

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The sum of coefficients of x^{499} and x^{500} in the binomial expansion of $(1+x)^{1000} + (1+x)^{999}(x) + x^2(1+x)^{998} + \dots + x^{1000}$ is
- (1) $^{1002}C_{501}$ (2) $^{1002}C_{500}$
(3) $^{1001}C_{500}$ (4) $^{1001}C_{501}$

Answer (2)

Sol. $S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000}$

$$\Rightarrow 8 = (1+x)^{1000} \left[\frac{1 - \left(\frac{x}{1+x}\right)^{1001}}{1 - \left(\frac{x}{1+x}\right)} \right]$$

$$= (1+x)^{1001} \left[1 - \left(\frac{x}{1+x}\right)^{1001} \right]$$

$$= (1+x)^{1001} - x^{1001}$$

Coefficient's sum of x^{499} and x^{500}

$$= ^{1001}C_{499} + ^{1001}C_{500} = ^{1002}C_{500}$$

2. If $\sum_{r=1}^{25} \frac{r}{r^4 + r^2 + 1} = \frac{p}{q}$, where p and q are coprime positive integer, then $p+q$ is equal to
- (1) 841 (2) 976
(3) 984 (4) 890

Answer (2)

Sol. $\sum_{r=1}^{25} \frac{r}{(r^2+1)^2 - r^2} = \sum_{r=1}^{25} \frac{r}{(r^2+1+r)(r^2+1-r)}$

$$\begin{aligned} &= \frac{1}{2} \sum_{r=1}^{25} \left(\frac{1}{r^2+1-r} - \frac{1}{r^2+1+r} \right) \\ &= \frac{1}{2} \left[\left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{7} \right) + \dots + \left(\frac{1}{601} - \frac{1}{657} \right) \right] \\ &= \frac{1}{2} \left[1 - \frac{1}{651} \right] \\ &= \frac{1}{2} \left[\frac{650}{651} \right] \\ &= \frac{325}{657} \end{aligned}$$

$$\therefore p+q = 976$$

3. $\frac{6}{3^{26}} + \frac{10}{3^{25}} + \frac{10.2}{3^{24}} + \frac{10.2^2}{3^{23}} + \dots + \frac{10.2^{24}}{3}$ is equal to
- (1) 2^{26} (2) 2^{25}
(3) 3^{26} (4) 3^{25}

Answer (1)

Sol. $\frac{6}{3^{26}} + \frac{10.1}{3^{25}} + \frac{10.2}{3^{24}} + \dots + \frac{10.2^{24}}{3}$

Let $S = 10 \left[\frac{1}{3^{25}} + \frac{2}{3^{24}} + \frac{2^2}{3^{23}} + \dots + \frac{2^{24}}{3} \right]$

$$S = 10 \frac{[6^{25} - 1]}{5 \times 3^{25}}$$

$$= \frac{2}{3^{25}} [6^{25} - 1]$$

$$\text{Now sum} = \frac{6}{3^{26}} + \frac{2}{3^{25}} [6^{25} - 1]$$

$$= \frac{6}{3^{26}} + \frac{6}{3^{26}} [6^{25} - 1]$$

$$= \frac{6^{26}}{3^{26}} = 2^{26}$$

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4. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12(3+[x])}{3+[\sin x]+[\cos x]} dx$, where $[\]$

represents greatest integer function, is equal to

- (1) $3 + 10\pi$ (2) $11\pi + 4$
 (3) $10\pi + 2$ (4) $11\pi + 2$

Answer (4)

Sol. $\int_{-\frac{\pi}{2}}^{-1} \frac{12(3-2)}{3-1+0} dx + \int_{-1}^0 \frac{12(3-1)}{3-1+0} dx +$
 $\int_0^1 \frac{12(3+0)}{3+0+0} dx + \int_1^{\frac{\pi}{2}} \frac{12(3+1)}{3+0+0} dx$
 $= \int_{-\frac{\pi}{2}}^{-1} 6 dx + \int_{-1}^0 12 dx + \int_0^1 12 dx + \int_1^{\frac{\pi}{2}} 16 dx$
 $= 6\left(-1 + \frac{\pi}{2}\right) + 12(1) + 12(1) + 16\left(\frac{\pi}{2} - 1\right)$
 $= -6 + 3\pi + 12 + 12 + 8\pi - 16$
 $= 2 + 11\pi$

5. By the principal of inverse trigonometric function, the value of $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to

- (1) $\frac{33}{56}$ (2) $\frac{31}{55}$
 (3) $\frac{32}{59}$ (4) $\frac{38}{55}$

Answer (1)

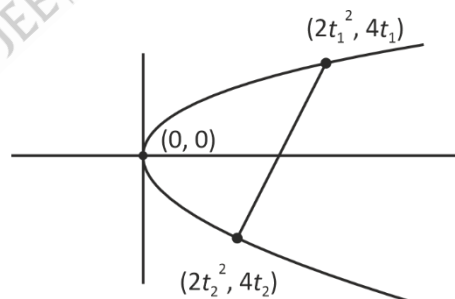
Sol. $\therefore \tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$
 $= \tan\left(2\tan^{-1}\left(\frac{2}{3}\right) - 2\tan^{-1}\left(\frac{1}{3}\right)\right)$
 $= \tan\left(2\tan^{-1}\left(\frac{\frac{2}{3} - \frac{1}{3}}{1 + \frac{2}{9}}\right)\right)$
 $= \tan\left(2\tan^{-1}\left(\frac{3}{11}\right)\right)$
 $= \tan\left(\tan^{-1}\left(\frac{\frac{6}{11}}{1 - \frac{9}{14}}\right)\right)$
 $= \frac{66}{112} = \frac{33}{56}$

6. Let a triangle ABC such that $A \equiv (0, 0)$ and vertices B and C lie on the parabola $y^2 = 8x$ such that $\left(\frac{7}{3}, \frac{4}{3}\right)$ is the centroid of the $\triangle ABC$ then $(BC)^2$ is equal to

- (1) 90 (2) 120
 (3) 150 (4) 110

Answer (2)

Sol.



$$\Rightarrow \frac{2(t_1^2 + t_2^2)}{3} = \frac{7}{3} \Rightarrow t_1^2 + t_2^2 = \frac{7}{2}$$

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$$\frac{4(t_1 + t_2)}{3} = \frac{4}{3} \Rightarrow t_1 + t_2 = 1$$

$$\Rightarrow (t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$$

$$\text{also, } (t_1 + t_2)^2 = \frac{7}{2} + 2t_1t_2 = 1$$

$$\Rightarrow t_1t_2 = \frac{-5}{4}$$

$$\Rightarrow (t_1 - t_2)^2 = 1 - 4\left(\frac{-5}{4}\right) = 6$$

$$\text{Now, } BC = \sqrt{(2t_1^2 - 2t_2^2)^2 + (4(t_1 + t_2))^2}$$

$$BC = 2|(t_1 - t_2)|\sqrt{(t_1 + t_2)^2 + 4}$$

$$\Rightarrow BC = 2\sqrt{6}(\sqrt{5})$$

$$\Rightarrow BC^2 = 4 \times 6 \times 5 = 120$$

7. Let $A = \{Z \in \mathbb{C} : |Z - 2| \leq 4\}$ and $B = \{Z \in \mathbb{C} : |Z - 2| + |Z + 2| \leq 4\}$ then $\max\{Z_1 - Z_2 : Z_1 \in A \text{ and } Z_2 \in B\}$ is equal to

- (1) 6 (2) 8
(3) 4 (4) 5

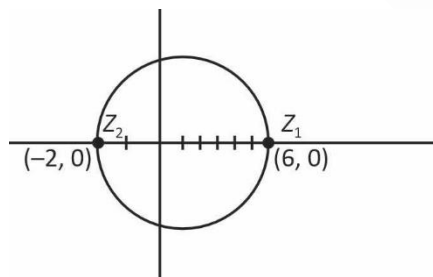
Answer (2)

Sol. $A \Rightarrow (x - 2)^2 + y^2 \leq 16$

$B \Rightarrow$ Line segment

$$\sqrt{(x - 2)^2 + y^2} + \sqrt{(x + 2)^2 + y^2} \leq 4$$

$$\Rightarrow y = 0, x \in (-2, 2)$$



The max $|z - z_2|$ = diameter of circle
= 8

8. If the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{5}{16}$ and $a, 4, \alpha, \beta$ are in increasing A.P. then both the roots of the equation $ax^2 - ax + 2(\alpha - 2\beta) = 0$ lie between

- (1) $(-3, 0)$
(2) $(-2, 3)$
(3) $(0, 3)$
(4) $(-3, 1)$

Answer (2)

Sol. Let $4 - d, 4, 4 + d, 4 + 2d$ are in A.P.

$$\Rightarrow \frac{1}{2} \left(\frac{1}{4 - d} + \frac{1}{4 + 2d} \right) = \frac{5}{10}$$

$$\Rightarrow 5d^2 - 6d - 8 = 0$$

$$\Rightarrow d = 2, \frac{-4}{5}$$

$$\Rightarrow d = 2$$

$$a = 2, \beta = 8, \alpha = 6$$

$$6x^2 - 2x + 2(6 - 16) = 0$$

$$\Rightarrow 3x^2 - x - 10 = 0$$

$$\Rightarrow x = 2, \frac{-5}{3}$$

9. The range of $f(x) = \text{sgn}(\sin x) + \text{sgn}(\cos x) + \text{sgn}(\tan x) + \text{sgn}(\cot x)$, $x \neq \frac{n\pi}{2}, n \in \mathbb{I}$,

$$\text{where } \text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$

- (1) $\{4, -4, 2, -2\}$ (2) $\{-2, 0, 4\}$
(3) $\{4, -4, 0, -2\}$ (4) $\{2, -2, 0, 4, -4\}$

Answer (2)

Sol. $f(x) = \text{sgn}(\sin x) + \text{sgn}(\cos x) + \text{sgn}(\tan x) + \text{sgn}(\cot x)$,

$$x \neq \frac{n\pi}{2}, n \in \mathbb{I},$$

The value of

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$$f(x) = \begin{cases} 1+1+1+1, & \text{for } x \in \left(0, \frac{\pi}{2}\right) \\ 1-1-1-1, & \text{for } x \in \left(\frac{\pi}{2}, \pi\right) \\ -1-1+1+1, & \text{for } x \in \left(\pi, \frac{3\pi}{2}\right) \\ -1+1-1-1, & \text{for } x \in \left(\frac{3\pi}{2}, 2\pi\right) \end{cases}$$

Hence range of $f(x) = \{-2, 0, 4\}$

10. **Statement I:** $25^{13} + 20^{13} + 8^{13} + 3^{13}$ is divisible by 7.

Statement II: The integral value of $(7+4\sqrt{3})^{25}$ is an odd number

- (1) Neither statements are correct
(2) Only statement I is correct
(3) Only statement II is correct
(4) Both the statements are correct

Answer (4)

Sol. Statement 1 :

$(25^{13} + 3^{13}) + (20^{13} + 8^{13})$ is divisible by 28 as

$(a+b) | a^n + b^n$ for $n \in$ odd natural numbers

or using congruence modular arithmetic,

$$25^{13} + 3^{13} + 20^{13} + 8^{13}$$

$$\equiv (4^{13} + 3^{13} + (-1)^{13} + 1^{13}) \pmod{7}$$

$$\equiv ((-3)^{13} + 3^{13}) \pmod{7}$$

$$\equiv 0 \pmod{7} \quad \Rightarrow 7 \text{ divides the sum}$$

Statement 2

$$\text{Let } I + f_1 = (7 + 4\sqrt{3})^{25}$$

$$f_2 = (7 - 4\sqrt{3})^{25} \text{ as } 0 < 7 - 4\sqrt{3} < 1$$

$$\Rightarrow I + f_1 + f_2 =$$

$$2 \left({}^{25}C_1 (4\sqrt{3})^{24} 7^1 + {}^{25}C_3 (4\sqrt{3})^{22} 7^3 + \dots + {}^{25}C_{25} (4\sqrt{3})^0 7^{25} \right)$$

$$= 2k \text{ for some integer } k$$

$$\Rightarrow \text{Since } f_1 + f_2 \in (0, 2)$$

$$\Rightarrow I + f_1 + f_2 = 2k \Rightarrow I = 2k - 1$$

as only integer in

$$(0, 2) \text{ is } 1 \Rightarrow f_1 + f_2 = 1 \Rightarrow I \text{ is odd.}$$

11. Let $y = y(x)$ be the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \cot x, \quad x \in (0, \pi). \text{ If } y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}, \text{ then}$$

$$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right) \text{ is}$$

- (1) 2π (2) -3π
(3) $-\pi$ (4) π

Answer (3)

$$\text{Sol. } x \frac{dy}{dx} - y = x^2 \cot x$$

$$\frac{dy}{dx} - \frac{y}{x} = x \cot x$$

$$e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

$$\frac{y}{x} = \int \cot x dx$$

$$\frac{y}{x} = \int \ln |\sin x| + c$$

$$y = x \ln |\sin x| + cx$$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

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$$\Rightarrow c = 1$$

$$\Rightarrow y = x \ln |\sin x| + x$$

$$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$$

$$= 6\left[\frac{\pi}{6}\ln\left(\frac{1}{2}\right) + \frac{\pi}{6}\right] - 8\left[\frac{\pi}{4}\ln\left(\frac{1}{\sqrt{2}}\right) + \frac{\pi}{4}\right]$$

$$\pi - 2\pi = -\pi$$

12. Let $f(x) = \lim_{\theta \rightarrow 0} \frac{\cos \pi x - (x^{2/\theta}) \sin(x-1)}{1 + (x^{2/\theta}) \sin(x-1)}, x \in R$. Then

which of the following is correct.

- (1) f is continuous at $x = 1$ and $f(1) = -1$
- (2) f is discontinuous at $x = -1$ and $f(1) = -1$
- (3) f is continuous at $x = 1$ and $f(1) = 1$
- (4) f is discontinuous at $x = 1$ and $f(1) = 1$

Answer (1)

Sol. $f(x) = \lim_{\theta \rightarrow 0} \frac{\cos \pi x - (x^2)^{1/\theta} \sin(x-1)}{1 + (x^2)^{1/\theta} \sin(x-1)}, x \in R$

$$f(x) = \begin{cases} \frac{-\sin(x-1)}{(x-1)}, & x > 1 \\ -1, & x = 1 \\ \cos(\pi x), & x < 1 \end{cases}$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = -1$$

13. **Statement I:** The function F defined from $R \rightarrow R$

$$F(x) = \frac{x}{1+|x|} \text{ is one-one}$$

Statement II: The function F defined from $R \rightarrow R$

$$F(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18} \text{ is many-one}$$

- (1) Statement I is correct but statement II is not correct
- (2) Statement I and statement II both are correct
- (3) Statement I is incorrect but statement II is correct
- (4) Both statement are incorrect

Answer (2)

Sol. Statement 1: $F(x) = \frac{x}{1+|x|}$

$$F(x) = \begin{cases} \frac{x}{1-x} & x < 0 \\ \frac{x}{1+x} & x \geq 0 \end{cases}$$

$$F'(x) = \begin{cases} \frac{1}{(1-x)^2} & x < 0 \\ \frac{1}{(1+x)^2} & x \geq 0 \end{cases}$$

$\Rightarrow F(x)$ is increasing in R and $f(x)$ is continuous $\forall x \in R$

$\Rightarrow F(x)$ is one-one

Statement 2: $F(x) = \frac{x^2 + 4x - 30}{x^2 - 8x + 18}$

$$F'(x) = \frac{(x^2 - 8x + 18)(2x + 4) - (x^2 + 4x - 30)(2x - 8)}{(x^2 - 8x + 18)^2}$$

$$= \frac{(2x^3 - 16x^2 + 36x + 4x^2 - 32x + 72) - (2x^3 + 8x^2 - 60x - 8x^2 - 32x + 240)}{(x^2 - 8x + 18)^2}$$

$$= -\frac{12x^2 + 96x - 72}{(x^2 - 8x + 18)^2}$$

$$= -\frac{12(x^2 - 8x + 6)}{(x^2 - 8x + 18)^2}$$

$F'(x)$ will become 0 at 2 distinct points and also $F(x)$ is continuous.

$\Rightarrow F(x)$ is many-one function.

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14. Consider the data:

$$x: 4k \quad \frac{30}{7}k \quad \frac{32}{7}k \quad \frac{34}{7}k \quad \frac{36}{7}k \quad \frac{38}{7}k \quad \frac{40}{7}k \quad 6k$$

$$p(x): \frac{2}{15} \quad \frac{1}{15} \quad \frac{2}{15} \quad \frac{1}{5} \quad \frac{1}{15} \quad \frac{2}{15} \quad \frac{1}{5} \quad \frac{1}{15}$$

If $E(x) = \frac{263}{15}$, then $P(x < 20)$ is equal to

- (1) $\frac{1}{15}$ (2) $\frac{8}{15}$
 (3) $\frac{4}{15}$ (4) $\frac{14}{15}$

Answer (4)

Sol. $E(x) = \frac{263}{15}$

$$\Rightarrow \sum x p(x) = \frac{263}{15}$$

$$\Rightarrow \frac{526}{105}k = \frac{263}{15}$$

$$\Rightarrow k = \frac{7}{2}$$

$$P(x < 20) = 1 - P(6k)$$

$$= 1 - P(21)$$

$$= 1 - \frac{1}{15}$$

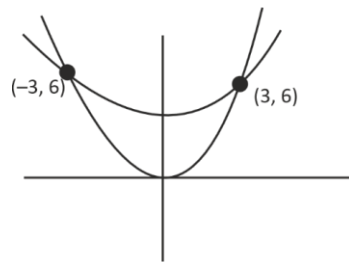
$$= \frac{14}{15}$$

15. Let $P_1: y = 4x^2$ and $P_2: x^2 + 27$ be two parabolas. If the area bounded by region enclosed by P_1 and P_2 is 6 times the area bounded by P_1 and $y = \alpha x, \alpha > 0$, then α is equal to

- (1) 8 (2) 12
 (3) 13 (4) 6

Answer (2)

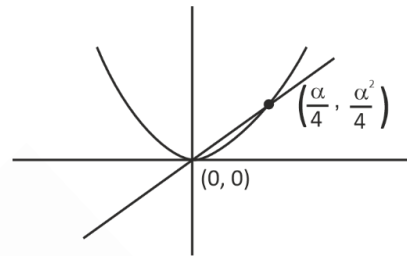
Sol.



$A_1 = \text{Area between } P_1 \text{ and } P_2$

$$\Rightarrow \int_{-3}^3 [(x^2 + 27) - 4x^2] dx = 108$$

$\Rightarrow \text{Area between } P_1 \text{ and } y = \alpha x \text{ is}$



$$\Rightarrow A_2 = \int_0^{\alpha/4} (\alpha x - 4x^2) dx$$

$$= \left[\frac{\alpha x^2}{2} - \frac{4x^3}{3} \right]_0^{\alpha/4}$$

$$= \frac{\alpha^3}{32} - \frac{4}{3} \times \frac{\alpha^3}{64} = \frac{108}{6} = 18$$

$$= \frac{\alpha^3}{32} \left(1 - \frac{2}{3} \right) = 18$$

$$\Rightarrow \alpha^3 = 18 \times 32 \times 3 = 27 \times 64$$

$$\Rightarrow \alpha = 3 \times 4 = 12$$

16. Let Q be the image of the point $P(3, 2, 1)$ in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$, then the distance of Q from the line

$$\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2} \text{ is}$$

- (1) 3 (2) 4
 (3) 5 (4) 7

Answer (4)

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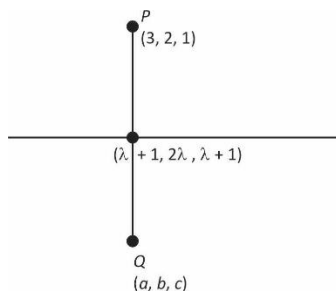
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Sol.



$PQ \perp$ line

$$\Rightarrow (\lambda + 1 - 3)(1) + (2\lambda - 2)(2) + (\lambda + 1 - 1)(1) = 0$$

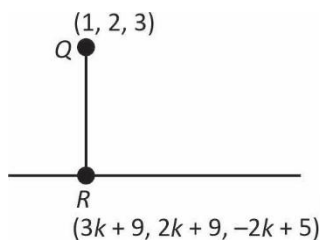
$$\Rightarrow (\lambda - 2) + 4(\lambda - 1) + \lambda = 0$$

$$6\lambda = 6 \Rightarrow \lambda = 1$$

$\Rightarrow$ Image point is $Q(1, 2, 3)$.

The Distance of Point Q from a Second Line

$$\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$$



$$(3k + 9 - 1)(3) + (2k + 9 - 2)(2) + (-2k + 5 - 3)(-2) = 0$$

$$\Rightarrow k = -2$$

$$R \equiv (3k + 9, 2k + 9, -2k + 5), \text{ for } k = -2$$

$$\Rightarrow R \equiv (3, 5, 9)$$

$$QR = \sqrt{2^2 + 3^2 + 6^2} = 7$$

17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be a 2×2 matrix such that $A^{100} = 1008 + I$, then sum of all elements of B^{100} is

Answer (0)

Sol. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3 - 4 & -3 \cdot 4 + 1 \cdot 4 \\ 1 \cdot 3 - 1 & -4 + 1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}, A^3 = \begin{bmatrix} 11 & -10 \\ 5 & -9 \end{bmatrix}, \dots$$

$$\therefore A^n = \begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$$

$$\Rightarrow A^{100} = \begin{bmatrix} 201 & -400 \\ 100 & -199 \end{bmatrix} = 100B + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 200 & -400 \\ 100 & -200 \end{bmatrix} = 100B$$

$$\Rightarrow B = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Sum of elements of $B = 0$

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22. Ellipse $\frac{x^2}{144} + \frac{y^2}{169} = 1$ and hyperbola $\frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$

have same focus and e and L denotes the eccentricity and length of latus rectum of hyperbola then $24(e + L)$ is

Answer (296)

Sol. $\frac{x^2}{144} + \frac{y^2}{169} = 1$

$$e^2 = 1 - \frac{144}{169}$$

$$= \frac{25}{169} \Rightarrow e = \frac{5}{13}$$

$$F_{\text{ellipse}} = \left(0, \pm \frac{5}{13} \times 13 \right)$$

$$= (0, \pm 5)$$

$$\frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$$

$$e_H^2 = 1 + \frac{16}{\lambda^2}$$

$$F = \left(0, \pm \lambda \sqrt{1 + \frac{16}{\lambda^2}} \right)$$

$$\Rightarrow \lambda \sqrt{1 + \frac{16}{\lambda^2}} = 5$$

$$\lambda^2 + 16 = 25$$

$$\lambda^2 = 9$$

$$\text{Now } e_H = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

$$L(\text{LR}) = \frac{2a^2}{b} = \frac{2 \times 16}{3} = \frac{32}{3}$$

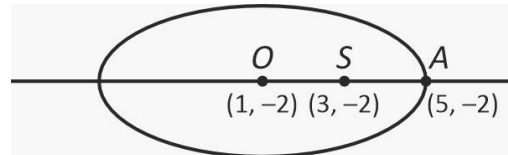
$$24(e + L) = 24 \left(\frac{5}{3} + \frac{32}{3} \right) = 24 \left(\frac{37}{3} \right)$$

$$= 8 \times 37 = 296$$

23. An ellipse has centre at $(1, -2)$ and one of the focus at $(3, -2)$ and one vertex at $(5, -2)$, then the length of its latus rectum is

Answer (6)

Sol.



$$\therefore ae = 2 \text{ and } a = 4$$

$$\therefore e = \frac{1}{2}$$

$$b^2 = a^2(1 - e^2)$$

$$b^2 = 16 \left(1 - \frac{1}{4} \right) = 12$$

$$\therefore \text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 12}{4} = 6$$

24. $F(x) = \int \frac{dx}{x^{2/3} + 2x^{1/2}}$ be such that $F(0) = -26 + 24\ln 2$. If $F(1) = a + b\ln 3$, then $a + b$ is equal to

Answer (13)

Sol. $\int \frac{dx}{x^{2/3} + 2x^{1/2}}$

$$\text{Let } x = t^6$$

$$= dx = 6t^5 dt$$

$$= 6 \int \frac{t^5 dt}{t^4 + 2t^3}$$

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$$\begin{aligned}
 &= 6 \int \frac{t^2}{t+2} dt \\
 &= 6 \int \frac{t^2 - 4 + 4}{t+2} dt \\
 &= 6 \int t + 2 \frac{+4}{t+2} dt \\
 &= 6 \left[\frac{t^2}{2} + 2t + 4 \ln|t+2| \right] + c
 \end{aligned}$$

$$F(x) = 6 \left[\frac{x^{1/3}}{2} + 2x^{1/3} + 4 \ln|x^{1/6} + 2| \right] + c$$

$$F(0) = 24 \ln 2 + c = -26 + 24 \ln 2$$

$$c = -26$$

$$F(1) = \left[\frac{1}{2} + 2 + 4 \ln 3 \right] - 26 \equiv a + b \ln 3$$

$$-11 + 24 \ln 3 = a + b \ln 3$$

$$a + b = 24 - 11 = 13$$

25.



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