Code Number: **A**

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Medical | IIT-JEE | Foundations

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Mock Test Paper for Class XII

MATHEMATICS

Time: 3 hrs.

Max. Marks: 80

Answers & Solutions

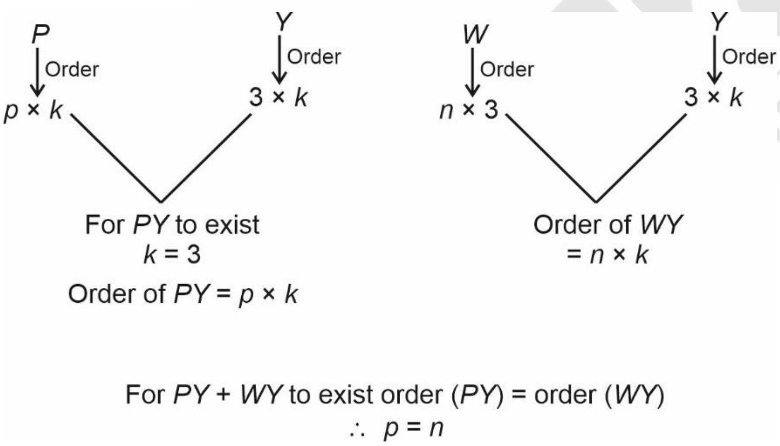
1. Answer (d) **[1]**

For a square matrix A of order $n \times n$, we have $A.(adj\ A) = |A|I_n$, where I_n is the identity matrix of order $n \times n$.

So, $A.(\text{adj } A) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix} = 2025I_3 \Rightarrow |A| = 2025\ \&\ |\text{adj} A| = |A|^{3-1} = (2025)^2$

$\therefore$ $|A| + |\text{adj } A| = 2025 + (2025)^2$.

2. Answer (a) **[1]**



3. Answer (c) **[1]**

$y = e^x \Rightarrow \dfrac{dy}{dx} = e^x$

In the domain (R) of the function, $\dfrac{dy}{dx} > 0$, hence the exhaustive set of values of x in which function is strictly increasing in $(-\infty, \infty)$

4. Answer (b) **[1]**

$|A| = 5, |B^{-1}AB|^2 = \left(|B^{-1}||A||B|\right)^2 = |A|^2 = 5^2$

5. Answer (b) **[1]**

A differential equation of the form $\frac{dy}{dx} = f(x, y)$ is said to be homogeneous, if $f(x, y)$ is a homogeneous function of degree 0.

Now, $x^n \frac{dy}{dx} = y\left(\log_e \frac{y}{x} + \log_e e\right) \Rightarrow \frac{dy}{dx} = \frac{y}{x^n}\left(\log_e e\left(\frac{y}{x}\right)\right)$ = $f(x, y)$; (Let). $f(x, y)$ will be a homogeneous function of degree 0, if $n = 1$.

6. Answer (a) **[1]**

Method 1 : (Short cut)

When the points (x_1, y_1), (x_2, y_2) and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_1 - (x_1 + x_2) & y_1 - (y_1 + y_2) \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ -x_2 & -y_2 \end{vmatrix} = (-x_1y_2 + x_2y_2 + x_2y_1 - x_2y_2) = 0$$

$\Rightarrow x_2y_1 = x_1y_2$.

Method 2 :

When the points (x_1, y_1), (x_2, y_2) and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_1 + x_2 & y_1 + y_2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1.(x_2y_1 + x_2y_2 - x_1y_2 - x_2y_2) - 1(x_1y_1 + x_1y_2 - x_1y_1 - x_2y_1) + (x_1y_2 - x_2y_1) = 0$

$\Rightarrow x_2y_1 = x_1y_2$.

7. Answer (a) **[1]**

$$A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$$

When the matrix A is skew symmetric then $A^T = -A \Rightarrow a_{ij} = -a_{ji}$;

$\Rightarrow c = -2$; $a = 0$ and $b = 3$

So, $a + b + c = 0 + 3 - 2 = 1$

8. Answer (c) **[1]**

$$P\left(\bar{A}\right) = \frac{1}{2}; P\left(\bar{B}\right) = \frac{2}{3}; P(A \cap B) = \frac{1}{4}$$

$$\Rightarrow P(A) = \frac{1}{2}; P(B) = \frac{1}{3}$$

We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$

$$P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P\left(\bar{A} \cap \bar{B}\right)}{P\left(\bar{B}\right)} = \frac{P(\overrightarrow{A \cup B})}{P(\bar{B})} = \frac{1 - P(A \cup B)}{P\left(\bar{B}\right)} = \frac{1 - \frac{7}{12}}{\frac{2}{3}} = \frac{5}{8}$$

9. Answer (b) **[1]**

For obtuse angle, $\cos\theta < 0 \Rightarrow \vec{p}\cdot\vec{q} < 0$

$2\alpha^2 - 3\alpha + \alpha < 0 \Rightarrow 2\alpha^2 - 2\alpha < 0 \Rightarrow \alpha \in (0, 1)$

10. Answer (c) **[1]**

$|\vec{a}| = 3, |\vec{b}| = 4, |\vec{a}+\vec{b}| = 5$

We have, $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left(|\vec{a}|^2 + |\vec{b}|^2\right) = 2(9+16) = 50 \Rightarrow |\vec{a}-\vec{b}| = 5$

11. Answer (b) **[1]**

Corner point	Value of the objective function $Z = 4x + 3y$
1. $O(0, 0)$	$z = 0$
2. $R(40, 0)$	$z = 160$
3. $Q(30, 20)$	$z = 120 + 60 = 180$
4. $P(0, 40)$	$z = 120$

Since, the feasible region is bounded so the maximum value of the objective function $z = 180$ is at $Q(30, 20)$

12. Answer (A) **[1]**

$$\int \frac{dx}{x^3(1+x^4)^{1/2}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{1/2}}$$

(Let $1+x^{-4} = 1+\frac{1}{x^4} = t,\ dt = -4x^{-5}dx = -\frac{4}{x^5}dx \Rightarrow \frac{dx}{x^5} = -\frac{1}{4}dt$)

$= -\frac{1}{4}\int \frac{dt}{t^{1/2}} = -\frac{1}{4}\times 2\times\sqrt{t} + c$, where 'c' denotes any arbitrary constant of integration

$$= -\frac{1}{2}\sqrt{1+\frac{1}{x^4}} + c = -\frac{1}{2x^2}\sqrt{1+x^4} + c$$

13. Answer (a) **[1]**

We know, $\int_0^{2a} f(x)dx = 0$, if $f(2a - x) = -f(x)$

Let $f(x) = \text{cosec}^7 x$

Now, $f(2\pi - x) = \text{cosec}^7(2\pi - x) = -\text{cosec}^7 x = -f(x)$

$\therefore \int_0^{2\pi} \text{cosec}^7 x\, dx = 0;$ Using the property $\int_0^{2a} f(x)dx = 0,$ if $f(2a - x) = -f(x)$

14. Answer (b) **[1]**

The given differential equation $e^{y'} = x \Rightarrow \frac{dy}{dx} = \log x$

$dy = \log x dx \Rightarrow \int dy = \int \log x\, dx$

$y = x\log x - x + c$

hence the correct option is (b)

15. Answer (b) **[1]**

The graph represents $y = \cos^{-1}x$ whose domain is [–1, 1] and range is [0, π]

16. Answer (d) **[1]**

Since the inequality $Z = 18x + 10y < 134$ has no point in common with the feasible region hence the minimum value of the objective function $Z = 18x + 10y$ is 134 at $P(3, 8)$

17. Answer (d) **[1]**

The graph of the function $f : R \to R$ defined by $f(x) = [x]$; (where [.] denotes G.I.F) is a straight line $\forall x \in (2.5 - h, 2.5 + h)$, '$h$' is an infinitesimally small positive quantity. Hence, the function is continuous and differentiable at $x = 2.5$.

18. Answer (b) **[1]**

The required region is symmetric about the y-axis

So, required area (in sq. units) is $= \left|2\int_0^4 2\sqrt{y}\,dy\right| = 4\left[\dfrac{y^{3/2}}{\frac{3}{2}}\right]_0^4 = \dfrac{64}{3}$

19. Answer (a) **[1]**

Both (A) and (R) are true and (R) is the correct explanation of (A)

20. Answer (a) **[1]**

Both (A) and (R) are true and (R) is the correct explanation of (A)

21. Let $y = \sin^{-1}x$ & $z = \sin x$

$\dfrac{dy}{dx} = \dfrac{d(\sin^{-1}x)}{dx}$ & $\dfrac{dz}{dx} = \dfrac{d}{dx}\sin x$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$ …..Equation (1) & $\dfrac{dz}{dx} = \cos x$

$\dfrac{dx}{dz} = \dfrac{1}{\cos x} = \sec x$ …..Equation (2) **[1]**

Multiply (1) & (2)

i.e. $\dfrac{dy}{dx} \times \dfrac{dx}{dz} = \dfrac{1}{\sqrt{1-x^2}} \times \sec x$

$\boxed{\dfrac{dy}{dz} = \dfrac{\sec x}{\sqrt{1-x^2}}}$ **[1]**

22. Given $\frac{dr}{dt} = 3 \text{ cm/s}$

To find $\frac{dS}{dt} = ?$ at $r = 10$ cm **[½]**

Sol. : $S = 4\pi r^2$

$\frac{dS}{dt} = \frac{d}{dt}(4\pi r^2)$ **[½]**

$\frac{dS}{dt} = 4\pi 2r\frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$ **[½]**

$\frac{dS}{dt} = 8\pi(10)3$

$\frac{dS}{dt} = 240\pi \text{ cm}^2/\text{s}$ **[½]**

OR

Let two numbers are x & y respectively

Given $x + y = 36$

$x^2 + y^2 = Z$ where Z is minimum value

To find value of x & y

i.e. two numbers **[½]**

Sol. : $Z = x^2 + y^2$ where $x + y = 36$

$y = 36 - x$

$Z = x^2 + (36 - x)^2$

$\frac{dZ}{dx} = 2x + 2(36 - x)(-1)$ **[½]**

$\frac{d^2Z}{dx^2} = 2 - 2(-1)$

$\frac{d^2Z}{dx^2} = 4 > 0$ *i.e.* minimum **[½]**

To find critical points put $\frac{dZ}{dx} = 0$

$2x - 2(36 - x) = 0$

$x - 36 + x = 0$

$2x = 36$

$x = 18 \Rightarrow y = 18$

Hence required numbers are 18 & 18 **[½]**

23. $f(x) = \begin{cases} kx+1, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases}$

$\text{LHL} f(\pi) = \lim_{h \to 0} k(\pi - h) + 1 = k\pi + 1$

$\text{RHL} f(\pi) = \lim_{h \to 0} \cos(\pi + h) = -1$ **[½]**

For $k = \frac{\pi}{2}$; $\text{LHL} f(\pi) = \frac{\pi^2}{2} + 1$ & $\text{RHL} f(\pi) = -1$

$\frac{\pi^2}{2} + 1 \ne -1$

$\therefore$ at $k = \frac{\pi}{2}$ not continuous **[½]**

In order to make it continuous

$\text{LHL} f(\pi) = \text{RHL} f(\pi)$

$k\pi + 1 = -1$ **[1]**

$k\pi = -2$

$\boxed{k = \frac{-2}{\pi}}$

24. $R_1 = \{(x, y); x^2 - 3xy + 2y^2 = 0, x, y \in R\}$

$x^2 - 3xy + 2y^2 = 0$

$x^2 - 2xy - xy + 2y^2 = 0$

$x(x - 2y) - y(x - 2y) = 0$

$(x - y)(x - 2y) = 0$

$\boxed{x = y}$ $\boxed{x = 2y}$ **[1]**

i.e. $(x, y) = (x, x) \in R$ Hence it is reflexive

But $(x, y) \in R$

& $(y, x) \notin R$ *i.e.*

let $(1, 2) \in R \Rightarrow 1 - 6 + 8 = 3$

$(2, 1) \in R \Rightarrow 4 - 6 + 2 = 0$

$3 \ne 0$

$\therefore$ Not symmetric **[1]**

25. $f(t) = 24t - 6t^2$ cm/hr $\leftarrow$ flow rate

Volume of water = $\int_0^4 f(t)dt$

= $\int (24t - 6t^2)dt$

$= \left(24\frac{t^2}{2} - 6\frac{t^3}{3}\right)\Big|_0^4$

$= \left(12t^2 - 2t^3\right)\Big|_0^4$ **[1]**

$= \left(12t^2 - 2t^3\right)\Big|_0^4$

$= 2t^2(6-t)\Big|_0^4$

$= 2(16)(6 – 4) – 0$

$V = 64 \text{ cm}^3$ **[1]**

OR

$\int \frac{\sqrt{\cos\theta}}{\sin\theta}\,d\theta$

$= \int \frac{\sqrt{\cos\theta}\cdot\sin\theta}{\sin^2\theta}\,d\theta$

$= \int \frac{\sqrt{\cos\theta}\cdot\sin\theta}{(1-\cos^2\theta)}\,d\theta$

$= -\int \frac{2t^2dt}{1-t^4}$ Let $t^2 = \cos\theta$

$2tdt = -\sin\theta d\theta$

$-2tdt = \sin\theta d\theta$ **[1]**

$= -\int \frac{(1+t^2)-(1-t^2)dt}{(1-t^2)(1+t^2)}$

$= -\int \frac{1}{1-t^2}dt + \int \frac{1}{1+t^2}dt$

$= -\frac{1}{2}\log\left|\frac{1+t}{1-t}\right| + \tan^{-1}(t)$

$= \tan^{-1}(t) - \frac{1}{2}\log\left|\frac{1+t}{1-t}\right|$

$= \boxed{\tan^{-1}\sqrt{\cos\theta} - \frac{1}{2}\log\left|\frac{1+\sqrt{\cos\theta}}{1-\sqrt{\cos\theta}}\right|}$ **[1]**

26. $S = \{(a, b); a, b \in R \text{ and } a \le b^3\}$

Reflexive if $(x, x) \in R \Rightarrow R$ is reflexive

Lets check for $\left(\frac{1}{2}, \frac{1}{2}\right)$ *i.e.* $\frac{1}{2} \le \left(\frac{1}{2}\right)^3$

$\frac{1}{2} \le \frac{1}{8}$

$0.5 \not\le 0.125$ **[1]**

Hence $(x, x) \notin R$ $\therefore$ 'S' is not reflexive

Symmetric if $(a, b) \in R$ & $(b, a) \in R$ $= R$ is symmetric

Lets check for $(-2, 3)$ *i.e.* $-2 \le (3)^3$ true

But $(3, -2)$ *i.e.* $3 \le (-2)^3$ false

$\therefore$ S is not symmetric **[1]**

Transitive if $(a, b) \in R$ & $(b, c) \in R$ & $(a, c) \in R$

Then R is transitive

Lets check for $\left(3, \frac{3}{2}\right)$ *i.e.* $3 \le \left(\frac{3}{2}\right)^3$ true

$\left(\frac{3}{2}, \frac{4}{3}\right)$ *i.e.* $\frac{3}{2} \le \left(\frac{4}{3}\right)^3$ true

But $\left(3, \frac{4}{3}\right)$ *i.e.* $3 \le \left(\frac{4}{3}\right)^3$ false

$\therefore$ 'S' is not transitive **[1]**

OR

$f : R \to R$

$f(x) = 4x^3 + 7$

Bijective functions are these function which follow conditions for injective & surjective. **[½]**

Injective (one-one)

Let $f(x_1) = f(x_2)$

i.e. $4x_1^3 + 7 = 4x_2^3 + 7$

$4x_1^3 = 4x_2^3$

$x_1^3 - x_2^3 = 0$

$(x_1 - x_2)(x_1^2 + x_2^2 + x_1x_2) = 0$

i.e. $\boxed{x_1 = x_2}$ & $\boxed{x_1^2 + x_2^2 + x_1x_2 = 0}$

$\therefore$ $f(x)$ is one-one It gives complex roots **[1]**

Surjective (onto)

Let $f(x) = y$

i.e. $4x^3 + 7 = y$

$4x^3 = y - 7$

$x^3 = \frac{y-7}{4}$

$x = \left(\frac{y-7}{4}\right)^{1/3}$ **[1]**

Now, put value of 'x' in $f(x)$ we get

$f\left\{\left(\frac{y-7}{4}\right)^{1/3}\right\} = 4\left\{\left(\frac{y-7}{4}\right)^{1/3}\right\}^3 + 7$

$= \not{4}\frac{(y-7)}{\not{4}} + 7$

$= y - 7 + 7$

$f\left(\left(\frac{y-7}{4}\right)^{1/3}\right) = y$ **[½]**

$\therefore$ $f(x)$ is bijective as it is injective & subjective

27. Given $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$ **[1]**

So, $A^3 - 3A - 2I$

$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **[1]**

$\therefore$ $A^3 - 3A - 2I = 0$

Now, $|A| = 0(0 - 1) - 1(0 - 1) + 1(1 - 0)$

$|A| = 1 + 1$

$\boxed{|A| = 2}$ **[1]**

OR

Let three numbers x, y & z respectively.

According to question

$$\begin{array}{l} x+y+z=6 \\ xx+y+3z=11 \\ x-2y+z=0 \end{array} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$AX = B$$

Multiply both side by A^{-1} i.e. $A^{-1}AX = A^{-1}B$

$$\boxed{X = A^{-1}B} \quad \ldots(1)$$

$$\boxed{A^{-1} = \frac{\text{adj}(A)}{|A|}} \quad \ldots(2)$$ **[1]**

$C_{11} = 7 = 7$; $C_{21} = -(3) = -3$; $C_{31} = 2 = 2$

$C_{12} = -(-3) = 3$; $C_{22} = 0 = 0$; $C_{32} = -(3) = -3$

$C_{13} = -1 = -1$; $C_{23} = -(-3) = 3$; $C_{33} = 1 = 1$

$$\text{adj}(A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \;\&\; |A| = 1(1+6) - 1(0-3) + 1(-1)$$

$$|A| = 7 + 3 - 1$$

$$|A| = 9$$

By using formula $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

We get $A^{-1} = \dfrac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ **[1]**

Put value of A^{-1} in equation (1) we get

$$X = A^{-1} \cdot B$$

$$X = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42-33+0 \\ 18+0+0 \\ -6+33+0 \end{bmatrix}$$

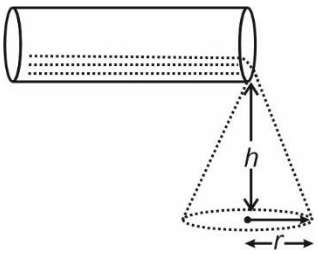
$$X = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ i.e. } \boxed{x=1};\ \boxed{y=2}\ \&\ \boxed{z=3}$$ **[1]**

28. $\dfrac{dV}{dt} = 12\text{ cm}^3/\text{s}$

$$h = \frac{r}{6}$$

$\dfrac{dh}{dt} = ?$ at $h = 4$ cm



Let r is radius of cone & h is hight of cone

Volume of cone is $\frac{1}{3}\pi r^2 h$ **[1]**

$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3}(6h)^2 \cdot h \text{ as } h = \frac{r}{6} \text{ i.e. } r = 6h$$

$$V = \frac{\pi}{3} 36h^3$$

$$V = 12\pi h^3$$

$$\frac{dV}{dt} = 12\pi 3h^2 \frac{dh}{dt}$$ **[1]**

$$\frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$12 = 36\pi(4)^2 \frac{dh}{dt}$$

$$\frac{12}{36\pi 16} = \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{48\pi} \text{ cm/s}}$$ **[1]**

29. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ & $\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

$\bar{b}_1 = 2\hat{i} + 3\hat{j} + 6k$ & $\bar{b}_2 = 4\hat{i} + 6\hat{j} + 12k$

But $\vec{b}_1 = 2\vec{b}_2$

$\therefore$ Lines are parallel **[1]**

Hence we will use $\text{S.D} = \frac{\left|(\bar{a}_2 - \bar{a}_1) \times \vec{b}\right|}{\left|\bar{b}\right|}$

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4k$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5k$

$(\bar{a}_2 - \bar{a}_1) = 2\hat{i} + \hat{j} - k$ & $\vec{b} = 2\hat{i} + 3\hat{j} + 6k$ **[1]**

$$\therefore \text{ S.D} = \frac{\left|\left(2\hat{i}+\hat{j}-k\right)\times\left(2\hat{i}+3\hat{j}+6k\right)\right|}{\sqrt{2^2+3^2+6^2}}$$

$$\text{S.D} = \frac{\left|9\hat{i}-14\hat{j}+4k\right|}{\sqrt{49}} = \frac{\sqrt{81+196+16}}{\sqrt{49}} = \sqrt{\frac{293}{49}}$$

$$\boxed{\text{S.D} = \frac{\sqrt{293}}{7}} \text{ units}$$ **[1]**

OR

$\left|\overrightarrow{OP}\right| = 3$ units

DRs *i.e*. Direction ratios $-k, 2k, -2k$ **[1]**

$$\left|\overrightarrow{OP}\right| = \sqrt{k^2+4k^2+4k^2} = 3$$

$$\sqrt{9k^2} = 3$$

$\pm 3k = 3$

$k = \pm 1$ **[1]**

$k = \pm 1$ where $P(-k, 2k, -2k)$

$\therefore$ $P(-1, 2, -2)$ & $P(1, -2, 2)$

Co-ordinate of 'P' (–1, 2, –2) or (1, –2, 2) **[1]**

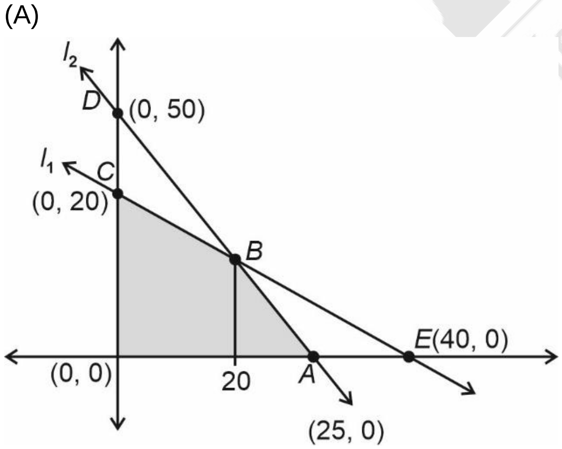
30.

$$P\left(\frac{A}{W}\right) = \frac{P\left(\frac{W}{A}\right)P(A)}{P(W)}$$ **[1]**

Where $P(W) = P\left(\frac{W}{A}\right)P(A) + P\left(\frac{W}{B}\right)P(B)$ **[1]**

i.e. $P\left(\frac{A}{W}\right) = \dfrac{\frac{1}{2}\times\frac{4}{9}}{\frac{1}{2}\times\frac{4}{9}+\frac{1}{2}\times\frac{5}{9}} = \frac{4}{9}$ **[1]**

31. (A)



Equation of line (l_1)

i.e. $\frac{x}{40}+\frac{y}{20}=1$

$x+2y-40\le 0$

Equation of line (l_2)

i.e. $\frac{x}{25}+\frac{y}{50}=1$

$\Rightarrow 2x+y-50\le 0$ **[1]**

Therefore constraints are

$x+2y-40\le 0$

$2x+y-50\le 0$ & $x, y\ge 0$ **[½]**

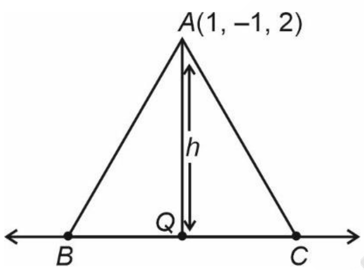
(B) $Z = x + y$

Corner pt.	$z = x + y$
C (0, 20)	z = 20
O (0 , 0)	z = 0
A (25, 0)	z = 25
B (20, 10)	z = 30 max. value

[1]

$\therefore$ Maximum value of z is '30' at (20, 10) **[½]**

32. $|\overrightarrow{BC}|=5$ units



$$\frac{x+2}{2}=\frac{y-1}{1}=\frac{z-0}{4}$$ **[1]**

Let 'h' be height of point A for base line BC. Height is perpendicular to line BC.

$$\frac{x+2}{2}=\frac{y-1}{1}=\frac{z-0}{4}=\lambda$$

So, $x=2\lambda-2$

$y=\lambda+1$

$z=4\lambda$

Co-ordinate of Q is $(2\lambda-2, \lambda+1, 4\lambda)$ **[1]**

Co-ordinate of $Q(2\lambda-2, \lambda+1, 4\lambda)$

DRs of $AQ(2\lambda-3, \lambda+2, 4\lambda-2)$

DRs of $BC(2, 1, 4)$

as line AQ is perpendicular to BC.

$\therefore$ DRs of AQ. DRs of $BC = 0$

i.e. $2(2\lambda - 3) + 1(\lambda + 2) + 4(4\lambda - 2) = 0$

$4\lambda - 6 + \lambda + 2 + 16\lambda - 8 = 0$

$21\lambda - 12 = 0$

$\boxed{\lambda = \frac{4}{7}}$ **[1]**

$\therefore$ Coordinates of $Q\left(2 \times \frac{4}{7} - 2, \frac{4}{7} + 1, 4 \times \frac{4}{7}\right)$

$$Q\left(\frac{-6}{7}, \frac{11}{7}, \frac{16}{7}\right)$$

Hence $|\bar{A}Q| = \sqrt{\left(1 + \frac{6}{7}\right)^2 + \left(-1 - \frac{11}{7}\right)^2 + \left(2 - \frac{16}{7}\right)^2}$

$|AQ| = \sqrt{\left(\frac{13}{7}\right)^2 + \left(-\frac{18}{7}\right)^2 + \left(\frac{-2}{7}\right)^2}$

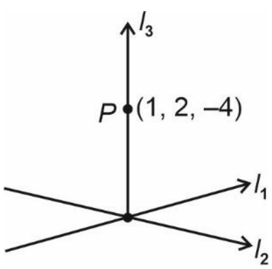
$|AQ| = \sqrt{\frac{497}{49}}$

$|AQ| = \sqrt{\frac{71}{7}}$ **[1]**

Area of $\Delta ABC = \frac{1}{2} BC \times AQ = \frac{1}{2} \times 5 \times \sqrt{\frac{71}{7}}$

$= \frac{5}{2}\sqrt{\frac{71}{7}}$ sq. units **[1]**

OR



Let equation of line l_1 is $\vec{r} = \left(8\hat{i} - 19\hat{j} + 10k\right) + \lambda\left(3\hat{i} - 16\hat{j} + 7k\right)$

& equation of line l_2 is

$\vec{r} = \left(15\hat{i} + 29\hat{j} + 5k\right) + \mu\left(3\hat{i} + 8\hat{j} - 5k\right)$ **[1]**

Direction of line l_1 is $\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7k$

Direction of line l_2 is $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5k$

Direction of line l_3 is $\vec{b}_3 = \vec{b}_1 \times \vec{b}_2$

$$\vec{b}_3 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$\vec{b}_3 = 24\hat{i} + 36\hat{j} + 72k$ **[2]**

Equation of line 'l_3' which passing through the point 'P'.

$\vec{r}_3 = \vec{a}_3 + \lambda\vec{b}_3$

$\vec{r}_3 = \left(\hat{i} + 2\hat{j} - 4k\right) + \lambda\left(24\hat{i} + 36\hat{j} + 72k\right)$ **[1]**

$\vec{r}_3 = \left(\hat{i} + 2\hat{j} - 4k\right) + \mu\left(2\hat{i} + 3\hat{j} + 6k\right)$ where $\mu = 12\lambda$ **[1]**

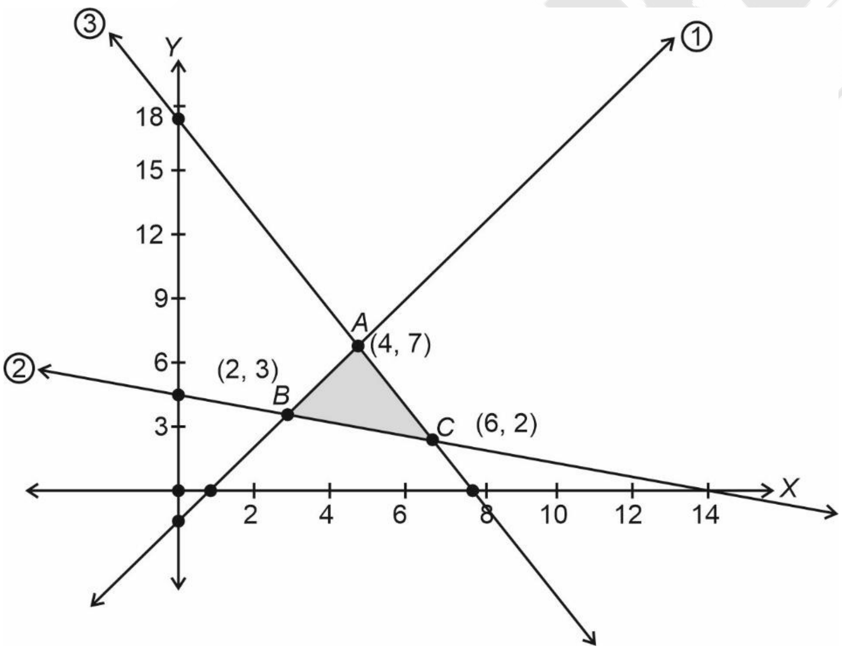
33. $2x - y \geq 1$; $x + 4y \geq 14$; $5x + 2y \leq 34$

x	0	$\frac{1}{2}$
y	–1	0

x	0	14
y	$\frac{7}{2}$	0

x	0	6.8
y	17	0

[1]



[1]

Co-ordinate of A *i.e*. point of intersection of line - l_1 & l_3.

i.e. $5x+2y=34 \Rightarrow 5x+2y=34$

$2(2x-y=1)$ $\quad$ $\underline{4x-2y=2}$

$9x=36$

$\boxed{x=4} \Rightarrow \boxed{y=7}$

i.e. $A(4, 7)$

Co-ordinates of B i.e. point of intersection of line $-l_1$ & l_2

i.e. $4(2x-y=1) \Rightarrow 8x-4y=4$

$x+4y=14$ $\quad$ $\underline{x+4y=14}$

$9x=38$

$\boxed{x=2} \Rightarrow \boxed{y=3}$ **[1]**

i.e. $B(2, 3)$

Co-ordinates of C i.e. point of intersection of line $-l_2$ & l_3

i.e., $5(x+4y=14) \Rightarrow$ $\quad$ $5x+20y=70$

$5x+2y=34$ $\quad$ $5x+2y=34$

$\quad - \quad - \quad -$

$18y=36$

i.e. $C(6, 2)$ $\quad$ $\boxed{y=2} \Rightarrow \boxed{x=6}$

Area of shaded region $ABC=\int_2^4 AB\,dx+\int_4^6 AC\,dx-\int_2^6 BC\,dx$

$$=\int_2^4(2x-1)dx+\int_4^6\left(\frac{34-5x)}{2}\right)dx-\int_2^6\left(\frac{14-x}{4}\right)dx$$

$$=\left(\frac{2x^2}{2}-x\right)\Big|_2^4+\left(\frac{34x-\frac{5x^2}{2}}{2}\right)\Bigg|_4^6-\left(\frac{14x-\frac{x^2}{2}}{4}\right)\Bigg|_2^6$$

$$=(x^2-x)\big|_2^4+\frac{1}{2}\left(34x-\frac{5}{2}x^2\right)\Big|_4^6-\frac{1}{4}\left(14x-\frac{x^2}{2}\right)\Big|_2^6$$

$$=16-4-(4-2)+\frac{1}{2}(204-90-136+40)-\frac{1}{4}(84-18-28+2)$$

$$=10+\frac{18}{2}-\frac{40}{4}$$

$= 10 + 9 - 10$ **[2]**

Area of shaded region ABC = 9 sq. unit

34. Let radius of sphere is 'r'. l, b, h of cuboid is $\frac{x}{3}, x, 2x$ respectively,

According to question Surface area of sphere + Surface area of cuboid = constant i.e., $$4\pi r^2 + 2(lb + bh + hl) = K$$

$$4\pi r^2 + 2\left(\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3}\right) = K$$

$$4\pi r^2 + \frac{2}{3}(9x^2) = K$$ **[1]**

$$\boxed{4\pi r^2 + 6x^2 = K} \quad \ldots(1)$$

Sum of volumes of sphere & cuboid is

$$V = \frac{4}{3}\pi r^3 + lbh$$

$$V = \frac{4}{3}\pi r^3 + \frac{x}{3} \times x \times 2x$$

$$V = \frac{4}{3}\pi r^3 + \frac{2}{3}x^3$$

$V = \frac{2}{3}\left(2\pi r^3 + x^3\right)$ from equation (i) $x = \left(\frac{K - 4\pi r^2}{6}\right)^{1/2}$ **[1]**

$$V = \frac{2}{3}\left[2\pi r^3 + \left(\frac{K - 4\pi r^2}{6}\right)^{3/2}\right]$$

$$\therefore \quad x^3 = \left(\frac{K - 4\pi r^2}{6}\right)^{3/2}$$

$$\frac{dv}{dr} = \frac{2}{3}\left[6\pi r^2 + \frac{3}{2}\left(\frac{K - 4\pi r^2}{6}\right)^{1/2} \cdot \left(\frac{-8\pi r}{6}\right)\right]$$

$$\frac{dv}{dr} = \frac{2}{3}\left[6\pi r^2 + \frac{-8\pi r}{4}\left(\frac{K - 4\pi r^2}{6}\right)^{1/2}\right]$$ **[1]**

Put $\frac{dv}{dr} = 0$

i.e. $\left(\frac{K - 4\pi r^2}{6}\right)^{1/2}$

$$3r = \left(\frac{K - 4\pi r^2}{6}\right)^{1/2}$$

Squaring both sides, $9r^2 = \frac{(K - 4\pi r^2)}{6}$

$$54r^2 = K - 4\pi r^2$$

$$54r^2 + 4\pi r^2 = K$$

$54r^2 + 4\pi r^2 = (4\pi r^2 + 6x^2)$ from equation (1)

$9r^2 = x^2$

$\boxed{x = 3r}$ Hence proved **[1]**

$$\frac{dv}{dr} = \frac{2}{3}\left[6\pi r^2 - 2\pi r\left(\frac{K-4\pi r^2}{6}\right)^{1/2}\right]$$

$$\frac{dv}{dr} = \frac{2}{3}\times 2\pi r\left[3r - \left(\frac{K-4\pi r^2}{6}\right)^{1/2}\right]$$

$$\frac{d^2v}{dr^2} = \frac{4\pi r}{3}\left[3 - \frac{1}{2}\left(\frac{K-4\pi r^2}{6}\right)^{-1/2}\cdot\left(\frac{-8\pi r}{6}\right)\right] + \frac{4\pi}{3}\left[3r - \left(\frac{K-4\pi r^2}{6}\right)^{1/2}\right]$$

From equation (1) $4\pi r^2 = K - 54r^2$

i.e., $\frac{d^2v}{dr^2} = \frac{4}{3}\pi r\left[3 - \frac{1}{2}(3r)^{-1/2}\left(\frac{-8\pi r}{6}\right)\right] + \frac{4\pi}{3}[3r - 3r]$

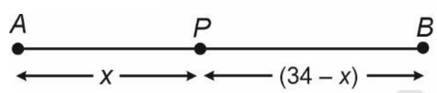
$$\frac{d^2v}{dr^2} = \frac{4}{3}\pi r\left[3 + \frac{8\pi r}{2(3r)^{1/2}\cdot 6}\right]$$

$$\frac{d^2v}{dr^2} = \frac{4}{3}\pi r\left[3 + \frac{8\pi r}{2\sqrt{3r}}\right] > 0$$ **[1]**

Volume of combined figure is minimum

OR

Length of given wire is 34 m

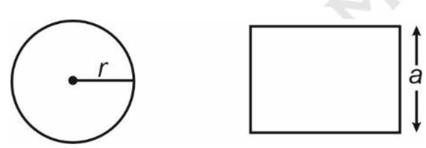


According to Question

'x' is perimeter of circle i.e., $2\pi r = x$ …(1)

& '$(34 - x)$' is perimeter of square i.e., $4a = 34 - x$ …(2)

Let a be side of square & 'r' radius of circle **[1]**

i.e., 

Combined area $A = \pi r^2 + a^2$

$$A = \pi\frac{x^2}{4\pi^2} + \left(\frac{34-x}{4}\right)^2 \quad \text{from (1) \& (2)}$$

$$A = \frac{x^2}{4\pi} + \frac{1}{16}(34-x)^2$$ **[1]**

$$\frac{dA}{dx}=\frac{2x}{4\pi}+\frac{1}{16}2(34-x)(-1)$$

$$\frac{dA}{dx}=\frac{x}{2\pi}-\frac{1}{8}(34-x)$$

$$\frac{d^2A}{dx^2}=\frac{1}{2\pi}-\frac{1}{8}(-1)=\frac{1}{2\pi}+\frac{1}{8}>0 \text{ minimum}$$

Put $\frac{dA}{dx}=0$ **[1]**

i.e., $\frac{x}{2\pi}=\frac{1}{8}(34-x) \Rightarrow 4x=34\pi-\pi x$

$$(4+\pi)x=34\pi$$

$$\boxed{x=\frac{34\pi}{4+\pi}}$$ **[1]**

$$A=\frac{x^2}{4\pi}+\frac{1}{16}(34-x)^2$$

$$A=\frac{(34\pi)^2}{4\pi(4+\pi)^2}+\frac{1}{16}\left(34-\frac{34\pi}{4+\pi}\right)^2$$

$$A=\frac{(34\pi)^2}{4\pi(4+\pi)^2}+\frac{1}{16}\left(\frac{136}{4+\pi}\right)^2$$

$$A=\frac{(34)^2}{4(4+\pi)^2}\left[\frac{\pi^2}{\pi}+\frac{1}{4}\times 16\right]$$

$$A=\frac{(34)^2}{4(4+\pi)^2}\cdot(\pi+4)$$

$\boxed{A=\frac{289}{(\pi+4)}}$ square unit **[1]**

35. (A) $y-x\frac{dy}{dx}=x+y\frac{dy}{dx}$

$$y-x=x\frac{dy}{dx}+y\frac{dy}{dx}$$

$$(y-x)=(x+y)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{y-x}{x+y} \quad \ldots(1)$$

$$\frac{dy}{dx}=\frac{x(y/x-1)}{(1+y/x)}=x^0 f(y/x)$$ **[½]**

$\therefore$ Given differential equation is a homogenous with degree zero.

Now put $y = vx$ in equation (1)

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$ **[½]**

As $\dfrac{dy}{dx}=\dfrac{y-x}{x+y}$

$$v+x\frac{dv}{dx}=\frac{vx-x}{x+vx}$$

$$v+x\frac{dv}{dx}=\frac{v-1}{1+v}$$

$$x\frac{dv}{dx}=\frac{v-1}{1+v}-v$$

$$x\frac{dv}{dx}=\frac{\cancel{v}-1-\cancel{v}-v^2}{1+v}$$

$$x\frac{dv}{dx}=-\frac{(v^2+1)}{1+v}$$ **[1]**

$$\int\frac{1}{1+v^2}\,dv+\int\frac{v}{1+v^2}\,dv=-\int\frac{dx}{x}$$

$$\tan^{-1}(v)+\frac{1}{2}\log|1+v^2|=-\log x+C$$

$$\tan^{-1}\left(\frac{y}{x}\right)+\frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right|+\log x=C$$

$$\boxed{\tan^{-1}\left(\frac{y}{x}\right)+\frac{1}{2}\log|x^2+y^2|=C}$$ **[1]**

(B) $\left(\dfrac{dy}{dx}\right)^3+\left(\dfrac{dy}{dx}\right)+\cos^2 y=0$

Order of D.E. is order of highest derivative.

Degree of D.E. is exponent of highest order derivative time.

$\therefore$ Order of given D.E. is '1' **[1]**

& Degree of given D.E. is '3' **[1]**

36. P(A fails) = 0.2

P(B fails alone) = 0.15

P(A and B fail) = 0.15

(A) P(B fails) = P(B fails alone) + P(A and B fail)

$= 0.15 + 0.15$

$\boxed{P(B \text{ fails}) = 0.30}$ **[1]**

(B) $P(A$ fails alone$) = P(A$ fails$) - P(A$ and B fail$)$

$= 0.2 - 0.15$

$\boxed{P(A \text{ fails alone}) = 0.05}$ **[1]**

(C) $P(A$ or B fails$) = P(A$ fails$) + P(B$ fails$) - P(A$ and B fail$)$

$= 0.20 + 0.30 - 0.15$

$= 0.50 - 0.15$

$\boxed{P(A \text{ or } B \text{ fails}) = 0.35}$ **[2]**

OR

$P(B \text{ fails}/A \text{ fails}) = \dfrac{P(B \text{ and } A \text{ fail})}{P(A \text{ fails})}$

$= \dfrac{0.15}{0.20}$

$= \dfrac{3}{4}$

$\boxed{P(B \text{ fails}/A \text{ fails}) = 0.75}$ **[2]**

37. (A) $\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$ where *A, B, C* are cost included by organization for village *X*, *Y*, *Z* respectively

[1]

(B) $\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} \frac{2}{100} \\ \frac{4}{100} \\ \frac{20}{100} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where *x, y, z* are No. of toilets expected in village *X*, *Y*, *Z* respectively **[1]**

(C) $A = 400 \times 50 + 300 \times 20 + 100 \times 40 = 30{,}000$

$B = 300 \times 50 + 250 \times 20 + 75 \times 40 = 23{,}000$

$C = 500 \times 50 + 400 \times 20 + 150 \times 40 = 39{,}000$

Total amount spend by organization = 92,000 **[2]**

OR

$x = 400 \times \dfrac{2}{100} + 300 \times \dfrac{4}{100} + 100 \times \dfrac{20}{100} = 40$

$$y = 300 \times \frac{2}{100} + 250 \times \frac{4}{100} + 75 \times \frac{20}{100} = 31$$

$$z = 500 \times \frac{2}{100} + 400 \times \frac{4}{100} + 150 \times \frac{20}{100} = 56$$

Total number of toilets = 127 **[2]**

38. Position vector of sail's boat is $\vec{r}(t)$

i.e., $\vec{r}(t) = (2t+1)\hat{i} + (t^2-2)\hat{j}$

(A) $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = \dfrac{d}{dt}\left[(2t+1)\hat{i} + (t^2-2)\hat{j}\right]$

$\vec{v}(t) = 2\hat{i} + 2t\hat{j}$

At t = 2 hr. $\vec{v}(2) = 2\hat{i} + 4\hat{j}$

Speed = $|\vec{v}(2)| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Speed of sailboat at t = 2 hr is $2\sqrt{5}$ unit per hour **[2]**

(B) Direction of motion of sailboat at t = 2 hr in times of a unit vector.

i.e. $\hat{v}(2) = \dfrac{\vec{v}(2)}{|\vec{v}(2)|} = \dfrac{2\hat{i} + 4\hat{j}}{2\sqrt{5}}$

$\hat{v}(2) = \dfrac{2}{2\sqrt{5}}\hat{i} + \dfrac{4}{2\sqrt{5}}\hat{j}$

$\boxed{\hat{v}(2) = \dfrac{1}{\sqrt{5}}\hat{i} + \dfrac{2}{\sqrt{5}}\hat{j}}$ **[2]**

also $\dfrac{d\vec{v}(t)}{dt} \neq 0$, so sailboat is not moving in straight line.

❑ ❑ ❑