s)$

Net reaction :

$2Al(s) + 6HCl(aq) \rightarrow Al_2Cl_6(aq) + 3H_2(g)$

$3H_2(g) + 3Cl_2(g) \rightarrow 6HCl(g)$

$6HCl(g) + aq \rightarrow 6HCl(aq)$

$Al_2Cl_6(aq) \rightarrow Al_2Cl_6(s) + aq$

Net reaction : $2Al(s) + 3Cl_2(g) \rightarrow Al_2Cl_6(s)$

So, ΔH_f = (i) + (ii) × 3 + (iii) × 6 – (iv)

= –1200 + (–3 × 164) + (–83 × 6) – (–663)

– 1200 – 492 + (–498) + (663)

= –1692 –498 + 663

= –2190 + 663

= –1527 kJ/mol

55. 19.5 g of fluoro acetic acid (molar mass = 78 g mol^{-1}) is dissolved in 500 g of water at 298 K. The depression in the freezing point of water was 1°C . What is K_a of fluoro acetic acid? (For water, $K_f = 1.86$ K kg mol^{-1}). Assume molarity and molality to have same values.

(1) 10^{-6} (2) 4×10^{-4}

(3) 3×10^{-5} (4) 3×10^{-3}

Answer (3)

Sol. $n_{acid} = \frac{19.5}{78} = 0.25$

And, $\Delta T_f = i \times K_f \times m = (1 + \alpha)(1.86) \times 0.25 \times \frac{1000}{500}$

$\left.\frac{1}{0.93} = 1+\alpha\right] \Rightarrow \alpha = 0.075$

$K_a = \frac{\alpha^2 c}{1-\alpha} = \frac{(0.075)^2 \times 0.5}{1-0.075} \approx 3\times10^{-3}\ (\text{approx})$

56. The solubility product constants of Ag_2CrO_4 and AgBr are 32x and 4y respectively at 298 K. The value of $\left(\frac{\text{molarity of } Ag_2CrO_4}{\text{molarity of AgBr}}\right)$ can be expressed as

(1) $\frac{2\sqrt[3]{x}}{y}$

(2) $2\sqrt{\frac{x}{y}}$

(3) $\sqrt{\frac{x}{y}}$

(4) $\frac{\sqrt[3]{x}}{\sqrt{y}}$

Answer (4)

Sol. $Ag_2CrO_4(s) \rightleftharpoons \underset{2s}{2Ag^+(aq)} + \underset{s}{CrO_4^{2-}(aq)}$

$K_{sp} = (2s)^2(s) = 4s^3 = 32x$

$s = (8x)^{1/3}$

And for AgBr,

$K_{sp} = (s')^2 \Rightarrow 4y = (s')^2$

$s' = 2(y)^{1/2}$

$\frac{s}{s'} = \frac{(8x)^{1/3}}{2(y)^{1/2}} = \frac{(x)^{1/3}}{(y)^{1/2}}$

57. An electrochemical cell is constructed using half cells in the direction of spontaneous change

$Fe(OH)_2(s) + 2e^- \rightarrow Fe(s) + 2OH^-(aq)$ $E° = -0.88$ V

And $AgBr(s) + e^- \rightarrow Ag(s) + Br^-(aq)$ $E° = +0.07$ V

Which of the following option is correct?

(1) Overall reaction $Fe(s) + 2OH^-(aq) + 2AgBr(s) \rightleftharpoons Fe(OH)_2(s) + 2Ag(s) + 2Br^-(aq)$

(2) $E^o_{cell} = -0.95$ V

(3) Fe is reduced in the electrochemical cell

(4) E^o_{cell} is an extensive property

Answer (1)

Sol. Anode : $Fe(s) + 2OH^-(aq) \rightarrow Fe(OH)_2(s) + 2e^-$

Cathode : $2AgBr(s) + 2e^- \rightarrow 2Ag(s) + 2Br^-(aq)$

Net reaction :

$Fe(s) + 2OH^-(aq) + 2AgBr(s) \rightarrow Fe(OH)_2(s) + 2Ag(s) + 2Br^-(aq)$

$E^o_{cell} = 0.07 - (-0.88) = +0.95$ V

E^o_{cell} is not extensive property

58. $t_{100\%}$ is the time required for the 100% completion of the reaction while $t_{1/2}$ is the time required for 50% of the reaction to be completed. Which of the following option correctly represents the relation between $t_{100\%}$ and $t_{1/2}$ for zero and first order reactions respectively?

(1) $t_{100\%} = (t_{1/2})^2$ and $t_{100\%} = (t_{1/2})^{-\infty}$

(2) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (t_{1/2})^{\infty}$

(3) $t_{100\%} = 2t_{1/2}$ and $t_{100\%} = (2t_{1/2})^2$

(4) $t_{100\%} = (t_{1/2})^{\infty}$ and $t_{100\%} = 2t_{1/2}$

Answer (3)

Sol. For zero order reaction,

$A_0 - A_t = kt$

$t_{1/2} = \frac{A_0}{2k_0}$ (k_0 is rate constant for zero order reaction) …(i)

$\Rightarrow t_{100\%} = 2t_{50\%}$

For first order reaction,

$A_t = A_0 \cdot e^{-kt}$

$t_{1/2} = \frac{\ln 2}{k_1}$ (k_1 is rate constant for first order reaction) …(ii)

So, $t_{100\%} = (t_{1/2})^{\infty} = \infty$

59. Given below are two statements :

Statement (I) : The first ionisation enthalpy of the elements Na, Mg, Cl and Ar follows the order Na > Mg > Cl > Ar.

Statement (II) : Among Ca, Al, Fe and B, the third ionisation enthalpy is very high for Ca.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (4)

Sol. S-I : First IE of Ar is very high $\Rightarrow$

So, S-I is false.

S-II : $Ca \xrightarrow{IE_1} Ca^+ \xrightarrow{IE_2} Ca^{2+} \xrightarrow{IE_3} Ca^{3+}$

(2, 8, 8, 2) (2, 8, 8, 1) (2, 8, 8) (2, 8, 7)

Ca^{2+} (2, 8, 8) → Stable configuration ⇓ Very high IE

IE_3 for Ca is very high.

So, S-II is true.

60. Given below are two statements :

Statement (I) : Oxidising power of halogens decreases in the order $F_2 > Cl_2 > Br_2 > I_2$, which is the basis of "Layer test".

Statement (II) : "Layer test" to identify Br_2 and I_2 in aqueous solution involves the oxidation of bromide or iodide into Br_2 or I_2 respectively with Cl_2, which is a type of displacement redox reaction.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (1)

Sol. S-I : Oxidising power : $F_2 > Cl_2 > Br_2 > I_2$

Layer test is used for detection of Br^-, I^- in a solution and oxidising power is the fundamental basis.

S-II : $2I^- + Cl_2 \rightarrow \underset{\text{Violet colour}}{I_2} + 2Cl^-$

$2Br^- + Cl_2 \rightarrow \underset{\text{Orange-yellow layer}}{Br_2} + 2Cl^-$

So, both S-I and S-II are true.

61. Which of the following sets includes all the species that will change the orange colour of $K_2Cr_2O_7$ in acidic medium?

(1) $Fe^{2+}, Sn^{2+}, I^-, S^{2-}$

(2) $S^{2-}, Fe^{3+}, I^-, C_2O_4^{2-}$

(3) $Fe^{2+}, NO_2^-, SO_2, Sn^{4+}$

(4) $Fe^{3+}, SO_4^{2-}, S^{2-}, Sn^{4+}$

Answer (1)

Sol. Oxidised by $K_2Cr_2O_7$

$$\left.\begin{array}{l} Fe^{2+} \rightarrow Fe^{3+} \\ Sn^{2+} \rightarrow Sn^{4+} \\ I^- \rightarrow I_2 \\ S^{2-} \rightarrow S \end{array}\right]$$

62. Match **List-I** with **List-II**.

List-I Chromium (III) Complexes (en = ethylene diamine)		**List-II $\Delta_o(cm^{-1})$**	
A.	$[Cr(CN)_6]^{3-}$	I.	15,060
B.	$[CrF_6]^{3-}$	II.	17,400
C.	$[Cr(H_2O)_6]^{3+}$	III.	22,300
D.	$[Cr(en)_3]^{3+}$	IV.	26,600

Choose the **correct** answer from the options given below.

(1) A-I, B-II, C-III, D-IV (2) A-II, B-III, C-IV, D-I

(3) A-III, B-IV, C-I, D-II (4) A-IV, B-I, C-II, D-III

Answer (4)

Sol. Ligand field strength order

$F^- < H_2O < en < CN^-$

Order of Δ_o =

$$\underset{15,060\ cm^{-1}}{[CrF_6]^{3-}} < \underset{17,400\ cm^{-1}}{[Cr(H_2O)_6]^{3+}} < \underset{22300\ cm^{-1}}{[Cr(en)_3]^{3+}} < \underset{26,600\ cm^{-1}}{[Cr(CN)_6]^{3-}}$$

63. Given below are two statements:

Statement (I) : 1,2,3-Trihydroxypropane can be separated from water by simple distillation.

Statement (II) : An azeotropic mixture cannot be separated by fractional distillation.

In the light of the above statements, choose the **correct answer** from the options given below .

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Answer (4)

Sol. SI: 1,2,3-Trihydroxypropane can be separated from water by distillation under reduced pressure.

SI: false.

SII: true

Azeotropic mixture cannot be separated by fractional dist.

64. Given below are two statements :

Statement (I) : Benzyl chloride reacts faster in S_N1 mechanism than ethyl chloride.

Statement (II) : Ethyl carbocation intermediate is less stabilized by hyperconjugation than benzyl carbocation by resonance.

In the light of the above statements, choose the **correct answer** from the options given below.

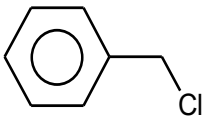
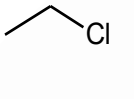
(1) Both **Statement I** and **Statement II** are true

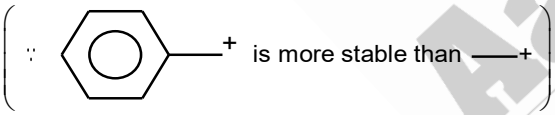
(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

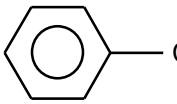
(4) **Statement I** is false but **Statement II** is true

Answer (1)

Sol. SI.  >  (S_N1)



SI → true

SII  stabilised by resonance.

$CH_3 - CH_2^+$ stabilised by hyperconjugation.

SII → true ($\because$ stability due to hyper conjugation is less)

65. In IUPAC nomenclature, the **correct** order of decreasing priority of functional group is

(1) $-CONH_2$, $>C=O$, $-CHO$, $-NH_2$, $-C \equiv C-$

(2) $-CONH_2$, $-COOCH_3$, $-CHO$, $-NH_2$, $-OH$

(3) $-CONH_2$, $-CHO$, $>C=O$, $-NH_2$, $-C \equiv C-$

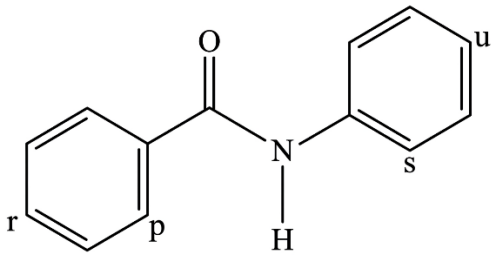
(4) $-CONH_2$, $-CHO$, $-CN$, $-NH_2$, $-C \equiv C-$

Answer (3)

Sol. Priority order

$-CONH_2 > -CHO > >C=O > -NH_2 > -C \equiv C-$

66. For the given molecule, "x", the preferred site for the attack of the electrophile is

$x =$ 

(1) Predominantly at "r" (2) "r" and "u"

(3) "p" and "s" (4) Predominantly at "u"

Answer (4)

Sol. Attack takes place at 'u' (most activated position due to activation via resonance and less steric hindrance (+M of nitrogen).

67. Match **List-I** with **List-II**.

	List-I Mixture of Compounds		**List-II Reagent used to distinguish**
A.	Diethyl amine + Ethyl amine	I.	Bromine water
B.	Acetaldehyde + Acetone	II.	$CHCl_3$ + KOH, Δ
C.	Ethanol + Phenol	III.	Neutral $FeCl_3$
D.	Benzoic acid + Cinnamic acid	IV.	Ammonical silver nitrate

Choose the **correct** answer from the options given below.

(1) A-IV, B-II, C-I, D-III (2) A-IV, B-II, C-III, D-I

(3) A-II, B-IV, C-I, D-III (4) A-II, B-IV, C-III, D-I

Answer (4)

Sol. A. Diethyl amine + Ethyl amine - $CHCl_3 + KOH, \Delta$

Ethyl amine (1° amine) gives isocyanide (carbyl amine test)

B. $CH_3CHO + CH_3 - \underset{\underset{O}{\|}}{C} - CH_3 \rightarrow$

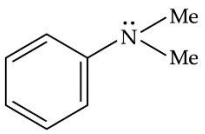
Ammoniacal $AgNO_3$ (Tollen's reagent) oxidises CH_3CHO to CH_3COOH.

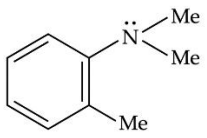
C. Ethanol + phenol — neutral. $FeCl_3$ test

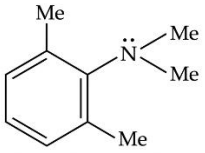
(phenol gives violet colour)

D. Benzoic acid + cinnamic acid — Br_2 water. Cinnamic acid decolorises Br_2 water, due to C = C.

68. Consider the three aromatic molecules (P, Q and R) whose structures have been given below:

P. 

Q. 

R. 

The **correct** order regarding the reactivity of these compounds with $Ph - \underset{(+)}{N} \equiv NCl^{(-)}$ under optimum but slightly acidic medium is

(1) P > Q > R (2) R > P > Q

(3) R > Q > P (4) P > R > Q

Answer (1)

Sol. Order of reactivity: P > Q > R.

Rate of S_EAr α e^- density in benzene ring.

69. Match **List-I** with **List-II**.

	List-I Vitamin		**List-II Name**
A.	Vitamin B_1	I.	Pyridoxine
B.	Vitamin B_2	II.	Ascorbic acid
C.	Vitamin B_6	III.	Thiamine
D.	Vitamin C	IV.	Riboflavin

Choose the **correct** answer from the options given below:

(1) A-II, B-I, C-III, D-IV (2) A-IV, B-III, C-II, D-I

(3) A-III, B-IV, C-I, D-II (4) A-I, B-III, C-II, D-IV

Answer (3)

Sol. Vitamin B_1 - Thiamine

Vitamin B_2 - Riboflavin

Vitamin B_6 - Pyridoxine

Vitamin C - Ascorbic acid

70. A salt with few drops of conc. HCl gives apple green colour in flame test. The group precipitate of the salt is dissolved in acetic acid and treated with K_2CrO_4 to give yellow precipitate. When the sodium carbonate extract of the salt solution is heated with conc. HNO_3 and ammonium molybdate, it resulted a canary yellow precipitate. The cation and anion present in the salt are respectively,

(1) Ca^{2+} and SO_4^{2-}

(2) Ba^{2+} and PO_4^{3-}

(3) Mn^{2+} and PO_4^{3-}

(4) Ba^{2+} and SO_4^{2-}

Answer (2)

Sol. $Ba^{2+} \rightarrow$ apple green in flame test.

$Ba^{2+} + CrO_4^{2-} \rightarrow BaCrO_4 \downarrow$ yellow ppt

PO_4^{3-} + conc. HNO_3 + $(NH_4)_2MoO_4 \rightarrow (NH_4)_3[P(Mo_3O_{10})_4] \downarrow$ Canary yellow ppt

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. 5.33 g of $CrCl_3 \cdot 6H_2O$, which is a 1 : 3 electrolyte, is dissolved in water and is passed through a cation exchanger. The chloride ions in the eluted solution, on treatment with $AgNO_3$ results in 8.61 g of AgCl. The ratio of moles of complex reacted and moles of AgCl formed is _____ $\times 10^{-2}$. (Nearest integer)

[Molar mass in g mol^{-1} Cr : 52, Ag : 108, Cl : 35.5, H : 1, O : 16]

Answer (33)

Sol. $CrCl_3 \cdot 6H_2O \longrightarrow Cl^-$ (Reaction scheme)

5.33 g

$\xrightarrow{AgNO_3}$ AgCl

8.61 g

$$n_{AgCl} = \frac{8.61}{143.5} = 0.06$$

$$n_{complex} = \frac{5.33}{266.5} = 0.02$$

$$\frac{n_{complex}}{n_{AgCl}} = \frac{0.02}{0.06} = \frac{1}{3} = 33.33 \times 10^{-2}$$

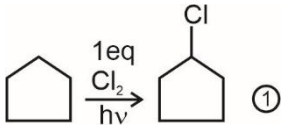
Complex $\rightarrow [Cr(H_2O)_6]Cl_3$

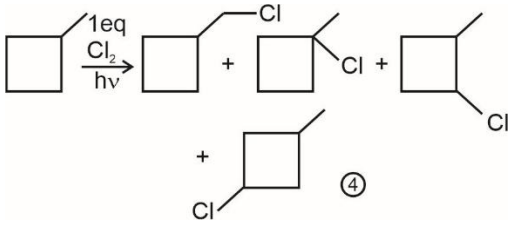
72. Consider the isomers of hydrocarbon with molecular formula C_5H_{10}. These isomers do not decolourise $KMnO_4$ solution. These isomers are subjected to chlorination with chlorine in presence of light to give monochloro compounds. The total number of monochloro compounds (structural isomers only) formed is _____.

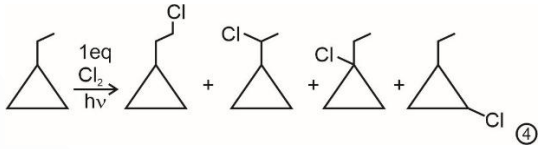
Answer (14)

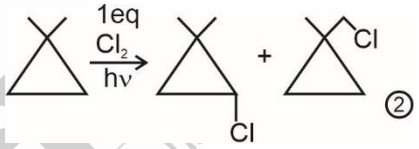
Sol. $C_5H_{10} \rightarrow$ do not decolorise $KMnO_4$

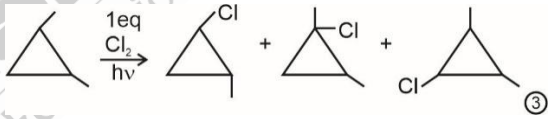
So, it is a cyclic compound, five isomers of C_5H_{10} are possible shown as 1, 2, 3, 4, 5

(1) 

(2) 

(3) 

(4) 

(5) 

73. One mole of an alkane (x) requires 8 mole oxygen for complete combustion. Sum of number of carbon and hydrogen atoms in the alkane (x) is _____.

Answer (17)

Sol. Alkane $\rightarrow C_nH_{2n+2}$

So, $n + \frac{2n+2}{4} = 8$

$6n + 2 = 8 \times 4]$ $\quad 6n = 30$

$n = 5$

So, alkane $\rightarrow C_5H_{12}$

74. For reaction A $\rightarrow$ P, rate constant k = 1.5 × 10^3 s^{-1} at 27°C

If activation energy for the above reaction is 60 kJ mol^{-1}, then the temperature (in °C) at which rate constant, k = 4.5 × 10^3 s^{-1} is _____. (Nearest integer)

Given : log2 = 0.30, log3 = 0.48, R = 8.3 J K^{-1} mol^{-1}, ln 10 = 2.3

Answer (41)

Sol. $\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{300} - \frac{1}{T_2}\right]$

$\log\frac{4.5\times10^{-3}}{1.5\times10^{-3}} = \log 3 = 0.48$

$0.48 = \frac{60\times10^3}{2.303\times8.3}\left[\frac{1}{300} - \frac{1}{T_2}\right]$

T_2 = 314.40 K

t = 41.4°C

75. At the transition temperature T, A $\rightleftharpoons$ B and ΔG° = 105 – 35 log T where A and B are two states of substance X. The transition temperature in °C when pressure is 1 atm is _____.

(Nearest integer)

Answer (727)

Sol. A $\rightleftharpoons$ B ΔG° = 105–35 logT

At transition, $\Delta G^\circ = 0$

$\frac{105}{35} = \log T \Rightarrow T = 1000$ K

t°C = 1000 – 273 = 727°C

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