Chapter 5

Mathematical Induction

FIRST PRINCIPLE OF MATHEMATICAL INDUCTION

The proposition $P(n)$ involving a natural number n is assumed to be true for all $n \in N$, follows the following three steps:

Step -I (Verification step)

Actual verification of the proposition $P(n)$ for the starting value of $n = 1$

Step -II (Induction step)

Assuming that if $P(n)$ is true for $n = k$; $k \geq 1$, prove that it is also true for $n = k + 1$.

Step -III (Generalization step)

Combining the above two steps leads to the conclusion that $P(n)$ is true for all integers $n \in N$.

SECOND PRINCIPLE OF MATHEMATICAL INDUCTION

(Extended Principle)

Sometimes, the first principle of mathematical induction does not suffice. In such cases we use the extended principle as below:

Step -I (Verification step)

We verify that $P(n)$ is true for 1 and 2 both.

Step -II (Induction step)

Assume that $P(n)$ is true for $n = k$ and $n = k + 1$, $k \geq i$, and prove that $P(n)$ is true for $n = (k + 2)$.

Step -III (Generalization step)

Combining the above two steps leads to the conclusion that $P(n)$ is thus true $\forall\ n \in N$.

Note : *The second principle of mathematical induction is useful to prove recurrence relations which involve three successive terms e.g.,* $\boxed{pT_{n+1} = qT_n + rT_{n-1}}$.

APPLICATION OF MATHEMATICAL INDUCTION

I. Identities Type Problems

II. Divisibility Type Problems

To prove that $P(n)$ is divisible by x, following procedure is followed

(i) First we show that $P(1)$ is divisible by x.

(ii) Assuming that $P(m)$ is divisible by x, it is proved that $P(m + 1)$ is also divisible by x. For this either divide $P(m + 1)$ with $P(m)$ and show that remainder is divisible by x.

or

Split $P(m + 1) = P(m) . I + R$; $I \in Z$ and show that R is divisible by x.

III. Inequalities Type Problems

IV. Problems Based on Extended Principle of Mathematical Induction

If the given problem cannot be solved by direct use of principle mathematical induction, try to use the extended principle of mathematical induction.

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