# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. The maximum value of $n$ for which $40^n$ divides 60! is equal to

(1) 11 (2) 12

(3) 13 (4) 14

**Answer (4)**

**Sol.** $40 = 2^3 \times 5$

Let $60! = 2^a \cdot 3^b \cdot 5^c \ldots$

$$a = \left[\frac{60}{2}\right]+\left[\frac{60}{4}\right]+\left[\frac{60}{8}\right]+\left[\frac{60}{16}\right]+\left[\frac{60}{32}\right]+\left[\frac{60}{64}\right]$$

$= 30 + 15 + 7 + 3 + 1$

$= 56$

$\therefore\ 2^a = (8)^{a/3}$

$\Rightarrow$ max power of 8 $= \left[\dfrac{56}{3}\right] = 18$

$$c = \left[\frac{60}{5}\right]+\left[\frac{60}{25}\right]+\left[\frac{60}{125}\right]$$

$= 12 + 2 = 14$

$$\min\left(c, \frac{a}{3}\right) = c = 14$$

$\therefore$ 14

2. If $z = (1 + i)(1 + 2i)(1 + 3i).....(1 + ni)$, $n \in N$ and $|z|^2 = 44200$, then $n$ is equal to

(1) 6 (2) 5

(3) 8 (4) 4

**Answer (2)**

**Sol.** $z = (1 + i)(1 + 2i)(1 + 3i)......(1 + ni)$

$z\bar{z} = (1 + i)(1 - i)(1 + 2i)(1 - 2i)........(1 + ni)(1 - ni)$

$|z|^2 = (2)(5)(1 + 9)......(1 + n^2)$

$\Rightarrow 2 \times 5 \times 10 \times........(1 + n^2) = 44200$

$$\prod_{r=1}^{n}\left(1+r^2\right) = 44200$$

Since $210 < \sqrt{44200} < 211$

$\Rightarrow$ $n$ will be small

$\Rightarrow$ Since $13|44200 \Rightarrow (5^2 + 1)|44200$

$\Rightarrow$ checking small values of n, $\displaystyle\prod_{r=1}^{5}\left(1+r^2\right) = 44200$

$\Rightarrow n = 5$

3. The image of the parabola $x^2 = 4y$ in the line $x - y = 1$ is

(1) $(y - 1)^2 = 4(x + 1)$

(2) $(y + 1)^2 = 4(x - 1)$

(3) $(y + 1)^2 = 4(x + 1)$

(4) $(y - 1)^2 = 4(x - 1)$

**Answer (2)**

**Sol.** Any point $P : (2t, t^2)$

Image of point $P$ in the line $x - y = 1$ is

$$\frac{x-2t}{1} = \frac{y-t^2}{-1} = \frac{-2(2t-t^2-1)}{2}$$

$$\Rightarrow\ x - 2t = \frac{y-t^2}{-1} = t^2 - 2t + 1$$

$\Rightarrow\ x = t^2 + 1$

$y = 2t - 1$

$$x = \left(\frac{y+1}{2}\right)^2 + 1 \ \Rightarrow\ 4(x-1) = (y+1)^2$$

4. The value of sum

$$s=\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\left(\frac{1}{3}\right)^2+\left(\frac{4}{7}\right)^2+\left(\frac{1}{3}\right)\left(\frac{4}{7}\right)\right)+$$
$$\left(\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^2\left(\frac{4}{7}\right)+\left(\frac{1}{3}\right)\left(\frac{4}{7}\right)^2+\left(\frac{4}{7}\right)^3\right)+\ldots,$$

then $s$ is equal to

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{1}{2}$ (4) 2

**Answer (2)**

**Sol.** Let $\frac{1}{3}=a, \frac{4}{7}=b$

then

$s=(a+b)+(a^2+b^2+ab)+(a^3+a^2b+ab^2+b^3)+\ldots$

multiplying by $(a-b)$

$s(a+b)=(a^2-b^2)+(a^3-b^3)+(a^4-b^4)+\ldots$

$$\Rightarrow s=\frac{1}{(a-b)}\left((a^2+a^3+a^4+\ldots-(b^2+b^3+b^4+\ldots)\right)$$

$$\Rightarrow s=\frac{1}{(a-b)}\left[\frac{a^2}{1-a}-\frac{b^2}{1-b}\right]$$

$$=\frac{1}{(a-b)}\left[\frac{a^2-a^2b-b^2+ab^2}{(1-a)(1-b)}\right]$$

$$=\frac{1}{(a-b)}\left[\frac{a^2-b^2}{(1-a)(1-b)}+\frac{ab(-a+b)}{(1-a)(1-b)}\right]$$

$$=\frac{a+b}{(1-a)(1-b)}-\frac{ab}{(1-a)(1-b)}$$

$$=\frac{\frac{19}{21}}{\frac{2}{3}\times\frac{3}{7}}-\frac{\frac{4}{21}}{\frac{6}{21}}=\frac{19}{6}-\frac{4}{6}=\frac{15}{6}=\frac{5}{2}$$

5. Let the equation $x^4 - ax^2 + 9 = 0$ have four real and distinct roots. Then the least integral value of $a$ is

(1) 5 (2) 6

(3) 7 (4) 8

**Answer (3)**

**Sol.** $x^4 - ax^2 + 9 = 0$

$a=x^2+\frac{9}{x^2}$

$\because$ A.M $\geq$ G.M

$$\Rightarrow \frac{x^2+\frac{9}{x^2}}{2}\geq\sqrt{x^2\cdot\frac{9}{x^2}}$$

$\Rightarrow a \geq 6$

If $a = 6$, then $x^2+\frac{9}{x^2} \Rightarrow x^4 = 9 \Rightarrow x^2 = 3$

$\Rightarrow x=\pm\sqrt{3}$ (two distinct roots)

$\because$ all roots are distinct and real

$\Rightarrow a > 6$

$\Rightarrow$ minimum integral value of $a$ is 7

6. The domain of $\sin^{-1}\left(\frac{1}{x^2-2x-1}\right)$ is $(-\infty,\alpha]\cup[\beta,\delta]\cup[\lambda,\infty)$. The value of $\alpha+\beta+\delta+\lambda$ is equal to

(1) 17 (2) 4

(3) 3 (4) 6

**Answer (2)**

**Sol.** $-1\leq\frac{1}{x^2-2x-1}\leq 1 \Rightarrow \frac{1}{x^2-2x-1}\in[-1,1]$

$\Rightarrow x^2-2x-1\in(-\infty,-1]\cup[1,\infty)$

$x^2-2x-1\leq -1$ $\qquad$ $x^2-2x-1\geq -1$

$\Rightarrow x^2-2x\leq 0$ $\qquad$ $\Rightarrow x^2-2x-2\geq 0$

$x\in[0,2]$ $\qquad$ $x\in(-\infty,1-\sqrt{3}]\cup[1+\sqrt{3},\infty)$

Domain : $(-\infty,1-\sqrt{3}]\cup[0,2]\cup[1+\sqrt{3},\infty)$

$\alpha = 1-\sqrt{3}$

$\beta = 0$

$\delta = 2$

$\lambda = 1+\sqrt{3}$

$\therefore \alpha+\beta+\delta+\lambda = 4$

7. Consider these statements regarding the function $f(x) = |\ln x| - |x-1|$

Statement 1 : $f(x)$ is differentiable for all $x > 0$

Statement 2 : $f(x)$ is increasing in $(1, \infty)$

Statement 3 : $f(x)$ is decreasing in $(0, 1)$

(1) Statement 1 and statement 3 is true

(2) All Statement are correct

(3) Statement 2 and statement 3 are correct

(4) Statement 1 and statement 2 are correct

**Answer (1)**

**Sol.** $f(x) = |\ln x| - |x-1|$

$$= \begin{cases} \ln x - (x-1), & x \ge 1 \\ -\ln x + (x-1), & 0 < x < 1 \end{cases}$$

$$= \begin{cases} \ln x - x + 1, & x \ge 1 \\ -\ln x + x - 1, & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x} - 1, & x \ge 1 \\ -\frac{1}{x} + 1, & 0 < x < 1 \end{cases}$$

$0 = f'(1^+) = f'(1^-)$

$\Rightarrow f(x)$ is differentiable $\forall\, x > 0$

$f'(x) < 0 \;\; \forall\, x > 1$

$f'(x) < 0 \;\; \forall\, 0 < x < 1$

$\Rightarrow f(x)$ is decreasing $\forall\, x \in (0, 0)$

8. $\lim\limits_{x \to 0} \dfrac{\tan(\tan x) - \tan(\sin x)}{\tan x - \sin x}$ is equal to

(1) 1 (2) 2

(3) –1 (4) $\frac{1}{2}$

**Answer (1)**

**Sol.** $\lim\limits_{x \to 0} \dfrac{[\tan(\tan x) - \tan(\sin x)][1 + \tan(\tan x)\tan(\sin x)]}{(\tan x - \sin x)(1 + \tan(\tan x)\tan(\sin x))}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\tan(\tan x - \sin x)}{(\tan x - \sin x)} \times (1 + \tan(\tan x)\tan(\sin x))$

$\Rightarrow (1) \times (1) = 1$

9. If $4x^2 + y^2 < 52$, $x, y, \in Z$, then the number of ordered pairs $(x, y)$ is

(1) 67 (2) 87

(3) 77 (4) 38

**Answer (3)**

**Sol.** $4x^2 + y^2 < 52$

$y^2 < 52 - 4x^2$

$y^2 < 4(13 - x^2)$

| $x$ | $y^2 <$ | $y$ | Number of possible values of y |
|---|---|---|---|
| 0 | 52 | –7 to 7 | 15 |
| ±1 | 48 | –6 to 6 | 13 each |
| ±2 | 36 | –5 to 5 | 11 each |
| ±3 | 16 | –3 to 3 | 7 each |

$\Rightarrow$ number of possible order pairs $(x, y)$

$= 15 + 2 \times 13 + 2 \times 11 + 2 \times 7 = 77$

10. If $\displaystyle\int \frac{7x^{10} + 9x^8}{\left(1 + x^2 + 2x^9\right)^2} dx = f(x) + c$ and $f(1) = \frac{1}{4}$. Then $f(x)$ is

(1) $\dfrac{x^9}{2x^2 + 9 + x^9}$ (2) $\dfrac{x^9}{2 + x^2 + x^9}$

(3) $\dfrac{x^9}{1 + x^2 + 2x^9}$ (4) $\dfrac{x^9}{1 + x^9 + 2x^2}$

**Answer (3)**

**Sol.** $\because I = \displaystyle\int \frac{7x^{10} + 9x^8}{(1 + x^2 + 2x^9)^2} dx = \int \frac{\frac{7}{x^8} + \frac{9}{x^{10}}}{\left(\frac{1}{x^9} + \frac{1}{x^7} + 2\right)^2} dx$

Let $\dfrac{1}{x^9} + \dfrac{1}{x^7} + 2 = t$

$$-9x^{-10} - 7x^{-8} = \frac{dt}{dx}$$

$$\left(\frac{7}{x^8} + \frac{9}{x^{10}}\right)dx = -dt$$

$$\therefore \ I = \int \frac{-dt}{t^2} = \frac{1}{t} + C = \frac{1}{\frac{1}{x^9} + \frac{1}{x^7} + 2} + C$$

Here $f(1) = \frac{1}{4}$ hence C=0.

and $f(x) = \frac{1}{\frac{1}{x^9} + \frac{1}{x^7} + 2} = \frac{x^9}{1 + x^2 + 2x^9}$

11. Let $X = \{1, 2, 3, .... 19\}$. If new data $Y = \{y_i : y_i = x_i + b, x_i \in X\}$. Such that mean and variance of $y$ is 30 and 120 respectively, then the sum of value(s) of $b$ is

(1) 50

(2) 60

(3) 40

(4) 30

**Answer (2)**

**Sol.** $y_i = ax_i + b$

$$\sum y_i = a\sum x_i + 19b$$

$$\Rightarrow \sum y_i = \frac{a \times 19 \times 20}{2} + 19b$$

$$\Rightarrow \frac{\sum y_i}{19} = 10a + b = 30$$

original variance

$$\Rightarrow \frac{\sum x_i^2}{19} - \left(\frac{\sum x_i}{19}\right)^2$$

$$= \frac{19 \times 20 \times 39}{19 \times 6} - (10)^2 = \frac{780}{6} - 100 = 30$$

Variance of $y_i$ = $30(a^2)$ = 120

$\Rightarrow a = \pm 2$

$\Rightarrow$ If $a = 2$, $b = 10$

If $a = -2$, $b = 50$

12. Let $A = \{a, b, c, d, e\}$ and $P(A) = S$, where $P(A)$ is the power set of $A$. Number of elements $(P, Q)$ in $S \times S$ such that $(P \cap Q) = \phi$ is equal to

(1) 245 (2) 343

(3) 233 (4) 243

**Answer (4)**

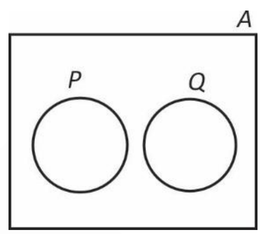
**Sol.** $S = P(A)$

$= \{\phi, \{a\}, \{b\}........\{a, b, c, d, e\}\}$

$n(S) = 32$

$n(S \times S) = 32 \times 32$

we want $P$ and $Q$ from $S \times S$ such $P \cap Q = \phi$




**Method-1 :**

Each element has three ways to chose from.

Therefore total number of element is equal to $(3)^5 = 243$

**Method-2 :**

Let $n(P) = a$, $n(Q) = 6$

and $n((P \cup Q)^c) = c$

$\Rightarrow a + b + c = 5$, since $P \cap Q = \phi$

$\sum_{a=0}^{5} {}^5C_a \, {}^{5-a}C_b$, where $a + b + c = 5$

For $c = 0$, $a + b = 5$

$$\sum_{a=0}^{5} {}^5c_a \, {}^{5-a}c_{5-a} = 2^5 = 32$$

For $c = 1$, $\sum_{a=0}^{4} {}^5c_a \, {}^{5-a}c_{4-a} = 80$

For $c = 2$, $\sum_{a=0}^{3} {}^5c_a \, {}^{5-a}c_{3-a} = 80$

For $c = 3$, $\sum_{a=0}^{2} {}^5c_a \, {}^{5-a}c_{2-a} = 40$

For $c = 4$, $\sum_{a=0}^{1} {}^{5}C_{a}\ {}^{5-a}C_{1-a} = 10$

for $c = 5$, only 1 case

$\Rightarrow 243$

13. Let $\langle a_i \rangle$ be an A.P of natural numbers with common difference $l$ such that $a_1 + a_2 + a_3 + a_4 = 18$ and $a_1a_2a_3a_4 + l^4 = 361$. Then max $\{a_1, a_2, a_3, a_4\}$ is equal to

(1) 6

(2) 18

(3) 14

(4) 12

**Answer (6)**

**Sol.** Notic that $361 = 1 + 3 \times 4 \times 5 \times 6$

and if $l = 1$, then

terms will be 3, 4, 5, 6

$\Rightarrow \max\{a_1, a_2, a_3, a_4\} = 6$

14.

15.

16.

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If all the words with or without meaning made using all the letters of the word "UDAYPUR" are arranged as in a dictionary, then the rank of the word "UDAYPUR" is ______

**Answer (1922)**

**Sol.** UDAYPUR

A, D, P, R, U, U, Y

words starting with A = $\frac{6!}{2!}$ = 360

words starting with D = $\frac{6!}{2!}$ = 360

words starting with P = $\frac{6!}{2!}$ = 360

words starting with R = $\frac{6!}{2!}$ = 360

words starting with UA = 5! = 120

words starting with UDAP = 3! = 6

words starting with UDAR = 3! = 6

words starting with UDAU = 3! = 6

Then next word is UDAYPRU

and next word is UDAYPUR

$\Rightarrow$ Rank of UDAYPUR is $(360) \times 4 + 120 + (6) \times 3 + 1 + 1$

$= 1580$

22. Let point $(h, k)$ lies on $x^2 + y^2 = 4$, and $(2h + 1, 3k + 2)$ lies on ellipse having eccentricity $e$. Then the value of $\frac{5}{e^2}$ is

**Answer (9)**

**Sol.** $\because$ $(h, k)$ lies on $x^2 + y^2 = 4$

$\therefore$ $h = 2\cos\theta$, $k = 2\sin\theta$

then $(2h + 1, 3k + 2) \equiv (4\cos\theta + 1$ and $6\sin\theta + 2)$

Here let $4\cos\theta + 1 = x$ and $6\sin\theta + 2 = y$

$\therefore$ Required ellipse is:

$$\frac{(x-1)^2}{4^2} + \frac{(y-2)^2}{6^2} = 1$$

$\therefore$ eccentricity $= e^2 = 1 - \frac{4^2}{6^2} = \frac{20}{36} = \frac{5}{9}$

$\therefore \frac{5}{e^2} = 9$

23. Given $P = [p_{ij}]_{3\times 3}$ and $Q = [q_{ij}]_{3\times 3}$ are 3 × 3 matrices, where $q_{ij} = 2^{i+j-1}\, p_{ij}$. If $|Q| = 2^{10}$, then the value of $|\text{adj}(\text{adj}(P))|$ is

**Answer (16)**

**Sol.** $q_{ij} = 2^{i+j-1}p^{ij}$, $|Q| = 2^{10}$

$$\Rightarrow Q = \begin{bmatrix} 2p_{11} & 2^2 p_{12} & 2^3 p_{13} \\ 2^2 p_{21} & 2^3 p_{22} & 2^4 p_{24} \\ 2^3 p_{31} & 2^4 p_{32} & 2^5 p_{33} \end{bmatrix}$$

$$\Rightarrow Q = 2\cdot 2^2\cdot 2^3 \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{22} & 2p_{22} & 2p_{24} \\ 2^2 p_{31} & 2^2 p_{32} & 2^2 p_{33} \end{bmatrix}$$

$= 2^{1+2+3+1+2}|P|$

$|Q| = 2^9|P|$

$\Rightarrow |P| = 2$

$\Rightarrow |\text{adj}(\text{adj}(P))| = |\text{adj}(P)|^2 = |P|^4 = 2^4 = 16$

24.

25.

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