



Code Number:

A

Aakash

Medical | IIT-JEE | Foundations

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Gurugram, Haryana - 122015

Mock Test Paper for Class XII

Time: 3 hrs.

MATHEMATICS

Max. Marks: 80

Roll No.

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GENERAL INSTRUCTIONS

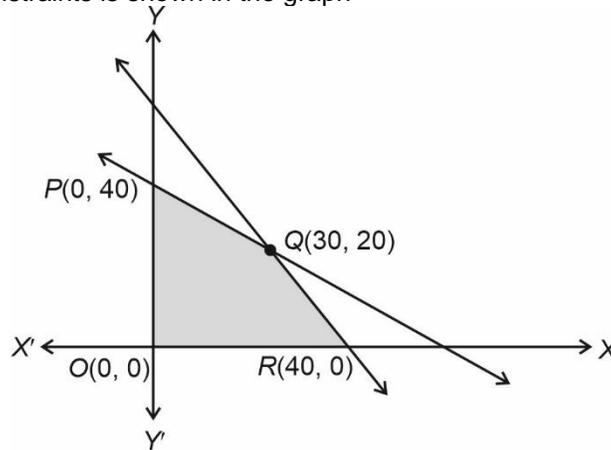
Read the following instructions carefully and follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D and E**.
- (iii) In **Section A**, Questions no. **1 to 18** are **Multiple Choice type questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.
- (iv) In **Section B**, Questions no. **21 to 25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.
- (v) In **Section C**, Questions no. **26 to 31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.
- (vi) In **Section D** Questions no. **32 to 35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.
- (vii) In **Section E** Questions no. **36 to 38** are **Case study-based questions**, carrying **4 marks each**.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Sections B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
- (ix) Use of calculators in **not** allowed.

SECTION-A

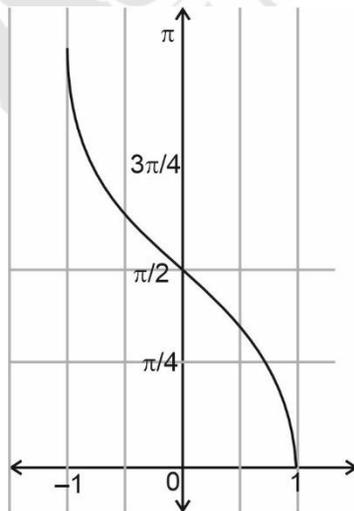
1. If for a square matrix A , $A(\text{adj}A) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix}$, then the value of $|A| + |\text{adj}A|$ is equal to
- (a) 1 (b) $2025 + 1$
(c) $(2025)^2 + 45$ (d) $2025 + (2025)^2$
2. Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively. Then the restriction on n, k and p so that $PY + WY$ will be defined are
- (a) $k = 3, p = n$ (b) k is arbitrary, $p = 2$
(c) p is arbitrary, $k = 3$ (d) $k = 2, p = 3$
3. The exhaustive set of values of x in which the function f defined by $f(x) = e^x$ is strictly increasing, is
- (a) $[1, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$
4. If A and B are non-singular matrices of same order with $\det(A) = 5$, then $\det(B^{-1}AB)^2$ is equal to
- (a) 5 (b) 5^2
(c) 5^4 (d) 5^5
5. The value of 'n', such that the differential equation $x^n \frac{dy}{dx} = y(\log y - \log x + 1)$ (where $x, y \in R^+$) is homogeneous, is
- (a) 0 (b) 1
(c) 2 (d) 3
6. If the points $(x_1, y_1), (x_2, y_2)$ and $(x_1 + x_2, y_1 + y_2)$ are collinear, then $x_1 y_2$ is equal to
- (a) $x_2 y_1$ (b) $x_1 y_1$
(c) $x_2 y_2$ (d) $x_1 x_2$
7. If $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ is a skew-symmetric matrix then the value of $a + b + c =$
- (a) 1 (b) 2
(c) 3 (d) 4
8. For any two events A and B , if $P(\bar{A}) = \frac{1}{2}, P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ equals
- (a) $\frac{3}{8}$ (b) $\frac{8}{9}$
(c) $\frac{5}{8}$ (d) $\frac{1}{4}$
9. The interval of value of α if the angle between $\vec{p} = 2\alpha\hat{i} - 3\alpha\hat{j} + k$ and $\vec{q} = \hat{i} + \hat{j} + \alpha k$ is obtuse, is
- (a) $R - [0, 1]$ (b) $(0, 1)$
(c) $[0, \infty)$ (d) $[1, \infty)$
10. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}| =$
- (a) 3 (b) 4
(c) 5 (d) 8

11. For the linear programming problem (LLPP), the objective function is $z = 4x + 3y$ and the feasible region determined by a set of constraints is shown in the graph



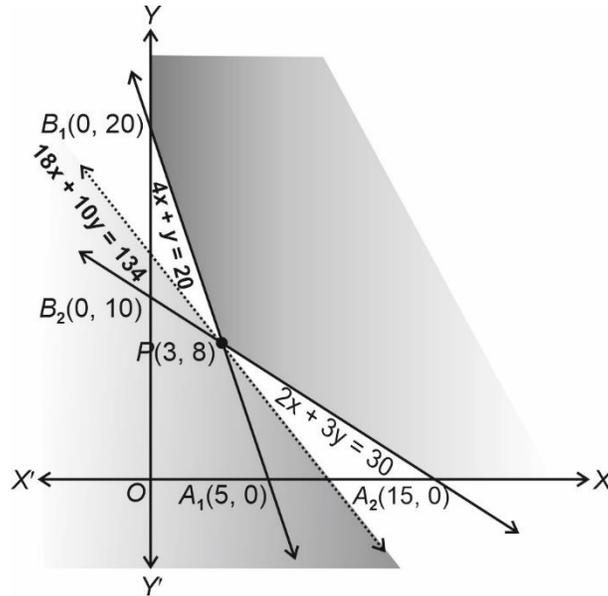
Which of the following statements is true?

- (a) Maximum value of Z is at $R(40, 0)$
 - (b) Maximum value of Z is at $Q(30, 20)$
 - (c) Value of Z at $R(40, 0)$ is less than the value of z at $P(0, 40)$
 - (d) The value of Z at $Q(30, 20)$ is less than the value of z at $R(40, 0)$
12. $\int \frac{dx}{x^3(1+x^4)^{1/2}}$ equals
- (a) $-\frac{1}{2x^2}\sqrt{1+x^4} + c$
 - (b) $\frac{1}{2x}\sqrt{1+x^4} + c$
 - (c) $-\frac{1}{4x}\sqrt{1+x^4} + c$
 - (d) $\frac{1}{4x^2}\sqrt{1+x^4} + c$
13. $\int_0^{2\pi} \operatorname{cosec}^7 x dx =$
- (a) 0
 - (b) 1
 - (c) 4
 - (d) 2π
14. What is the general solution of the differential equation $e^{y'} = x$? (where $y' = \frac{dy}{dx}$)
- (a) $y = x \log x + c$
 - (b) $y = x \log x - x + c$
 - (c) $y = x \log x + x + c$
 - (d) $y = x + c$
15. The graph drawn below depicts



- (a) $y = \sin^{-1}x$
- (b) $y = \cos^{-1}x$
- (c) $y = \operatorname{cosec}^{-1}x$
- (d) $y = \cot^{-1}x$

16. A linear programming problem (LPP) along with the graph of its constraints is shown below. The corresponding objective function is $Z = 18x + 10y$, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point $(3, 8)$.



(Note : The figure is not to scale)

- The optimal solution of the above linear programming problem _____.
- (a) Does not exist as the feasible region is unbounded
 (b) Does not exist as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region
 (c) Exists as the inequality $18x + 10y > 134$ has infinitely many points in common with the feasible region
 (d) Exists as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region
17. The function $f : \mathbb{R} \rightarrow \mathbb{Z}$ defined by $f(x) = [x]$; where $[.]$ denotes the greatest integer function, is
- (a) Continuous at $x = 2.5$ but not differentiable at $x = 2.5$
 (b) Not continuous at $x = 2.5$ but differentiable at $x = 2.5$
 (c) Not continuous at $x = 2.5$ and not differentiable at $x = 2.5$
 (d) Continuous as well as differentiable at $x = 2.5$
18. A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is parabolic shape given by $x^2 = 4y$. Then the area (in sq units) of the region bounded by parabola $x^2 = 4y$ and the line $y = 4$ is
- (a) $\frac{32}{3}$ (b) $\frac{64}{3}$
 (c) $\frac{128}{3}$ (d) $\frac{256}{3}$

ASSERTION-REASON BASED QUESTIONS

Questions numbers 19 to 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given – one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (c) Assertion (A) is true, but Reason (R) is false.
 (d) Assertion (A) is false and Reason (R) is also false.

19. **Assertion (A)** : Consider the function defined as $f(x) = |x| + |x-1|, x \in R$. Then $f(x)$ is not differentiable at $x = 0$ and $x = 1$.

Reason (R) : Suppose f be defined and continuous on (a, b) and $c \in (a, b)$, then $f(x)$ is not differentiable at

$$x = c \text{ if } \lim_{h \rightarrow 0^-} \frac{f(c+h) - f(c)}{h} \neq \lim_{h \rightarrow 0^+} \frac{f(c+h) - f(c)}{h}.$$

20. **Assertion (A)** : The function $f : R - \left\{ (2n+1)\frac{\pi}{2}; n \in Z \right\} \rightarrow (-\infty, -1] \cup [1, \infty)$ defined by $f(x) = \sec x$ is not one-one function in its domain

Reason (R) : The line $y = 2$ meets the graph of the function at more than one point

SECTION-B

21. Find the derivative of $\sin^{-1}x$ with respect to $\sin x$.
22. A balloon is being inflated, and its volume increases uniformly. The radius of the balloon is expanding at a rate of 3 cm/sec. At the moment when the radius is 10 cm, how fast is the surface area of the balloon increasing?

OR

Find two positive numbers whose sum is 36 and the sum of whose squares is minimum.

23. Check if function f defined by $f(x) = \begin{cases} kx+1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$ for $k = \frac{2}{\pi}$. If not then, what is the value of k for which it is continuous?
24. Given a relation R_1 on the set R of real numbers as $R_1 = \{(x, y) : x^2 - 3xy + 2y^2 = 0, x, y \in R\}$. Is the relation R_1 reflexive or symmetric?
25. A river's flow rate can be represented by the function $f(t) = 24t - 6t^2$ cm/h, where t is the time in hours. Find the total volume of water that flows past the certain point in the river from $t = 0$ hour to $t = 4$ hours.

OR

Evaluate : $\int \frac{\sqrt{\cos \theta}}{\sin \theta} d\theta$

SECTION-C

26. Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive, nor symmetric and nor transitive

OR

If $f : R \rightarrow R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijective function

27. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then evaluate $A^3 - 3A - 2I$. Also, find $|A|$

OR

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers we get double of the second number. Represent it algebraically and find the numbers using matrix method

28. Sand is pouring from the pipe at the rate $12 \text{ cm}^3/\text{s}$. The falling sand forms a cone on a ground in such a way that the height of cone is always one-sixth of the radius of the base. How fast is the height of sand cone increasing when the height is 4 cm.

29. Ravi is setting up two support cables for a structure and needs to make sure they are correctly spaced apart. The positions of the cables are given by the equations.

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Evaluate the distance between two cables.

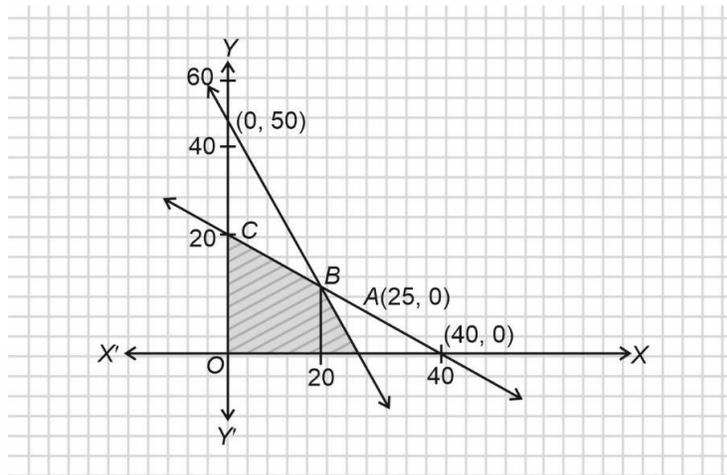
OR

Prashant is mapping out the coordinates for new features in his designed project. If O is the origin and OP is 3 units long bar with direction ratios proportional to $-1, 2, -2$, what are the co-ordinates of point P ?

30. A bag A contains 4 white and 5 red balls and a bag B contains 5 white and 4 red balls. One ball is drawn at random from one of the bags and it is found to be white. If the probability, that it was drawn from the bag A, is

$\frac{p}{q}$ where p and q are coprime, then value of $p + q$.

31. Feasible region of an LPP is given as



- (A) Write the constraints in terms of x and y .
 (B) Find the coordinates of B and C and maximise, $z = x + y$.

SECTION-D

32. Vertices B and C of $\triangle ABC$ lie along the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$. Find the area of the triangle given that A has coordinates $(1, -1, 2)$ and the line segment BC has length 5 units.

OR

Find the equation of a line passing through the point $(1, 2, -4)$ and perpendicular to two lines :

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \text{ and } \vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

33. Make a rough sketch of the area bounded by $2x - y \geq 1$, $x + 4y \geq 14$ and $5x + 2y \leq 34$ and find its area using integration.
34. The sum of surface areas of a sphere and a cuboid with sides $\frac{x}{3}$, x and $2x$ is constant. Show that the sum of their volumes is minimum if x is equal to three times the radius of sphere.

OR

An advertisement is supplied with decorative wire pieces of 34 m each and are asked to cut the wire into two pieces. From one piece a circular sign board is to be made and from another a square one. The idea is to keep the sum of the areas enclosed by a circle and square minimum for writing slogans. If wire is cut at ' x ' m from one end and made into a circle and r represents the radius of the circle. What is the minimum combined area of the window?

35. (A) Prove that the general solution of the differential equation $y - x \frac{dy}{dx} = x + y \frac{dy}{dx}$ is $\tan^{-1} \frac{y}{x} + \frac{1}{2} \log |x^2 + y^2| = C$, where C is constant of integration.

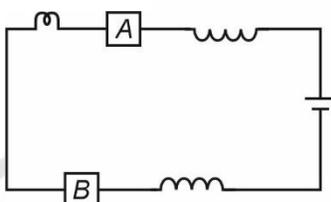
(B) Write order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \frac{dy}{dx} + \cos^2 y = 0$.

SECTION-E

36. Case Study-1

An electric circuit includes a device that gives energy to the charged particles constituting the current, such as a battery or a generator; devices that use current, such as lamps, electric motors, or computers; and the connecting wires or transmission lines.

An electric circuit consists of two subsystems say A and B as shown below:



For previous testing procedures, the following probabilities are assumed to be known.

$$P(A \text{ fails}) = 0.2, P(B \text{ fails alone}) = 0.15, P(A \text{ and } B \text{ fail}) = 0.15$$

Based on the given information, answer the following questions:

- (A) What is the probability that B fails?
 (B) What is the probability that A fails alone?
 (C) Find the probability that whole of the electric system fails.

OR

Find the conditional probability that B fails when A has already failed.

37. Case Study-1

To promote the usage of house toilets in villages especially for women, an organisation tried to generate awareness among the villagers through (i) house calls (ii) letters, and (iii) announcements

The cost for each mode per attempt is given below

- (i) ₹50 (ii) ₹20 (iii) ₹40



The number of attempts made in the villages X, Y and Z are given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also the change of making of toilets corresponding to one attempt of given modes is :

(i) 2% (ii) 4% (iii) 20%

Let A, B, C be the cost incurred by organisation in three villages, respectively.

Based on the given information, answer the following questions :

- (A) Form a matrix related to cost incurred by organisation for villages X, Y and Z.
 (B) Form a matrix related to the number of toilets expected in villages X, Y, Z after the promotion campaign
 (C) What is total amount spent by the organisation in all three villages X, Y and Z?

OR

What is the total no. of toilets expected after the promotion campaign?

38. Case Study-3

Through the ocean, a sailor is steering a sailboat. The sailboat's position at any given time t can be depicted using vector $\vec{r}(t) = (2t+1)\hat{i} + (t^2 - 2)\hat{j}$, where t is the time in hours, \hat{i} and \hat{j} are unit vectors along the x and y-axes, respectively.



Based on the given information, answer the following questions:

- (A) Find the speed of the sailboat at $t = 2$ hours if the speed is given by the magnitude of the velocity vector.
 (B) Find the direction of motion of the sailboat at $t = 2$ hours in terms of a unit vector. Also verify whether the sailboat is moving in straight line by determining if the velocity vector remains constant over time.

