MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Consider the 10 observations 2, 3, 5, 10, 11, 13, 15, 21, a and b such that mean of observations in 9 and variance is 34.2 . Then the mean deviation about median is

(1) 3 (2) 5

(3) 6 (4) 7

Answer (2)

Sol. Mean = $\frac{80+9+6}{10} = 9 \Rightarrow a+b = 100$

Also,

Variance = $[(2-9)^2 + (3-9)^2 + (5-9)^2 + (10-9)^2 + (11-9)^2 + (13-9)^2 + (15-9)^2 + (21-9)^2 + (a-9)^2 + (b-a)^2]/10 = 34.2$

$\Rightarrow$ Solving for a and b,

we get $a = 3$ or 7

$b = 7$ or 3

Deviation about median $= \frac{10+7}{2} = 8.5$

$= \frac{\Sigma|x_i - 8.5|}{10} = 5$

2. If α, β are roots of quadratic equation $\lambda x^2 - (\lambda + 3)x + 3 = 0$ and $\alpha < \beta$ such that $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$, then find sum of all possible values of λ.

(1) 3 (2) 2

(3) 4 (4) 6

Answer (4)

Sol. $\because \frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$

$\therefore 9(\alpha - \beta)^2 = \alpha^2\beta^2$ ...(1)

$\because \alpha + \beta = \frac{\lambda+3}{\lambda}$ and $\alpha \cdot \beta = \frac{3}{5}$

From eq. (1)

$$9\left\{\left(\frac{\lambda+3}{\lambda}\right)^2 - \frac{12}{\lambda}\right\} = \frac{9}{\lambda^2}$$

$\therefore (\lambda + 3)^2 - 12\lambda = 1$

or $\lambda^2 - 6\lambda + 8 = 0$

Sum of all values of $\lambda = 6$

3. If 3 balls are taken from the box without replacement and found to be all black. If all configuration of red balls and black balls are equally likely then the probability that box contained 1 red and 9 black ball is $\frac{p}{q}$ for some coprime natural number p and q then, $p + q$ is

(1) 59 (2) 69

(3) 57 (4) 79

Answer (2)

Sol. $$P\left(\frac{1R+9B}{BBB}\right) = \frac{P(1R+9B)P\left(\frac{BBB}{1R+9B}\right)}{P(BBB)}$$

$$= \frac{\left(\frac{1}{10}\right)\cdot\frac{^9C_3}{^{10}C_3}}{\sum_{k=0}^{10} P\left(\frac{BBB}{kR+(10-k)B}\right)\cdot P(kR+(10-k)B)}$$

$$= \frac{\frac{1}{10}\cdot\frac{^9C_3}{^{10}C_3}}{\frac{1}{10}\sum_{k=0}^{10}\frac{^{10-k}C_3}{^{10}C_3}} = \frac{^9C_3}{\sum_{k=0}^{10} {}^{10-k}C_3}$$

$$= \frac{{}^{9}C_3}{{}^{10}C_3 + {}^{9}C_3 + \ldots\ldots + {}^{3}C_3} = \frac{{}^{9}C_3}{{}^{11}C_4} = \frac{14}{55}$$

$\Rightarrow \boxed{69}$

4. Find the value of $\sum_{K=1}^{\infty} \frac{(-1)^{K+1} K(K+1)}{K!}$

(1) $\frac{2}{e}$ (2) $\frac{3}{e}$

(3) $\frac{1}{e}$ (4) e

Answer (3)

Sol. $\because \sum_{K=1}^{\infty} \frac{(-1)^{K+1} K(K+1)}{K!}$

$$= \sum_{K=1}^{\infty} \frac{(-1)^{K+1}\left((K+1)+2\right)}{(K!)!}$$

$$= \sum_{K=1}^{\infty} \frac{(-1)^{K+1}}{(K-2)!} + \frac{2.(-1)^{K+1}}{(K-1)!}$$

$$= \left(-1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots\right) + 2\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\infty\right)$$

$$= -e^{-1} + 2e^{-1} = \frac{1}{e}$$

5. If f be a real valued function such that $f(x^2 + 1) = x^4 + 5x^2 + 2$, then $\int_0^3 f(x)dx$ is equal to

(1) 16 (2) $\frac{31}{2}$

(3) $\frac{33}{2}$ (4) 14

Answer (3)

Sol. If $f(x^2 + 1) = x^4 + 5x^2 + 2$

Let $x^2 + 1 = t$

$\Rightarrow f(t) = (t-1)^2 + 5(t-1) + 2$

$= t^2 - 2t + 1 + 5t - 3$

$= t^2 + 3t - 2$

$$\Rightarrow \int_0^3 f(x)dx = \left.\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right|_0^3$$

$$= \frac{27}{3} + \frac{27}{2} - 6 = 3 + \frac{27}{2} = \frac{33}{2}$$

6. $\lim_{x \to 0} \frac{\ln\left(\sec(ex)\sec(e^2x)\sec(e^3x)\ldots\sec(e^{10}x)\right)}{e^2 - e^2\cos x}$

(1) $\frac{e^{18}-1}{e^2-1}$ (2) $\frac{e^{20}-1}{e^2-1}$

(3) $\frac{e^{16}-1}{e^2-1}$ (4) $\frac{e^{22}-1}{e^2-1}$

Answer (2)

Sol. $$\lim_{x\to 0} \frac{\ln(\sec(ex)) + \ln(\sec(e^2x)) + \ln(\sec(e^3x)) + \ldots + \ln(\sec(e^{10}x))}{e^2(1-\cos x)}$$

$$\lim_{x\to 0} \frac{e\tan(ex) + e^2\tan(e^2x) + e^3\tan(e^3x) + \ldots + e\tan(e^{10}x)}{\left(\frac{e^2\sin x}{x}\right)x}$$

$$\Rightarrow \frac{1}{e^2}[e^2 + e^4 + e^6 + e^8 + \ldots + e^{20}]$$

$1 + e^2 + e^4 + \ldots + e^{18}$

$$\frac{e^{2\times 10}-1}{e^2-1} = \frac{e^{20}-1}{e^2-1}$$

7. Consider a circle C_1 passing through origin and lying in region $x \geq 0$ only, with diameter 10. Consider a chord PQ of C_1 with equation $x = y$ and another circle C_2 which has PQ as diameter. A chord is drawn to C_2 passing through (2, 3) such that distance of chord from centre of C_2 is maximum has equation $x + ay + b = 0$ then $(b - a)$ is equal to

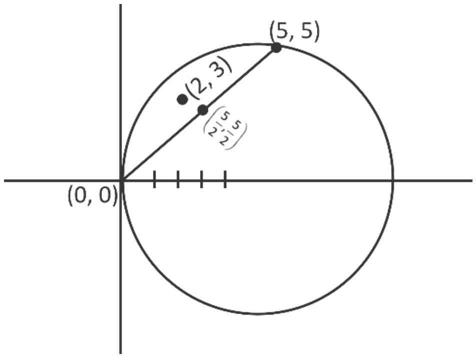
(1) 4 (2) 2

(3) 3 (4) 5

Answer (2)

Sol. $X \geq 0$ and touching at x (0, 0)

$C_1 : (x - 5)^2 + y^2 = 25$



$C_2 = (x - 5)(x) + y(y - 5) = 0$

Since the distance of chord is maximum from centre $\left(\frac{5}{2}, \frac{5}{2}\right)$

$\Rightarrow (y - 3) = (1)(x - 2)$

$\Rightarrow (y - 3) = (1)(x - 2) = x - 2$

$\Rightarrow$ x – y + 1 = 0

a = – 1

b = 1

b – a = 2

8. If $y = f(x)$ satisfies the differential equation $x\frac{dy}{dx} - \sin^2 y = x^3\cos^2 y$ and $y(1) = \frac{\pi}{4}$, then $y\left(\frac{\pi}{3}\right)$ is

(1) 1

(2) $\tan^{-1}\left(\frac{\pi}{4}\right)$

(3) $\tan^{-1}\left(\frac{\pi^3}{27}\right)$

(4) Zero

Answer (3)

Sol. $x\frac{dy}{dx} - \sin 2y = x^3\cos^2 y$

$\frac{dy}{dx} - \frac{\sin 2y}{x} = x^2\cos^2 y$

$\sec^2 y\frac{dy}{dx} - \frac{2\sin y\cos y}{x\cos^2 y} = x^2$

$\sec^2 y\frac{dy}{dx} - \frac{2}{x}\tan xy = x^2$

Let tan $y = t$

$\sec^2 y\frac{dy}{dx} - \frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = x^2$

If = $e^{-\int\frac{2}{x}dx}$ = $e^{-2\ln x}$ = $\frac{1}{x^2}$

$\frac{t}{y^2} = \int 1dx$

$\frac{\tan y}{x^2} = x + c$

$\tan y = x^3 + x^2c$

$\because y(1) = \frac{\pi}{4}$

$\Rightarrow c = 0$

$\Rightarrow \tan y = x^3$

Now $y\left(\frac{\pi}{3}\right) = \tan^{-1}\left(\frac{\pi^3}{27}\right)$

9. Let z be a complex number lying in the first quadrant such that $|z - 6| = 5$ and $|z - 3i + 5| = 7$, then $z^3 - 7z^2 + 25z + 16$ is equal to

(1) 45

(2) 55

(3) 35

(4) 25

Answer (2)

Sol. $|z - 6| = 5$ and $|z + 5 - 3i| = 7$

$\Rightarrow (x - 6)^2 + y^2 = 25$

$(x + 5)^2 + (y - 3)^2 = 49$

$\Rightarrow x^2 + y^2 - 12x + 11 = 0$

$x^2 + y^2 + 10x - 6y - 15 = 0$

$\Rightarrow -22x + 6y + 26 = 0$

Solving we get $x = 2$, $y = 3$ (in Ist quadrant)

$z = 2 + 3i$

$$(z-2)=3i \Rightarrow z^2-4z+13=0$$

$$z^3-7z^2+25z-39=0$$

10. Consider polynomial functions $f(x)$ and $g(x)$ such that $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$ such that $4g(f(x)) = 3x^2 - 32x + 72$ then $f(g(2))$ is equal to

(1) $-\frac{25}{6}$ (2) $\frac{25}{6}$

(3) $-\frac{7}{2}$ (4) $\frac{7}{2}$

Answer (4)

Sol. $g(f(x))=3\left(\frac{x}{2}\right)^2-8x+18$

Notice that $g(x)$ is Quadratic, hence $f(x)$ has to be linear with leading coefficient $\frac{1}{2}$

$$\Rightarrow f(x)=\frac{x}{2}+c$$

$$\because f(0)=-3 \Rightarrow f(x)=\frac{x-6}{2}$$

Verify :

$$g(f(x))=3\left(\frac{x-6}{2}\right)^2+2\left(\frac{x-6}{2}\right)-3$$

$$=\frac{1}{4}\left[3x^2-36x+108+4x-24-12\right]$$

$$=\frac{1}{4}(3x^2-32x+72)$$

Now, $f(g(2))=f(13)=\frac{7}{2}$

11. Let three unit vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$ then the value of natural number k such that $|2\vec{a}+k\vec{b}+k\vec{c}|^2 = 9$ is

(1) 9 (2) 6

(3) 3 (4) 5

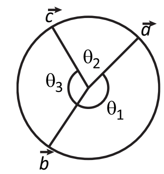
Answer (4)

Sol. $|\vec{a}|=1=|\vec{b}|=|\vec{c}|$

$$|\vec{a}-\vec{b}|^2=(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$=2-2\vec{a}\cdot\vec{b}$$

$$\Rightarrow \sum 2-2\vec{a}\cdot\vec{b}=9$$



$$\Rightarrow \cos\theta_1+\cos\theta_2+\cos\theta_3=-\frac{3}{2}$$

$$\Rightarrow \theta_1=\theta_2=\theta_3=\frac{2\pi}{3}$$

$$\Rightarrow \vec{a}\cdot\vec{b}=-\frac{1}{2},\ \vec{b}\cdot\vec{c}=-\frac{1}{2},\ \vec{c}\cdot\vec{a}=-\frac{1}{2}$$

$$|2\vec{a}+k\vec{b}+k\vec{c}|^2$$

$$=4+k^2+k^2+2(2k+2k+k^2)\left(-\frac{1}{2}\right)$$

$$=2k^2-4k+4=(k-2)^2=9$$

$$\Rightarrow k=5$$

12. Let $A=\begin{bmatrix}12 & -5\\ 10 & 6\end{bmatrix}$, $B=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}12\\ -6\end{bmatrix}$ such that $B = (I + A)^{-1}$, then $x_1 - x_2$ is equal to

(1) 27 (2) 108

(3) 21 (4) 54

Answer (2)

Sol. $B=(I+A)^{-1}$

$$\Rightarrow (I+A)^{-1}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}12\\ -6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}12\\ -6\end{bmatrix}\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}12 & -5\\ 10 & 6\end{bmatrix}\right)$$

$$=\begin{bmatrix}12\\ -6\end{bmatrix}=\begin{bmatrix}13 & -5\\ 10 & 7\end{bmatrix}$$

$$=\begin{bmatrix}186\\ 78\end{bmatrix}$$

$$\Rightarrow x_1-x_2=108$$

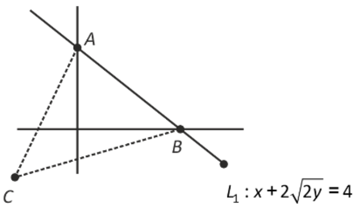
13. Let a line $L: x+2\sqrt{2}y-4=0$ cuts x-axis and y-axis at A and B respectively. Consider an equilateral triangle ABC, such that (0, 0) is the orthocentre of ΔABC. If $C \equiv (\alpha, \beta)$ then $\left|\alpha+\sqrt{2}\beta\right|$ is

(1) 4　　(2) 6

(3) 2　　(4) 3

Answer (2)

Sol.



$A\left(0,\sqrt{2}\right)$ $B(4, 0)$

ABC is an equilateral Δ

$\Rightarrow$ orthocentre = centroid

$\Rightarrow C(\alpha, \beta) \equiv \left(-4,-\sqrt{2}\right)$

$\Rightarrow \alpha = -4,\ \beta = -\sqrt{2}$

$\left|\alpha+\sqrt{2}\beta\right| = \left|-4-2\right| = 6$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $k = \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{2}{3}\right)+\tan^{-1}\left(\frac{1}{2}\sin^{-1}\frac{2}{3}\right)$. Then number of solutions of the equation $\sin^{-1}(kx-1) = \sin^{-1}x-\cos^{-1}x$ is

Answer (1)

Sol. $K = \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{2}{3}\right)+\tan\left(\frac{1}{2}\sin^{-1}\frac{2}{3}\right)$

$= \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{2}{3}\right)+\tan\left(\frac{1}{2}\left(\frac{\pi}{2}-\cos^{-1}\frac{2}{3}\right)\right)$

$= \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{2}{3}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

Let $\frac{1}{2}\cos^{-1}\frac{2}{3} = \theta$

$= \tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$

$= \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{2(1+\tan^2\theta)}{1-\tan^2\theta} = \frac{2}{\cos^2\theta}$

$= \frac{2}{2/3} = 3$

Now $\sin^{-1}(kx-1) = \sin^{-1}x-\cos^{-1}x$

$\sin^{-1}(3x-1) = \sin^{-1}x-\frac{\pi}{2}+\sin^{-1}x$

$\sin^{-1}(3x-1)+\frac{\pi}{2} = 2\sin^{-1}x$

$\therefore$ only positive value g x is zero

$\therefore$ No. of solution = 1

22. Product of first three term of G.P is 27, then the range of sum of these terms is $R-(a, b)$ then a^2+b^2 is

Answer (90)

Sol. Let first three terms be $\frac{a}{r}, a, ar$

$\frac{a}{r}\cdot a\cdot ar = 27$

$a^3 = 27$

$a = 3$

Now $S=\frac{a}{r}+a+ar$

$\Rightarrow S=\frac{3}{r}+3+3r$

If $r>0 \Rightarrow r+\frac{1}{r}\geq 2$

$r<0 \Rightarrow r+\frac{1}{r}\leq -2$

Case I:

If $r>0$

$S=3\left[r+\frac{1}{r}+1\right]$

$S\geq 9$

Case II

If $r<0$

$S=3\left(r+\frac{1}{r}+1\right)$

$S\leq -3$

$S\in(-\infty,-3]\cup[9,\infty)$

$S\in R-(-3,9)$

$a=-3 \quad b=9$

$a^2+b^2=90$

23. The common difference of AP a_1, a_2, a_3.....a_m is 13 times the common difference of AP b_1, b_2, b_3....b_n. Also a_{78} = 327, b_{43} = –385, b_{31} = –277, then a_1 is equal to

Answer (9336)

Sol. Let common difference be d_1 for

$a_1, a_2, ...a_m$

let common difference be d_2 for

$b_1, b_2, ...b_n$

Now $d_1 = 13d_2$

$a_{78} = a_1 + 77d_1 = 327$

$b_{43} = b_1 + 42d_2 = -385$

$b_{31} = b_1 + 30d_2 = -277 \Rightarrow b_1 = -7, d_2 = -9$

$\because d_2 = -9$

$\Rightarrow d_1 = -13 \times 9 = -117$

$a_1 + 77(-117) = 327$

$\Rightarrow a_1 = 9336$

24.

25.

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