

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given set = $\{1, 2, 3, \dots, 50\}$

One number is selected randomly from set. Find probability that number is multiple of 4 or 6 or 7.

- (1) $\frac{21}{50}$ (2) $\frac{18}{50}$
(3) $\frac{8}{25}$ (4) $\frac{21}{25}$

Answer (1)

Sol. Take $P(A)$ = Probability that number is multiple of 4

$P(B)$ = Probability that number is multiple of 6

$P(C)$ = Probability that number is multiple of 7

$$P(A) = \frac{12}{50}, P(B) = \frac{8}{50}, P(C) = \frac{7}{50}$$

$$P(A \cap B) = \frac{4}{50} \text{ (Multiple of 12)}$$

$$P(B \cap C) = \frac{1}{50} \text{ (Multiple of 42)}$$

$$P(A \cap C) = \frac{1}{50} \text{ (Multiple of 28)}$$

$$P(A \cap B \cap C) = 0 \text{ (Multiple of 84)}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0$$

$$= \frac{21}{50}$$

2. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x} dx$ is

- (1) $\sqrt{2} - \sqrt{3} + 1$ (2) $2\sqrt{2} - \sqrt{3} - 1$
(3) $2\sqrt{2} + \sqrt{3} - 1$ (4) $\sqrt{2} + \sqrt{3} - 1$

Answer (2)

Sol. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x| dx$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x) dx$$

$$= (\sin x + \cos x) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} + (-\sin x - \cos x) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - \left(\sin \frac{\pi}{6} + \cos \frac{\pi}{6} \right) \right] +$$

$$\left[\left(-\sin \frac{\pi}{3} - \cos \frac{\pi}{3} \right) - \left(-\sin \frac{\pi}{4} + \cos \frac{\pi}{4} \right) \right]$$

$$= \left[\sqrt{2} - \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \right] + \left[-\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right]$$

$$= 2\sqrt{2} - \sqrt{3} - 1$$

3. $A = \{1, 2, 3, 4\}$ minimum number of elements added to make it equivalence relation on set A containing (1, 3) and (1, 2) in it.

- (1) 8 (2) 9
(3) 12 (4) 16

Answer (1)

Sol. Set $A = \{1, 2, 3, 4\}$

For reflexive relation

We need to have (1, 1), (2, 2), (3, 3), (4, 4).

For symmetric,

(1, 3) $\in A$

So (3, 1) should be added

And (1, 2) $\in A$

So (2, 1) should be added

set has become $\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (1, 2), (2, 1)\}$

Now (3, 1) $\in A$

(1, 2) $\in A$

So (3, 2) should be added (for transitive)

Then (2, 3) should be added (for symmetric)

So set becomes

$\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (1, 2), (2, 1), (3, 2), (2, 3)\}$

So minimum 8 elements are added

4. If $\ln a, \ln b, \ln c$ are in AP and $\ln a - \ln 2b, \ln 2b - \ln 3c, \ln 3c - \ln a$ are in AP then $a : b : c$ is

- (1) $1 : 2 : 3$ (2) $7 : 7 : 4$
(3) $9 : 9 : 4$ (4) $4 : 4 : 9$

Answer (3)

Sol. $\ln a, \ln b, \ln c \rightarrow AP$

$$\Rightarrow b^2 = ac \quad \dots(i)$$

$$\ln \frac{a}{2b}, \ln \frac{2b}{3c}, \ln \frac{3c}{a} \rightarrow AP$$

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

$$\frac{4b^2}{9c^2} = \frac{3c}{2b}$$

$$8b^3 = 27c^3$$

$$\boxed{2b = 3c} \quad \dots(ii) \Rightarrow \boxed{4b = 9c}$$

$$4b^2 = 9c^2$$

$$4ac = 9c^2$$

$$\Rightarrow \boxed{4a = 9c} \quad \dots(iii)$$

From (ii) & (iii)

$$4a = 9c = 4b = k$$

$$a = \frac{k}{4}, b = \frac{k}{4}, c = \frac{k}{9}$$

$$a : b : c = \frac{1}{4} : \frac{1}{4} : \frac{1}{9}$$

$$a : b : c = 9 : 9 : 4$$

5. If $r = |z|$, $\theta = \arg(z)$ and $z = 2 - 2i \tan\left(\frac{5\pi}{8}\right)$ then find

(r, θ)

(1) $\left(2 \sec \frac{5\pi}{8}, \frac{3\pi}{8}\right)$ (2) $\left(2 \sec \frac{3\pi}{8}, \frac{3\pi}{8}\right)$

(3) $\left(2 \tan \frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (4) $\left(2 \tan \frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Answer (2)

Sol. $z = 2 - 2i \frac{\sin \frac{5\pi}{8}}{\cos \frac{5\pi}{8}}$

$$= \frac{2}{\cos \frac{5\pi}{8}} \left(\cos \frac{5\pi}{8} - i \sin \frac{5\pi}{8} \right)$$

$$= \frac{2}{\cos \frac{5\pi}{8}} e^{i \frac{(-5\pi)}{8}}$$

$$= 2 \sec \left(\frac{5\pi}{8} \right) e^{i \frac{(-5\pi)}{8}}$$

$$= 2 \sec \left(\frac{3\pi}{8} \right) e^{ir} e^{i \frac{(-5\pi)}{8}}$$

$$= 2 \sec \frac{3\pi}{8} e^{i \frac{(3\pi)}{8}}$$

$$\theta = \frac{3\pi}{8}, r = 2 \sec \frac{3\pi}{8}$$

6. In which interval the function $f(x) = \frac{x}{x^2 - 6x - 16}$ is increasing?

- (1) ϕ (2) $\left[1, \frac{3}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$
(3) $\left(\frac{5}{4}, \infty\right)$ (4) $\left[\frac{3}{4}, \frac{5}{4}\right]$

Answer (1)

Sol. $f(x) = \frac{x}{x^2 - 6x - 16}$

$$f'(x) = \frac{(x^2 - 6x - 16) - (x)(2x - 6)}{(x^2 - 6x - 16)^2}$$

$$\Rightarrow \frac{-x^2 - 16}{(x^2 - 6x - 16)^2} < 0 \quad \forall x \in D_f$$

$$\therefore x \in \phi$$

7. (α, β) lie on the parabola $y^2 = 4x$ and (α, β) also lie on chord with mid-point $\left(1, \frac{5}{4}\right)$ of another parabola

$x^2 = 8y$, then value of $|(8 - \beta)(\alpha - 28)|$ is

- (1) 192 (2) 92
(3) 64 (4) 128

Answer (1)

Sol. Chord with point, $T = S_1$

$$\Rightarrow xx_1 - 4(y + y_1) = x_1^2 - 8y_1$$

$$(x_1, y_1) \equiv \left(1, \frac{5}{4}\right) \Rightarrow x - 4\left(y + \frac{5}{4}\right) = \frac{1 - 8 \times 5}{4}$$

$$x - 4y - 5 = -9$$

$$\Rightarrow x - 4y + 4 = 0 \quad (L1)$$

(α, β) lie on $(L1)$ and also $y^2 = 4x$

$$\Rightarrow \alpha - 4\beta + 4 = 0$$

$$\beta^2 = 4\alpha$$

$$\beta^2 = 4(4\beta - 4)$$

$$\beta^2 - 16\beta + 16 = 0$$

$$\Rightarrow (\beta - 8)^2 = 64 - 16 = 48$$

$$\Rightarrow \beta = 8 \pm 4\sqrt{3}$$

$$\alpha = 4\beta - 4$$

$$= 28 \pm 16\sqrt{3}$$

$$(28 + 16\sqrt{3}, 8 + 4\sqrt{3}) \text{ and } (28 - 16\sqrt{3}, 8 - 4\sqrt{3})$$

$$(8 - \beta)(\alpha - 28)$$

$$\Rightarrow (-4\sqrt{3})(16\sqrt{3})$$

$$= -192$$

8. Unit vector $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$ makes angles

$$\frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3} \text{ with } \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}\right), \left(\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right),$$

$$\left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right) \text{ respectively and}$$

$$\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k} \text{ find } |\vec{u} - \vec{v}|.$$

$$(1) \sqrt{\frac{5}{2}}$$

$$(2) \sqrt{\frac{7}{2}}$$

$$(3) \sqrt{\frac{2}{5}}$$

$$(4) \sqrt{\frac{2}{7}}$$

Answer (1)

Sol. $\frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0 \quad \dots(1)$

$$\frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \quad \dots(2)$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{-1}{2} \quad \dots(3)$$

$$\Rightarrow y = 0, z = \frac{1}{\sqrt{2}}, x = \frac{-1}{\sqrt{2}}$$

$$\vec{v} - \vec{u} = \sqrt{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$|\vec{v} - \vec{u}| = \sqrt{2 + \frac{1}{2}}$$

$$= \sqrt{\frac{5}{2}}$$

9. If first term of non-constant GP be $\frac{1}{8}$ and every

term is AM of next two, then $\sum_{r=1}^{20} T_r - \sum_{r=1}^{18} T_r$ is

$$(1) 2^{15}$$

$$(2) -2^{15}$$

$$(3) -2^{18}$$

$$(4) 2^{18}$$

Answer (2)

Sol. $a_1 = \frac{1}{8}$

$$a, ar, ar^2, ar^3, \dots$$

$$2ar = ar^2 + ar^3$$

$$2 = r + r^2$$

$$r^2 + r - 2 = 0$$

$$(r + 2)(r - 1) = 0$$

$$r \neq 1$$

$$\Rightarrow r = -2$$

$$\sum_{r=1}^{20} T_r - \sum_{r=1}^{18} T_r$$

$$= \frac{a(1-r^{20})}{1-r} - \frac{a(1-r^{18})}{1-r}$$

$$= \frac{1}{8} \left[\frac{1}{3} [1 - r^{20} - 1 + r^{18}] \right]$$

$$= \frac{1}{24} 2^{18} [1 - 4]$$

$$= -\frac{2^{18}}{8} \Rightarrow -2^{15}$$

10. The mean of 5 observations is $\frac{24}{5}$ and variance is

$\frac{194}{25}$. If the mean of first four observations is $\frac{7}{2}$, then the variance of first four observations is

$$(1) \frac{3}{2}$$

$$(2) \frac{5}{2}$$

$$(3) \frac{5}{4}$$

$$(4) \frac{2}{3}$$

Answer (3)

Sol. $\sum_{i=1}^5 x_i = 24$

$$\frac{\sum x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$

$$\Rightarrow \sum x_i^2 = \frac{770}{25} \times 5 = 154$$

$$5^{\text{th}} \text{ observation} = 24 - \frac{7}{2} \times 4 = 10$$

$$\begin{aligned} \text{New variance} &= \frac{\sum_{i=1}^4 x_i^2}{4} - \left(\frac{7}{2}\right)^2 \\ &= \frac{154 - 100}{4} - \frac{49}{4} \\ &= \frac{5}{4} \end{aligned}$$

- 11.
- 12.
- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The remainder when $64^{32^{32}}$ is divided by 9 is

Answer (1)

Sol. $64 \equiv 1 \pmod{9}$

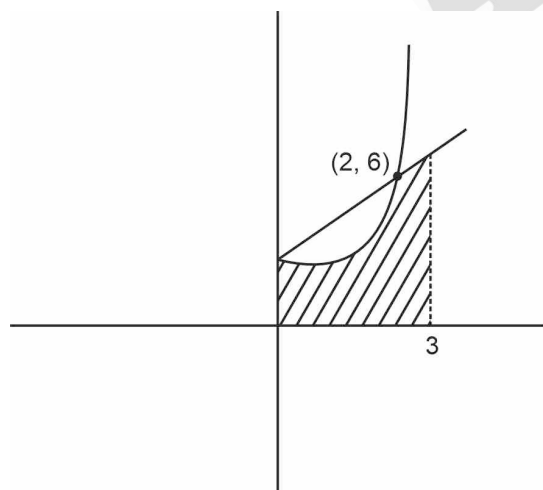
$$64^{32^{32}} \equiv 1^{32^{32}} \pmod{9}$$

$\Rightarrow$ Remainder = 1

22. Area bounded by $0 \leq y \leq \min\{x^2 + 2, 2x + 2\}$, $x \in [0, 3]$ is A, then $12A$ is

Answer (164)

Sol. $\min\{x^2 + 2, 2x + 2\} \begin{cases} x^2 + 2 & 0 \leq x \leq 2 \\ 2x + 2 & 2 \leq x \leq 3 \end{cases}$



$$\text{Area} = A = \int_0^2 (x^2 + 2) dx + \frac{1}{2} [6 + 8] \times 1$$

$$= \left[\frac{x^3}{3} + 2x \right]_0^2 + 7$$

$$\frac{8}{3} + 4 + 7 = \left(\frac{8}{3} + 11 \right) \text{ unit}$$

$$12A = 12 \left(\frac{8}{3} + 11 \right) = 164$$

23. The number of ways to distribute 8 identical books into 4 distinct bookshelf is (where any bookshelf can be empty)

Answer (165)

Sol. $x_1 + x_2 + x_3 + x_4 = 8$

$$\text{Number of ways} = \binom{8+4-1}{4-1}$$

$$= \binom{11}{3}$$

$$= 165$$

24. If $f(x) = \ln\left(\frac{1-x^2}{1+x^2}\right)$ then value of $225(f'(x) - f''(x))$

$$\text{at } x = \frac{1}{2}$$

Answer (736)

Sol. $f(x) = \ln(1 - x^2) - \ln(1 + x^2)$

$$f'(x) = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2}$$

$$= -2x \left[\frac{2}{1-x^4} \right]$$

$$f'(x) = \frac{4x}{x^4 - 1}$$

$$f''(x) = 4 \left[\frac{(x^4 - 1) - 4x^4}{(x^4 - 1)^2} \right]$$

$$= 4 \left[\frac{-3x^4 - 1}{(x^4 - 1)^2} \right]$$

$$f'(x) - f''(x) = 4 \left[\frac{x}{x^4 - 1} + \frac{3x^4 + 1}{(x^4 - 1)^2} \right]$$

$$\text{At } x = \frac{1}{2}$$

$$225[f'(x) - f''(x)] = 736$$

25. $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$, then find sum of roots,

Answer (1)

Sol. $\therefore$

$$\frac{\cos 2x(3 + \cos^2 2x)}{(\cos^2 x - \sin^2 x)[\sin^4 x + \cos^4 x + \sin^2 x \cos^2 x]},$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$= \frac{3 + \cos^2 2x}{1 - \sin^2 x \cos^2 x} = 4 \left(\frac{3 + \cos^2 2x}{4 - \sin^2 2x} \right) = 4$$

$$\Rightarrow x^3 - x^2 + 6 = 4$$

$$\Rightarrow x^3 - x^2 + 2 = 0$$

$\therefore$ therefore sum of roots = 1

26. $x \left(\cos \left(\frac{y}{x} \right) \right) \frac{dy}{dx} = y \cos \left(\frac{y}{x} \right) + x$

where $\sin \left(\frac{y}{x} \right) = \ln |x| + \frac{\alpha}{2}$ and $f(1) = \frac{\pi}{3}$

Find α^2 .

Answer (3)

Sol. $\therefore \left(\cos \frac{y}{x} \right) \frac{dy}{dx} = \frac{y}{x} \cos \frac{y}{x} + 1$

Putting $y = vx$

$$\Rightarrow \frac{dy}{dx} = x \frac{dv}{dx} + v$$

$$\Rightarrow \cos v \left(x \frac{dv}{dx} + v \right) = v \cos v + 1$$

$$\Rightarrow \int \cos v dv = \int \frac{dx}{x}$$

$$\Rightarrow \sin \frac{y}{x} = \ln |x| + c$$

where $c = \frac{\alpha}{2}$

putting initial condition,

$$2 \sin \frac{\pi}{3} = \alpha$$

$$\Rightarrow \alpha = \sqrt{3}$$

$$\Rightarrow \alpha^2 = 3$$

27. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, and area of $\triangle OAC$ is S and a parallelogram with sides parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ and diagonal $\overrightarrow{OB} = 12\vec{a} + 4\vec{b}$, has area equal to B , then $\frac{B}{S}$ is equal to

Answer (96)

Sol. $S = \frac{1}{2} |\vec{a} \times \vec{b}|$

$$B = |12\vec{a} \times 4\vec{b}|$$

$$\Rightarrow \frac{B}{S} = \frac{48 |\vec{a} \times \vec{b}|}{\frac{1}{2} |\vec{a} \times \vec{b}|} = 96$$

28.

29.

30.

