Aakash
Medical | IIT-JEE | Foundations

Phase-2
5th April 2026
(Morning Shift)

Answer & Solutions

for

JEE MAIN 2026

TRUST Aakash

Stellar Ranks in JEE (Main) 2025

2 Ranks in Top 10 AIR*

12 Ranks in Top 100 AIR*

39 Ranks in Top 500 AIR*

*Includes students of classroom, digital & distance across all categories.

Harsh Jha AIR 23

Kushagra Baingaha AIR 7

Shreyas Lohiya AIR 6

A Gupta AIR 15

Devya Rustagi AIR 28

Shreyas Lohiya AIR 68

Rujul Garg AIR 41

Advay Mayank AIR 36

Arush Anand AIR 64

Kotha D Reddy AIR 74

Top Ranks in JEE (Advanced) 2025

13 Ranks in Top 100 AIR*

51 Ranks in Top 500 AIR*

103 Ranks in Top 1000 AIR*

*Includes students of classroom, digital & distance across all categories.

Scan the QR Code to access JEE Cut-off 2025

Aakashians Create History in International Olympiads

Our Olympiads Results

378 Classroom Students **Aakashians Qualified in NSEs 2025-26**

777 Classroom Students **Aakashians Qualified in IOQM 2025**

134 Classroom Students **Aakashians Qualified in RMO 2025**

Aarav Gupta
Gold Medalist
66th International Mathematical Olympiad (IMO) 2025

Yug Gandhi
Gold Medalist
Singapore Math Olympiad 2025

Arjun Tyagi
Gold Medalist
International Olympiad in Artificial Intelligence (IOAI) 2025

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $a, b \in \mathbb{C}$. Let α, β be the roots of the equation $x^2 + ax + b = 0$.

If $\beta - \alpha = \sqrt{11}$ and $\beta^2 - \alpha^2 = 3i\sqrt{11}$, then $(\beta^3 - \alpha^3)^2$ is equal to:

(1) 160 (2) 176

(3) 194 (4) 187

Answer (2)

Sol. $\beta - \alpha = \sqrt{11}$

$\beta^2 - \alpha^2 = 3i\sqrt{11}$

$(\beta - \alpha)(\beta + \alpha) = 3i\sqrt{11}$

$\alpha + \beta = \dfrac{3i\sqrt{11}}{\sqrt{11}} = 3i$

Now, $(\beta - \alpha)^2 = (\beta + \alpha)^2 - 4\alpha\beta$

$\Rightarrow 11 = (3i)^2 - 4\alpha\beta$

$\Rightarrow \alpha\beta = -5$

$\beta^3 - \alpha^3 = (\beta - \alpha)\left(\beta^2 + \alpha\beta + \alpha^2\right)$

$= \sqrt{11}\left[(\beta + \alpha)^2 - \alpha\beta\right]$

$= \sqrt{11}[-9 + 5]$

$= -4\sqrt{11}$

$\therefore \left(\beta^3 - \alpha^3\right)^2 = 16 \times 11 = 176$

2. Let the sum of the first n terms of an A.P. be $3n^2 + 5n$. Then the sum of squares of the first 10 terms of the A.P. is:

(1) 10220 (2) 12860

(3) 15220 (4) 19780

Answer (3)

Sol. $S_n = 3n^2 + 5n$

$t_n = S_n - S_{n-1}$

$= 3n^2 + 5n - (3(n-1)^2 + 5(n-1))$

$= 3(2n - 1) + 5 = 6n + 2$

$\sum_{r=1}^{10} t_r^2 = \sum_{r=1}^{10} 4(3r+1)^2$

$= 4\left(\sum_{r=1}^{10} 9r^2 + 6\sum_{r=1}^{10} r + 10\right)$

$= 4\left[9\left(\dfrac{10 \times 11 \times 21}{6}\right) + 6\left(\dfrac{10 \times 11}{2}\right) + 10\right]$

$= 4[3465 + 330 + 10] = 15220$

3. Let A be a 3×3 matrix such that

$A^T \begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}5\\2\\2\end{bmatrix}, A^T \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}3\\1\\1\end{bmatrix}, A\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}3\\4\\4\end{bmatrix}$ and $A\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\3\\1\end{bmatrix}$.

If $\det(A) = 1$, then $\det\left(adj\left(A^2 + A\right)\right)$ is equal to:

(1) 16 (2) 25

(3) 49 (4) 64

Answer (4)

Sol. Let

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & l \end{bmatrix}$

$$A=\begin{bmatrix}2&1&1\\1&e&3\\3&1&1\end{bmatrix}$$

$|A|=1$

$\Rightarrow e=2$

$$\therefore A=\begin{bmatrix}2&1&1\\1&2&3\\3&1&1\end{bmatrix}$$

$A^2+A=A(A+I)$

$$A+I=\begin{bmatrix}3&1&1\\1&3&3\\3&1&2\end{bmatrix}$$

$|A+I|=3(6-3)-1(2-9)+1(1-9)=8$

$\therefore |A^2+A|=1\times 8=8$

$\left|\text{adj}\,(A^2+A)\right|=|A^2+A|^{n-1}=(8)^{3-1}=64$

4. Consider the system of linear equations in x,y,z :

$x+2y+tz=0$,

$6x+y+5tz=0$,

$3x+t^2y+f(t)z=0$,

where $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function. If this system has infinitely many solutions for all $t\in\mathbb{R}$, then f

(1) is a constant function

(2) is strictly increasing on $\mathbb{R}$

(3) is strictly decreasing on $\mathbb{R}$

(4) has two critical points

Answer (2)

Sol. $x+2y+tz=0$

$6x+y+5tz=0$

$3x+t^2y+f(t)z=0$

$\Delta=0$

$$\begin{vmatrix}1&2&t\\6&1&5t\\3&t^2&f(t)\end{vmatrix}=0$$

$1\left(f(t)-5t^3\right)-2\left(6f(t)-15t\right)+t\left(6t^2-3\right)=0$

$\Rightarrow -11f(t)+t^3+27t=0$

$\Rightarrow 11f(t)=t^3+27t$

$\Rightarrow f(t)=\frac{1}{11}\left(t^3+27t\right)$

$f'(t)=\frac{1}{11}\left(3t^2+27\right)$

$=\frac{3}{11}\left(t^2+9\right)>0$

for all $t\in\mathbb{R}$

$\Rightarrow f$ is strictly increasing on $\mathbb{R}$

5. $\sum_{n=1}^{10}\left(\frac{528}{n(n+1)(n+2)}\right)$ is equal to:

(1) 65 (2) 130

(3) 220 (4) 440

Answer (2)

Sol. $S=\sum_{n=1}^{10}\frac{528}{n(n+1)(n+2)}$

$=264\sum_{n=1}^{10}\left[\frac{(n+2)-(n)}{n(n+1)(n+2)}\right]$

$=264\sum_{n=1}^{10}\left(\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right)$

$=264\left[\left(\frac{1}{1\times 2}-\frac{1}{2\times 3}\right)+\left(\frac{1}{2\times 3}-\frac{1}{3\times 4}\right)+\ldots\right.$

$\left.\ldots+\left(\frac{1}{10\times 11}-\frac{1}{11\times 12}\right)\right]$

$=264\left(\frac{1}{2}-\frac{1}{11\times 12}\right)=264\times\frac{(66-1)}{11\times 12}$

= 130

6. Let $\tan A, \tan B$, where $A, B \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, be the roots of the quadratic equation $x^2 - 2x - 5 = 0$. Then $20\sin^2\left(\frac{A+B}{2}\right)$ is equal to:

(1) $10 + \sqrt{10}$ (2) $10 - 2\sqrt{10}$

(3) $10 - 3\sqrt{10}$ (4) $10 - \sqrt{10}$

Answer (3)

Sol. $\tan A + \tan B = 2$

$\tan A \tan B = -5$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{2}{1-(-5)} = \frac{2}{6} = \frac{1}{3}$$

$$\cos(A+B) = \frac{3}{\sqrt{10}}$$

$$2\sin^2\left(\frac{A+B}{2}\right) = 1 - \cos(A+B)$$

$$2\sin^2\left(\frac{A+B}{2}\right) = 1 - \frac{3}{\sqrt{10}}$$

$$20\sin^2\left(\frac{A+B}{2}\right) = 10\left(1 - \frac{3}{\sqrt{10}}\right)$$

$$= 10 - 3\sqrt{10}$$

7. A letter is known to have arrived by post either from KANPUR or from ANANTPUR. On the envelope just two consecutive letters AN are visible. The probability, that the letter came from ANANTPUR, is:

(1) $\frac{7}{10}$ (2) $\frac{10}{17}$

(3) $\frac{12}{19}$ (4) $\frac{7}{19}$

Answer (2)

Sol. E_1 : The letter came from KANPUR

E_2 : The letter came from ANANTPUR

A : Visible letter is 'AN'

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = \frac{2}{7}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{2}\times\frac{2}{7}}{\frac{1}{2}\times\frac{1}{5} + \frac{1}{2}\times\frac{2}{7}} = \frac{10}{17}$$

8. The mean deviation about the mean for the data

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

is equal to:

(1) $\frac{40}{13}$ (2) $\frac{42}{13}$

(3) $\frac{44}{13}$ (4) $\frac{46}{13}$

Answer (3)

Sol. Here $\sum f_i = 26$

and $\sum f_i x_i = 234$

$\therefore$ Mean = $\frac{234}{36} = 9$

$$\text{Mean deviation} = \frac{\sum\left|\left(x_i - \bar{X}\right)f_i\right|}{\sum f_i}$$

$$= \frac{4.8 + 2.6 + 0.2 + 1.2 + 3.2 + 6.6}{26} = \frac{88}{26} = \frac{44}{13}$$

9. Let a focus of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be $S(4, 0)$ and its eccentricity be $\frac{4}{5}$. If the point $P(3, \alpha)$ lies on E and O is the origin, then the area of ΔPOS is equal to:

(1) $\frac{12}{5}$ (2) $\frac{14}{5}$

(3) $\frac{24}{5}$ (4) $\frac{48}{5}$

Answer (3)

Sol. Here $E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore \quad e^2=1-\frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2}=\frac{9}{25}$

$\therefore \quad ae = 4 \Rightarrow a = 5$ and $b = 3$

$\therefore \quad E: \frac{x^2}{25}+\frac{y^2}{9}=1$

$\because$ P lies on E, then $\frac{9}{25}+\frac{\alpha^2}{9}=1$

$\therefore \quad \alpha^2=\pm\frac{12}{5}$

$\because$ Area of $\Delta POS = \frac{1}{2}\times 4\times\frac{12}{5}=\frac{24}{5}$ sq. units

10. Let P be a moving point on the circle $x^2 + y^2 - 6x - 8y + 21 = 0$. Then, the maximum distance of P from the vertex of the parabola $x^2 + 6x + y + 13 = 0$ is equal to:

(1) 8

(2) 10

(3) 12

(4) 9

Answer (3)

Sol. $C: (x - 3)^2 + (y - 4)^2 = 4$

$P: (x + 3)^2 = -y - 13 + 9$

$(x + 3)^2 = -(y + 4)$

Vertex = (–3, –4)

Max distance $= \sqrt{(3+3)^2+(4+4)^2}+2$

$= \sqrt{36+64}+2$

= 10 + 2

= 12

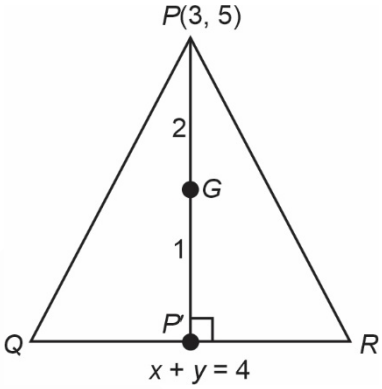
11. In an equilateral triangle PQR, let the vertex P be at (3, 5) and the side QR be along the line $x + y = 4$. If the orthocentre of the triangle PQR is (α, β), then $9(\alpha + \beta)$ is equal to:

(1) 16 (2) 27

(3) 36 (4) 48

Answer (4)

Sol. Foot of perpendicular $P' = \frac{x-3}{1}=\frac{y-5}{1}=-\frac{(8-4)}{2}$



$\Rightarrow x = 1$

$\Rightarrow y = 3$

$\therefore \quad P'(1,3)$

Centroid = orthocenter

2 1

P G P'

(3, 5) (1, 3)

$G: \left(\frac{3+2}{3},\frac{6+5}{3}\right)=\left(\frac{5}{3},\frac{11}{3}\right)$

$9(\alpha+\beta)=\frac{16}{3}\times 9=48$

12. The sum of all the integral values of p such that the equation $3\sin^2x + 12\cos x - 3 = p$, $x \in \mathbb{R}$, has at least one solution, is:

(1) –54 (2) –60

(3) –75 (4) –84

Answer (3)

Sol. $3\sin^2x + 12\cos x - 3 = p$

$3(1 - \cos^2x) + 12\cos x - 3 = p$

$-3\cos^2x + 12\cos x = p$

$-3[\cos^2x - 4\cos x + 4] + 12 = p$

$-3[(\cos x - 2)^2] + 12 = p$

$-15 \le \text{LHS} \le 9 \Rightarrow P \in [-15, 9]$

$\sum p = -15 - 14 ... + 0 + 1 + 2 + ... + 9$

$= \frac{25}{2}[-15 + 9] = -75$

13. The square of the distance of the point P(5, 6, 7) from the line $\frac{x-2}{2} = \frac{y-5}{3} = \frac{z-2}{4}$ is equal to:

(1) 3 (2) 5

(3) 6 (4) 8

Answer (3)

Sol. Any point on Line be $(2\lambda + 2, 3\lambda + 5, 4\lambda + 2)$

Now $< 2\lambda - 3, 3\lambda - 1, 4\lambda - 5 > \cdot < 2, 3, 4 >= 0$

$4\lambda - 6 + 9\lambda - 3 + 16\lambda - 20 = 0$

$\Rightarrow \lambda = 1$

Point on line will be (4, 8, 6)

Distance = $\sqrt{(5-4)^2 + (6-8)^2 + (7-6)^2}$

$= \sqrt{1+4+1} = \sqrt{6}$

Square of distance = 6

14. Let $\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + 2\hat{k}$. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = 0$ and $\vec{r} \cdot \vec{a} = 0$, then $|3\vec{r}|^2$ is equal to:

(1) 44 (2) 54

(3) 86 (4) 132

Answer (1)

Sol. $\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times \vec{a} - \vec{b} \times \vec{a} = 0$

$\Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \vec{r} - \vec{b} = \lambda\vec{a}$

$\Rightarrow \vec{r} - \lambda\vec{a} + \vec{b}$

$\Rightarrow \vec{r} \cdot \vec{a} = 0$

$\Rightarrow (\lambda\vec{a} + \vec{b}) \cdot \vec{a} = 0$

$\Rightarrow \lambda\vec{a} + \vec{b} \cdot \vec{a} = 0$

$\Rightarrow 9\lambda - 1 = 0$

$\Rightarrow \lambda = \frac{1}{9}$

$|\vec{r}| = (\lambda\vec{a} + \vec{b}).\vec{r}$

$= 0 + (\lambda\vec{a} + \vec{b}) \cdot \vec{b}$

$= \frac{1}{9}(-1) + 5 = \frac{44}{9}$

$9|\vec{r}|^2 = 44$

15. The square of the distance of the point of intersection of the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(a\hat{i} - \hat{j}), a \neq 0$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + a\hat{k})$ from the origin is:

(1) 5

(2) 10

(3) 17

(4) 26

Answer (3)

Sol. Let the point of intersection is A

$\therefore A(1 + \lambda a, 1 - \lambda, -1) \equiv (4 + 2\mu, 0, -1 + \mu a)$

$\therefore 1 - \lambda = 0 \Rightarrow \lambda = 1$

$\mu a = 0 \Rightarrow \mu = 0 \quad (\because a \neq 0)$

$1 + \lambda a = 4 + 2\mu \Rightarrow 2\mu - a = -3$

$\Rightarrow a = 3$

$\therefore A(4, 0, -1)$

$\therefore d^2 = \left(\sqrt{16+1}\right)^2 = 17$

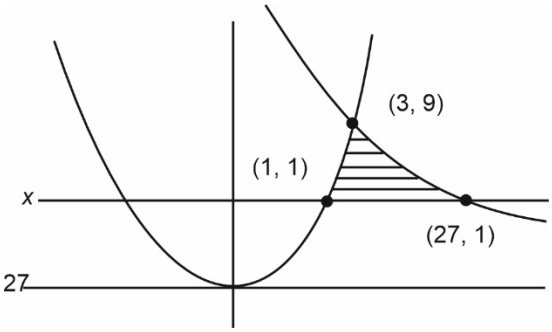
16. The area of the region
$R = \{(x,y) : xy \le 27, 1 \le y \le x^2\}$ is equal to:

(1) $78\log_e 3 - \frac{52}{3}$ (2) $54\log_e 3 - \frac{52}{3}$

(3) $54\log_e 3 - \frac{26}{3}$ (4) $54\log_e 3 + \frac{26}{3}$

Answer (3)

Sol. $A = \int_1^3 (x^2 - 1)dx + \int_3^{27}\left(\frac{27}{x} - 1\right)dx$



$= \left(\frac{x^3}{3} - x\right)\Big|_1^3 + 27\ln x - a\Big|_3^{27}$

$= (a-3) + \left(\frac{1}{3} - 1\right) + (81\ln 3 - 27) - (27\ln 3 - 3)$

$= \frac{20}{3} + 54\ln 3 - 24$

$= 54\ln 3 - \frac{52}{3}$

17. The product of all possible values of α, for which

$\lim_{x\to 0}\left(\frac{1-\cos(\alpha x)\cos((\alpha+1)x)\cos((\alpha+2)x)}{\sin^2((\alpha+1)x)}\right) = 2$,

is

(1) –2 (2) 1

(3) –1 (4) $\frac{5}{4}$

Answer (3)

Sol. $\lim_{x\to 0}\left(\frac{1-\cos(\alpha x)\cdot\cos((\alpha+1)x)\cdot\cos((\alpha+2)x)}{\sin^2((\alpha+1)x)}\right) = 2$

Using $\cos x \approx 1 - \frac{x^2}{2}$

$$\lim_{x\to 0}\frac{1-\left(1-\frac{\alpha^2x^2}{2}\right)\left(1-\frac{((\alpha+1)x)^2}{2}\right)\left(1-\frac{((\alpha+2)x)^2}{2}\right)}{((\alpha+1)x)^2}$$

$$\lim_{x\to 0}\frac{\frac{x^2}{2}(\alpha^2 + (\alpha+1)^2 + (\alpha+2)^2}{(\alpha+1)^2\cdot x^2}$$

$$\frac{1}{2}\frac{(\alpha^2 + (\alpha+1)^2 + (\alpha+2)^2)}{(\alpha+1)^2} = 2$$

$3\alpha^2 + 6\alpha + 5 = 4\alpha^2 + 8\alpha + 4$

$\alpha^2 + 2\alpha - 1 = 0$

$\alpha_1 \cdot \alpha_2 = -1$

18. The value of the integral $\int_0^\infty \frac{\log_e(x)}{x^2+4}dx$ is

(1) $\frac{\pi\log_e(2)}{2}$ (2) $\frac{\pi\log_e(2)}{4}$

(3) $1 + \pi\log_e(2)$ (4) $2 + \pi\log_e(2)$

Answer (2)

Sol. $I = \int_0^\infty \frac{\log_e(x)}{x^2+4}dx$

Put $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$

$$I = \int_0^{\frac{\pi}{2}} \frac{\ln(2\tan\theta)}{4\sec^2\theta}\cdot 2\sec^2\theta\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}}(\ln 2 + \ln(\tan\theta))\, d\theta$$

$$= \frac{\pi}{4}\ln 2 + \frac{1}{2}\int_0^{\frac{\pi}{2}}\ln(\tan\theta)\, d\theta$$

$$\because \quad I_1 = \int_0^{\frac{\pi}{2}}\ln(\tan\theta)\, d\theta = \int_0^{\frac{\pi}{2}}\ln(\cot\theta)\, d\theta$$

$$= -\int_0^{\frac{\pi}{2}} \ln(\tan\theta)\, d\theta$$

$\Rightarrow I_1 = 0$

$\Rightarrow I = \frac{\pi}{4}\ln 2$

19. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$ for all $x, y \in \mathbb{R}$ and $f'(0) = 3$. Then the minimum value of the function $g(x) = 3 + e^x f(x)$, is

(1) $3\left(\frac{e+1}{e}\right)$ (2) $3\left(\frac{e-1}{e}\right)$

(3) $\frac{3-e}{e}$ (4) $3e$

Answer (2)

Sol. $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$ ...(i)

Differentiating (i) w.r.t. to x

$$f'\left(\frac{x+y}{3}\right)\cdot\frac{1}{3} = \frac{1}{3}f'(x)$$

$\Rightarrow f'\left(\frac{x+y}{3}\right) = f'(x)$ ...(ii)

Since (ii) holds for all $y \Rightarrow f'(x)$ must be constant

$\Rightarrow f'(x) = c$, $\because f'(0) = 3 \Rightarrow c = 3 \Rightarrow f'(x) = 3$

$\Rightarrow f(x) = 3x + k$

From (i) $f(0) = \frac{2f(0)}{3} \Rightarrow f(0) = 0$

$\Rightarrow f(x) = 3x$

$g(x) = 3 + 3xe^x$

$g'(x) = 3xe^x + 3e^x = 3e^x(x + 1)$

$g''(x) = 3e^x + 3(x + 1)e^x = 3e^x(x + 2)$

For local maxima/minima $g'(x) = 0 \Rightarrow x = -1$

$\because$ $g''(-1) > 0 \Rightarrow x = -1$ is point of local minima

$$(g(x))_{min} = g(-1) = 3\left(1-\frac{1}{e}\right) = \frac{3(e-1)}{e}$$

20. The value of integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{4-\operatorname{cosec}^2 x}{\cos^4 x}\right) dx$ is

(1) $\frac{11}{\sqrt{3}}$ (2) $\frac{16}{\sqrt{3}}$

(3) $\frac{32}{3\sqrt{3}}$ (4) $\frac{64}{3\sqrt{3}}$

Answer (3)

Sol. Consider, $\frac{4-\operatorname{cosec}^2 x}{\cos^4 x} = \left(4-(1+\cot^2 x)\right)(\sec^2 x)^2$

$$= \left(3-\frac{1}{\tan^2 x}\right)(1+\tan^2 x)(\sec^2 x)$$

Put $u = \tan x$

$du = \sec^2 x\, dx$

$$\Rightarrow I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \left(3-\frac{1}{u^2}\right)(1+u^2)du$$

$$= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} (3u^2+2-u^{-2})du$$

$$= \left(\frac{3u^3}{3}+2u-\frac{u^{-1}}{-1}\right)\Bigg|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$= \frac{16\sqrt{3}}{3} - \frac{16\sqrt{3}}{9} = \frac{32\sqrt{3}}{9} = \frac{32}{3\sqrt{3}}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $A=\{1,2,3,4,5,6\}$. The number of one-one functions $f: A \to A$ such that $f(1) \geq 3, f(3) \leq 4$ and $f(2)+f(3)=5$, is _____ .

Answer (72)

Sol. $f(1) \geq 3$

$f(3) \leq 4$

$f(2) + f(3) = 5$

$f(4)$, $f(5)$, $f(6)$ has no conditions.

$(f(2), f(3))$	$f(1)$
(1, 4)	$\{3, 5, 6\} \to 3$ choices
(4, 1)	$\{3, 5, 6\} \to 3$ choices
(2, 3)	$\{4, 5, 6\} \to 3$ choices
(3, 2)	$\{4, 5, 6\} \to 3$ choices

$\therefore$ Total function $\to 3 \times 4 \times 3!$

$= 72$

22. Two players A and B play a series of games of badminton. The player, who wins 5 games first, wins the series. Assuming that no game ends in a draw, the number of ways, in which player A wins the series is _____ .

Answer (126)

Sol. A has to win 5 games

_ _ _ _ _ _ _ _ _

select any 5 out of these first 9 matches and place win for A will be required number or ways $= {}^9C_5 = 126$

23. If the sum of the coefficients of x^7 and x^{14} in the expansion of $\left(\frac{1}{x^3}-x^4\right)^n$, $x \neq 0$, is zero, then the value of n is _____.

Answer (21)

Sol. General term

$$T_{n+1} = {}^nC_r \cdot (-x^4)^r \cdot \left(\frac{1}{x^3}\right)^{n-r}$$

$$= {}^nC_r (-1)^r x^{4r-3(n-r)}$$

$$= {}^nC_r (-1)^r x^{(-3n+7r)}$$

Coefficient of $x^7 \Rightarrow -3x + 7r_1 = 7$

$$\Rightarrow r_1 = \frac{7+3x}{7}$$

Coefficient of $x^{14} \Rightarrow -3n + 7r_2 = 14$

$$\Rightarrow r_2 = \frac{14+3x}{7}$$

$$\Rightarrow {}^nC_{\frac{7+3n}{7}} (-1)^7 \cdot + {}^nC_{\frac{14+3n}{7}} (1)^{14} = 0$$

$$\Rightarrow {}^nC_{\frac{7+3n}{7}} = {}^nC_{\frac{14+3n}{7}}$$

$$\Rightarrow n = \frac{21+6n}{7}$$

$$\Rightarrow \boxed{n = 21}$$

24. If $\frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1}\left(\frac{2^{p-1}}{1+2^{2p-1}}\right) = \alpha$, then $\tan\alpha$ is equal to _____.

Answer (2048.00)

Sol. $\tan^{-1}\left(\frac{2^{p-1}}{1+2^p\cdot 2^{p-1}}\right)$

$= \tan^{-1}\frac{(2^p-2^{p-1})}{1+(2^p)(2^{p-1})}$

$= \tan^{-1}\left(2^p\right)-\tan^{-1}\left(2^{p-1}\right)$

$\sum_{p=1}^{11} = \tan^{-1}\left(\frac{2^{p-1}}{1+2^p\cdot 2^{p-1}}\right) = \tan^{-1}\left(2^1\right)-\tan^{-1}\left(2^0\right)$

$+\tan^{-1}(2^2)-\tan^{-1}(2^1)+\tan^{-1}\left(2^{11}\right)-\tan^{-1}\left(2^{10}\right)$

$= \tan^{-1}\left(2^{11}\right)-\tan^{-1}(1)$

$\frac{\pi}{4}+\tan^{-1}\left(2^{11}\right)-\tan^{-1}1 = \tan^{-1}\left(2^{11}\right) = \alpha$

$\Rightarrow \tan\alpha = 2^{11} = 2048$

25. Let $y = y(x)$ be the solution of the differential equation $x\sin\left(\frac{y}{x}\right)dy = \left(y\sin\left(\frac{y}{x}\right)-x\right)dx, y(1)=\frac{\pi}{2}$ and let $\alpha = \cos\left(\frac{y(e^{12})}{e^{12}}\right)$. Then the number of integral values of p, for which the equation $x^2 + y^2 - 2px + 2py + \alpha + 2 = 0$ represents a circle of radius $r \le 6$, is _____

Answer (6) Bonus

Sol. Let $\frac{y}{x} = t \Rightarrow y = tx$

$\Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\left(\frac{1}{t}\right)(\sin t)\left(t+x\frac{dt}{dx}\right) = \left(\sin t - \frac{1}{t}\right)$

$\Rightarrow \sin t + \frac{x\sin t}{t}\frac{dt}{dx} = \sin t - \frac{1}{t}$

$\Rightarrow \frac{dt}{dx}\times x\sin t = -1$

$\int \sin t\, dt = \oint \frac{dx}{-x}$

$\Rightarrow -\cos t = -\ln|x| + c$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \ln|x| + c$

$\cos\left(\frac{\frac{\pi}{2}}{(1)}\right) = 0 + c \Rightarrow c = 0$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \ln|x|$

$\cos\left(\frac{y(e^{12})}{e^{12}}\right) = \ln(e^{12}) = 12 = \alpha$

Note: $\cos\left(\frac{y(e^{12})}{e^{12}}\right)$ cannot be greater than 1

(bonus)

The radius of circle

$= \sqrt{p^2+p^2-(\alpha+2)} \le 6$

$\Rightarrow 0 \le 2p^2 - 14 \le 36$

$14 \le 2p^2 \le 50$

$7 \le p^2 \le 25$

p = Integral values, $p^2 = \{9, 16, 25\}$

$\Rightarrow p = \{-3, 3, 4, -4, 5, -5\}$

$\Rightarrow$ 6 such values

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. In a Vernier calipers, when both jaws touch each other, zero of the Vernier scale is shifted to the right of zero of the main scale and 7th Vernier division coincides with a main scale reading. If the value of 1 main scale division is 1 mm and there are 10 Vernier scale divisions, then the Vernier caliper has

(1) 0.07 cm negative zero error

(2) 0.7 cm negative zero error

(3) 0.07 cm positive zero error

(4) 0.7 cm positive zero error

Answer (3)

Sol. Least count of standard vernier = 0.1 mm

reading without object = 7 × LC

= 0.7 mm

Zero error is 0.07 cm with positive sign.

27. L, C and R represents physical quantities inductance, capacitance and resistance respectively. The dimensional formula $ML^2T^{-4}A^{-2}$ corresponds to _____.

(1) $\frac{R}{\sqrt{LC}}$ (2) $\frac{R}{LC}$

(3) $\frac{C}{\sqrt{LR}}$ (4) $\frac{1}{R}\sqrt{\frac{L}{C}}$

Answer (1)

Sol. $R = \frac{V}{I}$

$$[R] = \frac{ML^2T^{-3}A^{-1}}{A} = ML^2T^{-3}A^{-2}$$

$$\left[\sqrt{LC}\right] = T^1$$

$$\left[\frac{R}{\sqrt{LC}}\right] = ML^2T^{-4}A^{-2}$$

28. When one moves from a point 16 km below the earth's surface to a point 16 km above the earth's surface. The change in g is approximately α%. The value of α is _____.

(Take radius of the earth = 6400 km.)

(1) 0.12

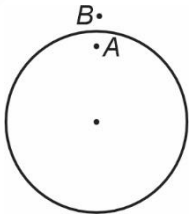
(2) 0.25

(3) 0.50

(4) 0.75

Answer (2)

Sol.



$$g_A = g_0\left(1-\frac{d}{R}\right)$$

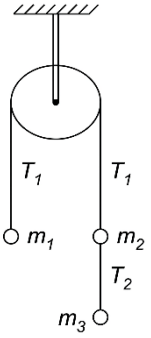
$$g_B = g_0\left(1-\frac{2H}{R}\right)$$

$$\frac{|g_B - g_A|}{g} \times 100$$

$$\frac{g_0 \times \frac{16}{6400}}{g_0} \times 100 = 0.25$$

29. Three masses m_1 = 4 kg, m_2 = 4 kg and m_3 = 6 kg are suspended from a fixed smooth frictionless pully as shown in the figure below. The value of $\frac{T_1}{T_2}$ is _____.

(take g = 10 m/s^2)

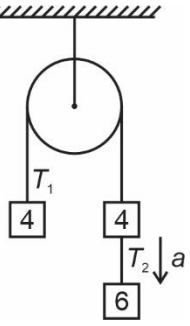


(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{3}{5}$ (4) $\frac{2}{5}$

Answer (1)

Sol.



$$a = \frac{6g}{14} = \frac{3g}{7}$$

$$T_1 - 4g = \frac{4.3g}{7}$$

$$T_1 = \frac{40g}{7}$$

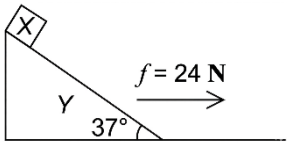
$$6g - T_2 = \frac{6.3g}{7}$$

$$T_2 = \frac{24g}{7}$$

$$\frac{T_1}{T_2} = \frac{5}{3}$$

30. A wedge Y with mass of 10 kg and all frictionless surfaces and the inclined surface making 37° with horizontal. A block X with mass 2 kg is placed at the highest point of the wedge as shown in figure is at rest. At t = 0 wedge (Y) is pulled toward right with constant force (f) of 24 N. Taking the block X at rest at t = 0, the time taken by it to slide down 8.8 m on the slope, while Y is on the move, is _____ s.

(take tan (37°) = 3/4 and g = 10 m/s^2)



(1) 2

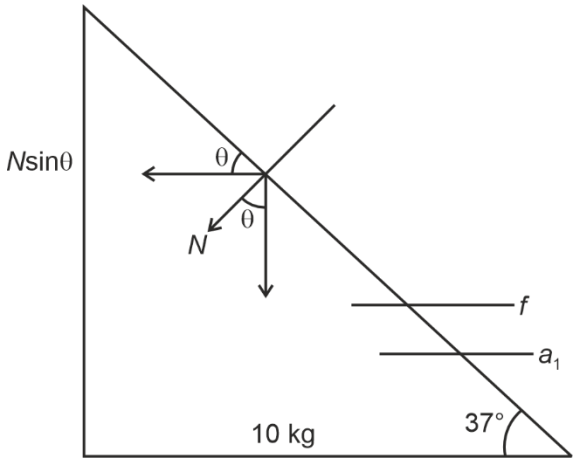
(2) 4

(3) $\sqrt{2}$

(4) $2\sqrt{2}$

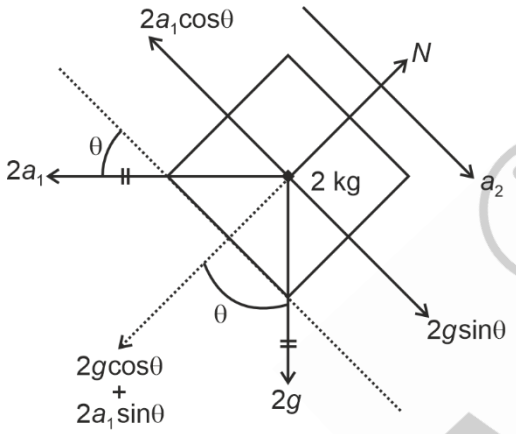
Answer (1)

Sol.



$$f - N\sin\theta = 10a_1 \quad \ldots(i)$$

w.r.t. wedge



$$2g\sin\theta - 2a_1\cos\theta = 2a_2$$

$$a_2 = g\sin\theta - a_1\cos\theta \quad \ldots(ii)$$

$$N = 2g\cos\theta + 2a_1\sin\theta \quad \ldots(iii)$$

From equation (i) and (iii)

$$24 - \left(2\times 10\times\frac{4}{5} + 2\times a_1\times\frac{3}{5}\right)\times\frac{3}{5} = 10a_1$$

$$a_1 = \frac{90}{67}\ \text{m/s}^2$$

From equation (ii)

$$a_2 = \frac{330}{67}\ \text{m/s}^2$$

$$t = \sqrt{\frac{25}{a_2}} = \sqrt{\frac{2\times 8.8\times 67}{330}} \approx 1.89\ \text{sec.}$$

31. The Young's modulus of steel wire of radius r and length L is Y. If the radius r and length L of the wire are doubled then the value of Y

(1) Increases by two times

(2) Reduces by half

(3) Remains unchanged

(4) Becomes one fourth

Answer (3)

Sol. Young's modulus is property of the material.

32. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Statement I: Change in internal energy of a system containing n mole of ideal gas can be written as $\Delta U = nC_v(T_f - T_i) = \frac{nR}{\gamma - 1}(T_f - T_i)$, where $\gamma = \frac{C_p}{C_v}$, T_i = initial temperature, T_f = final temperature.

Statement II: Relation between degree of freedom f and $\gamma\left(= C_p / C_v\right)$ is $\left(\gamma = 1 + \frac{2}{f}\right)$

Choose the ***correct*** answer from the options given below.

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(3) **A** is true but **R** is false

(4) **A** is false but **R** is true

Answer (2)

Sol. $\Delta U = nC_v\Delta T$

$$Y = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$$

$$Y = \frac{\frac{f}{2}R + R}{\frac{f}{2}R}$$

$$Y = 1 + \frac{2}{f}$$

33. Consider the following statements:

A. Zeroth law of thermodynamics gives concept of temperature.

B. First law of thermodynamics gives concept of internal energy.

C. In isothermal expansion of ideal gas, $\Delta Q \neq \Delta W$.

D. Product of intensive and extensive variables is extensive.

E. The ratio of any extensive variable to mass will be an extensive variable.

Choose the correct combination of statements from the options given below:

(1) C, D and E Only

(2) A, B and C Only

(3) A, B and D Only

(4) B, C and D Only

Answer (3)

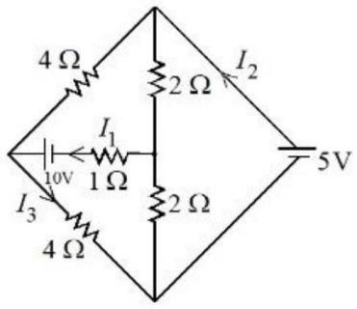
Sol. A. Zeroth law defines temperature.

B. First law introduces internal energy via energy conservation.

C. In isothermal $\Delta Q = \Delta W$

D. Product of intensive and extensive is extensive.

34. Refer to the figure given below. The values of I_1, I_2 and I_3 are ____.



(1) $I_1 = 2.5$ A, $I_2 = 1.875$ A, $I_3 = 1.875$ A

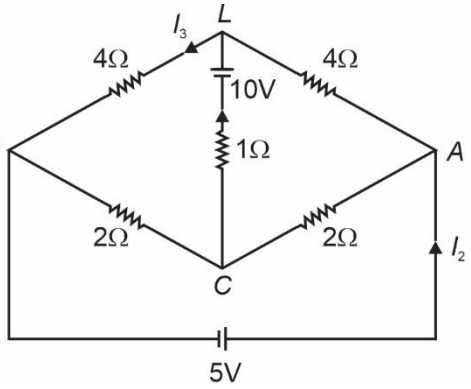
(2) $I_1 = 1.875$ A, $I_2 = 2.5$ A, $I_3 = 1.875$ A

(3) $I_1 = 1.875$ A, $I_2 = 1.875$ A, $I_3 = 2.5$ A

(4) $I_1 = 2.5$ A, $I_2 = 2.5$ A, $I_3 = 1.875$ A

Answer (1)

Sol.



$V_A - V_B = 5$

Take $V_B = 0 \Rightarrow V_A = 5$ V

At L: $\frac{V_L - 5}{4} + \frac{V_L - 0}{4} + I_{LC} = 0$ ...(1)

Also between L and C

$I_{LC} = \frac{V_L - V_C - 10}{1}$ ...(2)

At node C

$\frac{V_c - 5}{2} + \frac{V_c - 0}{2} - I_{LC} = 0$ ...(3)

From (1), (2) and (3)

$V_L = 7.5$, $V_C = 0$

$\therefore \quad I_1 = 2.5$A

$I_3 = \frac{V_L}{4} = \frac{7.5}{4} = 1.875$ A

$I_2 = 1.875$ A (using KCL)

35. An electron of mass m is moving in an electric field $\vec{E} = -2E_o\hat{i}$ (E_o = constant > 0), with an initial velocity $\vec{V} = v_o\hat{i}$ (v_o = constant > 0). If $\lambda_o = \frac{h}{4mv_o}$, its de Broglie wavelength at time t is ____.

(e = charge of electron)

(1) $\frac{4\lambda_o}{\left[1 - \frac{E_o e}{2m}\frac{t}{v_o}\right]}$

(2) $\frac{4\lambda_o}{\left[1 + \frac{E_o e}{2m}\frac{t}{v_o}\right]}$

(3) $\frac{4\lambda_o}{\left[1 + \frac{2E_o e}{m}\frac{t}{v_o}\right]}$

(4) $\frac{4\lambda_o}{\left[1 - \frac{2E_o e}{m}\frac{t}{v_o}\right]}$

Answer (4)

Sol. $2E_o \longleftarrow$ $\qquad \lambda_o = \frac{h}{4mV_o}$

$\longrightarrow V_o$

$v = v_o - \frac{e \cdot 2E_o}{m}t$

$mv = mv_o - 2eE_o t$

$\frac{h}{\lambda} = -2eE_o t$

$\frac{h}{\lambda} = \frac{h}{4\lambda_0} - 2eE_o t$

$\lambda = \frac{4\lambda_o}{1 - \frac{2eE_o t}{h}}$

$\lambda = \frac{4\lambda_o}{1 - \frac{2eE_o t}{mv_o}}$

36. In the hydrogen atom, the electron makes a transition from the higher orbit (i) to a lower orbit (*f*). The ratio of the radius of the orbits in given by $r_i : r_f$ = 16 : 4. The wavelength of photon emitted due to this transition is _____ nm. (Given Rydberg constant = 1.0973×10^7/m)

(1) 121 (2) 242

(3) 486 (4) 974

Answer (3)

Sol. $\gamma \frac{\alpha n^2}{z}$

$n_1 = 4, n_2 = 2$

$$\frac{1}{\lambda} = 1.0973 \times 10^7 \left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\frac{1}{\lambda} = 1.0973 \times 10^7 \frac{12}{4 \cdot 16}$$

$\lambda = 486$

37. A displacement current of 4.0 A can be set up in the space between two parallel plates of 6μ F capacitor. The rate of change of potential difference across the plates of the capacitor is nearly $\alpha \times 10^6$ V/s. The value of α is _____.

(1) 0.58 (2) 0.67

(3) 0.82 (4) 0.75

Answer (2)

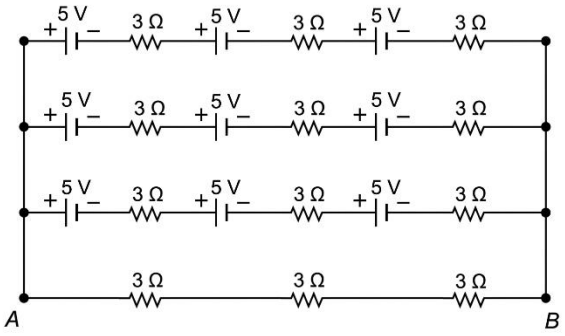
Sol. $Q = cv$

$$i = \frac{de}{d\tau} = c\frac{dv}{dt}$$

$$u = 6 \times 10^{-6} \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{2}{3} \times 10^6 \text{ v / s}$$

38. Refer to the figure given below, current between terminals *A* and *B* is _____ *A*.

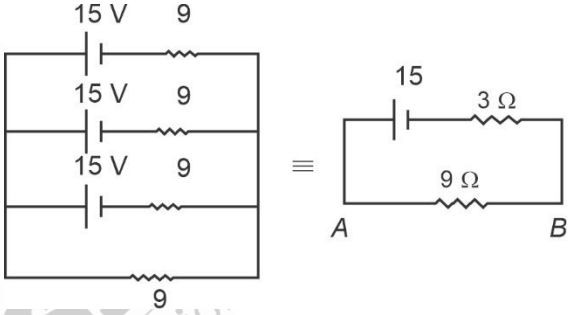


(1) 12.5 (2) 1.25

(3) 7.5 (4) 5

Answer (2)

Sol.



$$I = \frac{15}{12} = \frac{5}{4} = 1.25$$

39. In Young's double slit experiment, the fringe width of the interference pattern produced on the screen is 2.4μ m. If the experiment is carried out in another medium having refractive index 1.2, the fringe width will be _____ μm.

(1) 1.2 (2) 2

(3) 2.4 (4) 2.88

Answer (2)

Sol. $W = \frac{\lambda D}{d}$

Wavelength (λ') in medium = $\frac{\lambda}{\mu}$

$$W' = \frac{\lambda D}{\mu d} = \frac{W}{\mu} = \frac{2.4}{1.2} = 2 \ \mu m$$

40. A ray of light passing through an equilateral prism is having velocity 2.12×10^8 m/s in the prism material, then the minimum angle of deviation is _____ degrees.

(1) 45 (2) 30

(3) 28 (4) 58

Answer (2)

Sol. $\mu = \frac{c}{v} = \frac{3\times10^8}{2.12\times10^8} = 1.415 \simeq \sqrt{2}$

$$\mu = \frac{\sin\left(\frac{\delta_m + 60}{2}\right)}{\sin(30)}$$

$$\frac{\delta_m + 60}{2} = 45$$

$$\rho_m = 30$$

41. Light source having wavelength 331 nm is used to generate photo-electrons whose stopping potential is 0.2 V . The work function of the used metal in the experiment is $\alpha \times 10^{-19}$J. The value of α is _____ . ($h = 6.62 \times 10^{-34}$ J s, $e = 1.6 \times 10^{-19}$ C and $c = 3 \times 10^8$ m/s)

(1) 3.68 (2) 4.68

(3) 5.68 (4) 2.68

Answer (3)

Sol. $\phi = \frac{hc}{\lambda} - {}^eV_s$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{331\times10^{-9}} - 0.2$$

$$\phi = 6\times10^{-19}\,\text{J} - (0.2)\,\text{eV}$$

$= 6 \times 10^{-19}$ J $- 0.2 \times 1.6 \times 10^{-19}$ J

$= (6 - 0.32) \times 10^{-19}$

$$\phi = 5.68\times10^{-19}\,\text{J}$$

42. A compound microscope is designed with two symmetric biconvex lenses. The objective lens is cut vertically, creating two identical plano-convex lenses. One of them is used in place of original objective lens. To retain same magnification keeping the object distance unchanged, the tube length has to be

(1) increased two times

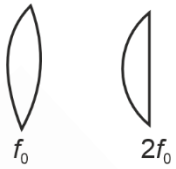
(2) increased $\frac{3}{2}$ times

(3) decreased two times

(4) decreased $\frac{3}{2}$ times

Answer (1)

Sol.

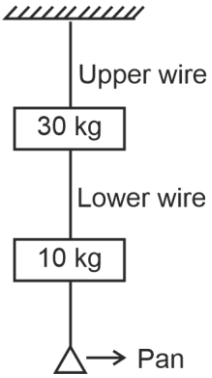


$$M = \frac{L}{f_0} \times \frac{D}{f_e}$$

So L should be double.

43. Two wires as shown in the figure below, made of steel and have breaking stress of 12×10^8 N/m^2. Area of cross-section of upper wire is 0.008 cm^2 and of lower wire is 0.004 cm^2. The maximum mass that can be added to pan without breaking any wire is ___ kg.

(take g = 10 m/s^2)



(1) 56 (2) 38

(3) 96 (4) 5.6

Answer (2)

Sol. Maximum tension (T_1) in upper wire

$= 12 \times 10^8 \times 8 \times 10^{-7} = 960$

Maximum tension (T_2) in lower wire

$= 12 \times 10^8 \times 4 \times 10^{-7} = 480$

$T_2 = 100 + mg$

$m = 38$

$T_1 = 40 + mg$

$m = 56$

So maximum value of m can be 38 kg.

44. An a.c. source of angular frequency ω is connected across a resistor R and a capacitor C in series. The current is observed as I. Now the frequency of the source is changed to $\omega/4$, (keeping the voltage unchanged) the current is found to be $I/3$. The ratio of resistance to reactance at frequency ω is

(1) $\sqrt{\frac{6}{7}}$ (2) $\sqrt{\frac{3}{5}}$

(3) $\sqrt{\frac{7}{8}}$ (4) $\sqrt{\frac{3}{4}}$

Answer (3)

Sol. $I_2 = \frac{I_1}{3}$

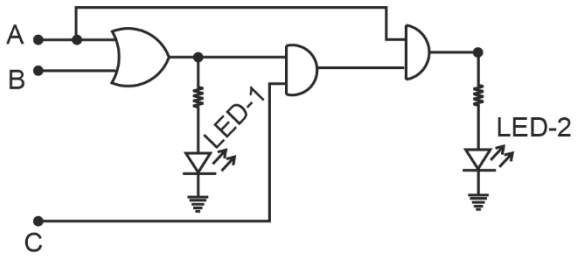
$z_2 = 3z_1$

$$R^2 + \frac{16}{\omega^2 C^2} = 9\left[R^2 + \frac{1}{\omega^2 c^2}\right]$$

$$\frac{7}{\omega^2 C^2} = 8R^2$$

$$R\omega C = \sqrt{\frac{7}{8}}$$

45. For the given logic circuit, which of the following inputs combination will make both LED-1 and LED-2 to glow?



(1) A = 0, B = 1, C = 1 (2) A = 1, B = 0, C = 0

(3) A = 1, B = 0, C = 1 (4) A = 1, B = 1, C = 0

Answer (3)

Sol. To glow LED-1 possible values of A, B ae (0, 1), (1, 0), (1, 1)

To glow LED-2

A must be 1

C must be 1

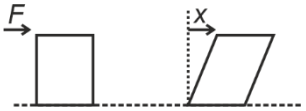
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A cube has side length 5 cm and modulus of rigidity 10^5 N/m^2. The displacement produced by a force of 10 N in the upper face of cube is _____ mm.

Answer (2)

Sol.



$$\frac{F}{l^2} = 10^5 \cdot \frac{x}{l}$$

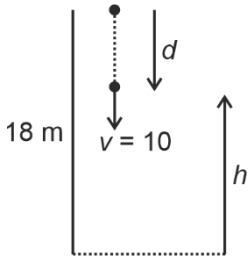
$$x = \frac{F}{10^5 \cdot l} = \frac{10}{10^5 \times 5 \times 10^{-2}} = 2 \times 10^{-3}$$

47. From 18 m height above the ground a ball is dropped from rest. The height above the ground at which the magnitude of velocity equal to the magnitude of acceleration (in the same set of units) due to gravity is _____ m.

(Take g = 10 m/s^2 and neglect the air resistance)

Answer (13)

Sol.



$v^2 = 0 + 2 \times 10 \times d$

d = 5 m

$h = 18 - d$

= 13 m

48. A transverse wave on a string is described by $y = 3\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$. Where x, y are in cm and t in seconds. The least distance between the two successive crests in the wave is _____ cm. (Nearest integer)

(π = 3.14)

Answer (349)

Sol. $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018} = 348.8$

49. The charged particle moving in a uniform magnetic field of $\left(3\hat{i} + 2\hat{j}\right)$ T has an acceleration $\left(4\hat{i} - \frac{x}{2}\hat{j}\right)$ m/s^2. The value of x is

Answer (12)

Sol. $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$

$\vec{a} = \frac{q}{m}\left(\vec{v} \times \vec{B}\right)$

$\vec{a} \perp \vec{B}$

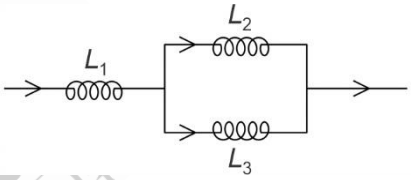
$\vec{a} \cdot \vec{B} = 0$

$12 - x = 0$

$x = 12$

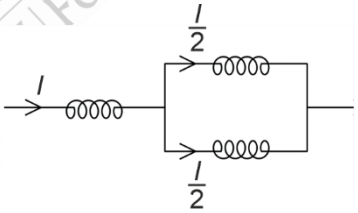
50. In the given circuit below inductance values of L_1, L_2 and L_3 are same. The magnetic energy stored in the entire circuit is (U_t) and that stored in the L_2 inductor is (U_l). $\frac{U_t}{U_l}$ is _____.

(Ignore the mutual inductance if any)



Answer (6)

Sol.



$U_t = \frac{1}{2}LI^2 + \frac{1}{2}L \cdot \left(\frac{I}{2}\right)^2 + \frac{1}{2}L\left(\frac{I}{2}\right)^2$

$U_l = \frac{1}{2}L\left(\frac{I}{2}\right)^2$

$\frac{U_t}{U_l} = 6$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. How many grams of residue is obtained by heating 2.76 g of silver carbonate?

(Given: Molar mass of C,O and Ag are 12, 16 and 108 g mol^{-1} respectively)

(1) 1.08 g (2) 2.16 g
(3) 3.24 g (4) 4.32 g

Answer (2)

Sol. $2Ag_2CO_3(s) \xrightarrow{\Delta} 4Ag(s) + 2CO_2(g) + O_2(g)$

$\therefore n_{Ag_2CO_3} = \frac{2.76}{276} = 0.01$

$\therefore n_{Ag}$ produced = 0.02

$\therefore$ mass of Ag(residue) = 0.02 ×108 = 2.16 g

52. Arrange the following atomic orbitals of multi electron atoms in order of increasing energy.

A. $n = 3, l = 2, m = +1$
B. $n = 4, l = 0, m = 0$
C. $n = 6, l = 1, m = 0$
D. $n = 5, l = 1, m = +1$
E. $n = 2, l = 1, m = +1$

Choose the correct answer from the options given below:

(1) C < D < B < A < E (2) B < A < E < C < D
(3) E < C < D < B < A (4) E < B < A < D < C

Answer (4)

Sol. Energy is decided by (n + l) rule.

$\Rightarrow$ if (n + l) value same $\Rightarrow n\uparrow \Rightarrow$ energy $\uparrow$

A n = 3, l = 2 $\Rightarrow$ 5
B n = 4, l = 0 $\Rightarrow$ 4
C n = 6, l = 1 $\Rightarrow$ 7
D n = 5, l = 1 $\Rightarrow$ 6
E n = 2, l = 1 $\Rightarrow$ 3

$\therefore$ order : C > D > A > B > E

53. Identify the correct statements from the following:

A. Heisenberg uncertainty principle is applicable to electrons.
B. The size of $2p_x$ orbital is less than the size of $3p_x$ orbital.
C. The energy of 2s orbital of H atom is equal to the energy of 2s orbital of Li.
D. The electronic configuration of Cr is $[Ar]3^5 4s^1$

Choose the correct answer from the options given below:

(1) A, B and C Only
(2) A, B and D Only
(3) B, C and D Only
(4) A, C and D Only

Answer (2)

Sol. Size of $2p_x$ orbital is less than $3p_x$

2s of Li has less energy compare to 2s of H ($Z\uparrow \Rightarrow F_{attraction} \uparrow$)

[Cr] = [Ar] $3d^5 4s^1$

54. What is the mole fraction of water in 10% by weight (w/w) of aqueous urea solution?

[Given: Molar mass of H, O, C and N are 1, 16, 12 and 14 g mol^{-1} respectively.]

(1) 0.825 (2) 0.032

(3) 0.867 (4) 0.967

Answer (4)

Sol. 10% by weight of aq. Urea

$\Rightarrow$ 100 g solution $\Rightarrow$ 10g urea

$\Rightarrow$ 80 g H_2O $\Rightarrow$ 10g urea

$$\therefore x_{urea} = \frac{\frac{10}{60}}{\frac{10}{60}+\frac{80}{18}} = \frac{0.17}{0.17+4.44}$$

$$= \frac{0.17}{4.6} = 0.036$$

$$\therefore x_{H_2O} = 1 - 0.036$$

= 0.964

55. M_3A_2 is a sparingly soluble salt of molar mass y g mol^{-1} and solubility x g L^{-1}.

The ratio of the molar concentration of the anion $\left(A^{3-}\right)$ to the solubility product of the salt is

(1) $\frac{1}{54}\cdot\frac{y^4}{x^4}$ (2) $\frac{y^5}{108x^4}$

(3) $108\cdot\frac{x^5}{y^5}$ (4) $\frac{1}{108}\frac{y^4}{x^4}$

Answer (1)

Sol. $M_3A_2(s) \rightleftharpoons \underset{3s}{3M^{2+}}(aq) + \underset{2s}{2A^{3-}}(aq)$

$$s = \frac{x}{y} mol L^{-1}$$

$$k_{sp} = [M^{2+}]^3[A^{3-}]^2$$

$= (3s)^3 \cdot (2s)^2 = 108\ s^5$

$$k_{sp} = 108\left(\frac{x}{y}\right)^5$$

$$\therefore \left[A^{3-}\right] = \frac{2x}{y}$$

$$\therefore \frac{K_{sp}}{[A^{3-}]} = \frac{108\times x^5\times y}{y^5\times 2x} = \frac{54x^4}{y^4}$$

$$\therefore \frac{\left[A^{3-}\right]}{K_{sp}} = \frac{y^4}{54x^4}$$

56. Arrange the following resultant mixtures in increasing order of their pH values.

A. 10 mL 0.2 M $Ca(OH)_2$ + 25 mL 0.1 M HCl

B. 10 mL 0.01 M H_2SO_4 + 10 mL 0.01 M $Ca(OH)_2$

C. 10 mL 0.1 M H_2SO_4 + 10 mL 0.1 M KOH

Choose the **correct** answer from the options given below.

(1) B < C < A (2) C < A < B

(3) C < B < A (4) A < C < B

Answer (3)

Sol. (A) $\underset{2\,mmol}{Ca(OH)_2} + \underset{2.5\,mmol}{HCl} \Rightarrow$ Basic solution

(B) $\underset{0.1\,mmol}{H_2SO_4} + \underset{0.1\,mmol}{Ca(OH)_2} \Rightarrow$ Complete neutralization

(C) $\underset{1\,mmol}{H_2SO_4} + \underset{1\,mmol}{KOH} \Rightarrow$ Acidic solution

pH : C < B < A

57. First order gas phase reaction

$A \rightarrow B + C$

p_i = initial pressure of gas A, p_t = total pressure of the reaction mixture at time t

Expression of rate constant (k) is

(1) $\frac{1}{t}\ln\frac{p_i}{2p_i - p_t}$ (2) $\frac{1}{t}\ln\frac{2p_i}{p_i - p_t}$

(3) $\frac{1}{t}\ln\frac{p_i}{3p_i - 2p_t}$ (4) $\frac{1}{t}\ln\frac{3p_i}{4p_i - p_t}$

Answer (1)

Sol.

	A(g)	→	B(g)	+	C(g)
$t = 0$	p_0		–		–
$t = t$	$p_0 - x$		x		x
$t = \infty$	–		p_0		p_0

$\therefore$ $p_t = p_0 + x = p_i + x \Rightarrow x = p_t - p_i$

$p_i = p_0$

$\therefore$ $p_0 - x = p_i - p_t + p_i = 2p_i - p_t$

$\therefore$ $k = \frac{1}{t}\ln\frac{p_0}{p_0 - x} = \frac{1}{t}\ln\frac{p_i}{2p_i - p_t}$

58. Given below are two statements :

Statement I : The correct order of electronegativity of fluorine, oxygen and nitrogen is F > O > N.

Statement II : The oxidation state of oxygen in OF_2 is +2 and in Na_2O is –2.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (1)

Sol. EN : F > O > N $\Rightarrow$ S-I is correct.

$OF_2 \Rightarrow$ Oxygen in +2 state.

$Na_2O \Rightarrow$ Oxygen in –2 state.

S-II is correct.

59. Correct statements from the following are :

A. Nitrogen in oxidation states from +1 to +4 disproportionates in acid medium.

B. Nitrogen has the ability to form $d\pi$-$p\pi$ multiple bonds with itself and other elements with small size and high electronegativity.

C. N-N single bond is stronger than P-P single bond.

D. Nitrogen has highest density in its group due to small size.

E. The maximum covalency of nitrogen is four since it has only four valence orbitals for bonding.

Choose the **correct** answer from the options given below.

(1) B, C and D only (2) C, D and E only

(3) A, C and E only (4) A and E only

Answer (4)

Sol.

- A is correct.
- B is incorrect.

  N does not form $p\pi$-$d\pi$ bonds with itself.
- P–P > N–N (B.E) $\Rightarrow$ C is incorrect.
- N has lowest density in its group.

  A, E $\Rightarrow$ correct.

60. Which of the following is NOT a physical or chemical characteristics of interstitial compounds?

(1) They have high melting points, higher than those of pure metals

(2) They are very soft and ionic in nature

(3) They retain metallic conductivity

(4) They are chemically inert and usually non-stoichiometric

Answer (2)

Sol.

- Interstitial hydrides are not ionic in nature.
- They are very hard.

61. The **correct** statements about metal carbonyls are

A. The metal-carbon bonds in metal carbonyls possess both σ and π-character.

B. Due to synergic bonding interactions between metal and CO ligand, the metal-carbon bond becomes weak.

C. The metal-carbon σ bond is formed by the donation of lone pair of electrons on the carbonyl carbon into a vacant orbital of metal.

D. The metal-carbon π bond is formed by the donation of electrons from filled d-orbital of metal into vacant π^* orbital of CO.

Choose the correct answer from the options given below:

(1) A and B Only (2) A, C and D Only

(3) B and C Only (4) A and D Only

Answer (2)

Sol. In metal carbonyl, metal-carbon bonds become strong due to synergic bonding.

62. Given below are two statements:

Statement I: Each electron in e_g orbitals destabilizes the orbitals by $+0.6\Delta_o$ and each electron in the t_{2g} orbitals stabilizes the orbitals by $-0.4\Delta_0$ in an octahedral field on the basis of crystal field theory.

Statement II: All the d - orbitals of the transition metals have the same energy in their free atomic state but when a complex is formed the ligands destroy the degeneracy of these orbitals on the basis of crystal field theory.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are correct

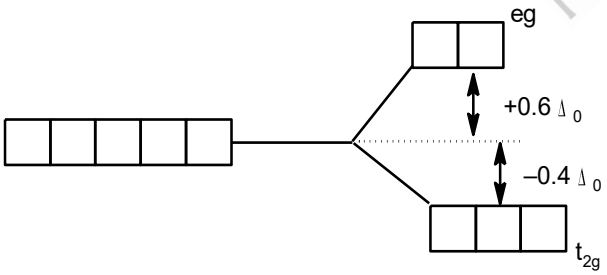
(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Answer (1)

Sol. In octahedral complex



63. Given below are two statements:

Statement I: On the basis of inductive effect, the order of stability of alkyl carbanions is $CH_3^- > CH_3 - CH_2^- > (CH_3)_2 CH^- > (CH_3)_3 C^-$.

Statement II: Allyl and benzyl carbanions are more stabilised by inductive effect and not by resonance effect.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Answer (3)

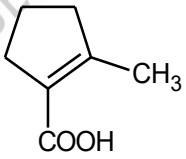
Sol. Stability of carbanion

$$CH_3^- > \underset{(1^\circ)}{CH_3 - CH_2^-} > \underset{(2^\circ)}{(CH_3)_2 CH^-} > \underset{(3^\circ)}{(CH_3)_3 C^-}$$

+I effect destabilises carbanion.

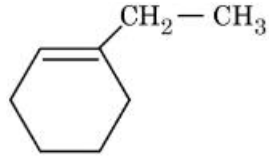
Allyl and benzyl carbanions are stabilised by resonance effect also.

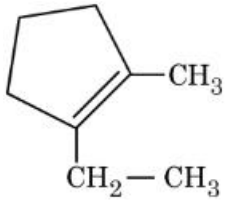
64. "P" is a hydrocarbon of molecular formula:- C_8H_{14}. On ozonolysis, "P" forms "Q". "Q" on treatment with alkali under reflux condition produces "R", which on treatment with I_2 / NaOH gives a yellow precipitate. Acidification of the solution gives "S". The structure of "S" is given below:-

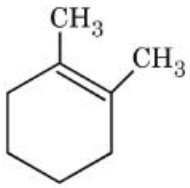


The correct structure of " P " is

(1) 

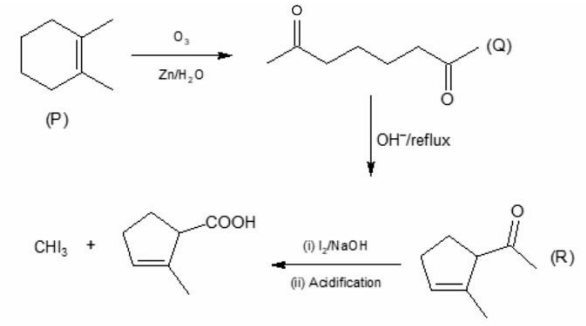
(2) 

(3) 

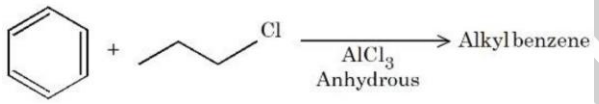
(4) 

Answer (4)

Sol. $P \xrightarrow[Zn/H_2O]{O_3} Q \xrightarrow[reflux]{OH^-} R \xrightarrow[NaOH]{I_2}$ yellow ppt + S



65. For the following Friedel Craft's alkylation reaction, which of the statements are correct?

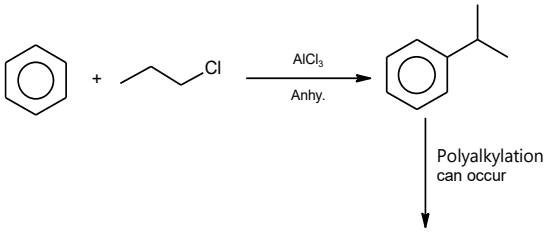


A. Major product is n-propyl benzene.
B. iso-propyl carbocation intermediate is also generated.
C. Multiple substitution is inevitable.
D. Introducing electron-donating substituent on benzene will not produce any alkyl benzene.

Choose the correct answer from the options given below:

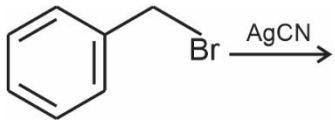
(1) A and D only (2) B and C only
(3) A and C only (4) B and D only

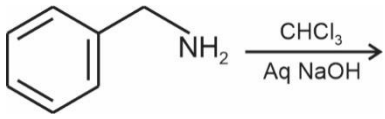
Answer (2)

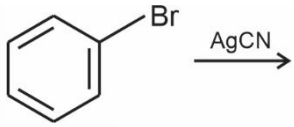
Sol. 

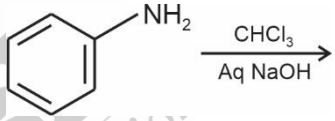
B, C are correct.

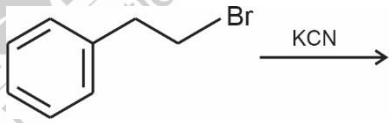
66. Benzyl isocyanide can be obtained from

A. 

B. 

C. 

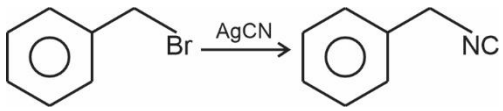
D. 

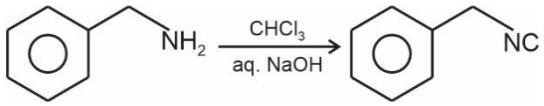
E. 

Choose the correct answer from the options given below:

(1) A and B Only
(2) A and C Only
(3) B and D Only
(4) D and E Only

Answer (1)

Sol. (A) 

(B) 

67. Consider compounds A, B and C with following structural formulae

$A = CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - OH$

$B = CH_2 = CH - CH_2 - CH_2 - CH_3$

$C = HO - CH_2 - CH_2 - CH(OH) - CH_3$

For the conversion of B from A, reagent (D) required is _____ and structural formula of product (E) obtained when C undergoes same reaction using excess reagent (D) is ____ .

(1)

D	E
Conc. H_2SO_4	$CH_2 = CH - CH(OH)CH_3$

(2)

D	E
PCC	$HO - CH_2 - CH_2 - CH = CH_2$

(3)

D	E
PCC	$CH_2 = CH - CH = CH_2$

(4)

D	E
Conc. H_2SO_4 or H_3PO_4	$CH_2 = CH - CH = CH_2$

Answer (4)

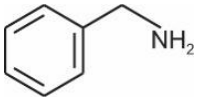
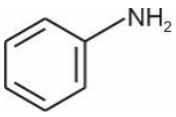
Sol. $CH_3CH_2CH_2CH_2CH_2OH \xrightarrow[\text{or } H_3PO_4]{\text{Conc. } H_2SO_4} CH_2 = CHCH_2CH_2CH_3$

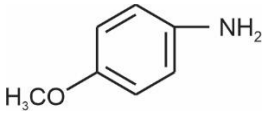
(A) (B)

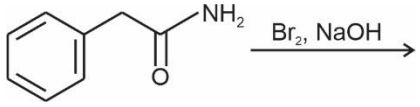
$CH_2(OH) - CH_2 - CH(OH) - CH_3 \xrightarrow[\text{or } H_3PO_4 \text{ (excess)}]{\text{Conc. } H_2SO_4} CH_2 = CH - CH = CH_2$

(C) (E)

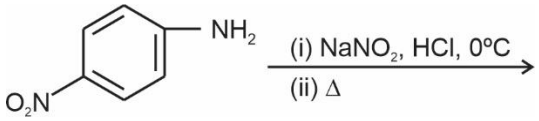
68. Identify the **incorrect** statements.

A.  is a stronger base than 

B.  can be synthesized by Gabriel phthalimide synthesis.

C. 

Primary aromatic amine

D. 

product will dissolve in NaOH

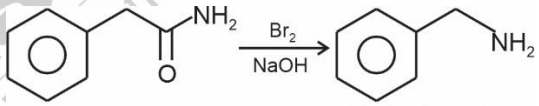
Choose the correct answer from the options given below:

(1) A and D Only (2) A and C Only

(3) B and C Only (4) A and B Only

Answer (1)

Sol.

- $PhCH_2NH_2 > PhNH_2$ (Base)
- Arylamine can't be synthesized by Gabriel phthalimide synthesis.
- 

(Aliphatic amine)

69. Identify the **correct** statements.

A. Glucose exists in two anomeric forms.

B. Anomers of glucose differ in configuration at C–1 in cyclic hemiacetal structure.

C. Melting point of α-anomer of glucose is greater than β-anomer.

D. Specific rotation of α - anomer is +19° while for β - anomer is +112°

E. α and β - anomers of glucose are prepared by crystallization of saturated glucose solution at 303 K and 371 K respectively.

Choose the correct answer from the options given below:

(1) A and B Only (2) B and C Only

(3) A, B and D Only (4) A, B and E Only

Answer (4)

Sol. Anomers of glucose differ in configuration at C-1 in cyclic hemiacetal structure.

$\therefore$ α-anomer $\Rightarrow$ m.p $\longrightarrow$ 419 K

β-anomer $\Rightarrow$ m.p $\longrightarrow$ 423 K

$\Rightarrow$ α and β-anomers of glucose are prepared by crystallization of saturated glucose solution at 303 K and 371 K respectively.

A, B, E $\Rightarrow$ correct

70. Given below are two statements:

Statement I: Sodium dichromate and potassium dichromate are classified as primary standards in titrimetric analysis.

Statement II: Phenolphthalein is a weak base, therefore it dissociates in acidic medium.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Both Statement I and Statement II are ture

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (2)

Sol.
- $K_2Cr_2O_7$ is classified as primary standard.
- Phenolphthalein is a weak acid.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Consider the following species :

BrF_5, XeF_5^-, BF_4^-, ICl_4^-, XeF_4, SF_4, NH_4^+, ClF_3, XeF_2, ICl_2^-

Number of species having sp^3d hybridized central atom is _____.

Answer (4)

Sol. $BrF_5 \Rightarrow 5\sigma + 1\ lp = 6 \Rightarrow sp^3d^2$

$XeF_5^- = 5\sigma + 2\ lp = 7 \Rightarrow sp^3d^3$

$BF_4^- = 4\sigma \Rightarrow sp^3$

$ICl_4^- = 4\sigma + 2\ lp \Rightarrow sp^3d^2$

$XeF_4 = 4\sigma + 2\ lp \Rightarrow sp^3d^2$

$SF_4 = 4\sigma + 1\ lp \Rightarrow sp^3d$

$NH_4^+ = 4\sigma \Rightarrow sp^3$

$ClF_3 = 3\sigma + 2\ lp \Rightarrow sp^3d$

$XeF_2 = 2\sigma + 3\ lp \Rightarrow sp^3d$

$ICl_2^- = 2\sigma + 3\ lp \Rightarrow sp^3d$

72. In an estimation of sulphur by Carius method 0.2 g of the substance gave 0.6 g of $BaSO_4$. The percentage of sulphur in the substance is ____ %.

(Given molar mass in g mol^{-1} S : 32, $BaSO_4$: 231)

Answer (42)

Sol. Mass of sulphur = $\frac{0.6}{231} \times 32$

$= 0.08311$

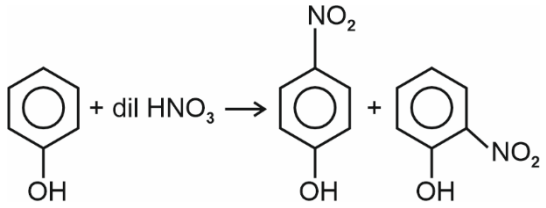
$\%\ S = \frac{0.08311}{0.2} \times 100$

$= 41.55$

73. One mole of phenol is treated with dilute HNO_3 at 298 K to give a mixture of products. The mixture is separated by steam distillation. The steam volatile compound (X) is separated. The increase in percentage of oxygen in (X) with respect to phenol is ____ $\times 10^{-1}\%$

(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16)

Answer (175)

Sol. 

O–nitro phenol is steam volatile

$$\% \text{ of oxygen in phenol} = \frac{16}{94}\times 100 = 17\%$$

$$\% \text{ of oxygen in o-nitrophenol} = \frac{48}{139}\times 100 = 34.53$$

$$\% \text{ of increase} = 34.53 - 17$$
$$= 17.53$$
$$= 175.3 \times 10^{-1}$$

74. The values of pressure equilibrium constant recorded at different temperatures for the following equilibrium reaction have been given below

$A(g) \rightleftharpoons B(g) + C(g)$

$\frac{1}{T}(K^{-1})$	$Log_{10}K_p$
0.05	3.5
0.06	2.5
0.07	1.5

The magnitude of $\frac{\Delta H^\circ}{R}$ calculated from the above data is _____. (Nearest integer)

Answer (230)

Sol. $$\log\frac{Kp_2}{Kp_1} = \frac{\Delta H^\circ}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$2.303\times(3.5-2.5) = \frac{\Delta H^\circ}{R}[0.06 - 0.05]$$

$$230.3 = \frac{\Delta H^\circ}{R}$$

75. If the half life of a first order reaction is 6.93 minutes then the time required for completion of 99% of the reaction will be ____ minutes.

(Given : log 2 = 0.3010)

Answer (46)

Sol. $t_{1/2} = 6.93$

$$k = \frac{0.693}{6.93} = 0.1 \text{ min}^{-1}$$

$$t_{99\%} = \frac{2.303}{0.1}\log\frac{100}{1}$$

$$= 2.303 \times 2 \times 10$$

$$= 46.06$$

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