Aakash
Medical | IIT-JEE | Foundations

Phase-2
5th April 2026
(Evening Shift)

Answer & Solutions

for

JEE MAIN 2026

TRUST Aakash

Stellar Ranks in JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR 23

Kushagra Baingaha AIR 7

Shreyas Lohiya AIR 6

A Gupta AIR 15

Devya Rustagi AIR 28

Shreyas Lohiya AIR 68

Rujul Garg AIR 41

Advay Mayank AIR 36

Arush Anand AIR 64

Kotha D Reddy AIR 74

Top Ranks in JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students **Aakashians Qualified in NSEs 2025-26**

777 Classroom Students **Aakashians Qualified in IOQM 2025**

134 Classroom Students **Aakashians Qualified in RMO 2025**

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots the equation $x^2 - 4x + q = 0$; $p, q \in \mathbf{Z}$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then $|p+q|$ equals:

(1) 16 (2) 32
(3) 34 (4) 38

Answer (3)

Sol. $x^2 - x + p = 0$

$\alpha + \beta = 1, \alpha\beta = p$

$x^2 - 4x + 2 = 0$

$\gamma + \delta = 4$

$\gamma\delta = 2$

$\alpha, \beta, \gamma, \delta$ are in G.P

Let the terms be a, ar, ar^2, ar^3

$a + ar = 1 \Rightarrow a(1+r) = 1$

$ar^2 + ar^3 = 4$

$\Rightarrow ar^2(1+r) = 4$

$\frac{ar^2(1+r)}{a(1+r)} = \frac{4}{1} \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

Case 1: $r = 2$

$a(1+2) = 1$

$\Rightarrow a = \frac{1}{3}$

$\therefore \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \gamma = \frac{4}{3}, \delta = \frac{8}{3}$

$\therefore p = \alpha\beta = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$q = \gamma\delta = \frac{4}{3} \times \frac{8}{3} = \frac{32}{9}$

Case 2: $r = -2$

$a = \alpha = -1, \beta = 2, \gamma = -4, \delta = 8$

$p = \alpha\beta = -2$

$q = \gamma\delta = -32$

$\therefore |p+q| = 34$

2. Let $z_1, z_2 \in \mathbb{C}$ be the distinct solutions of the equation $z^2 + 4z - (1+12i) = 0$.

Then $|z_1|^2 + |z_2|^2$ is equal to:

(1) 18 (2) 22
(3) 29 (4) 34

Answer (4)

Sol. $z^2 + 4z - (1+12i) = 0$

$z = \frac{-4 \pm \sqrt{16 + 4(1+12i)}}{2}$

$z = \frac{-4 \pm \sqrt{20 + 48i}}{2}$

$z = -2 \pm \sqrt{5 + 12i}$

Let $\sqrt{5+12i} = x + iy$

Squaring both sides

$x^2 - y^2 + 2xyi = 5 + 12i$

$\therefore x^2 - y^2 = 5$ ...(I)

$2xy = 12$ ...(II)

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$= 25 + 144 = 169$

$x^2 + y^2 = 13$ ...(III)

From (I) & (III)

$x = \pm 3, y = \pm 2$

$xy = 6$ (from II)

$\therefore \sqrt{5+12i} = \pm(3+2i)$

$z_1 = -2 + 3 + 2i = 1 + 2i$

$z_2 = -2 - 3 - 2i = -5 - 2i$

$|z_1|^2 + |z_2|^2 = 1 + 4 + 25 + 4$

$= 34$

3. If $f:\mathbf{N}\to\mathbf{Z}$ is defined by

$$f(n)=\begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix},\ k\in N,$$

and $\sum_{n=1}^{k} f(n)=98$, then k is equal to:

(1) 3 (2) 4
(3) 5 (4) 6

Answer (1)

Sol. $$f(n)=\begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}$$

$$\sum_{n=1}^{k} f(n)=\begin{vmatrix} \sum n & -1 & -5 \\ -2\sum n^2 & 3(2k+1) & 2k+1 \\ -3\sum n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}$$

$$=\frac{k(k+1)}{12}\begin{vmatrix} 6 & -1 & -5 \\ -4(2k+1) & 3(2k+1) & 2k+1 \\ -9k(k+1) & 3k(2k+1) & 3k^2+6k+1 \end{vmatrix}$$

$$=\frac{k(k+1)(2k+1)}{12}\begin{vmatrix} 6 & -1 & -5 \\ -4 & 3 & 1 \\ -9(k^2+k) & 3(2k^2+k) & 3k^2+6k+1 \end{vmatrix}$$

$$=\frac{k(k+1)(2k+1)}{12}\begin{vmatrix} 0 & -1 & -5 \\ 0 & 3 & 1 \\ 1 & 3(2k^2+k) & 3k^2+6k+1 \end{vmatrix}$$

$$=\frac{7}{6}k(k+1)(2k+1)$$

$\because \frac{7}{6}k(k+1)(2k+1)=98$

$\therefore$ $k(k+1)(2k+1)=6\times14=3\times4\times7$

$\therefore$ $k=3$

4. Let M be a 3 × 3 matrix such that

$$M\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix},\ M\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\1\\2\end{pmatrix}\text{ and }M\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\1\\1\end{pmatrix}.$$

If $M\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\7\\11\end{pmatrix}$, then $x+y+z$ equals:

(1) 4 (2) 5
(3) 7 (4) 11

Answer (2)

Sol. $$M\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\1\\2\end{bmatrix}\text{ and }M\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}-1\\1\\1\end{bmatrix}$$

$$\Rightarrow M=\begin{bmatrix}1&0&-1\\2&1&1\\3&2&1\end{bmatrix}$$

$$M\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\7\\11\end{bmatrix}$$

$x-z=1$ ...(1)

$2x+y+z=7$ ...(2)

$3x+2y+z=11$ ...(3)

Form (1) take $z=x-1$

$2x+y+(x-1)=7\Rightarrow 3x+y=8$ ...(4)

$3x+2y+(x-1)=11\Rightarrow 4x+2y=12$ ...(5)

from (4) and (5)

$x=2,\ y=2$

$\Rightarrow z=1$

$x+y+z=2+2+1=5$

5. If the sum of the first 10 terms of the series $\frac{1}{1+1^4\times4}+\frac{2}{1+2^4\times4}+\frac{3}{1+3^4\times4}+\frac{4}{1+4^4\times4}+\ldots\ldots$ is $\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to :

(1) 256
(2) 264
(3) 276
(4) 284

Answer (3)

Sol. $T_\gamma = \dfrac{\gamma}{1+4\gamma^4}$

$= \dfrac{\gamma}{1+4\gamma^4-4\gamma^2+4\gamma^2}$

$= \dfrac{\gamma}{\left(2\gamma^2+1\right)^2-4\gamma^2}$

$= \dfrac{\gamma}{\left(2\gamma^2+1+2\gamma\right)\left(2\gamma^2+1-2\gamma\right)}$

$= \dfrac{1}{4}\left[\dfrac{\left(2\gamma^2+2\gamma+1\right)-\left(2\gamma^2-2\gamma+1\right)}{\left(2\gamma^2+1+2\gamma\right)\left(2\gamma^2-2\gamma+1\right)}\right]$

$= \dfrac{1}{4}\left[\dfrac{1}{2\gamma^2-2\gamma+1}-\dfrac{1}{2\gamma^2+2\gamma+1}\right)$

$S_{10} = \sum_{\gamma=1}^{10} T_\gamma = \dfrac{1}{4}\left(\dfrac{1}{1}-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{13}+\ldots-\dfrac{1}{221}\right)$

$= \dfrac{1}{4}\left[1-\dfrac{1}{221}\right]$

$= \dfrac{55}{221}$

m + n = 55 + 221 = 276

Option (3) is correct

6. Let $A_1, A_2, A_3, \ldots\ldots\ldots, A_{39}$ be 39 arithmetic means between the numbers 59 and 159. Then the mean of A_{25}, A_{28}, A_{31} and A_{36} is equal to :

(1) 129 (2) 136

(3) 131.50 (4) 134

Answer (4)

Sol. $d = \dfrac{b-a}{n+1}$

$= \dfrac{159-59}{39+1} = \dfrac{100}{40} = \dfrac{5}{2}$

k^{th} Arithmetic mean, $A_k = a + kd$

$\dfrac{A_{25}+A_{28}+A_{31}+A_{36}}{4}$

$= \dfrac{a+25d+a+28d+a+31d+a+36d}{4}$

$= \dfrac{4a+120d}{4}$

$= a + 30d$

$= 59 + 30\left(\dfrac{5}{2}\right) = 134$

7. The coefficient of x^2 in the expansion of $\left(2x^2+\dfrac{1}{x}\right)^{10}, x \neq 0$, is :

(1) 3240 (2) 3360

(3) 3480 (4) 3600

Answer (2)

Sol. $T_{r+1} = {}^{10}C_r (2x^2)^{10-r}\left(\dfrac{1}{x}\right)^r$

$\Rightarrow$ 20 – 2r – r = 2

r = 6

coff of $x^2 = {}^{10}C_6(2)^4 = 3360$

8. The probabilities that players A and B of a team are selected for the captaincy for a tournament are 0.6 and 0.4, respectively. If A is selected the captain, the probability that the team wins the tournament is 0.8 and if B is selected the captain, the probability that the team wins the tournament is 0.7. Then the probability, that the team wins the tournament, is:

(1) 0.74 (2) 0.76

(3) 0.72 (4) 0.78

Answer (2)

Sol. $\because P(A) = 0.6$ and $P(B) = 0.4$ be probability for captain *A* and *B* respectively.

$P\left(\dfrac{\omega}{A}\right) = 0.8$ and $P\left(\dfrac{\omega}{B}\right) = 0.7$.

The probability that team wins the tournament

$$= P(A)\cdot P\left(\frac{\omega}{A}\right) + P(B)\cdot P\left(\frac{\omega}{B}\right)$$
$$= 0.6\times 0.8 + 0.4\times 0.7$$
$$= 0.76.$$

9. A box contains 5 blue, 6 yellow and 4 red balls. The number of ways, of drawing 8 balls containing at least two balls of each colour, is:

(1) 4100 (2) 4140

(3) 4140 (4) 4230

Answer (1)

Sol. The method to select the balls

= (2 blue, 2 yellow, 4 red), (2 blue, 4 yellow, 2 red), (4 blue, 2 yellow, 2 red), (3 blue, 3 yellow, 2 red), (3 blue, 2 yellow, 3 red) or (2 blue, 3 yellow, 3 red)

$= {}^5C_2.{}^6C_2.{}^4C_4 + {}^5C_2.{}^6C_4.{}^4C_2 + {}^5C_4.{}^6C_2.{}^4C_2 + {}^5C_3.{}^6C_3.{}^4C_2 + {}^5C_3.{}^6C_2.{}^4C_3 + {}^5C_2.{}^6C_3.{}^4C_3.$

= 150 + 900 + 450 + 1200 + 600 + 800

= 4100

10. A variable X takes values 0, 0, 2, 6, 12, 20,…,$n(n-1)$ with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ${}^nC_3$, ${}^nC_4$, ${}^nC_5$,…,${}^nC_n$, respectively. If the means of this data is 60, then its median is:

(1) 56 (2) 42

(3) 72 (4) 90

Answer (1)

Sol. $\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$

$$= \frac{0.{}^nC_0 + 0.{}^nC_1 + 2\,{}^nC_2 + 6\,{}^nC_3 + \ldots.. + (n(n-1))\,{}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots.. + {}^nC_n} = 60$$

$$= \frac{\sum_{r=0}^{n}(r(r-1))\,{}^nC_r}{2^n}$$

$$= \frac{\sum_{r=2}^{n} n(n-1)\left({}^{n-2}C_{r-2}\right)}{2^n} = 60$$

$$\frac{n(n-1)2^{n-2}}{2^n} = 60$$

$n(n-1)\,2^{-2} = 60$

$\Rightarrow n(n-1) = 240$

$\Rightarrow n = 16$

Median will be at 2^{15} position

Mean of the median will correspond to

$x_8 = 8(8-1) = 56$

11. Let the point P be the vertex of the parabola $y = x^2 - 6x + 12$. If a line passing through the point P intersects the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the points R and S, then the maximum value of $(PR + PS)^2$ is:

(1) 10

(2) 20

(3) 25

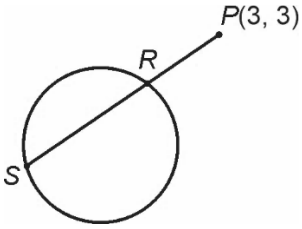
(4) 5

Answer (2)

Sol. $(x-3)^2 = y - 3$

Vertex = (3, 3)

$C : (x-1)^{-2} + (y-2)^2 = 2$



$$(PR+PS)^2_{\max} = \left[\sqrt{(3-1)^2+(3-2)^2} + \sqrt{2} + \sqrt{(3-1)^2+(3-2)^2} - \sqrt{2}\right]^2$$

$$= \left(2\sqrt{5}\right)^2 = 20$$

12. Let the directrix of the parabola $P : y^2 = 8x$, cut x-axis at the point A. Let $B(\alpha, \beta)$, $\alpha > 1$, be a point on P such that the slope of AB is $\frac{3}{5}$. If BC is a focal chord of P, then six times the area of ΔABC is:

(1) 80 (2) 160

(3) 174 (4) 192

Answer (2)

Sol. $y^2 = 8x$

Directrix = $x + 2 = 0$

$\Rightarrow A(-2, 0)$

$B(2t^2, 4t) = (\alpha, \beta)$

$$\frac{4t}{2t^2+2} = \frac{3}{5}$$

$20t = 6t^2 + 6$

$6t^2 - 20t + 6 = 0$

$3t^2 - 10t + 3 = 0$

$$\Rightarrow t = \frac{1}{3}, 3$$

$\Rightarrow (\alpha, \beta) \equiv (18, 12)$

$\because$ BC is focal chord $\Rightarrow t_1t_2 = -1$

$3t_2 = -1$

$$t_2 = \frac{-1}{3}$$

$$C\left(\frac{2}{9}, \frac{-4}{3}\right)$$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 18 & 12 & 1 \\ \frac{2}{9} & \frac{-4}{3} & 1 \end{vmatrix}\right| = \frac{80}{3}$$

6 area = 160

13. Let the eccentricity e of a hyperbola satisfy the equation $6e^2 - 11e + 3 = 0$. If the foci of the hyperbola are $(3, 5)$ and $(3, -4)$, then the length of its latus rectum is :

(1) $\frac{11}{3}$ (2) $\frac{17}{3}$

(3) $\frac{15}{2}$ (4) $\frac{17}{2}$

Answer (3)

Sol. $6e^2 - 11e + 3 = 0$

$6e^2 - 9e - 2e + 3 = 0$

$(3e - 1)(2e - 3) = 0$

$$\Rightarrow e = \frac{1}{2} \text{ or } \frac{3}{2}$$

$\because$ e is for hyperbola $\Rightarrow e > 1$

$$\Rightarrow e = \frac{3}{2}$$

Also $2ae = \sqrt{(3-3)^2 + (5+4)^2}$

$$2 \times a \times \frac{3}{2} = 9$$

$\boxed{a = 3}$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$\frac{9}{4} = 1 + \frac{b^2}{9}$$

$$\frac{5}{4} = \frac{b^2}{9} \Rightarrow b^2 = \frac{45}{4}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times \frac{45}{4}}{3}$$

$$= \frac{15}{2}$$

14. Let a triangle PQR be such that P and Q lie on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ and are at a distance of 6 units from $R(1, 2, 3)$. If (α, β, γ) is the centroid of ΔPQR, then $\alpha+\beta+\gamma$ is equal to:

(1) 4 (2) 5

(3) 6 (4) 8

Answer (3)

Sol. Let $P(8k-3, 2k+4, 2k-1)$

$(PR)^2 = 36$

$(8k-4)^2+(2k+2)^2+(2k-4)^2=36$

Solving

$k = 0, 1$

$k = 0;\quad P(-3, 4, -1)$

$k = 1;\quad Q(5, 6, 1)$

Centroid: $\left(\frac{-3+5+1}{3}, \frac{4+6+2}{3}, \frac{-1+1+3}{3}\right)=(1,4,1)$

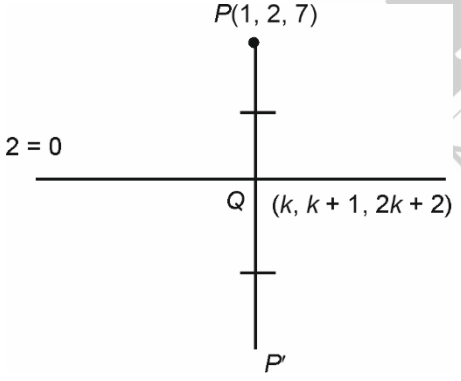
15. If the distance of the point $(a, 2, 5)$ from the image of the point $(1, 2, 7)$ in the line $\frac{x}{1}=\frac{y-1}{1}=\frac{z-2}{2}$ is 4, then the sum of all possible values of a is equal to :

(1) 11 (2) 9

(3) 6 (4) 4

Answer (3)

Sol. $(k-1)\cdot 1+(k-1)\cdot 1+(2k-5)\cdot 2=0$



$\Rightarrow \quad 6k = 12$

$\Rightarrow \quad k = 2$

$Q(2, 3, 6)$

$\therefore \quad P'(3, 4, 5)$

$(\text{distance})^2 = (3-a)^2+4+0=16$

$= (3-a)^2 = 12$

$\Rightarrow a^2-6a-3=0$

Sum = 6

16. Let O be the origin, $\overrightarrow{OP}=\vec{a}$ and $\overrightarrow{OQ}=\vec{b}$. If R is the point on $\overrightarrow{OP}$ such that $\overrightarrow{OP}=5\overrightarrow{OR}$, and M is the point such that $\overrightarrow{OQ}=5\overrightarrow{RM}$, then $\overrightarrow{PM}$ is equal to :

(1) $\frac{1}{5}\left(\vec{a}-4\vec{b}\right)$

(2) $\frac{1}{5}\left(\vec{b}-4\vec{a}\right)$

(3) $\frac{1}{5}\left(-\vec{a}+4\vec{b}\right)$

(4) $\frac{1}{5}\left(-\vec{b}+4\vec{a}\right)$

Answer (2)

Sol. $\overrightarrow{OP}=\vec{a}, \overrightarrow{OQ}=\vec{b}$ ss

$\overrightarrow{OP}=5\overrightarrow{OR} \Rightarrow \overrightarrow{OR}=\frac{1}{5}\vec{a}$

$\overrightarrow{OQ}=5\overrightarrow{RM} \Rightarrow \overrightarrow{RM}=\frac{1}{5}\vec{b}$

$\overrightarrow{OM}=\overrightarrow{OR}+\overrightarrow{RM}=\frac{1}{5}\left(\vec{a}+\vec{b}\right)$

$\overrightarrow{PM}=\overrightarrow{OM}-\overrightarrow{OP}$

$=\frac{1}{5}\vec{a}+\frac{1}{5}\vec{b}-\vec{a}$

$=\frac{1}{5}\vec{b}-\frac{4}{5}\vec{a}$

$=\frac{1}{5}\left(\vec{b}-4\vec{a}\right)$

17. Let $f(x)=\lim\limits_{y\to 0}\dfrac{(1-\cos(xy))\tan(xy)}{y^3}$. Then the number of solutions of the equation $f(x)=\sin x$, $x\in\mathbb{R}$ is:

(1) 0 (2) 2

(3) 3 (4) 1

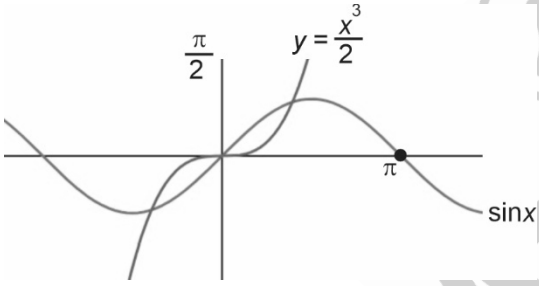
Answer (3)

Sol. $f(x)=\lim\limits_{y\to 0}\dfrac{(1-\cos xy)\tan(xy)}{y^3}$

$$=\lim_{y\to 0}\frac{2\sin^2\left(\frac{xy}{2}\right)}{\left(\frac{xy}{2}\right)^2\times\left(\frac{4}{x^2}\right)}\times\frac{\tan(xy)}{(xy)\times\frac{1}{x}}$$

$$=\lim_{y\to 0}\frac{x^2}{2}\times\frac{1}{\frac{1}{x}}=\frac{x^3}{2}$$

$$f(x)=\frac{x^3}{2}$$



18. Let $(2^{1-a}+2^{1+a}), f(a), (3^a+3^{-a})$ be in A.P. and α be the minimum value of $f(a)$. Then the value of the integral $\int_{\log_e(\alpha-1)}^{\log_e(\alpha)}\dfrac{dx}{(e^{2x}-e^{-2x})}$ is :

(1) $\frac{1}{2}\log_e\left(\frac{4}{3}\right)$ (2) $\frac{1}{4}\log_e\left(\frac{4}{3}\right)$

(3) $\frac{1}{2}\log_e\left(\frac{8}{5}\right)$ (4) $\frac{1}{4}\log_e\left(\frac{8}{5}\right)$

Answer (2)

Sol. $2f(a)=2^{1-a}+2^{1+a}+3^a+3^{-a}$

$$f(a)=\frac{2^{1-a}+2^{1+a}+3^a+3^{-a}}{2}$$

$\because\ \dfrac{2^{1-a}+2^{1+a}}{2}\geq 2$

$\Rightarrow\ (2^{1-a}+2^{1+a})_{\min}=4\quad$ when $a=0$

$\because\ \dfrac{3^a+3^{-a}}{2}\geq 1$

$\Rightarrow\ (3^a+3^{-a})_{\min}=2\quad$ when $a=0$

$\Rightarrow\ (f(a))_{\min}=3\quad$ when $a=0\ \Rightarrow\ \alpha=3$

$$I=\int_{\ln 2}^{\ln 3}\frac{dx}{(e^{2x}-e^{-2x})}$$

Put $e^{2x}=t\Rightarrow 2e^{2x}dx=dt$

$$\Rightarrow\ I=\frac{1}{2}\int_4^9\frac{dt}{t^2-1}=\frac{1}{2}\times\left[\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right]_4^9=\frac{1}{4}\ln\frac{4}{3}$$

19. Let $f:[1,\infty)\to\mathbb{R}$ be a differentiable function defined as $f(x)=\int_1^x f(t)dt+(1-x)(\log_e x-1)+e$.

Then the value of $f(f(1))$ is:

(1) $(1+e^e)$ (2) $(1+e)$

(3) $(1+e+e^e)$ (4) $1+2e$

Answer (1)

Sol. $f(x)=\left(\int_1^x f(t)dt\right)+(1-x)(\ln x-1)+e$

$$f'(x)=f(x)+\frac{(1-x)}{x}+(\ln x-1)(-1)$$

$$\frac{dy}{dx}=y+\frac{1}{x}-1-\ln x+1$$

$$\frac{dy}{dx}+(-1)y=\frac{1}{x}-\ln x$$

$I.F = e^{-x}$

$y(e^{-x}) = \int\left(\frac{e^{-x}}{x} - e^{-x}\ln x\right)dx$

$= \int\frac{e^{-x}}{x}dx - \left(-\ln x e^{-x} + \int\frac{e^{-x}}{x}dx\right) + c$

$y(e^{-x}) = e^{-x}\ln x + c$

$y = \ln x + ce^{x}$

$y(1) = e$

$\Rightarrow \boxed{c = 1}$

$y = e^{x} + \ln x$

$f(x) = e^{x} + \ln x$

$f(1) = e$

$f(f(1)) = e^{e} + \ln(e^{e})$

$= e^{e} + 1$

20. Let f(x) and g(x) be twice differentiable functions satisfying $f''(x) = g''(x)$ for all $x \in \mathbb{R}$, $f'(1) = 2g'(1) = 4$ and $g(2) = 3f(2) = 9$. Then $f(25) - g(25)$ is equal to :

(1) 20 (2) 40

(3) –20 (4) –40

Answer (2)

Sol. $f''(x) = g''(x) \Rightarrow f'(x) = g'(x) + C$

$\because\ f'(1) = 4, g'(1) = 2 \Rightarrow C = 2 \Rightarrow f'(x) = g'(x) + 2$

$\Rightarrow\ f(x) = g(x) + 2x + K$

$\because\ f(2) = 3, g(2) = 9 \Rightarrow K = -10$

$\Rightarrow\ f(x) = g(x) + 2x - 10$

$\Rightarrow\ f(25) - g(25) = 40$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $A = \{1, 4, 7\}$ and $B = \{2, 3, 8\}$. Then the number of elements, in the relation

$R = \{((a_1, b_1),(a_2, b_2)) \in ((A \times B) \times (A \times B)) : a_1 + b_2$ divides $a_2 + b_1\}$ is _____.

Answer (18)

Sol. $A = \{1, 4, 7\}, B = \{2, 3, 8\}$

$R = \{((a_1, b_1),(a_2, b_2)) \in ((A \times B) \times (A \times B))$
$: a_1 + b_2$ divides $a_2 + b_1\}$

$a_1 + b_2$ divides $a_2 + b_1, a_1, a_2 \in A, b_1, b_2 \in B$

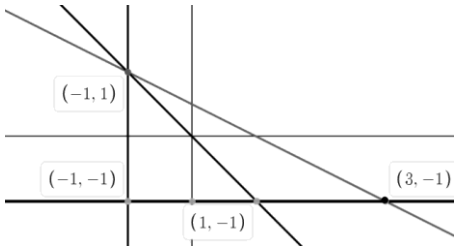
$\Rightarrow$
- $1+2\,|\,1+2, 1+8, 4+2, 4+8, 7+2, 7+8$
- $1+3\,|\,1+3, 4+8$
- $1+8\,|\,1+8, 7+2$
- $4+2\,|\,4+2, 4+8$
- $4+3\,|\,4+3$
- $4+8\,|\,4+8$
- $7+2\,|\,1+8, 7+2$
- $7+3\,|\,7+3$
- $=7+8\,|\,7+8$

$\Rightarrow$ Number of elements in R = 18

22. From the point $(-1,-1)$, two rays are sent making angles of 45° with the line $x + y = 0$. These rays get reflected from the mirror $x + 2y = 1$. If the equations of the reflected rays are $ax + by = 9$ and $cx + dy = 7, a,b,c,d \in Z$, then the value of $ad + bc$ is _____ .

Answer (07.00)

Sol.



Notice that the lines are

$x = -1, y = -1$

Image of $P(-1, -1)$ about the line mirror

$$\frac{x+1}{1} = \frac{y+1}{2} = -2\frac{(-1)+2(-1)-1}{1^2+2^2}$$

$$= \frac{8}{5}$$

$$\Rightarrow x = \frac{3}{5},\ y = \frac{11}{5}$$

$$\Rightarrow\ Q \equiv \left(\frac{3}{5}, \frac{11}{5}\right)$$

Line 1 : line passing through points

$\left(\frac{3}{5}, \frac{11}{5}\right)$ and $(3, -1)$

Line 2 : line passing through points

$\left(\frac{3}{5}, \frac{11}{5}\right)$ and $(-1, 1)$

$$L_1 : (y+1) = (x-3)\frac{\left(\frac{16}{5}\right)}{\left(\frac{-12}{5}\right)} = \frac{-4}{3}(x-3)$$

and $L_2 : (y-1) = (x+1)\dfrac{\left(\frac{6}{5}\right)}{\frac{8}{5}} = (x+1)\frac{3}{4}$

$\Rightarrow\ L_1 : 3y + 4x = 9 \Rightarrow\ 4\times 4\ +\ +(3)(-3)$

$L_2 : 4y - 3x = 7\ \ = 16 - 9 = 7$

23. If $\left\{\theta \in [-\pi, \pi] : \cos\theta\cos\frac{5\theta}{2} = \cos 7\theta\cos\frac{7\theta}{2}\right\}$, then $n(S)$ is equal to_____.

Answer (19)

Sol. $\cos\theta\cos\frac{5\theta}{2} = \cos 7\theta\cos\frac{7\theta}{2}$

$$\cos\left(\theta + \frac{5\theta}{2}\right) + \cos\left(\theta - \frac{5\theta}{2}\right) = \cos\left(7\theta + \frac{7\theta}{2}\right) + \cos\left(7\theta - \frac{7\theta}{2}\right)$$

$$\cos\frac{7\theta}{2} + \cos\frac{3\theta}{2} = \cos\cos\frac{21\theta}{2} + \cos\frac{7\theta}{2}$$

$$\cos\frac{3\theta}{2} = \cos\frac{21\theta}{2}$$

$$\frac{21\theta}{2} = 2k\pi \pm \frac{3\theta}{2}$$

$$\frac{21\theta}{2} = 2k\pi + \frac{3\theta}{2} \Rightarrow \theta = \frac{2k\pi}{9},\ k \in I$$

$$\frac{21\theta}{2} = 2k\pi - \frac{3\theta}{2} \Rightarrow \theta = \frac{k\pi}{6},\ k \in I$$

when $\theta = \dfrac{2k\pi}{9}$

$$-\pi \le \frac{2k\pi}{9} \le \pi \Rightarrow -\frac{9}{2} \le k \le \frac{9}{2}$$

$k \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\} \rightarrow 9$ sol^n

for $\theta = \dfrac{k\pi}{6}$

$$-\pi \le \frac{k\pi}{6} \le \pi \Rightarrow -6 \le k \le 6$$

$k \in \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\} \rightarrow 13$ sol^n

$\theta = \left\{-\frac{2\pi}{3}, 0, \frac{2\pi}{3}\right\}$ is common

$\therefore$ Total Sol^n = 19

24. Let $f:\mathbf{R}\to\mathbf{R}$ be a function such that $f(x)+3f\left(\frac{\pi}{2}-x\right)=\sin x, x\in\mathbf{R}$. Let the maximum value of f on $\mathbf{R}$ be α. If the area of the region bounded by the curves $g(x)=x^2$ and $h(x)=\beta x^3,\beta>0$, is α^2, then $30\beta^3$ is equal to ____ .

Answer (16)

Sol. $f(x)+3f\left(\frac{\pi}{2}-x\right)=\sin x$ ..(1)

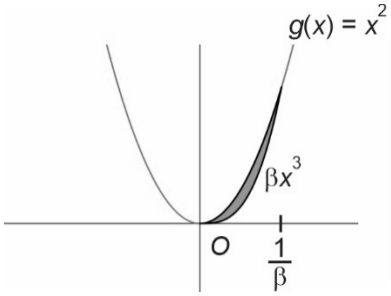
$x\to\frac{\pi}{2}-x$

$\Rightarrow f\left(\frac{\pi}{2}-x\right)+3f(x)=\cos x$ ..(2)

$\Rightarrow f(x)=\frac{3\cos x-\sin x}{8}$

$\alpha=\max(f(x))=\frac{\sqrt{3^2+(-1)^2}}{8}=\frac{\sqrt{10}}{8}=\sqrt{\frac{5}{32}}$

$\Rightarrow \alpha^2=\frac{5}{32}$



$\Rightarrow \text{Area}=\int_0^{1/\beta}\left|(\beta x^3-x^2)\right|d$

$=\left|\frac{\beta x^4}{4}-\frac{x^3}{3}\right|_0^{1/\beta}=\left|\frac{\beta}{4}\times\frac{1}{\beta^4}-\frac{1}{3\beta^3}\right|=\frac{5}{32}$

$\Rightarrow\left|\frac{1}{\beta^3}\left(\frac{1}{4}-\frac{1}{3}\right)\right|=\frac{5}{32}$

$\Rightarrow\frac{1}{12\beta^3}=\frac{5}{32}\Rightarrow\beta^3=\frac{32}{5\times12}=\frac{8}{15}$

25. Let $y = y(x)$ be the solution of the differential equation

$(\tan x)^{1/2}dy=(\sec^3 x-(\tan x)^{3/2}y)dx,$

$y\left(\frac{\pi}{4}\right)=\frac{6\sqrt{2}}{5}$. If $y\left(\frac{\pi}{3}\right)=\frac{4}{5}\alpha$, then α^4 equals

Answer (48.00)

Sol. $\frac{dy}{dx}=\frac{\sec^3 x-(\sqrt{\tan x})^3}{\sqrt{\tan x}}$

$\frac{dy}{dx}+(\tan x)y=(\sec^2 x)\frac{\sec x}{\sqrt{\tan x}}$

I.F. $=e^{\int\frac{\sin x}{\cos x}dx}=e^{-\ln|\cos x|}=\sec x$

$\Rightarrow y\sec x=\int\frac{(\sec x)^2(\sec x)^2}{\sqrt{\tan x}}dx$

Let $\tan x = t^2$

$(\sec^2 x)dx = 2t\,dt$

$y\sec x=\int\frac{(1+t^4)(2t\,dt)}{t}$

$=2t+\frac{2t^5}{5}+C$

$y\sec x=2\sqrt{\tan x}+\frac{2}{5}(\sqrt{\tan x})^5+C$

$y\left(\frac{\pi}{4}\right)\times\sqrt{2}=2+\frac{2}{5}+C\Rightarrow\frac{6\sqrt{2}\times\sqrt{2}}{5}=\frac{12}{5}+C$

$\Rightarrow C=0$

$y\left(\frac{\pi}{3}\right)(2)=2\times\sqrt{(\sqrt{3})}\left[1+\frac{1}{5}(\sqrt{3})^4\right]$

$=2\sqrt{\sqrt{3}}\times\left(1+\frac{3}{5}\right)$

$\Rightarrow y\left(\frac{\pi}{3}\right)=3^{\frac{1}{4}}\times\frac{8}{5}=\frac{4\alpha}{5}\Rightarrow\alpha=2\cdot3^{\frac{1}{4}}$

$\Rightarrow \alpha^4 = 48$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. Match **List-I** with **List-II**.

List-I	List-II
A. Meter (L)	I. $\sqrt{\frac{hc}{G}}$
B. Second (S)	II. $\sqrt{\frac{Gh}{c^5}}$
C. Kilogram (M)	III. $\sqrt{\frac{K^2L^2c^3}{Gh}}$
D. Kelvin (K)	IV. $\sqrt{\frac{Gh}{c^3}}$

where h (Planck's constant), G (gravitational constant) and c (speed of light in vacuum) as fundamental units.

Choose the **correct** answer from the options given below :

(1) A-II, B-IV, C-I, D-III (2) A-IV, B-II, C-I, D-III

(3) A-IV, B-I, C-II, D-III (4) A-III, B-I, C-II, D-IV

Answer (2)

Sol. $[h] = ML^2T^{-1}$

$[c] = LT^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$$\frac{hc}{G} = \frac{ML^3T^{-2}}{M^{-1}L^3T^{-2}} = M^2$$

$$\frac{Gh}{c^5} = \frac{L^5T^{-3}}{L^5T^{-5}} = T^2$$

$$\frac{Gh}{c^3} = \frac{L^5T^{-3}}{L^3T^{-3}} = T^2$$

27. In an experiment to determine the resistance of a given wire using Ohm's law, the voltmeter and ammeter readings are noted as 10 V and 5 A , respectively. The least counts of voltmeter and ammeter are 500 mV and 200 mA , respectively. The estimated error in the resistance measurement is _____ Ω

(1) 0.25

(2) 2

(3) 2.5

(4) 0.18

Answer (4)

Sol. $V = 10$ V $I = 5$ A

$\Delta V = 500$ mV $\Delta I = 200$ mA

$$R = \frac{V}{I} = 2$$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\frac{\Delta R}{2} = \frac{0.5}{10} + \frac{0.2}{5}$$

$\Delta R = 0.18$

28. A mass of 1 kg is kept on a inclined plane with 30° inclination with respect to horizontal plane and it is at rest initially. Then the whole assembly is moved up with constant velocity of 4 m/s. The work done by the frictional force in time 2 s is _____ J. (Take $g = 10$ m/s^2)

(1) 20

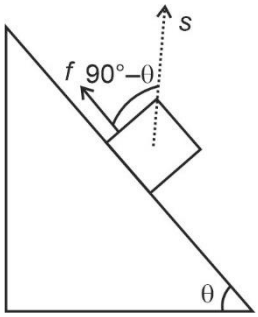
(2) 25

(3) 30

(4) 10

Answer (1)

Sol.



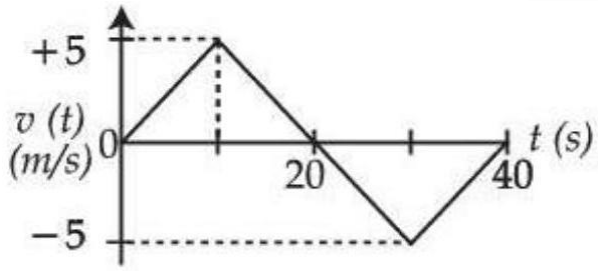
$f = mg\sin\theta$

$\omega = f \cdot s \cdot \cos(90 - \theta)$

$W = mg\sin\theta \cdot 8 \cdot \sin\theta$

$= 80\sin^2\theta = 20$

29. The velocity (v) versus time (t) plot of a particle is shown in the figure, for a time interval of 40 s . The total distance travelled by the particle and the average velocity during this period are, respectively _____.



(1) 25 m and zero

(2) 50 m and zero

(3) 100 m and zero

(4) 100 m and 2.5 m/s

Answer (3)

Sol. Displacement $= \frac{1}{2}\times 20\times 5 - \frac{1}{2}\times 20\times 5 = 0$

Average velocity = 0

Distance $= \frac{1}{2}\times 20\times 5 + \frac{1}{2}\times 20\times 5 = 100$

30. A wheel initially at rest is subjected to a uniform angular acceleration about its axis. In the first 2 s it rotates through an angle θ_1 and in the next 2 s it rotates through an angle θ_2. The ratio $\frac{\theta_2}{\theta_1}$ is _____.

(1) 6 (2) 3

(3) 4 (4) $\frac{1}{3}$

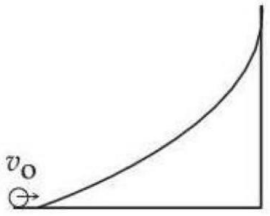
Answer (2)

Sol. $\theta_1 = \frac{1}{2}\alpha\cdot(2)^2 = 2\alpha$

$\theta_1 + \theta_2 = \frac{1}{2}\cdot\alpha\cdot(4)^2 = 8\alpha$

$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$

31. An object of uniform density rolls up the curved path with the initial velocity v_o as shown in the figure. If the maximum height attained by an object is $\frac{7v_o^2}{10\ g}$ (g = acceleration due to gravity), the object is a _____.



(1) solid cylinder (2) ring

(3) disc (4) solid sphere

Answer (4)

Sol. $(KE + PE)_{\text{lowest point}} = (PE + KE)_{\text{Highest point}}$

$\frac{1}{2}mv^2 + \frac{1}{2}I.\omega^2 = mgh$

$$\frac{1}{2}mv_0^2+\frac{1}{2}I.\frac{v_0^2}{R^2}=\frac{7}{10}mv_0^2$$

$$I=\frac{2}{5}mR^2$$

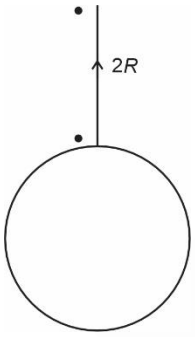
32. A body of mass m is taken from the surface of earth to a height equal to twice the radius of earth (R_e). The increase in potential energy will be _____. (g is acceleration due to gravity at the surface of earth)

(1) $\frac{1}{2}mgR_e$ (2) $\frac{3}{4}mgR_e$

(3) $\frac{1}{4}mgR_e$ (4) $\frac{2}{3}mgR_e$

Answer (4)

Sol. $\Delta U=U_2-U_1$



$$=-\frac{GMm}{3R}+\frac{GMm}{R}$$

$$=\frac{2}{3}mgR$$

33. Eight mercury drops, each of radius r, coalesce to form a bigger drop. The surface energy released in this process is _____. (S is the surface tension of mercury).

(1) $8\pi r^2S$ (2) $16\pi r^2S$

(3) $64\pi r^2S$ (4) $4\pi r^2S$

Answer (2)

Sol. Let radius of bigger drop is R.

$$8.\frac{4\pi}{3}r^3=\frac{4\pi}{3}R^3$$

$$R=2r$$

$$\Delta E=8\cdot4\pi r^2\cdot S-4\pi R^2S$$

$$=16\pi r^2S$$

34. An ideal gas at pressure P and temperature T is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is _____.

(1) $\frac{2}{T}$ (2) $\frac{1}{T}$

(3) $\frac{4}{T}$ (4) $\frac{3}{T}$

Answer (3)

Sol. $PT^3=k$

$$V=k_1T^4$$

$$\frac{dV}{V}=\frac{k_1.4T^3.dT}{k_1T^4}$$

$$\frac{dV}{V}=\left(\frac{4}{T}\right)dT$$

35. Match **List - I** with **List - II**.

	List – I		**List – II**
A.	$\sin^2\omega t$	I.	Periodic with time period $T=\frac{\pi}{\omega}$ but not simple harmonic motion (SHM)
B.	$\sin^3(2\omega t)$	II.	Periodic with time period $T=\frac{2\pi}{\omega}$ but Not SHM

C. $\sin(\omega t)+\cos(\pi\omega t)$ III. Periodic with time period $T=\frac{\pi}{\omega}$ and SHM

D. $\cos\omega t+\cos 2\omega t$ IV. Non-periodic

Choose the **correct** answer from the options given below :

(1) A-III, B-I, C-IV, D-II

(2) A-II, B-I, C-III, D-IV

(3) A-III, B-II, C-IV, D-I

(4) A-II, B-I, C-IV, D-III

Answer (1)

Sol. $\sin^2(\omega t)=\frac{1-\cos(2\omega t)}{2}$

$T=\frac{2\pi}{\omega}$

$\sin^3(2\omega t)=\frac{3}{4}\sin(2\omega t)-\frac{1}{4}\sin(6\omega t)$

$T_1=\frac{\pi}{\omega}, T_2=\frac{\pi}{3\omega}$

T = LCM of T_1 and $T_2 \Rightarrow T=\frac{\pi}{\omega}$

Period but not SHM.

$\sin(\omega t)+\cos(\pi\omega t)$

$T_1=\frac{2\pi}{\omega}, T_2=\frac{2\pi}{\pi\omega}=\frac{2}{\omega}$

LCM does not exist of T_1 and T_2

So non periodic.

$\cos(\omega t)+\cos(2\omega t)$

$T_1=\frac{2\pi}{\omega}, T_2=\frac{2\pi}{2\omega}$ [LCM (T_1, T_2) = $\frac{2\pi}{2\omega}$

Periodic but not SHM.

36. A metal rod of length L rotates about one end at origin with a uniform angular velocity ω. The magnetic field radially falls off as $B(r) = B_0e^{-\lambda r}$; λ being a positive constant. The emf induced (neglecting the centripetal force on electrons in the rod) is:

(1) $B_0\omega\left[\frac{1}{\lambda^2}-e^{-\lambda L}\left(\frac{1}{\lambda^2}+\frac{L}{\lambda}\right)\right]$

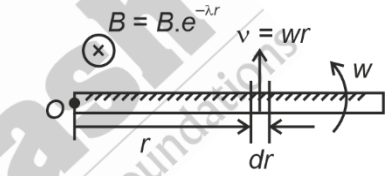
(2) $B_0\omega\left[\frac{1}{\lambda^2}+e^{-\lambda L}\left(\frac{1}{\lambda^2}+\frac{L}{\lambda}\right)\right]$

(3) $B_0\omega\left[\frac{4}{\lambda^2}-e^{-2\lambda L}\left(\frac{1}{\lambda^2}+\frac{2L}{\lambda}\right)\right]$

(4) $B_0\omega\left[\frac{3}{\lambda^2}-e^{-3\lambda L}\left(\frac{3}{\lambda^2}+\frac{L}{\lambda}\right)\right]$

Answer (1)

Sol. $d\varepsilon = vBdr$



$\int_0^{\varepsilon} d\varepsilon=\int_0^{l} wrB_0e^{-\lambda r}dr$

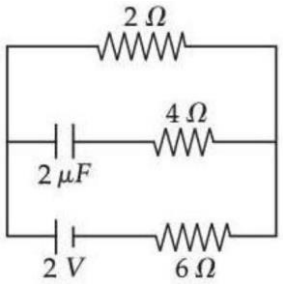
$\varepsilon=wB_0\int_0^{l} re^{-\lambda r}dr$

$\varepsilon=wB_0\left[-e^{-\lambda r}\left(\frac{r}{\lambda}+\frac{1}{\lambda^2}\right)\right]^{l}$

$\varepsilon=wB_0\left[-\left\{e^{-\lambda L}\left(\frac{L}{\lambda}+\frac{1}{\lambda^2}\right)-e^{-\lambda\times 0}\left(\frac{0}{\lambda}+\frac{1}{\lambda^2}\right)\right\}\right]^{l}$

$\varepsilon=wB_0\left\{\frac{1}{\lambda^2}-e^{-\lambda L}\left(\frac{L}{\lambda}+\frac{1}{\lambda^2}\right)\right\}$

37. Under steady state condition the potential difference across the capacitor in the circuit is _____V.

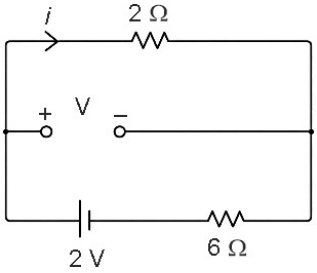


(1) 0.5 (2) 1.5

(3) 0 (4) 2

Answer (1)

Sol.



$$i = \frac{2}{8} = \frac{1}{4}$$

$$v = i \times 2 = \frac{1}{2}$$

38. A particle of charge q and mass m is projected from origin with an initial velocity $\vec{v} = \left(\frac{v_0}{\sqrt{2}}\hat{x} + \frac{v_0}{\sqrt{2}}\hat{y}\right)$.

There exists a uniform magnetic field $\vec{B} = B_0\hat{z}$ and a space varying electric field $\vec{E} = E_0 e^{-\lambda x}\hat{x}$ within the region $0 \leqslant x \leqslant L$. After travelling a distance such that x-coordinate has changed from $x = 0$ to $x = L$, the change in the kinetic energy is _____.

(1) $\frac{qE_0}{\lambda}\left[1 - e^{-\lambda L}\right]$

(2) $\left(\frac{v_0 qB_0}{2\lambda}\right)\left[2 - e^{-2\lambda L}\right]$

(3) $\frac{qE_0}{\lambda}\left[1 + e^{-\lambda L}\right]$

(4) $q\left(\frac{E_0 + v_0 B_0}{\lambda}\right)\left[1 - e^{-\lambda L/2}\right]$

Answer (1)

Sol. Applying work energy theorem.

$W_E + W_B = \Delta$K.E.

$$W_E = \int_0^L qE_0 e^{-\lambda x}dx = qE.\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^L$$

$$W_E = \frac{qE_0}{\lambda}\left(1 - e^{-xL}\right)$$

$W_B = 0$

$$\Delta\text{K.E.} = \frac{qE_0}{\lambda}\left(1 - e^{-\lambda L}\right)$$

39. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A): The electromagnetic wave exerts pressure on the surface on which they are allowed to fall.

Reason (R): There is no mass associated with the electromagnetic waves.

In the light of the above statements, choose the **correct answer** from the options given below:

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(3) **(A)** is true but **(R)** is false

(4) **(A)** is false but **(R)** is true

Answer (2)

Sol. Rest mass of photons is zero.

40. A thin convex lens and a thin concave lens are kept in contact and are co-axial. Which of the following statements is correct for this combination of two lenses?

(1) behaves as concave lens if $|f_{convex}| > |f_{concave}|$

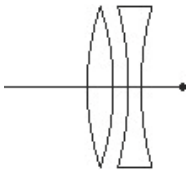
(2) behaves as concave lens if $|f_{convex}| < |f_{concave}|$

(3) behaves as convex lens if $|f_{convex}| > |f_{concave}|$

(4) Focal length of the lens system will change if the positions of two lenses are interchanged

Answer (1)

Sol.



$P = P_1 + P_2$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$|f_{convex}| > |f_{concave}|$

$P < 0 \rightarrow$ Concave behavior.

41. An object AB is placed 15 cm on the left of a convex lens P of focal length 10 cm. Another convex lens Q is now placed 15 cm right of lens P. If the focal length of lens Q is 15 cm, the final image is _____.

(1) virtual, formed at 7.5 cm right of lens Q, with a size bigger than that of AB

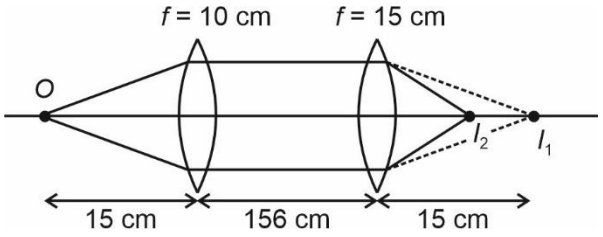
(2) real, formed at 7.5 cm right of lens Q, with a size same as that of AB

(3) formed at infinity

(4) real, formed at 7 cm right of lens Q, with a size smaller than that of AB

Answer (2)

Sol.



$$\frac{1}{v_1} - \frac{1}{-15} = \frac{1}{10} \qquad v_1 = 30$$

$$\frac{1}{v_2} - \frac{1}{15} = \frac{1}{15} \qquad v_2 = 7.5$$

$$m_1 = \frac{30}{-15} = -2 \qquad m_2 = \frac{7.5}{15} = \frac{1}{2}$$

$|m_1 m_2| = 1$

42. The maximum intensity in a Young's double slit experiment is I_o. Distance between the slits (d) is 5λ, where λ is the wavelength of light used. The intensity of the fringe, exactly opposite to one of the slits on the screen, placed at $D = 10d$ is _____.

(1) $\frac{I_o}{4}$ (2) $\frac{I_o}{2}$

(3) I_0 (4) $\frac{3I_o}{4}$

Answer (2)

Sol. $\Delta x = d\sin\theta$

$$= d \cdot \frac{(d/2)}{D}$$

$$= \frac{d^2}{2D}$$

$$= \frac{25\lambda^2}{2 \times 10 \times 5\lambda} = \frac{\lambda}{4}$$

$$\Delta\phi = \frac{\pi}{2}$$

$$I = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

43. An electron is travelling with a velocity v in free space and when it enters a medium, its velocity is reduced by 20%. The de Broglie wavelength of electron in the medium is $\alpha\lambda_0$, where λ_0 is its de Broglie wavelength in free space. The value of α is _____.

(1) 1.20

(2) 1.0

(3) 1.25

(4) 0.75

Answer (3)

Sol. $\lambda_0 = \frac{h}{mv_0}$

$\alpha\lambda_0 = \frac{h}{m \times 0.8\, v_0}$

$\alpha = 1.25$

44. Assuming the experimental mass of ${}^{12}_{6}C$ as $12u$, the mass defect of ${}^{12}_{6}C$ atom is _____ MeV/c^2. (Mass of proton $= 1.00727$ u. mass of neutron $= 1.00866$u, $1u = 931.5$ MeV/c^2 and c is the speed of the light in vacuum).

(1) 127.5

(2) 89.03

(3) 272.0

(4) 92.0

Answer (2)

Sol. $\Delta m = 6 \times (1.00727) + 6(1.00866) - 12$

= 0.04362 + 0.05196

= 0.0955

E = 0.0955 × 931.5 = 88.95

45. In a semiconductor p-n diode, the doping concentrations on p-side and n-side are 10^{15} atoms/cm^3 and 10^{18} atoms/cm^3, respectively. Which one of the following statements is true?

(1) Widths of depletion region on either side of the interface are equal

(2) The depletion region width is more on p-side compared to that in n-side

(3) The depletion region width is more on n-side compared to that in p-side

(4) No depletion region forms because of unequal doping concentrations on p and n-sides

Answer (2)

Sol. Depletion layer width (w) $\propto \frac{1}{\text{doping}}$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A copper wire of length 3 m is stretched by 3 mm by applying an external force. The volume of the wire is 600×10^{-6} m^3. The elastic potential energy stored in the wire in stretched condition would be _____ J.

(Given Young modulus of copper = 1.1 × 10^{11} N/m^2)

Answer (33)

Sol. ℓ = 3 m $\quad\quad$ $\Delta\ell$ = 3 mm

$A\ell$ = 600 × 10^{-6} Y = 1.1 × 10^{11}

$PU = \frac{1}{2} \cdot \frac{F}{A} \cdot \frac{\Delta\ell}{\ell} \cdot (\text{vol.})$

$$= \frac{Y}{2} \cdot \frac{\Delta \ell}{\ell} \cdot \frac{\Delta \ell}{\ell} (\text{vol.})$$

$$= \frac{1.1 \times 10^{11}}{2} \times \left(\frac{3 \times 10^{-3}}{3} \right)^2 \times 6 \times 10^{-4}$$

$= 3.3 \times 10^1$

$= 33$

47. The heat extracted out of x gram of water initially at 50°C to cool it down to 0°C is sufficient to evaporate (1000 – x) gram of water also initially at 50°C. The value of x (closest integer) is _____.

(Take latent heat of water 2256 kJ/kg. K, specific heat capacity of water 4200 J/kg. K)

Answer (922)

Sol. $x \times 4.2 \times 50 = (1000 - x)\, 4.2 \times 50 + (1000 - x)\, 2256$

$(2x - 1000) \times 4.2 \times 50 = (100 - x)\, 2256$

$420x - 210 \times 10^3 = 2256 \times 10^3 - 2256x$

$2676x = 2466 \times 10^3$

$x = 921.52$

48. A series LCR circuit with $R = 20\Omega$, $L = 1.6$ H and $C = 40\ \mu$F is connected to a variable frequency a.c. source. The inductive reactance at resonant frequency is _____ Ω.

Answer (200)

Sol. $\omega = \frac{1}{\sqrt{1.6 \times 40 \times 10^{-6}}} = \frac{1}{8 \times 10^{-3}}$

$X_L = \frac{1}{8 \times 10^{-3}} \times 1.6 = 200$

49. When an external resistance of 5Ω is connected across terminals of a cell, a current of 0.25 A flows through it. When the 5Ω resistor is replaced by a 2Ω resistor, a current of 0.5 A flows through it. The internal resistance of the cell is _____ Ω.

Answer (1)

Sol. $\frac{\epsilon}{r+5} = \frac{1}{4}$

$4\epsilon = r + 5$

$\frac{\epsilon}{r+2} = \frac{1}{2}$

$2\epsilon = r + 2$

$r + 5 = 2r + 4$

$r = 1$

50. A circular loop of radius 20 cm and resistance 2Ω is placed in a time varying magnetic field $\vec{B} = \left(2t^2 + 2t + 3\right) T$. At t = 0, for the plane of the loop being perpendicular to the magnetic field and the induced current in the loop at t = 3 s is $\frac{\alpha}{50}$ A. The value of α is _____.

(Take $\pi = 22/7$)

Answer (44)

Sol. $i = \frac{\epsilon}{R} = \frac{A}{R}\frac{dB}{dt} = \frac{\pi \cdot \frac{1}{25}}{2} \cdot (4t + 2)$

$= \frac{22}{7} \cdot \frac{1}{50} \cdot 14$

$= \frac{44}{50}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. What volume of hydrogen gas at STP would be liberated by action of 50 mL of H_2SO_4 of 50% purity (density $= 1.3\ g\ mL^{-1}$) on 20 g of zinc? Given : Molar mass of H, O, S, Zn are 1, 16, 32, 65 g mol^{-1} respectively.

(1) 5.824 L (2) 7.428 L
(3) 6.892 L (4) 8.375 L

Answer (2)

Sol. $Zn + H_2SO_4 \rightarrow ZnSO_4 + H_2$

Mass of H_2SO_4 = 50 × 1.3 × 0.5g

Moles of $H_2SO_4 = \frac{50 \times 1.3 \times 0.5}{98} = \frac{32.5}{98} = 0.3316$

= moles of H_2

$V_{H_2} = 7.428$ L

52. Which of the following statement(s) is/are true ?

A. If two orbitals have the same value of $(n+l)$, the orbital with lower value of n will have lower energy.

B. Energies of the orbitals in the same subshell increase with increase in atomic number.

C. The size of $2p_x$ orbital is less than the size of $3p_x$ orbital.

D. Among 5f, 6s, 4d, 5p and 5d orbitals, none of the orbitals have 2 radial nodes.

Choose the correct answer from the options given below :

(1) A, B and C only (2) A and C only
(3) C and D only (4) A only

Answer (2)

Sol. • Number of radial nodes = n – l – 1

	Number of radius nodes
5f	5 – 3 – 1 = 1
6s	6 – 0 – 1 = 5
4d	4 – 2 – 1 = 1
5p	5 – 1 – 1 = 3
5d	5 – 2 – 1 = 2

• Energy is inversely proportional to atomic number

Z increases $\Rightarrow E_n \downarrow$

53. The covalent radii of atoms A and B are r_A and r_B, respectively. The covalent bond length and total length of AB molecule are respectively

(1) $(r_A + r_B), 2(r_A + r_B)$

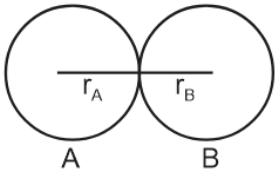
(2) $\frac{1}{2}(r_A + r_B), (r_A + r_B)$

(3) $(r_A + r_B), (r_A + r_B)$

(4) $2(r_A + r_B), \frac{1}{2}(r_A + r_B)$

Answer (1)

Sol.



Covalent bond length = $r_A + r_B$

Total bond length = $2(r_A + r_B)$

54. Consider the following data for the reaction $X_2(g)+Y_2(g)=2XY(g)$

at 600 K . The $\Delta_rG^{\ominus}$ (in $kJmol^{-1}$) for the reaction is :

Compound	$\Delta_fH^{\ominus}_{600\ K}$ $(kJmol^{-1})$	$S^{\ominus}_{600\ K}$ $(J\ mol^{-1}\ K^{-1})$
$XY(g)$	42	200
$X_2(g)$	8	140
$Y_2(g)$	80	250

(1) –21000 (2) –10

(3) –1000 (4) –9.012

Answer (2)

Sol. $\Delta_rG^{\circ}=\Delta_rH^{\circ}-T\Delta_rS^{\circ}$ at 600 K

$\Delta H^{\circ}=2\times42-[8+80]$

$=84-88=-4\times10^3\ J\ mol^{-1}$

$\Delta S^{\circ}=2\times200-[140+250]$

$=400-390=10\ J\ K^{-1}\ mol^{-1}$

$\Delta_rG^{\circ}=-4000-(600)\times10$

$=-10000=-10\ kJ/mol$

55. The correct order of molar heat capacities measured at 298 K and 1 bar is :

(1) Copper(s) > Bromine(l) > Helium(g)

(2) Bromine(l) > Copper(s) > Helium(g)

(3) Helium(g) > Bromine(l) > Copper(s)

(4) Helium(g) > Bromine(l) = Copper(s)

Answer (2)

Sol. **Molar heat capacity**

Bromine (l) $\approx$ 76 $J\ mol^{-1}\ K^{-1}$

Copper (S) $\approx$ 24.9 $J\ mol^{-1}\ K^{-1}$

He(g) $\approx \frac{5}{2}R$ = 20.8 $J\ mol^{-1}\ K^{-1}$

56. The reaction $A(g)\rightleftharpoons B(g)+C(g)$ was initiated with the amount ' a ' of $A(g)$. At equilibrium it is found that the amount of $A(g)$ remaining is $(a-x)$ at a total pressure of p .

The equilibrium constant Kp of the reaction can be calculated from the expression :

(1) $\frac{x^2}{a^2+x^2}\times p$ (2) $\frac{x^2}{a^2-x^2}\times p$

(3) $\frac{a+x^2}{x^2}\times p$ (4) $\frac{a^2-x^2}{x^2}\times p$

Answer (2)

Sol.

	A(g)	$\rightleftharpoons$	B(g)	+	C(g)
$t=0$	a		–		–
$t=t_{eq}$	$a-x$		x		x

$p_B=\frac{x}{a+x}\times p=p_c$

$p_A=\frac{a-x}{a+x}\times p$

$K_p=\frac{\left(\frac{x}{a+x}\times p\right)^2}{\frac{a-x}{a+x}\times p}=\frac{x^2p}{a^2-x^2}$

57. One half cell in a voltaic cell is constructed by dipping silver rod in $AgNO_3$ solution of unknown concentration, other half cell is Zn rod dipped in 1 molar solution of $ZnSO_4$. A voltage of 1.60 V is measured at 298 K for this cell. What is the concentration of Ag^+ ions used in terms of $\log x\left(x=\left[Ag^+\right]\right)$?

$E^{\ominus}_{Zn^{2+}/Zn}=-0.76\ V,\ E^{\ominus}_{Ag^+/Ag}=+0.80\ V,\frac{2.303RT}{F}=0.059\ V$

(1) $\frac{2}{3.9}$ (2) $\frac{4}{5.9}$

(3) $\frac{2.9}{2}$ (4) $\frac{5.9}{4}$

Answer (2)

Anode : $Zn \rightarrow Zn^2 + 2e^-$

Sol. Cathode : $2Ag^+ + 2e^- \rightarrow 2Ag$

Net reaction : $Zn + 2Ag^+ \rightarrow Zn^{2+} + 2Ag$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Ag^+]^2}$$

$$1.60 = 1.56 - \frac{0.059}{2}\log\frac{1}{(x)^2}$$

$$\frac{0.04 \times 2}{0.059} = 2\log x$$

$$\frac{4}{5.9} = \log x$$

58. Given below are two statements :

Statement I : The number of pairs among $[Al_2O_3, Cr_2O_3], [Cl_2O_7, Mn_2O_7], [Na_2O, V_2O_3]$ and $[CO, N_2O]$ that contain oxides of same nature (acidic, basic, neutral or amphoteric) is 4.

Statement II : Among Na_2O, Al_2O_3, CO and Cl_2O_7, the most basic and acidic oxides are Na_2O and Cl_2O_7, respectively.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (1)

Sol. $\left.\begin{matrix} Al_2O_3 \\ Cr_2O_3 \end{matrix}\right]$ both amphoteric

$\left.\begin{matrix} Cl_2O_7 \\ Mn_2O_7 \end{matrix}\right]$ highly covalent $\Rightarrow$ both acidic

$\left.\begin{matrix} Na_2O \\ V_2O_3 \end{matrix}\right]$ basic

$\left.\begin{matrix} CO \\ N_2O \end{matrix}\right]$ neutral

$Na_2O \rightarrow$ most basic

$Cl_2O_7 \rightarrow$ most acidic

S-I $\rightarrow$ true

S-II $\rightarrow$ true

59. Given below are two statements :

Statement I: Aluminium upon reaction with NaOH forms $[Al(OH)_6]^{3-}$ ion.

Statement II : The geometry of ICl_4^-, ClO_3^- and IBr_2^- is square planar, pyramidal and linear respectively.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

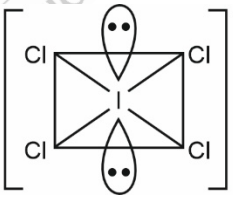
(4) Statement I is false but Statement II is true

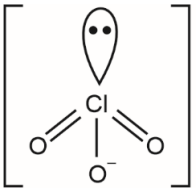
Answer (4)

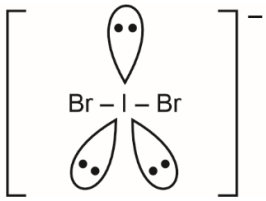
Sol. $2Al_{(s)} + 2NaOH_{(aq)} + 6H_2O_{(l)} \rightarrow$

$2Na^+[Al(OH)_4]^-(aq) + 3H_2O(g)$

S-I false

$ICl_4^- \longrightarrow$  Sq. planar

$ClO_3^- \longrightarrow$  Pyramidal

$IBr_2^- \longrightarrow$  Linear

S-II true

60. Given below are two statements :

Statement I: Presence of large number of unpaired electrons in transition metal atoms results in higher enthalpies of their atomisation.

Statement II: $d_{xy} = d_{xz} = d_{yz} < d_{x^2-y^2} = d_{z^2}$ and $d_{x^2-y^2} = d_{z^2} < d_{xy} = d_{xz} = d_{yz}$ are the d-orbital splittings in $\left[Fe(H_2O)_6\right]^{3+}$ and $[Ni(Cl)_4]^{2-}$ complex ions respectively.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Answer (1)

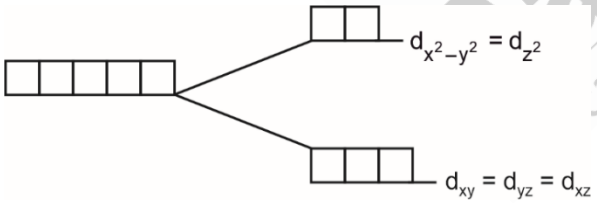
Sol. S-I true

More number of unpaired

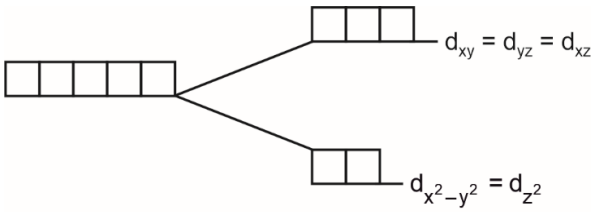
$e^- \Rightarrow$ Stronger metallic bonding

S-II true

Octahedral field in $[Fe(H_2O)_6]^{3+}$



Tetrahedral field in $[Ni(Cl_4)]^{2-}$



61. Identify the correct statements from the following

A. $\left[Fe(C_2O_4)_3\right]^{3-}$ is the most stable complex among $[Fe(OH)_6]^{3-}$, $\left[Fe(C_2O_4)_3\right]^{3-}$ and $[Fe(SCN)_6]^{3-}$

B. The stability of $\left[Cu(NH_3)_4\right]^{2+}$ is greater than that of $[Cu(en)_2]^{2+}$

C. The hybridization of Fe in $K_4[Fe(CN)_6]$ is d^2sp^3

D. $\left[Fe(NO_2)_3Cl_3\right]^{3-}$ exhibits linkage isomerism

E. NO_2^- and SCN^- ligands are NOT ambidentate ligands

Choose the correct answer from the options given below :

(1) A, B, C, D and E (2) B, C and D only

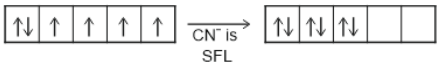
(3) A, C and D only (4) A, C and E only

Answer (3)

Sol. (A) $[Fe(C_2O_4)_3]^{3-}$ is most stable due to Chelation – Correct.

(B) $[Cu(NH_3)_4]^{2+} < [Cu(en)_2]^{3+}$ stability due to to chelation; so B is incorrect.

(C) $k_4[Fe(CN)_6]^{2+}$

Fe^{2+}  CN^- is SFL

So (Fe^{2+}) is d^2sp^3 hybridised

– Correct

(D) NO_2^- is ambident ligand $\Rightarrow$ linkage isomerism — correct.

(E) NO_2^-, SCN^- are ambident ligand; so E is incorrect

62. Match List - I with List - II.

	List-I Purification technique		List-II Used to separate
(A)	Simple distillation	I	Steam volatile compound
(B)	Fractional distillation	II	Two liquids with large difference in boiling points
(C)	Steam distillation	III	Liquid decomposing at its boiling point
(D)	Distillation under reduced pressure	IV	Two liquids with close boiling points

Choose the correct answer from the options given below :

(1) A-II, B-III, C-I, D-IV (2) A-II, B-IV, C-I, D-III

(3) A-II, B-IV, C-III, D-I (4) A-IV, B-III, C-II, D-I

Answer (2)

Sol. A simple distillation. (II) Two liquids with large difference in boiling point

B. Fractional distillation (IV) Two liquids with close boiling point

C. Steam distillation (I) Steam volatile compounds

D. Distillation under Red need pressure (III) Liquids decomposing at its boiling point

63. Find out the correct stability order.

A. 2-Methylbut-2-ene

B. cis-But-2-ene

C. 2,3-Dimethylbut-2-ene

D. Prop-1-ene

Choose the correct answer from the options given below :

(1) C > A > B > D (2) C > A > D > B

(3) B > D > A > C (4) A > B > C > D

Answer (1)

Sol. A. $CH_3—CH=\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}—CH_3$ (9 α H)

B. $\underset{\displaystyle CH_3}{\overset{\displaystyle |}{CH}}=\underset{\displaystyle CH_3}{\overset{\displaystyle |}{CH}}$ (6 α H)

C. $H_3C—\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}=\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}—CH_3$ (12 α H)

D. $CH_2=CH—CH_3$ (3 α H)

Stability order C > A > B > D

64. Identify the correct IUPAC name of hydrocarbon (x) containing three primary carbon atoms and with molar mass 72 g mol^{-1}.

(1) 1 – Dimethylcyclopropane

(2) 2,2 - Dimethylpropane

(3) 2 – Methylbutane

(4) n-pentane

Answer (3)

Sol. $CH_3—\underset{\displaystyle CH_3}{\overset{\displaystyle |}{C}}—CH_2—CH_3$ (C_5H_{12})

M. mass = 72 g/mol.

65. Complete the following reaction sequence and give the name of major product ' P' .

(i) $OH^- / H_2O / \Delta$ 

(ii) ⟶ (Major product)

(iii) Cl_2 / Red P

(iv) H_2O

(1) 2-Chloropropanoic acid

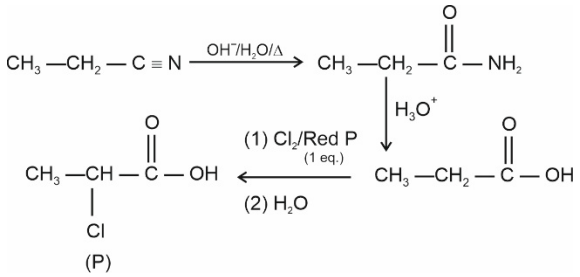
(2) 3-Chloropropanoic acid

(3) 1-Chloropropane

(4) 2-Chloropropane

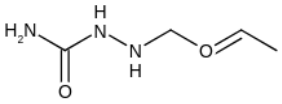
Answer (1)

Sol.

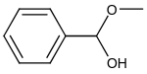


66. Given below are two statements :

Statement I: The condensation reaction between



Statement II: The molecule,



will generate $Ph-CH=O$ in the presence of dilute acid.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

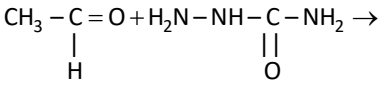
(3) Statement I is true but Statement II is false

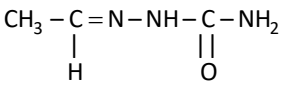
(4) Statement I is false but Statement II is true

Answer (4)

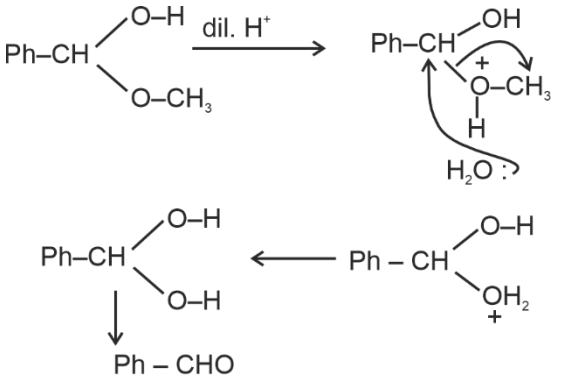
Sol.

SI : False





SII : True



67. Given below are two statements :

Statement I: Heating benzamide with bromine in an ethanolic solution of sodium hydroxide will give benzylamine.

Statement II: Nitration of aniline with HNO_3 / H_2SO_4 at 288 K produces m-nitroaniline in higher amount than o-nitroaniline (pH adjusted).

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

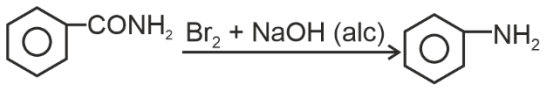
(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

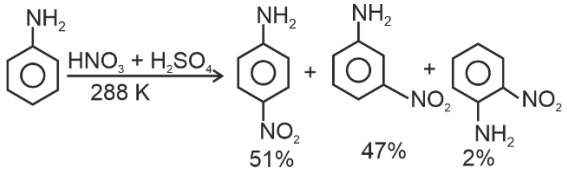
Answer (4)

Sol.

SI : False



SII : True



68. Identify the incorrect statement about tertiary structure of proteins.

(1) They can be fibrous or globular in structure.

(2) The main forces that stabilize the structure are hydrogen bonding, disulphide links, van der Waals and electrostatic forces of attraction.

(3) The structure remains intact when exposed to pH changes.

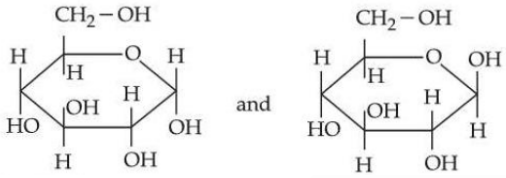
(4) A linear polypeptide chain will convert to a secondary structure and then further folding of the secondary structure will convert to tertiary structure.

Answer (3)

Sol. pH change causes denaturation

69. Given below are two statements :

Statement I :



are two anomers of D-(+)-glucose.

Statement II :

The open chain forms of D-glucose and D- fructose contain three similar chiral carbons at C_3, C_4 and C_5.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

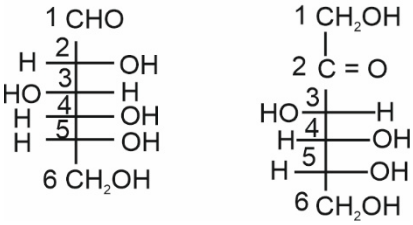
(4) Statement I is false but Statement II is true

Answer (1)

Sol. **S1 :** True (α and β-D-glucose).

SII : True

D-glucose D-fructose



70. A paper dipped in a dil. H_2SO_4 solution of ' X ' upon treatment with SO_2 gas turns into green. The compound ' X ' is :

(1) KI-starch (2) $KMnO_4$

(3) $Pb(CH_3COO)_2$ (4) $K_2Cr_2O_7$

Answer (4)

Sol. $X + SO_2 \xrightarrow[H_2SO_4]{} \underset{\text{Green}}{Cr_2(SO_4)_3}$

It turns green due to formation of $Cr_2(SO_4)_3$; x → $K_2Cr_2O_7$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The total number of unpaired electrons present in the d^3, d^4 (low spin) d^5 (high spin), d^6 (high spin) and d^7 (low spin) octahedral complex systems is ____ .

Answer (15)

Sol. d^3 (octahedral complex) ⇒ n = 3

d^4 (low spin, octahedral complex) ⇒ n = 2

d^5 (High spin, octahedral complex) ⇒ n = 5

d^6 (High spin, octahedral complex) ⇒ n = 4

d^7 (low spin, octahedral complex) ⇒ n = 1

Total unpaired electrons = 3 + 2 + 5 + 4 + 1 = 15

72. RMgI when treated with ice cold water liberated a gas which occupied $1.4 dm^3 / g$ at STP. The gas produced is further reacted with iodine in presence of HIO_3 to give compound (X). Compound (X) in presence of Na and dry ether produced compound (Y). Molar mass of compound (Y) is ____ $gmol^{-1}$. (Nearest integer)

Answer (30)

Sol. 1.4 L gas weighs1 g at STP

$$22.4 \text{ L/mol gas weigh } \frac{1}{1.4 \text{ L}} \times 22.4 = 16 \text{ g/mol}$$

The liberated gas should be $CH_4(g)$

$$CH_3MgI + H_2O \rightarrow CH_4 + Mg(OH)I$$

$$CH_4 + I_2 \xrightarrow{HIO_4} \underset{(X)}{CH_3 - I}$$

$$2\,CH_3I \xrightarrow{Na\ /\ Dry\ ether} \underset{(Y)}{CH_3 - CH_3}$$

Molar mass of Y = 30 g/mol

73. 20 g hemoglobin in a 1 L aqueous solution (A) at 300 K is separated from pure water by semi permeable membrane. At equilibrium the height of solution in a tube dipped in a solution (A) is found to be 80.0 mm higher than the tube dipped in water. The molar mass of hemoglobin is ____ $kgmol^{-1}$. (Nearest integer)

(Given : $g = 10\ m\ s^{-2}, R = 8.3\ k\ Pa\ dm^3\ K^{-1}\ mol^{-1}$, density of solution = $1000\ kgm^{-3}$)

Answer (62)

Sol. $\pi = h\rho g$

$h = 80 \text{ mm} = 80 \times 10^{-3} \text{ m}$

$\rho = 1000 \text{ kg/m}^3$

$g = 10 \text{ ms}^{-2}$

$\pi = 80 \times 10^{-3} \times 1000 \times 10$

$= 800 \text{ Pa}$

$= 800 \text{ N m}^{-2}$

$\pi = 0.8 \text{ kPa}$

$$\pi = CRT = \frac{n}{V}RT$$

$$\pi = \frac{w}{MV}RT$$

$$M = \frac{w}{\pi V}RT$$

$$M = \frac{20 \times 8.3 \times 300}{0.8 \times 1}$$

$M = 62250 \text{ g/mol}$

$M = 62.25 \text{ kg/mol}$

≈ 62

74. At 298 K , the molar conductivity of $x\%(w/w)MX$ solution (aqueous) is $123.5\ S\ cm^2\ mol^{-1}$. The conductance of same solution is 1.9×10^{-3} S . The value of x is ____ $\times 10^{-2}$. (Given: cell constant $= 1.3\ cm^{-1}$; molar mass of MX is $75\ g\ mol^{-1}$, density of aqueous solution of MX at 298 K is $1.0\ g\ mL^{-1}$)

Answer (13)

Sol. $\kappa = \frac{1}{R} \cdot \frac{l}{A}$

$\kappa = G \cdot G^*$

$= 1.6 \times 10^{-3} \times 1.3$

$= 2.08 \times 10^{-3} S\ cm^{-1}$

$\Lambda_m = \frac{\kappa \times 1000}{M}$

$123.5 = \frac{2.08 \times 10^{-3} \times 1000}{M}$

$M = \frac{2.08}{123.5}$

$= 0.0168 mol\ L^{-1}$

0.0168 mol of MX in 1000 mL of solution

Mass of solution = $V \times d$

$= 1000 \times 1.0$

$= 1000g$

Mass of MX = 0.0168×75

$= 1.263g$

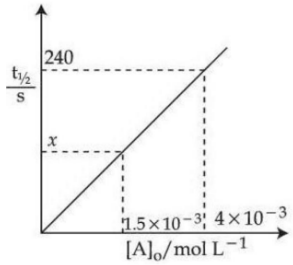
1000g of solution contain 1.263g solute

100g of solution will contain

12.63 g solute

% w/w $= 12.63 \approx 13$

75. For a reaction $A \rightarrow P$ at T K, the half life ($t_{1/2}$) is plotted as a function of initial concentration $[A]_0$ of A as given below.



The value of x in the given figure is ____ s (Nearest integer)

Answer (90)

Sol. For zero order

$t_{1/2} = \frac{A_0}{2k}$

$y = mx$

Straight line starting from origin

The given graph follow zero order kinetic

$t_{1/2} \propto A_0$

$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{(A_0)_1}{(A_0)_2}$

$\frac{240}{x} = \frac{4 \times 10^{-3}}{1.5 \times 10^{-3}}$

$x = \frac{240 \times 1.5}{4}$

$x = 90\ s$

❑ ❑ ❑