17/04/2025

Version Code - **B2**

Subject Code: **M**

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Medical | IIT-JEE | Foundations

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Answers & Solutions

for

Karnataka Common Entrance Test-2025

Time : 80 Minutes

(MATHEMATICS)

M.M. : 60

INSTRUCTIONS FOR CANDIDATES:

DOs:

1. Check whether the CET Number has been entered and shaded in the respective circles on the OMR answer sheet.
2. The version code of this question booklet should be entered on the OMR answer sheet and the respective circles should also be shaded completely.
3. The version Code and Serial Number of this question booklet should also be entered on the Nominal Roll without any mistakes.
4. Compulsorily sign at the bottom portion of the OMR answer sheet in the space provided.
5. Use the space provided on each page of the question booklet for rough work.

DON'Ts:

1. **THE TIMING AND MARKS PRINTED ON THE OMR ANSWER SHEET SHOULD NOT BE DAMAGED / MUTILATED / SPOILED.**
2. The **3rd Bell rings at 10.40 am, till then**
   - **Do not remove the seal present on the right hand side of this question booklet.**
   - **Do not look inside this question booklet or start answering on the OMR answer sheet.**
3. Do not use the OMR answer sheet for the same.

MATHEMATICS

Choose the correct answer:

1. The function $f(x)=\begin{cases} e^x+ax & , \ x<0 \\ b(x-1)^2 & , \ x\geq 0 \end{cases}$ is differentiable at $x = 0$. Then

(1) $a = 3, b = -1$ (2) $a = 1, b = 1$

(3) $a = 3, b = 1$ (4) $a = -3, b = 1$

Answer (4)

Sol. $f(x)=\begin{cases} e^x+ax & , \ x<0 \\ b(x-1)^2 & , \ x\geq 0 \end{cases}$

For continuity,

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)=f(0)$

$e^0 + a \times 0 = b(0-1)^2$

$\Rightarrow b = 1$

For differentiable

$\lim\limits_{x\to 0^-} e^x+a=\lim\limits_{x\to 0^+} 2b(x-1)$

$e^0 + a = 2(1)(0 - 1)$

$\Rightarrow 1 + a = -2$

$a = -3$

2. A function

$f(x)=\begin{cases} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & , \text{ if } x\neq 0 \\ 0 & , \text{ if } x=0 \end{cases}$, is

(1) differentiable at $x = 0$, but not continuous at $x = 0$

(2) continuous at $x = 0$

(3) not continuous at $x = 0$

(4) differentiable at $x = 0$

Answer (3)

Sol. $f(x)=\begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1} & , \ x\neq 0 \\ 0 & , \ x=0 \end{cases}$

$\lim\limits_{x\to 0^-} \dfrac{e^{1/x}-1}{e^{1/x}+1}=\dfrac{0-1}{0+1}=-1$

$\lim\limits_{x\to 0^+} \dfrac{e^{1/x}-1}{e^{1/x}+1}=\lim\limits_{x\to 0^+} \dfrac{1-e^{-1/x}}{1+e^{-1/x}}=\dfrac{1-0}{1+0}=1$

LHL ≠ RHL

Not continuous at $x = 0$

3. If $y = a\sin^3 t$, $x = a\cos^3 t$, then $\dfrac{dy}{dx}$ at $t=\dfrac{3\pi}{4}$ is

(1) 1 (2) -1

(3) $\dfrac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

Answer (1)

Sol. $y = a\sin^3 t$

$x = a\cos^3 t$

$\dfrac{dy}{dt}=a\left(3\sin^2 t\right).\cos t$

$\dfrac{dx}{dt}=a\left(3\cos^2 t\right)(-\sin t)$

$\therefore \dfrac{dy}{dx}=\dfrac{3a\sin^2 t\,.\cos t}{-3a\cos^2 t\,\sin t}$

$=-\dfrac{\sin t}{\cos t}=-\tan t$

$\therefore \left.\dfrac{dy}{dx}\right]_{t=\frac{3\pi}{4}}=-\tan\dfrac{3\pi}{4}$

$= -(-1) = 1$

4. The derivative of sin x with respect to log x is

(1) $\frac{\cos x}{x}$ (2) $\cos x$

(3) $x \cos x$ (4) $\frac{\cos x}{\log x}$

Answer (3)

Sol. $y = \sin x$

$z = \log x$

$\frac{dy}{dx} = \cos x, \frac{dz}{dx} = \frac{1}{x}$

$\frac{dy}{dz} = \frac{\cos x}{1/x} = x \cos x$

5. The minimum value of $1 - \sin x$ is

(1) 2 (2) 0

(3) -1 (4) 1

Answer (2)

Sol. $y = 1 - \sin x$

$-1 \le \sin x \le 1$

$-1 \le -\sin x \le 1$

$0 \le 1 - \sin x \le 2$

Minimum value $= 0$

6. The function $f(x) = \tan x - x$

(1) neither increases nor decreases

(2) always increases

(3) always decreases

(4) never increases

Answer (2)

Sol. $f(x) = \tan x - x$

$f'(x) = \sec^2 x - 1 = \tan^2 x$

$\tan^2 x$ is always positive

$\therefore f'(x) \ge 0$

$\therefore f(x)$ is always increasing.

7. The value of $\int \frac{dx}{(x+1)(x+2)}$ is

(1) $\log\left|\frac{x+1}{x+2}\right| + c$ (2) $\log\left|\frac{x-1}{x+2}\right| + c$

(3) $\log\left|\frac{x-1}{x-2}\right| + c$ (4) $\log\left|\frac{x+2}{x+1}\right| + c$

Answer (1)

Sol. $\int \frac{dx}{(x+1)(x+2)} = \int\left(\frac{(x+2)-(x+1)}{(x+1)(x+2)}\right)dx$

$= \int \frac{1}{x+1}dx - \int \frac{1}{x+2}dx + C$

$= \log|x+1| - \log|x+2| + C$

$= \log\left|\frac{x+1}{x+2}\right| + C$

8. The value of $\int_{-1}^{1} \sin^5 x \cos^4 x \, dx$ is

(1) 0 (2) $\frac{-\pi}{2}$

(3) π (4) $\frac{\pi}{2}$

Answer (1)

Sol. $I = \int_{-1}^{1} \sin^5 x \cos^4 x dx$

$I = \int_{0}^{1}\left(\sin^5 x \cos^4 x - \sin^5 x \cos^4 x\right)dx$

$I = 0$

$\left[\because \int_{-a}^{a} f(x)dx = \int_{0}^{a}(f(x)+f(-x))dx\right]$

9. The value of $\int_0^{2\pi}\sqrt{1+\sin\left(\frac{x}{2}\right)}dx$ is

(1) 0 (2) 8

(3) 4 (4) 2

Answer (2)

Sol. $I=\int_0^{2\pi}\sqrt{1+2\sin\frac{x}{4}\cos\frac{x}{4}}$

$=\int_0^{2\pi}\sqrt{\left(\sin x\left(\frac{x}{4}\right)\right)^2+\left(\cos\left(\frac{x}{4}\right)\right)^2+2\sin\frac{x}{4}\cos\frac{x}{4}}$

$\int_0^{2\pi}\sqrt{\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)^2}=\int_0^{2\pi}\left|\sin\frac{x}{4}+\cos\frac{x}{4}\right|dx$

Let, $\frac{x}{4}=t$, $\Rightarrow dx=4dt$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}|\sin t+\cos t|\,4dt$

$=4\left(-\cos t+\sin t\right)\Big|_0^{\frac{\pi}{2}}$

$=4[1-(-1+0)]=8$

10. $\int\frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals

(1) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (2) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(3) $\left(x^4+1\right)^{\frac{1}{4}}+c$ (4) $-\left(x^4+1\right)^{\frac{1}{4}}+c$

Answer (1)

Sol. $\int\frac{dx}{x^2(x^4+1)^{3/4}}$

$=\int\frac{dx}{x^2\left(x^3\left(1+\frac{1}{x^4}\right)^{3/4}\right)}$

$=\int\frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Let $1+\frac{1}{x^4}=t$

$-4x^{-5}dx=dt$

$\frac{dx}{x^5}=\frac{-dt}{4}$

$=\int\frac{-dt}{4t^{3/4}}=\frac{-1}{4}\left[\frac{t^{-3/4}+1}{\frac{-3}{4}+1}\right]+C$

$=-t^{\frac{1}{4}}+C$

$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$

$=-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+C$

11. $\int_0^1\log\left(\frac{1}{x}-1\right)dx$ is

(1) $\log_e\left(\frac{1}{2}\right)$ (2) 1

(3) 0 (4) $\log_e 2$

Answer (3)

Sol. $I_2 = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$ …(i)

$I = \int_0^1 \log\left(\frac{(1-(1-x)}{(1-x)}\right)dx$

$I = \int_0^1 \log\left(\frac{x}{1-x}\right)dx$ …(ii)

Add (i) and (ii)

$2I = \int_0^1 \left(\log\left(\frac{(1-x)}{x}\right) + \log\left(\frac{x}{1-x}\right)\right)dx$

$\therefore \quad I = 0$

12. The area bounded by the curve $y = \sin\left(\frac{x}{3}\right)$, x axis, the lines $x = 0$ and $x = 3\pi$ is

(1) 3 sq. units

(2) 9 sq. units

(3) $\frac{1}{3}$ sq. units

(4) 6 sq. units

Answer (4)

Sol. $y = \sin\left(\frac{x}{3}\right)$

Area = $\int_0^{3\pi} \sin\left(\frac{x}{3}\right)dx$

Let $\frac{x}{3} = t \Rightarrow dx = 3dt$

Area = $\int_0^{\pi} \sin t \cdot 3dt$

$= 3(-\cos t)_0^{\pi}$

$= 3(1 + 1) = 6$

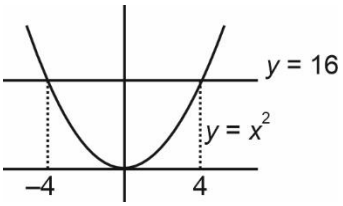
13. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

(1) $\frac{128}{3}$ sq. units (2) $\frac{32}{3}$ sq. units

(3) $\frac{256}{3}$ sq. units (4) $\frac{64}{3}$ sq. units

Answer (3)

Sol.



Required area $= 2\int_0^4 (16 - x^2)dx$

$= 2 \cdot \left(16x - \frac{x^3}{3}\right)\Big|_0^4$

$= 2\left(64 - \frac{64}{3}\right) = \frac{4}{3} \cdot 64 = \frac{256}{3}$ sq. units

14. General solution of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is

(1) $x \sec x = \tan y + c$ (2) $y \sec x = \tan x + c$

(3) $y \tan x = \sec x + c$ (4) $\text{cosec}\, x = y \tan x + c$

Answer (2)

Sol. $\frac{dy}{dx} + y\tan x = \sec x$

I.F. $e^{\int \tan x dx} = e^{\int x \sec x} = \sec x$

$y \cdot \sec x = \int \sec^2 x\, dx$

$= \tan x + c$

15. If 'a' and 'b' are the order and degree respectively of the differentiable equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$, then $a - b =$ _____

(1) 0 (2) 1

(3) 2 (4) –1

Answer (1)

Sol. $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$

Order = 2 = a, degree = 2 = b

$a - b = 0$

16. The distance of the point $P(-3, 4, 5)$ from yz plane is

(1) 3 units (2) 4 units

(3) 5 units (4) –3 units

Answer (1)

Sol. Distance from yz plane = absolute value of x coordinate

$= |-3| = 3$

17. If $A = \{x : x \text{ is an integer and } x^2 - 9 = 0\}$

$B = \{x : x \text{ is a natural number and } 2 \le x < 5\}$

$C = \{x : x \text{ is a prime number} \le 4\}$

Then $(B - C) \cup A$ is,

(1) {2, 3, 5} (2) {–3, 3, 4}

(3) {2, 3, 4} (4) {3, 4, 5}

Answer (2)

Sol. $A = \{-3, 3\}$

$B = \{2, 3, 4\}$

$C = \{2, 3\}$

$B - C = \{4\}$

$(B - C) \cup A = \{-3, 3, 4\} \Rightarrow 2$

18. A and B are two sets having 3 and 6 elements respectively.

Consider the following statements.

Statement (I): Minimum number of elements in $A \cup B$ is 3

Statement (II): Maximum number of elements in $A \cap B$ is 3

Which of the following is correct?

(1) Both statements (I) and (II) are false

(2) Statement (I) is true, statement (II) is false

(3) Statement (I) is false, statement (II) is true

(4) Both statements (I) and (II) are true

Answer (3)

Sol. Minimum number of elements in $A \cup B$ is 6, hence (I) is false.

Maximum number of elements in $A \cap B$ is 3, hence (II) is correct.

19. Domain of the function f, given by $f(x) = \frac{1}{\sqrt{(x-2)(x-5)}}$ is

(1) $(-\infty, 3] \cup (5, \infty)$ (2) $(-\infty, 2] \cup [5, \infty)$

(3) $(-\infty, 2) \cup (5, \infty)$ (4) $(-\infty, 3) \cup [5, \infty)$

Answer (3)

Sol. $(x - 2)(x - 5) > 0$

$\Rightarrow x \in (-\infty, 2) \cup (5, \infty)$

20. If $f(x) = \sin[\pi^2]x - \sin[-\pi^2]x$, where $[x]$ = greatest integer $\le x$, then which of the following is not true?

(1) $f(\pi) = -1$ (2) $f(0) = 0$

(3) $f\left(\frac{\pi}{2}\right) = 1$ (4) $f\left(\frac{\pi}{4}\right) = 1 + \frac{1}{\sqrt{2}}$

Answer (1)

Sol. $f(x) = \sin([\pi^2]x) - \sin([-\pi^2]x)$

$\because \quad 9 < \pi^2 < 10, -10 < -\pi^2 < -9$

$\Rightarrow \quad f(x) = \sin(9x) - \sin(-10x) = \sin(9x) + \sin(10x)$

$\Rightarrow \quad f(\pi) = 0, f(0) = 0, f\left(\frac{\pi}{2}\right) = 1, f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + 1$

21. Which of the following is not correct?

(1) $\tan45° = \tan(-315°)$ (2) $\cos5\pi = \cos4\pi$

(3) $\sin2\pi = \sin(-2\pi)$ (4) $\sin4\pi = \sin6\pi$

Answer (2)

Sol. (1) $\tan45° = 1, \tan(-315°) = -\tan(315°) = -\tan(360° - 45°) = 1$

(2) $\cos(5\pi) = -1, \cos(4\pi) = 1$

(3) $\sin2\pi = 0, \sin(-2\pi) = 0$

(4) $\sin(4\pi) = 0, \sin(6\pi) = 0$

22. If $\cos x + \cos^2 x = 1$, then the value of $\sin^2 x + \sin^4 x$ is

(1) 2 (2) –1

(3) 1 (4) 0

Answer (3)

Sol. $\because \quad \cos x + \cos^2 x = 1 \Rightarrow \cos x = \sin^2 x$

$\Rightarrow \quad \sin^2 x + \sin^4 x = \sin^2 x + \cos^2 x = 1$

23. The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is

(1) 4.03 (2) 10

(3) 3 (4) 8.5

Answer (3)

Sol. mean $= \bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = 10$

$\Rightarrow$ mean deviation about mean

$= \frac{1}{8}(|4 - 10| + |7 - 10| + |8 - 10| + |9 - 10| + |10 - 10| + |12 - 10| + |13 - 10| + |17 - 10|)$

$= \frac{1}{8}(6 + 3 + 2 + 1 + 0 + 2 + 3 + 7) = 3$

24. A random experiment has five outcomes w_1, w_2, w_3, w_4 and w_5. The probabilities of the occurrence of the outcomes w_1, w_2, w_4 and w_5 are respectively $\frac{1}{6}$, a, b and $\frac{1}{12}$ such that $12a + 12b - 1 = 0$. Then the probabilities of occurrence of the outcome w_3 is

(1) $\frac{1}{12}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{6}$

Answer (2)

Sol. Let $P(w_3) = x$

$\because \quad P(w_1) + P(w_2) + P(w_3) + P(w_4) + P(w_5) = 1$

$\Rightarrow \quad = \frac{1}{6} + a + x + b + \frac{1}{12} = 1 \Rightarrow 2 + 12x + 12(a + b) + 1 = 12$

$\because \quad 12(a + b) = 1$

$\Rightarrow \quad 2 + 12x + 1 + 1 = 12 \Rightarrow 12x = 8 \Rightarrow x = \frac{2}{3}$

25. A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If the die is rolled once, then $P(1 \text{ or } 3)$ is

(1) $\frac{1}{6}$ (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Answer (3)

Sol. $P(1) = \frac{2}{6}, P(2) = \frac{3}{6}, P(3) = \frac{1}{6}$

$P(1 \cup 3) = P(1) + P(3) - (1 \cap 3) = \frac{2}{6} + \frac{1}{6} - 0 = \frac{1}{2}$

Option (3) is correct.

26. Let $A = \{a, b, c\}$, then the number of equivalence relations on A containing (b, c) is

(1) 4 (2) 1

(3) 3 (4) 2

Answer (4)

Sol. $A = \{a, b, c\}$

Smallest equivalence relation containing (b, c) is

$R_1 = \{(a, a), (b, b), (c, c), (b, c), (c, b)\}$

Now we are left with (a, b), (a, c), (b, a), (c, a)

If we add any one pair [say (a, b)], then for symmetry we must add (b, a)

Also for transitivity we are required to add

(c, a), (a, c) which make it a universal relation.

$\Rightarrow$ Only 2 equivalence relation is possible. i.e. R_1 and universal relation.

27. Let the functions "f" and "g" be $f : \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = \sin x$ and $g : \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = \cos x$, where R is the set of real numbers

Consider the following statements:

Statement (I): f and g are one-one

Statement (II): $f + g$ is one-one

Which of the following are correct?

(1) Both statements (I) and (II) are false

(2) Statement (I) is true, statement (II) is false

(3) Statement (I) is false, statement (II) is true

(4) Both statements (I) and (II) are true

Answer (2)

Sol. $f : \left[0, \frac{\pi}{2}\right] \to R$

$f(x) = \sin x$

$g : \left[0, \frac{\pi}{2}\right] \to R$

$g(x) = \cos x$

Statement (I): $\because$ $f(x)$ is increasing in $\left[0, \frac{\pi}{2}\right]$ and attains the value from [0, 1]

$\Rightarrow$ $f(x)$ is one-one in $\left[0, \frac{\pi}{2}\right]$

Also,

$g(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and take the values from [0, 1]

$\Rightarrow$ $g(x)$ is also one-one in $\left[0, \frac{\pi}{2}\right]$

$\Rightarrow$ Statement (I) is correct.

Statement (II): $f + g \equiv \sin x + \cos x = h(x)$

$h^1(x) = \cos x - \sin x = 0$

$\Rightarrow \frac{\sin x}{\cos x} = 1 \Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}$

$h(x)$ attain its maxima at $x = \frac{\pi}{4}$

$\Rightarrow$ among $x = \frac{\pi}{4}$ the function $\uparrow$ ses or $\downarrow$ es

$\Rightarrow$ $h(x)$ is not one-one

28. $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3) =$

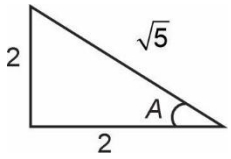
(1) 10 (2) 1

(3) 5 (4) 15

Answer (4)

Sol. $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$

$$E = \left[\sec(\tan^{-1}2)\right]^2 + \left[\text{cosec}(\cot^{-1}3)\right]^2$$

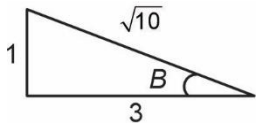


Let $\tan^{-1}2 = A$

$\Rightarrow \tan A = 2$

$\Rightarrow \sec A = \sqrt{5}$

$\Rightarrow A = \sec^{-1}\sqrt{5}$



$B = \cot^{-1}3$

$\cot B = 3$

$\text{cosec } B = \sqrt{10}$

$B = \text{cosec}^{-1}\sqrt{10}$

$\Rightarrow E = (\sec(\sec^{-1}\sqrt{5}))^2 + (\text{cosec}(\text{cosec}^{-1}\sqrt{10}))^2$

$= \left(\sqrt{5}\right)^2 + \left(\sqrt{10}\right)^2$

$= 15$

29. $2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is valid for all values of 'x' satisfying

(1) $\frac{1}{\sqrt{2}} \le x \le 1$ (2) $0 \le x \le \frac{1}{\sqrt{2}}$

(3) $-1 \le x \le 1$ (4) $0 \le x \le 1$

Answer (1)

Sol. $2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Let $\cos^{-1}x = y$

$0 \le \cos^{-1} \le \pi$

$0 \le 2y \le 2\pi$ …(i)

Also

$$\frac{-\pi}{2} \le \sin^{-1}\left(2x\sqrt{1-x^2}\right) \le \frac{\pi}{2}$$

$$\frac{-\pi}{2} \le \sin^{-1}(2y) \le \frac{\pi}{2} \quad \text{…(ii)}$$

From (i) and (ii)

$$0 \le 2y \le \frac{\pi}{2}$$

$$0 \le y \le \frac{\pi}{4}$$

$$\Rightarrow 0 \le \cos^{-1}x \le \frac{\pi}{4}$$

$$\boxed{\frac{1}{\sqrt{2}} \le x \le 1}$$

30. Consider the following statements:

Statement (I): In a LPP, the objective function is always linear.

Statement (II) : In a LPP, the linear inequalities on variables are called constraints.

Which of the following is correct?

(1) Statement (I) is false, Statement (II) is true

(2) Statement (I) is true, Statement (II) is true

(3) Statement (I) is true, Statement (II) is false

(4) Both Statements (I) and (II) are false

Answer (2)

Sol. Objective function will be linear.

$\Rightarrow$ Statement (I) is correct

Statement (II): In LPP, the linear constant on variable are called constraints.

$\Rightarrow$ Statement (II) is correct

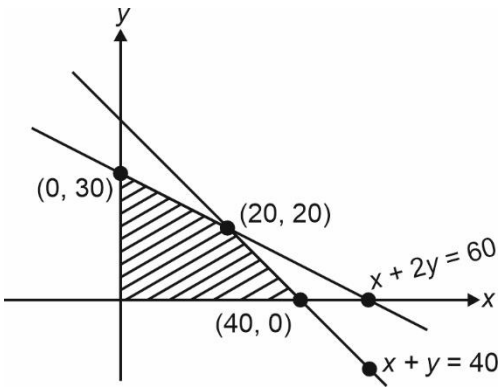
31. The maximum value of $z = 3x + 4y$, subject to the constraints $x + y \le 40$, $x + 2y \le 60$ and $x, y \ge 0$ is

(1) 40 (2) 130

(3) 120 (4) 140

Answer (4)

Sol.



$z = 3x + 4y$

$(0, 0) \rightarrow 0$

$(40, 0) \rightarrow 120$

$(20, 20) \rightarrow 140 \leftarrow$ Max

$(0, 30) \rightarrow 120$

32. Consider the following statements.

Statement (I) : If E and F are two independent events, then E' and F' are also independent.

Statement (II) : Two mutually exclusive events with non-zero probabilities of occurrence cannot be independent.

Which of the following is correct?

(1) Both the statements are false

(2) Statement (I) is true and statement (II) is false

(3) Statement (I) is false and statement (II) is true

(4) Both the statements are true

Answer (4)

Sol. Statement-I : If E and F are independent E' and F' are also independent.

This statement is correct.

Statement-II: $P(A \cap B) = 0$, but $P(A), P(B) \neq 0$

$\therefore P(A) \cdot P(B) \neq P(A \cap B)$

Statement-II is correct.

33. If A and B are two non-mutually exclusive events such that $P(A|B) = P(B|A)$, then

(1) $P(A) = P(B)$ (2) $A \subset B$ but $A \neq B$

(3) $A = B$ (4) $A \cap B = \phi$

Answer (1)

Sol. $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow P(A) = P(B)$

34. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(1) $P(A) = P(B)$

(2) $P(A \mid B) = \frac{P(B)}{P(A)}$

(3) $P(A|B) < P(A)$

(4) $P(A|B) \ge P(A)$

Answer (4)

Sol. $A \subset B$

$P(A) < P(B)$ Option 1 in incorrect

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A)}{P(B)}$

$P(B) < 1$

$\therefore P\left(\frac{A}{B}\right) = \frac{P(A)}{P(B)} \ge P(A)$

Option 4 is correct.

35. Meera visits only one of the two temples *A* and *B* in her locality. Probability that she visits temple *A* is $\frac{2}{5}$. If she visits temple A, $\frac{1}{3}$ is the probability that she meets her friend, whereas it is $\frac{2}{7}$ if she visits temple *B*. Meera met her friend at one of the two temples. The probability that she met her at temple *B* is

(1) $\frac{9}{16}$ (2) $\frac{7}{16}$

(3) $\frac{5}{16}$ (4) $\frac{3}{16}$

Answer (1)

Sol. *A* : Meera visits temple *A*

B : Meera visits temple *B*

F : She meets her friend

$$P(A)=\frac{2}{5};\ P(B)=\frac{3}{5}$$

$$P\left(\frac{F}{A}\right)=\frac{1}{3};\ P\left(\frac{F}{B}\right)=\frac{2}{7}$$

$$P\left(\frac{B}{F}\right)=\frac{P(B\cap F)}{P(F)}$$

$$=\frac{P(B)\cdot P\left(\frac{F}{B}\right)}{P(A)\cdot P\left(\frac{F}{A}\right)+P(B)\cdot P\left(\frac{F}{B}\right)}$$

$$=\frac{\frac{3}{5}\times\frac{2}{7}}{\frac{2}{5}\times\frac{1}{3}+\frac{3}{5}\times\frac{2}{7}}$$

$$=\frac{\frac{6}{7}}{\frac{2}{3}+\frac{6}{7}}$$

$$=\frac{9}{16}$$

36. If Z_1 and Z_2 are two non-zero complex numbers, then which of the following is not true?

(1) $|Z_1+Z_2|\geq|Z_1|+|Z_2|$ (2) $\overline{Z_1+Z_2}=\overline{Z_1}+\overline{Z_2}$

(3) $|Z_1Z_2|=|Z_1|\cdot|Z_2|$ (4) $\overline{Z_1\cdot Z_2}=\overline{Z_1}\cdot\overline{Z_2}$

Answer (1)

Sol. $|Z_1+Z_2|\leq|Z_1|+|Z_2|$

Using triangular inequality.

37. Consider the following statements:

Statement (I): The set of all solutions of the linear inequalities $3x + 8 < 17$ and $2x + 8 \geq 12$ are $x < 3$ and $x \geq 2$ respectively.

Statement (II): The common set of solutions of linear inequalities $3x + 8 < 17$ and $2x + 8 \geq 12$ is (2,3)

Which of the following is true?

(1) Both the statements are false

(2) Statement (I) is true but statement (II) is false

(3) Statement (I) is false but statement (II) is true

(4) Both the statements are true

Answer (2)

Sol. $3x + 8 < 17$

$= 3x < 9$

$= x < 3$

$2x + 8 \geq 12$

$= 2x \geq 4$

$x \geq 2$

Statement-I is true.

Statement-II is false

$(-\infty, 3) \cap [2, \infty) = [2, 3]$

38. The number of four digit even number that can be formed using the digits 0, 1, 2 and 3 without repetition is

(1) 12 (2) 6

(3) 10 (4) 4

Answer (3)

Sol. Case-I (when unit digit is 0)

$\_\_ \quad \_\_ \quad \_\_ \quad \underset{\_\_}{0} \longrightarrow 3! = 6$

Case-II (when unit digit is 2)

$\underset{\downarrow}{\_\_}_{2} \quad \underset{\downarrow}{\_\_}_{2} \quad \underset{\downarrow}{\_\_}_{1} \quad \underset{\_\_}{2} = 4$

$\therefore$ Total numbers = 6 + 4 = 10

39. The number of diagonals that can be drawn in an octagon is

(1) 30 (2) 15

(3) 20 (4) 28

Answer (3)

Sol. Number of diagonals in an convex polygon

$= {}^{n}C_2 - n$

for $n = 8$

Number of diagonals $= {}^{8}C_2 - 8$

$= 28 - 8 = 20$

40. If the number of terms in the binomial expansion of $(2x + 3)^{3n}$ is 22, then the value of n is

(1) 9 (2) 8

(3) 6 (4) 7

Answer (4)

Sol. Number of terms in $(x + y)^n$ is $n = 1$.

for $(2n = 3)^{3n}$, number of terms $= 3n + 1$

$\therefore \quad 2n + 1 = 22$

$\Rightarrow \quad n = 7$

41. If 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z respectively, then

(1) $y = \frac{x+z}{2}$ (2) $z = \sqrt{xy}$

(3) $y = \sqrt{xz}$ (4) $x = \sqrt{yz}$

Answer (3)

Sol. Let a be 1^{st} term and r be common ratio,

$$\begin{matrix} x = ar^3 \\ y = ar^9 \\ z = ar^{15} \end{matrix} \Rightarrow xz = a^2r^{18} = (ar^9)^2 = y^2 \Rightarrow y = \sqrt{xz}$$

42. If A is a square matrix such that $A^2 = A$, then $(I - A)^3$ is

(1) $-I - A$ (2) $I - A$

(3) $A - I$ (4) $I + A$

Answer (2)

Sol. $\because A^2 = A \ \therefore A^3 = A^2 = A$

$(I - A)^3 = I^3 - 3A + 3A^2 - A^3$

$= I - 3A + 3A - A = I - A$

43. If A and B are two matrices such that AB is an identity matrix and the order of matrix B is 3×4 then the order of matrix A is

(1) 4×4

(2) 3×4

(3) 3×3

(4) 4×3

Answer (4)

Sol. $AB = I$

For product to be valid the order of $A_{m \times n}$ $B_{n \times m}$ to be an identity matrix of order $m \times m$.

$\Rightarrow \quad n \times m \equiv (3 \times 4) \Rightarrow A_{m \times n} \equiv A_{4 \times 3}$ order of A will be 4×3

44. Which of the following statements is not correct?

(1) A skew symmetric matrix has all diagonal elements equal to zero

(2) A row matrix has only one row

(3) A diagonal matrix has all diagonal elements equal to zero

(4) A symmetric matrix A is a square matrix satisfying $A' = A$

Answer (3)

Sol. (2) A row matrix consist of only a now.

(1) For skew-symmetric matrix has all diagonal elements equal to 0.

(3) A diagonal matrix has non-diagonal element zero but there is no condition that diagonal elements should be zero.

Hence, option (C) is not correct.

(4) $A^T = A \Rightarrow$ Symmetrix matrix

45. If a matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^6 = kA'$, then the value of k is

(1) 6 (2) 32

(3) 1 (4) $\frac{1}{32}$

Answer (2)

Sol. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow |A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$ using characteristic equation, $(1-\lambda)^2 - 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda = 0$

$\Rightarrow A^2 - 2A = 0 \Rightarrow A^2 = 2A$

$\Rightarrow A^4 = 4A^2 = 4(2A) = 8A$

$A^6 = A^2 \cdot A^4 = (2A)(8A) = 16A^2$

$= 32A$

46. If $A = \begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix}$ and $|A^3| = 125$, then the value of k is

(1) –4 (2) ±2

(3) ±3 (4) –5

Answer (3)

Sol. $A = \begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix} \Rightarrow |A| = k^2 - 4$

$\Rightarrow |A^3| = |k^2 - 4|^3 = 125 = 5^3$

$\Rightarrow k^2 - 4 = 5 \Rightarrow k = \pm 3$

47. If A is a square matrix satisfying the equation $A^2 - 5A + 7I = 0$, where I is the identity matrix and 0 is null matrix of same order, then $A^{-1} =$

(1) $\frac{1}{5}(7I - A)$ (2) $\frac{1}{7}(5I - A)$

(3) $\frac{1}{7}(A - 5I)$ (4) $7(5I - A)$

Answer (2)

Sol. If $A^2 - 5A + 7I = 0$

Melting A^{-1} to RHS both side

$A^T(A^2 - 5A + 7I) = A^{-1}\,0$

$A^{-1}AA - 5A^{-1}A + 7A^{-1}I = 0$

$\Rightarrow A - 5I + 7A^{-1} = 0 \Rightarrow A^{-1} = \frac{1}{7}(5I - A)$

48. If A is a square matrix of order 3 × 3, det A = 3, then the value of det $(3A^{-1})$ is

(1) 9 (2) $\frac{1}{3}$

(3) 3 (4) 27

Answer (A)

Sol. $|A| = 3$

$|3A^{-1}| = 3^3|A^{-1}| = 3^3 \cdot \frac{1}{|A|} = \frac{3^3}{3} = 9$

49. If $B=\begin{bmatrix}1 & 3\\ 1 & \alpha\end{bmatrix}$ be the adjoint of a matrix A and $|A| = 2$, then the value of α is

(1) 3 (2) 4

(3) 5 (4) 2

Answer (3)

Sol. $|B| = (1)(\alpha) - (3)(1) = \alpha - 3$

$|B| = |A|^{2-1} = |A|$

$\therefore \alpha - 3 = 2$

$\Rightarrow \alpha = 5$

50. The system of equations $4x + 6y = 5$ and $8x + 12y = 0$ has

(1) Only two solutions

(2) No solution

(3) Infinitely many solutions

(4) A unique solution

Answer (4)

Sol. $4x + 6y = 5$ … (i)

$8x + 12y = 0$ … (ii)

Multiply eq. (i) by 2

Both are same equations.

$\therefore$ Infinitely many solutions.

51. If $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then the value of λ is

(1) 0

(2) 1

(3) ±1

(4) –1

Answer (4)

Sol. $\vec{a}+\lambda\vec{b}=\hat{i}+2\hat{j}+\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$

$=(1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(1+4\lambda)\hat{k}$

$(\vec{a}+\lambda\vec{b}).\vec{c}=0$

$(1 + \lambda) + (2 - \lambda) + (1 + 4\lambda) = 0$

$\Rightarrow 4 + 4\lambda = 0$

$\Rightarrow \lambda = -1$

52. If $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}.\vec{b}=12$, then the value of $|\vec{a}\times\vec{b}|$ is

(1) 16 (2) 5

(3) 10 (4) 14

Answer (1)

Sol. $|\vec{a}|=10$; $|\vec{b}|=2$

$\vec{a}\,.\,\vec{b}=12$

$|\vec{a}|.|\vec{b}|\cos\theta=12$

$\cos\theta=\frac{12}{20}=\frac{3}{5}$

$\therefore |\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta=(10)(2)\times\frac{4}{5}=20\times\frac{4}{5}=16$

53. Consider the following statements:

Statements (I) : If either $|\vec{a}|=0$ or $|\vec{b}|=0$, then $\vec{a}.\vec{b}=0$.

Statements (II) : If $\vec{a}\times\vec{b}=0$, then $\vec{a}$ is perpendicular to $\vec{b}$.

Which of the following is correct?

(1) Both Statement (I) and Statement (II) are false

(2) Statement (I) is true but Statement (II) is false

(3) Statement (I) is false but Statement (II) is true

(4) Both Statement (I) and Statement (II) are true

Answer (2)

Sol. Statements (I) :

If $|\vec{a}| = 0$ or $|\vec{b}| = 0$, then $\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|\cos\theta = 0$.

Statement (I) is true.

Statements (II) :

If $\vec{a} \times \vec{b} = 0$, then $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 0$

This means $\sin\theta = 0$, $|\vec{a}| = 0$ or $|\vec{b}| = 0$

If $\sin\theta = 0$, $\theta = 0$ or π, meaning $\vec{a}$ and $\vec{b}$ are parallel or antiparallel.

Either, $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

$\therefore$ Statement (II) is false.

54. If a line makes angles 90°, 60° and θ with x, y and z axes respectively, where θ is acute, then the value of θ is

(1) $\pi/2$ (2) $\pi/6$

(3) $\pi/4$ (4) $\pi/3$

Answer (2)

Sol. We know that,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 90° + \cos^2 60° + \cos^2\theta = 1$

$$0 + \frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

$$\therefore\ 0 < \theta < \frac{\pi}{2}$$

$$\theta < \frac{\pi}{6}$$

55. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is

(1) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$ (2) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

(3) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$ (4) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

Answer (3)

Sol. Let $\vec{a}_1 = 2\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{b}_1 = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{a}_1 \cdot \vec{b}_1 = 2a + 3b - 2c = 0$

(i) $2(3) + 3(-4) - 2(3) = 6 - 12 - 6 \neq 0$

(ii) $2(3) + 3(4) - 2(-3) = 6 + 12 + 6 \neq 0$

(iii) $2(-3) + 3(4) - 2(3) = -6 + 12 - 6 = 0$

$\therefore$ Option (3) is correct.

56. A line passes through (–1, –3) and perpendicular to $x + 6y = 5$. Its x intercept is

(1) 2 (2) $\frac{1}{2}$

(3) $-\frac{1}{2}$ (4) –2

Answer (4)

Sol. Given $x + 6y = 5$

$\Rightarrow\ 6y = 5 - x$

$$y = \frac{5}{6} - \frac{x}{6} \qquad \ldots(i)$$

$$\therefore\ m = \frac{-1}{6}$$

Let m_1 be the slope perpendicular to (i).

$\therefore\ m \cdot m_1 = -1$

$$m_1\left(\frac{-1}{6}\right) = -1$$

$m_1 = 6$

$\therefore$ Equation of line passing through (–1, –3) having slope 6 is

$(y + 3) = 6(x + 1)$

$\Rightarrow y + 3 = 6x + 6$

$\Rightarrow y = 6x + 3$

For x-intercept, put $y = 0$.

$0 = 6x + 3$

$\Rightarrow x = -2$

$\therefore$ Option (4) is correct.

57. The length of the latus rectum of $x^2 + 3y^2 = 12$ is

(1) 24 units (2) $\frac{2}{3}$ units

(3) $\frac{1}{3}$ units (4) $\frac{4}{\sqrt{3}}$ units

Answer (4)

Sol. Length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2\times 4}{\sqrt{12}}$

$= \frac{4}{\sqrt{3}}$

$\therefore$ Option (4) is correct.

58. $\lim\limits_{x\to 1}\frac{x^4-\sqrt{x}}{\sqrt{x}-1}$ is

(1) $\frac{1}{2}$ (2) 0

(3) 7 (4) Does not exist

Answer (3)

Sol. $\lim\limits_{x\to 1}\frac{x^4-\sqrt{x}}{\sqrt{x}-1}$

0/0 form, applying L'Hospital.

$$\lim_{x\to 1}\frac{\frac{d}{dx}(x^4-\sqrt{x})}{\frac{d}{dx}(\sqrt{x}-1)} = \lim_{x\to 1}\frac{4x^3-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}-0}$$

$$= \frac{4-\frac{1}{2}}{\frac{1}{2}}$$

$$= \frac{\frac{7}{2}}{\frac{1}{2}} = 7$$

$\therefore$ Option (3) is correct.

59. If $y = \frac{\cos x}{1+\sin x}$, then

(a) $\frac{dy}{dx} = \frac{-1}{1+\sin x}$

(b) $\frac{dy}{dx} = \frac{1}{1+\sin x}$

(c) $\frac{dy}{dx} = -\frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

(d) $\frac{dy}{dx} = \frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$

(1) Both (b) and (d) are correct

(2) Only (b) is correct

(3) Only (a) is correct

(4) Both (a) and (c) are correct

Answer (4)

Sol. $y = \frac{\cos x}{1+\sin x}$

Differentiable both sides w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \frac{(1+\sin x)\frac{d}{dx}(\cos x) - \cos x\left(\frac{d}{dx}(1+\sin x)\right)}{(1+\sin x)^2}$$

$$= \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$\Rightarrow \quad \frac{-1}{(1+\sin x)} = \frac{-1}{2}\left(\sec\left(\frac{\pi}{4} - \frac{\pi}{2}\right)\right)^2$$

$\therefore$ Option (4) is correct.

60. Match the following :

In the following, $[x]$ denotes the greatest integer less than or equal to x.

	Column-I		**Column-II**
(a)	$x\|x\|$	(i)	Continuous in $(-1, 1)$
(b)	$\sqrt{\|x\|}$	(ii)	Differentiable in $(-1, 1)$
(c)	$x + [x]$	(iii)	Strictly increasing in $(-1, 1)$
(d)	$\|x-1\| + \|x+1\|$	(iv)	Not differentiable at, at least one point in $(-1, 1)$

(1) a – iii, b – ii, c – iv, d – i

(2) a – i, b – ii, c – iv, d – iii

(3) a – iv, b – iii, c – i, d – ii

(4) a – ii, b – iv, c – iii, d – i

Answer (4)

Sol. (a) $x|x|$ is continuous in $(-1, 1)$ and differentiable in $(-1, 1)$.

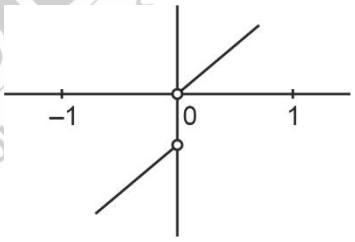
$\therefore$ (ii) is correct.

(b) $\sqrt{|x|}$ is not differentiable in $(-1, 1)$.

$\therefore$ (iv) is correct.

(c) $x + [x] = \begin{cases} x-1, & x \in (-1, 0) \\ x, & x \in [0, 1) \end{cases}$

$\therefore$ (iii) is correct.



(d) $|x-1| + |x+1|$ is continuous in $(-1, 1)$.

$\therefore$ (i) is correct.

So, option (4) is correct.

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