A
CODE

Corporate Office **:** Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Time : 3 hrs **MM : 264**

MOCK TEST - 1

for JEE (Advanced) - 2022

Paper - 1

ANSWERS

PHYSICS

1. (6)
2. (4)
3. (8)
4. (8)
5. (0)
6. (3)
7. (8)
8. (8)
9. (A, C)
10. (A, C)
11. (B, C)
12. (A, B)
13. (A, B, D)
14. (A, B, C, D)
15. (A, C)
16. (A, C)
17. (A, B, D)
18. (A, C, D)
19. $A \rightarrow$ (P, S)
    $B \rightarrow$ (S)
    $C \rightarrow$ (R, S)
    $D \rightarrow$ (P, T)
20. $A \rightarrow$ (Q, R, S)
    $B \rightarrow$ (P, S)
    $C \rightarrow$ (P)
    $D \rightarrow$ (Q, R)

CHEMISTRY

21. (5)
22. (2)
23. (8)
24. (6)
25. (2)
26. (7)
27. (6)
28. (4)
29. (A, B, D)
30. (C, D)
31. (A, B, C, D)
32. (A, C)
33. (A, B, C, D)
34. (A, B, D)
35. (A, B, C, D)
36. (A, B, C)
37. (A, B, D)
38. (A)
39. $A \rightarrow$ (P, S)
    $B \rightarrow$ (Q, R, T)
    $C \rightarrow$ (P, S)
    $D \rightarrow$ (Q, R, T)
40. $A \rightarrow$ (T)
    $B \rightarrow$ (P, S)
    $C \rightarrow$ (R, S)
    $D \rightarrow$ (Q)

MATHEMATICS

41. (4)
42. (2)
43. (5)
44. (5)
45. (0)
46. (2)
47. (1)
48. (2)
49. (A, B, D)
50. (A, B, D)
51. (A, D)
52. (B, C)
53. (A, B)
54. (A, C)
55. (B, C, D)
56. (C, D)
57. (A, D)
58. (A, D)
59. $A \rightarrow$ (P)
    $B \rightarrow$ (T)
    $C \rightarrow$ (S)
    $D \rightarrow$ (Q)
60. $A \rightarrow$ (Q)
    $B \rightarrow$ (R)
    $C \rightarrow$ (Q)
    $D \rightarrow$ (P, Q, R, S, T)

A
CODE

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Time : 3 hrs

MOCK TEST - I

MM : 264

for JEE (Advanced) – 2022

Paper - I

ANSWER & SOLUTIONS

PART – I : PHYSICS

1. Answer (6)

$$a = \frac{(V)(2\rho_0)(12) - V \times \rho_0 \times 10}{V \times 2\rho}$$

$= 6\ m/s^2$

2. Answer (4)

In one case image is virtual ($u = -15$ cm)

In another case image is real ($u = -40$ cm)

$$v_1 = \frac{uf}{u+f} = \frac{-10f}{-10+f}$$

$$v_2 = \frac{-10f}{-40+f}$$

In both situations, sign convention is opposite

$\Rightarrow \quad v_2 = v_1$

$$\Rightarrow \quad v_2 = \frac{-10f}{-10+f} = \frac{40f}{-40+f}$$

$f = 16$ cm

3. Answer (8)

$$\frac{dw}{dt} \propto v^2$$

$$P = (\ln\sqrt{2})v^2$$

$$Fv = (\ln\sqrt{2})v^2$$

$$m\frac{dv}{dt} = \ln\sqrt{2}\ v$$

$$\ln v\Big|_{v_0}^{2v_0} = \ln\sqrt{2}\,\frac{t}{m}$$

$t = 2m$

$t = 2 \times 4 = 8$ sec

4. Answer (8)

At null point potential gradient across

$$AB = \frac{1}{10}\frac{V}{\text{cm}}$$

If switch S_1 is closed, emf of 5 V battery is balanced by length $AJ = l_1$

$\therefore \quad 5 = \frac{1}{10} \times l_1$

$\Rightarrow \quad l_1 = 50$ cm

If switch S_2 is closed, emf of 3 V battery is balanced by length $BJ = l_2$

$\therefore \quad 3 = \frac{1}{10} \times l_2$

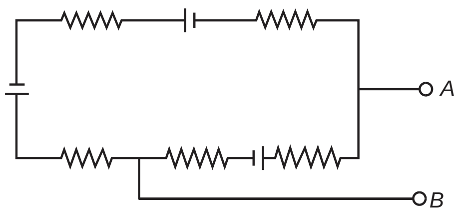
$l_2 = 30$ cm

$\therefore \quad l_1 + l_2 = 80$ cm

5. Answer (0)

$U_i = U_f \Rightarrow \Delta U = 0$

6. Answer (3)



For open circuit

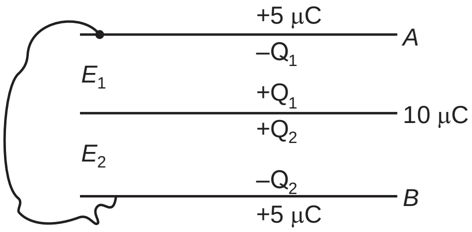
Let current in the circuit be I, applying Kirchhoff's law, we get

$5IR = 3 \times 15$

$$\Rightarrow \quad I = \frac{45}{5R} = \frac{45}{5 \times 100} = 9 \times 10^{-2}\,\text{A}$$

$V_{AB} = V_A - V_B = -2 \times 100 \times 9 \times 10^{-2} + 15 = -3$ V

7. Answer (8)



Since, A and B connected,

Hence, $|V_1| = |V_2|$

$E_1d_1 = E_2d_2$

$$\frac{\sigma_1 d_1}{\varepsilon_0} = \frac{\sigma_2 d_2}{\varepsilon_0}$$

$\sigma_1 d_1 = \sigma_2 d_2$ or $5\sigma_1 = 8\sigma_2$

$$\Rightarrow \frac{5\sigma_1}{\sigma_2} = 8$$

8. Answer (8)

$$62.1\,\text{eV} = \frac{hc}{\lambda_0} = \frac{1242\text{eV} - \text{nm}}{\lambda_0} = \lambda_0 = 20 \text{ nm}$$

$\therefore$ Photoelectric emission takes place only for λ_1 and λ_2

Now, for any wavelength

In time $t \rightarrow$ Energy incident = Pt

$\therefore$ In time $t \rightarrow$ No. of photon incident

$$= \frac{Pt}{\left(\frac{hc}{\lambda}\right)} = \frac{\lambda Pt}{hc}$$

$\therefore$ Number of photoelectrons emitted

$$= \frac{1}{2}\left(\frac{\lambda pt}{hc}\right)$$

$\therefore$ Charge developed = $\dfrac{1}{2}\dfrac{(\lambda pt)}{(hc)} \times (e)$

$\therefore$ Potential difference across the resistor

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda pte}{2hcr}$$

$$\therefore \quad V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda_1}{hc}\right)\left(\frac{pte}{2r}\right)$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda_2}{hc}\right)\left(\frac{pte}{2r}\right)$$

$$\therefore V_1 = \left(9 \times 10^9\right)\left(\frac{12.42 \text{ nm}}{1242 \text{ eV-nm}}\right)\frac{\left(10^{-3}\right)t(e)}{2 \times 10^{-3}}$$

$$= 9 \times 10^{-9} \times \left(\frac{1}{100}\right) \times \left(\frac{1}{2}\right) \times t = 4.5 \times 10^7 t$$

$$V_2 = \left(9 \times 10^9\right)\left(\frac{6.21\text{nm}}{1242\text{eV-nm}}\right)\frac{\left(10^{-3}\right)(t)(e)}{2 \times 10^{-3}}$$

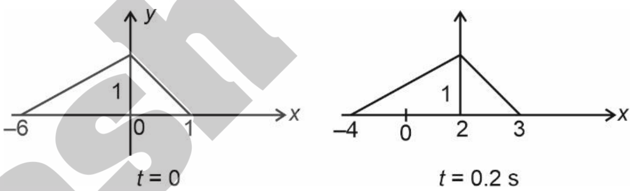
$$= 9 \times 10^{-9} \times \frac{1}{200} \times \left(\frac{t}{2}\right) = \frac{4.5}{2} \times 10^7 t$$

$$\therefore \quad V = V_1 + V_2 = \frac{3}{2} \times (4.5) \times 10^7 \times t$$

$$\therefore \quad I = \frac{V}{R}$$

9. Answer (A, C)

Shape of the string at $t = 0$ and $t = 0.2$ s has been shown below



$$\Rightarrow \quad y = \frac{5}{6}\text{cm}, \quad \lambda = 1$$

$$v = -(\text{wave velocity})\ \frac{\partial y}{\partial x}$$

$$= (-10)\frac{1}{6} = -\frac{5}{3} \text{ cm/s}$$

10. Answer (A, C)

$80 \times S_1 + 45S_2 = 0$

$S_2 - S_1 = 5$

$125S_1 + 225 = 0$

$m_1v_1 + m_2v_2 = 0$

$$\frac{50}{80} = a_1$$

$$v_1 = \frac{5}{8} \text{ m/s}$$

$$\frac{50}{45} = a_2$$

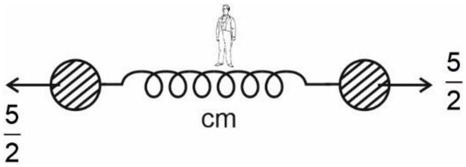
$$v_2 = \frac{10}{9} \text{ m/s}$$

$$v_{\text{rel}} = \frac{5}{8} + \frac{10}{9} = \frac{125}{72}$$

11 Answer (B, C)

Pressure will decrease and hence variable force has to be applied and gas is expanding, so heat absorbed by the gas.

12. Answer (A, B)



$$v_{cm} = \frac{-4+1}{2} = -\frac{3}{2}$$

$$v_1 = \frac{5}{2} + \frac{3}{2} = 4 \text{ m/s}$$

$v_2 = 0$

13. Answer (A, B, D)

Let V_D be the potential of D, then

$$\frac{V_A - V_D}{10} + \frac{V_B - V_D}{20} + \frac{V_C - V_D}{30} = 0$$

$\Rightarrow \quad V_D = 40$ V

Also, ratio of current in AD, DB and DC are $\frac{70-40}{10} : \frac{40}{20} : \frac{40-10}{30}$

i.e. 3 : 2 : 1

Also, total power network draws, $P = \sum I^2R = 200$ W

14. Answer (A, B, C, D)

f_{max} on A = 0.3 × 5 × 10 = 15 N

f_{max} on B = 0.5 × (5 + 15) × 10 = 100 N

If block B does not start sliding on the ground surface, then the friction between A and B will be zero.

15. Answer (A, C)

For missing wavelength, minima is formed

$$\therefore \quad Dx = \frac{\left(\frac{d}{2}\right) \times d}{D} = \left(n + \frac{1}{2}\right)\lambda$$

For $n = 0$ $\qquad \lambda = \frac{d^2}{D}$

$$\frac{d}{2D} = (2n+1)\frac{\lambda}{2}$$

$n = 1$ $\qquad \lambda = \frac{d^2}{3D}$

16. Answer (A, C)

$$W_{O\to A\to C} = y^2dy = \frac{y^3}{3} = \frac{1}{3}$$

$$W_{O\to C} = \frac{2}{5}(1) = \frac{2}{5}\text{ J}$$

$$= 2x^4dx = \frac{2}{5}x^5$$

17. Answer (A, B, D)

$$a = kx^2 \Rightarrow \frac{da}{dx} = 2kx$$

at $\quad x = 1,\ \frac{da}{dx} = 1 \Rightarrow k = \frac{1}{2}$

$$\Rightarrow \quad a = \frac{kx^2}{2} \Rightarrow v\frac{dv}{dx} = \frac{x^2}{2}$$

$$\Rightarrow \quad \int_0^v vdv = \int_0^x \frac{x^2}{2}dx \Rightarrow \frac{v^2}{2} = \frac{x^3}{6}$$

$$\Rightarrow \quad v^2 = \frac{x^3}{3}$$

18. Answer (A, C, D)

Impulse due to normal reaction is finite from the ground. So friction force gives finite impulse. Therefore frictional torque causes a finite angular impulse about center of mass of system so angular momentum about center of mass of system will change.

19. Answer (A $\rightarrow$ P, S); (B $\rightarrow$ S); (C $\rightarrow$ R, S); (D $\rightarrow$ P, T)

20. Answer (A $\rightarrow$ Q, R, S); (B $\rightarrow$ P, S); (C $\rightarrow$ P); (D $\rightarrow$ Q, R)

PART – II : CHEMISTRY

21. Answer (5)

$2NO(g) + O_2(g) \rightarrow 2NO_2(g) \rightarrow N_2O_4(s)$

Initial moles of NO = $\frac{1200 \times 0.25}{760 \times R \times 220} = \frac{300}{760 \times R \times 220}$

Initial moles of O_2 = $\frac{900 \times 0.15}{760 \times R \times 220}$

$$= \frac{135}{760 \times R \times 220}$$

$\therefore$ O_2 is the limiting reactant

$\therefore$ Moles of NO remaining after the reaction

$$= \frac{300 - 2 \times 135}{760 \times R \times 220} = \frac{30}{760 \times R \times 220}$$

P, pressure of remaining NO gas =

$$\frac{30 \times 760 \times P \times 220}{760 \times P \times 220 \times 0.40} = 75 \text{ torr}$$

$$\frac{P}{15} = 5$$

22. Answer (2)

Suppose the mol. wt. of the solute is M

Mole of solute $= \frac{10}{M}$

Mole of solvent (H_2O) $= \frac{180}{18} = 10$

Mole fraction of solute $= \dfrac{\frac{10}{M}}{\frac{10}{M} + 10} = \frac{1}{M + 1}$

We know, relative lowering of vapour pressure = mole fraction of solute

$$0.005 = \frac{1}{M + 1}$$

M = 199

23. Answer (8)

$$MnO_4^- + X^{n+} + H^+ \rightarrow Mn^{2+} + XO_3^- + H_2O$$

n-factor of $MnO_4^- = 5$

n-factor of X^{n+} = (5 – n)

Equivalents of MnO_4^- = Equivalents of X^{n+}

$1.61 \times 10^{-3} \times 5 = 2.68 \times 10^{-3} \times (5 - n)$

$\Rightarrow$ n = 2

Equivalent weight of XCl_2 = 84 g equiv^{-1}

Molecular mass of XCl_2 = 84 × 2 = 168 g mol^{-1}

Atomic mass of X = 168 – 71 = 97 = 9 × 10 + 7

$\therefore$ a = 9 and b = 7

$$\frac{a + b}{n} = \frac{9 + 7}{2} = 8$$

24. Answer (6)

$$\underset{1 \text{ mol}}{\text{Calcite}} \rightarrow \underset{(1 \text{ mol})}{\text{Aragonite}}$$

ΔU = 210 J mol^{-1}

$\Delta H = \Delta U + \Delta PV$ [P = 1 bar]

$= 210 + P\left[\frac{M}{d_2} - \frac{M}{d_1}\right] \times 100$ [$\because$ 1 L bar = 100 J]

$$= 210 + 100\left[\frac{100}{2.93 \times 10^3} - \frac{100}{2.71 \times 10^3}\right] = 210 - 0.277$$

$= 209.723$ J mol^{-1} ; $\frac{\Delta H}{35} \simeq 6$ J mol^{-1}

25. Answer (2)

It is a zero order reaction

Conc. of H^+ ions in a drop $= \frac{6 \times 10^{-7}}{.05 \times 10^{-3}} = 1.2 \times 10^{-2}$ M

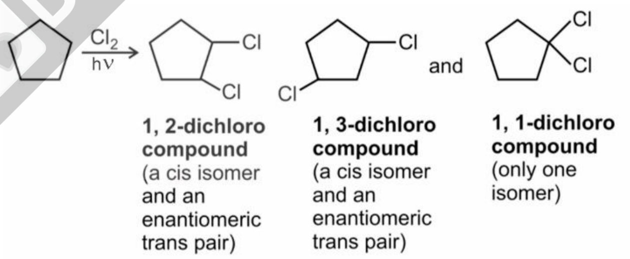
Rate constant, k = 6×10^5 mol L^{-1} s^{-1}

Time, $t = \frac{1.2 \times 10^{-2}}{6.0 \times 10^5} = 2 \times 10^{-8}$ sec $= x \times 10^{-8}$ sec

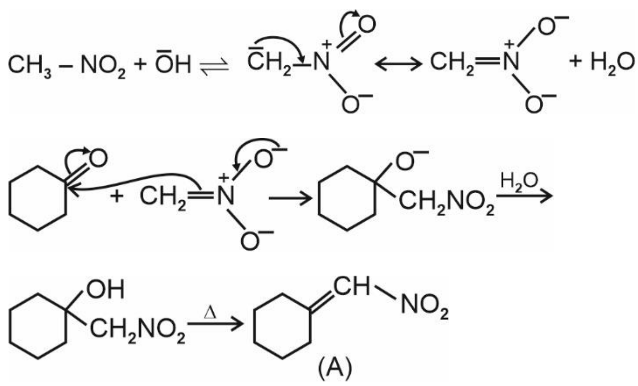
$\therefore$ x = 2

26. Answer (7)

Dichlorosubstituted products



27. Answer (6)



Molecular mass of (A) = 141

Sum of the digits of M = 1 + 4 + 1 = 6

28. Answer (4)

$x = 12$

$y = 12$

$z = 6$

$w = 6$

29. Answer (A, B, D)

Factual

30. Answer (C, D)

Decomposition of $N_2O_5(g)$ follows first order kinetics

$$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$$

Initial conc. 2 M

Conc. at time t $2(1 - 0.40)$ 1.6 0.40 [$\alpha = 40\%$]

$$k = 6.2 \times 10^{-4}\ sec^{-1}$$

$$t_{1/2} = \frac{0.693}{6.2 \times 10^{-4}} = 1117.7\ sec$$

$$\frac{\text{Initial moles of reaction mix.}}{\text{No. of moles of reaction mix. after 40\% dissociation}}$$

$$= \frac{2}{3.2} = \frac{5}{8}$$

$$t = \frac{2.303}{6.2 \times 10^{-4}} \log\frac{10}{6} = 824.6\ sec.$$

Rate = k [N_2O_5]

If volume of reaction mixture is doubled, the rate of reaction becomes half of initial rate.

31. Answer (A, B, C, D)

$CH_3CH_2COOK \rightarrow K^+ + CH_3CH_2COO^-$

$$H_2O \rightleftharpoons H^+ + OH^-$$

Cathode : $2H^+ + 2e \rightarrow H_2$

Anode : $CH_3CH_2COO^- \rightarrow CH_3CH_2-\overset{O}{\overset{\|}{C}}-O + e$

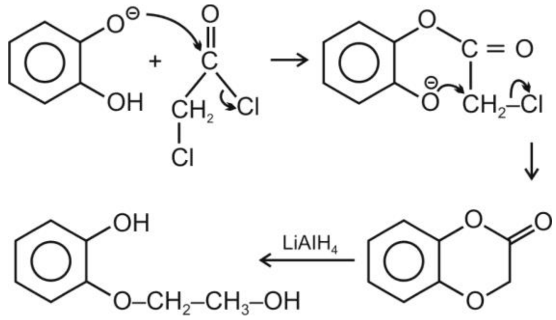
$$CH_3CH_2-\overset{O}{\overset{\|}{C}}-O\cdot \longrightarrow CH_3\dot{C}H_2 + CO_2$$

$$CH_3\dot{C}H_2 + \dot{C}H_2CH_3 \longrightarrow \underset{\text{(n-butane)}}{CH_3CH_2CH_2CH_3}$$

$$CH_3\dot{C}H_2 + H-\dot{C}H_2-\dot{C}H_2 \longrightarrow CH_3CH_3 + CH_2 = CH_2$$

$$CH_3\dot{C}H_2 + \dot{O}-\overset{O}{\overset{\|}{C}}-CH_2CH_3 \rightarrow C_2H_5-\overset{O}{\overset{\|}{C}}-OC_2H_5$$

32. Answer (A, C)



33. Answer (A, B, C, D)

γ – radiation is emitted from the excited daughter nuclei

34. Answer (A, B, D)

Consider the Van't Hoff factor

35. Answer (A, B, C, D)

Ethanol is weaker than any of these acids.

36. Answer (A, B, C)

(A) Oxidation state of N in the given compounds

$$\overset{-2}{N}_2H_4 < \overset{-1}{N}H_2OH < \overset{+1}{N}_2O < \overset{+3}{N}_2O_3$$

(B) Oxidising power of halogens is decided by their SRP values and it decreases down the group.

(C) Boiling point of hydrides of group-15 elements is mainly decided by their molecular mass as well as H-bonds in liquid NH_3. PH_3 has the lowest boiling point and it increases with the increase in molecular mass except NH_3 whose boiling point lies between AsH_3 and SbH_3 due to H-bonding.

(D) Among alkali metals Na is the weakest and Li is the strongest reducing agent.

37. Answer (A, B, D)

(A) Thermal stability of alkali metal hydrides is decided by their lattice energy. Higher the lattice energy, higher will be the stability.

(B) KO_2 is paramagnetic as O_2^{-1} has one unpaired electron

(C) Milk of magnesia is used as an antacid

(D) $BeCl_2$ in the solid state exists as linear polymeric chain

38. Answer (A)

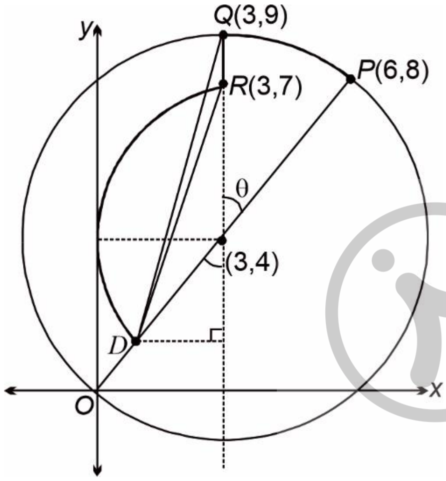
Compound (A) is neither phenol (no colour with $FeCl_3$) nor has a carboxylic functional group (does not dissolve in aq.$NaHCO_3$)

39. Answer (A → P, S); (B → Q, R, T); (C → P, S); (D → Q, R, T)

40. Answer (A → T); (B → P, S); (C → R, S); (D → Q)

PART – III : MATHEMATICS

41. Answer (4)



Point on C_1 : $|z - 3 - 4i| = 5$

where $|z|$ is maximum is

$P \equiv 6 + 8i$

Let complex number corresponding to point Q be z_2

Taking rotation of $6 + 8i$ about $3 + 4i$, we get

$$\frac{z_2 - (3+4i)}{6+8i-(3+4i)} = e^{i\tan^{-1}\frac{3}{4}}$$

$$z_2 = (3 + 4i) + (3 + 4i)\left(\cos\left(\tan^{-1}\frac{3}{4}\right) + i\sin\left(\tan^{-1}\frac{3}{4}\right)\right)$$

$$= 3 + 4i + (3 + 4i)\left(\frac{4}{5} + i\frac{3}{5}\right)$$

$$= 3 + 4i + \frac{1}{5}(3 + 4i)(4 + 3i) = 3 + 9i$$

∴ Complex number corresponding to R, $z_3 = 3 + 7i$

42. Answer (2)

We have,

$$S_{100} = \frac{0}{({}^{100}C_0)^5} + \frac{1}{({}^{100}C_1)^5} + \frac{2}{({}^{100}C_2)^5} + ... + \frac{100}{({}^{100}C_{100})^5} \quad ......(1)$$

Also,

$$S_{100} = \frac{100}{({}^{100}C_0)^5} + \frac{(100-1)}{({}^{100}C_1)^5} + \frac{(100-2)}{({}^{100}C_2)^5} + ... + \frac{0}{({}^{100}C_{100})^5} \quad ......(2)$$

∴ On adding (1) and (2), we get

$$2S_{100} = 100\, t_{100} \Rightarrow \frac{S_{100}}{100t_{100}} = \frac{1}{2}$$

Hence, $\sec\left(\cos^{-1}\left(\frac{S_{100}}{100t_{100}}\right)\right) = \sec\left(\cos^{-1}\frac{1}{2}\right)$

$= \sec\left(\frac{\pi}{3}\right) = 2$

43. Answer (5)

$$l = \lim_{x\to\infty} x\ln\left(\frac{e(1+(1/x))}{(1+(1/x))^x}\right) \quad (\infty \times 0 \text{ form})$$

$$= \lim_{x\to\infty} x\left(1+\ln\left(1+\frac{1}{x}\right) - x\ln\left(1+\frac{1}{x}\right)\right)$$

put $x = \frac{1}{t}$; as $x \to \infty$, $t \to 0$

Hence $l = \lim_{t\to 0}\frac{1}{t}\left(1+\ln(1+t) - \frac{\ln(1+t)}{t}\right)$

$$= \lim_{t\to 0}\left[\ln(1+t)^{1/t} + \frac{t-\ln(1+t)}{t^2}\right]$$

$= 1 + \lim_{y\to 0}\left(\frac{e^y - 1 - y}{y^2}\right)$ where $\ln(1 + t) = y$; $1 + t = e^y$, hence $t = e^y - 1$

$$= 1 + \frac{1}{2} = \frac{3}{2} = \frac{m}{n}$$

$\Rightarrow (m + n) = 5$

44. Answer (5)

Expression

$$= \cos^2\frac{\pi}{11} + \cos^2\frac{2\pi}{11} + \cos^2\frac{3\pi}{11} + \cos^2\frac{4\pi}{11} + \cos^2\frac{5\pi}{11}$$

$$= \frac{1}{2}\left[\left(1+\cos\frac{2\pi}{11}\right) + \left(1+\cos\frac{4\pi}{11}\right) + ... + \left(1+\cos\frac{10\pi}{11}\right)\right]$$

$$= \frac{1}{2}\left[5 + \underbrace{\left(\cos\frac{2\pi}{11} + \cos\frac{4\pi}{11} + \cos\frac{6\pi}{11} + ... + \cos\frac{10\pi}{11}\right)}_{S}\right]$$

On multiplying and dividing the series S by $2\sin\frac{\pi}{11}$, we get

$$\frac{1}{2\sin\frac{\pi}{11}}\left[\left(\sin\frac{3\pi}{11}-\sin\frac{\pi}{11}\right)+\left(\sin\frac{5\pi}{11}-\sin\frac{3\pi}{11}\right)+.......+\left(\sin\pi-\sin\frac{9\pi}{11}\right)\right]$$

$= -\frac{1}{2}$

Hence expression = $\frac{1}{2}\left[\left(5-\frac{1}{2}\right)\right]$ = $\frac{9}{4}$ = $\frac{p}{q}$

$\Rightarrow\ |p-q| = 5$

45. Answer (0)

$BA = I - AC - BC$

$BAC = C - AC^2 - BC^2 = C - (A + B)C^2$

$C - BAC = (A + B)C^2$

$\Rightarrow\ A + B + C - BAC = A + B + (A + B)C^2 = (A + B)(I + C^2)$

$\det(A + B + C - BAC) = \det(A + B)\det(I + C^2) = 0$

46. Answer (2)

Let E_i be the event of getting i on the die.

Obviously, $\sum_{i=1}^{6} P(E_i) = 1$

$\sum_{i=1}^{6}\lambda_i^2 = 1 \Rightarrow \lambda = \frac{1}{91}$

let A be the event of not getting an even number

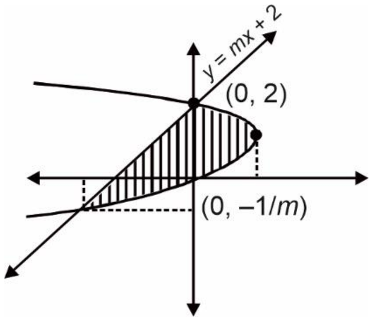
$\Rightarrow A = E_1 \cup E_3 \cup E_5$

$P(A) = P(E_1) + P(E_3) + P(E_5) = 35\lambda$

$\therefore$ Required probability = $P(E_5/A)$=

$\frac{P(E_5/A)}{P(A)} = \frac{P(E_5)}{P(A)} = \frac{25\lambda}{35\lambda} = \frac{5}{7} = \frac{m}{n}$.

47. Answer (1)



$y = mx + 2 \Rightarrow x = \left(\frac{y-2}{m}\right)$ ...(1)

$x = 2y - y^2$ ...(2)

$(y - 1)^2 = -(x - 1)$ vertex (1, 1)

From (1) and (2), $\frac{y-2}{m} = 2y - y^2$

$\Rightarrow\ my^2 + (1 - 2m)y - 2 = 0 \quad \alpha\beta = -\frac{2}{m}$

$\alpha = 2,\ \beta = -\frac{1}{m}$

Area = $\int_{-1/m}^{2}\left[\left(2y-y^2\right)-\frac{(y-2)}{m}\right]dy$

$\frac{9}{2} = \left[\frac{2y^2}{2}-\frac{y^3}{3}-\frac{1}{m}\frac{y^2}{2}+\frac{2y}{m}\right]_{-1/m}^{2}$

$\frac{9}{2} = \left(\frac{4}{3}+\frac{2}{m}+\frac{1}{6m^3}+\frac{1}{m^2}\right)$

$m = 1$ satisfy the equation

$\Rightarrow\ m = 1$

48. Answer (2)

Since $[x]$ is an integer

$\therefore x + 1 = 2k$; $k \in I$

$\Rightarrow\left[\frac{3(2k-1)^2-2(2k-1)+1}{2}\right] = k$

$\Rightarrow\left[\frac{12k^2-12k+3-4k+2+1}{2}\right] = k$

$\Rightarrow[6k^2 - 8k + 3] = k \Rightarrow 6k^2 - 8k + 3 = k$

$\therefore 6k^2 - 9k + 3 = 0$

$\Rightarrow 2k^2 - 3k + 1 = 0 \Rightarrow (k - 1)(2k - 1) = 0$

$\therefore k = 1$; $k = \frac{1}{2}$ (reject as $k \in I$) $\Rightarrow\ k = 1$

$\therefore x = 1$ so, $n = 1$

Hence $\frac{1}{\pi}\left(\frac{\pi}{2}+\frac{\pi}{4}+\pi+\frac{\pi}{4}\right) = 2$.

49. Answer (A, B, D)

$\frac{dy}{dx} + y = f(x)$

I.F. = e^x

$ye^x = \int e^x f(x)dx + C$

now if $0 \le x \le 2$ then $ye^x = \int e^x e^{-x}dx + C$

$\Rightarrow ye^x = x + C$

$x = 0,\ y(0) = 1, \quad C = 1$

$\therefore\ ye^x = x + 1 \qquad ....(1)$

$y = \dfrac{x+1}{e^x};\ y(1) = \dfrac{2}{e} \Rightarrow$ **(A)** is correct

$y' = \dfrac{e^x - (x+1)e^x}{e^{2x}}$

$y'(1) = \dfrac{e - 2e}{e^2} = \dfrac{-e}{e^2} = -\dfrac{1}{e}$

$\Rightarrow$ **(B)** is correct

if $x > 2$

$ye^x = \int e^{x-2}\,dx$

$ye^x = e^{x-2} + C$

$y = e^{-2} + Ce^{-x}$

as y is continuous

$\therefore\ \lim\limits_{x\to 2}\dfrac{x+1}{e^x} = \lim\limits_{x\to 2}\left(e^{-2} + Ce^{-x}\right)$

$3e^{-2} = e^{-2} + Ce^{-2} \Rightarrow C = 2$

$\therefore$ for $x > 2$

$y = e^{-2} + 2e^{-x}$ hence $y(3) = 2e^{-3} + e^{-2}$

$= e^{-2}(2e^{-1} + 1)$

$y' = -2e^{-x}$

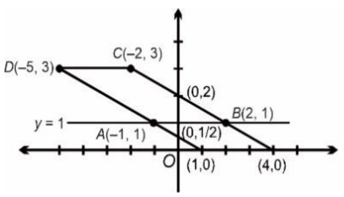
$y'(3) = -2e^{-3} \Rightarrow$ **(D)** is correct

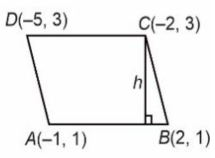
50. Answer (A, B, D)

$y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$

$(y - 3)(y - 1)$ $(x + 2y - 1)(x + 2y - 4) = 0$

$y = 1,\ y = 3$





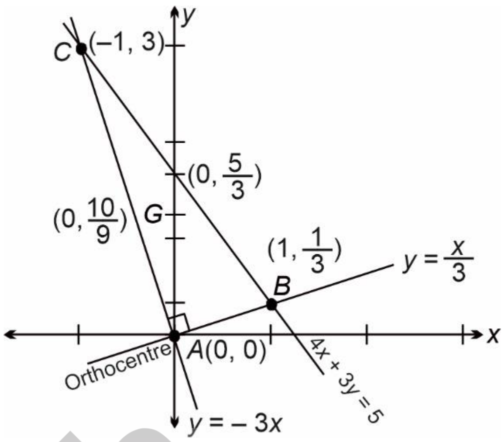
$\ell(AB) = 3$ and $h = 2$

Area of parallelogram $= 3 \times 2 = 6$

$\therefore\ AC = \sqrt{1^2 + 2^2} = \sqrt{5}$, $BD = \sqrt{7^2 + 2^2} = \sqrt{53}$

51. Answer (A, D)

Figure is self explanatory.



52. Answer (B, C)

We have, $f(2x) - f(2x)\, f\left(\dfrac{1}{2x}\right) + f(16x^2y) = f(-2) - f(4xy)$

Replacing y by $\dfrac{1}{8x^2}$, we get

$f(2x) - f(2x)\, f\left(\dfrac{1}{2x}\right) + f(2) = f(-2) - f\left(\dfrac{1}{2x}\right)$

$f(2x) + f\left(\dfrac{1}{2x}\right) = f(2x)\, f\left(\dfrac{1}{2x}\right)$ (As $f(x)$ is even)

$\therefore\ f(2x) = 1 \pm (2x)^n$

$\Rightarrow f(x) = 1 \pm x^n$

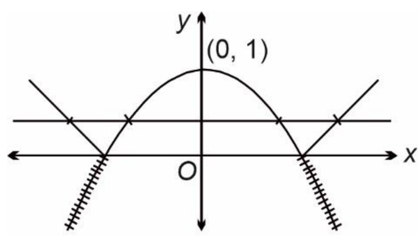
Now $f(4) = 1 \pm 4^n = -255$ (Given)

Taking negative sign, we get $256 = 4^n$

$\Rightarrow\ n = 4$

Hence $f(x) = 1 - x^4$, which is even function.

Now $|f(x)| = k - 2$



Graph of $f(x)$

$\Rightarrow 0 < k - 2 < 1 \Rightarrow 2 < k < 3$

Clearly $f(x)$ has local maximum at $x = 0$.

Also $\int_0^1 f(x)dx = \int_0^1 (1-x^4)dx = \left(1-\frac{x^5}{5}\right)_0^1 =$

$1-\frac{1}{5} = \frac{4}{5}$.

53. Answer (A, B)

We have, $\cot^{-1}\frac{1}{x} = \begin{bmatrix} \pi+\tan^{-1}x, & x<0 \\ \tan^{-1}x, & x>0 \end{bmatrix}$ and

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \quad \forall x \in R$

Now, let $J = \int_{-1}^{2} (\cot^{-1}\frac{1}{x} + \cot^{-1}x)dx$

$= \int_{-1}^{0} (\cot^{-1}\frac{1}{x} + \cot^{-1}x)dx + \int_{0}^{2} (\cot^{-1}\frac{1}{x} + \cot^{-1}x)dx$

$= \frac{3\pi}{2} + \pi = \frac{5\pi}{2}$

And

$K = \int_{-2\pi}^{7\pi} \frac{\sin x}{|\sin x|}dx = \int_{6\pi}^{7\pi} 1.dx = \pi$

54. Answer (A, C)

Clearly, from the given figure,

$a < 0$, $c > 0$.

Also, $\frac{-b}{2a} > 0 \quad \Rightarrow \quad b > 0$

So, $abc < 0$

Also, $f(-1) = a - b + c = 0$ and $f(3)$

$= 9a + 3b + c = 0$

Clearly, $f\left(\frac{1}{3}\right) > 0$

$\Rightarrow \frac{a}{9} + \frac{b}{3} + c > 0$

$\Rightarrow a + 3b + 9c > 0.$

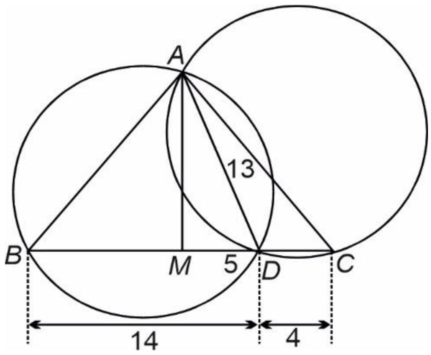
Also $f\left(\frac{-1}{3}\right) > 0$

$\Rightarrow \frac{a}{9} - \frac{b}{3} + c > 0$

$\Rightarrow a - 3b + 9c > 0$

Now, verify alternatives.

55. Answer (B, C, D)



The fact that the two circumcircles are congruent means the chord AD must subtend the same angle in both the circles.

i.e. $\angle ABC = \angle ACB$

$\Rightarrow \Delta ABC$ is isosceles.

Now AM is the altitude of ΔABC

$AM = 12 \Rightarrow$ Area $= \frac{18\cdot 12}{2} = 108$

($\Delta = \frac{1}{2}$(base)(altitude)

Also $\tan B = \tan C = \frac{12}{9}$

$\Rightarrow B = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore \angle A = \pi - 2\tan^{-1}\left(\frac{4}{3}\right)$

$= \pi - \left[\pi + \tan^{-1}\frac{2\cdot(4/3)}{1-(16/9)}\right]$

$= \tan^{-1}\left(\frac{24}{7}\right)$

56. Answer (C, D)

$(a-1)(x^2 + \sqrt{3}x + 1)^2 - (a+1)[(x^2+1)^2 - (x\sqrt{3})^2) \le 0$

or $(a-1)(x^2 + \sqrt{3}x + 1)^2 - (a+1)[x^2 + x\sqrt{3} + 1)(x^2 - x\sqrt{3} + 1) \le 0$

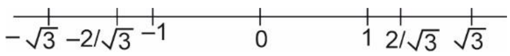
$(x^2 + \sqrt{3}x + 1)\ [(a-1)(x^2 + \sqrt{3}x + 1) - (a+1)(x^2 - \sqrt{3}x + 1)] \le 0 \quad \forall x \in R$

$\Rightarrow -2(x^2+1)+2a\sqrt{3}\,x \le 0$

$\Rightarrow x^2 - a\sqrt{3}\,x + 1 \ge 0 \;\; \forall\, x \in R$

$\Rightarrow 3a^2 - 4 \le 0 \quad (D \le 0)$

$\Rightarrow a \in \left[-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right]$



$\Rightarrow$ Number of possible integral value of a is

$\{-1, 0, 1\}$

$\Rightarrow 3$

and sum of all integral values of a is $-1+0+1=0$

57. Answer (A, D)

Let the plane is

$(2x+3y-z+1)+\lambda(x+y-2z+3)=0$ ...(1)

$(2+\lambda)x+(3+\lambda)y-(1+2\lambda)z+1+3\lambda=0$

$\Rightarrow \; 3(2+\lambda)-(3+\lambda)+2(1+2\lambda)=0$

$+\,6\lambda+5=0 \;\Rightarrow\; \lambda=-5/6$

Putting value of λ in (1)

$7x+13y+4z-9=0 \Rightarrow \alpha=9$

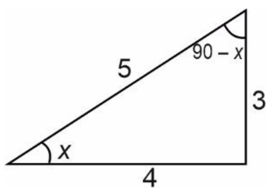
Now image of (1, 1, 1) in plane π is

$$\frac{x-1}{7}=\frac{y-1}{13}=\frac{z-1}{4}=-2\left(\frac{7+13+4-9}{49+169+16}\right)$$

$$\frac{x-1}{7}=\frac{y-1}{13}=\frac{z-1}{4}=-\frac{15}{117}$$

$$x=\frac{12}{117},\; y=\frac{-78}{117},\; z=\frac{57}{117} \Rightarrow \beta=117.$$

58. Answer (A, D)



Squaring and adding the given equations,

$4+9+12\sin(x+y)=25$

$\Rightarrow \sin(x+y)=1=\sin\dfrac{\pi}{2}$

$\therefore\; x+y=2n\pi+\dfrac{\pi}{2}=(4n+1)\dfrac{\pi}{2} \quad n\in I \Rightarrow$ **(A)**

if $x+y=\dfrac{\pi}{2} \quad\Rightarrow\quad y=\dfrac{\pi}{2}-x$

$5\sin x=3 \quad\Rightarrow\quad \sin x=\dfrac{3}{5}$

or $\cos x=\dfrac{4}{5}$

also

$\cos y=\dfrac{3}{5}$ and $\sin y=\dfrac{4}{5}$; hence $y>x$

59. Answer (A$\to$P); (B$\to$T); (C$\to$ S); (D$\to$Q)

(A) $I_1=\displaystyle\int_0^\infty x^7\cdot e^{-x^2}dx = e^{-x^2}\cdot\frac{x^8}{8}\Big]_0^\infty - \int_0^\infty(-2x)e^{-x^2}\cdot\frac{x^8}{8}dx$

$=0+\dfrac{2}{8}\cdot I_2$

$\Rightarrow \dfrac{I_2}{I_1}=4$

(B) $x^4-13x^2+36\le 0$

$\Rightarrow (x^2-9)(x^2-4)\le 0$

$\Rightarrow\; x\in[-3,-2]\cup[2,3]$

Now, let

$f(x)=x^3-3x \;\Rightarrow\; f'(x)=3(x^2-1)>0 \;\;\forall\, x \in [-3,-2]\cup[2,3]$

$\therefore\quad f_{\max.}\,(x=3)=(3)^3-3(3)=27-9=18.$

(C) Any circle through (2, 2) and (9, 9) is

$(x-2)(x-9)+(y-2)(y-9)+\lambda(y-x)=0$ ...(1)

For the point of intersection with x-axis, we put $y=0$ in (1), we get

$(x-2)(x-9)+18-\lambda x=0$

Put disc. $=0 \;\Rightarrow\; (11+\lambda)^2-4\cdot 36=0$

$\Rightarrow \quad \lambda = -23, 1$

$\therefore \quad x = \dfrac{11+\lambda}{2} = \pm 6$

So, the absolute value of the difference of x-coordinate of the point of contact = $|6 - (-6)| = 12$

(D) $y = \cos^{-1}(3x - 4x^3) = \dfrac{\pi}{2} - \sin^{-1}(3x - 4x^3) = \dfrac{\pi}{2} - \left(\pi - 3\sin^{-1}x\right)$

because $\sin^{-1}(3x - 4x^3) = \pi - 3\sin^{-1}x$ if $x \in \left[\dfrac{1}{2}, 1\right]$

Hence $\dfrac{dy}{dx} = \dfrac{3}{\sqrt{1-x^2}}$

$\left.\dfrac{dy}{dx}\right]_{x=\frac{\sqrt{3}}{2}} = \dfrac{3}{\sqrt{1-\frac{3}{4}}} = 6$

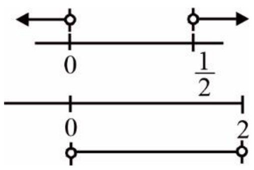
Alternatively (D): Clearly,

$\dfrac{dy}{dx} = \dfrac{-1\times(3-12x^2)}{\sqrt{1-(3x-4x^3)^2}}$

$\Rightarrow \left.\dfrac{dy}{dx}\right]_{x=\frac{\sqrt{3}}{2}} = \dfrac{3(4x^2-1)}{\sqrt{1-x(3-4x^2)}} = 6$

60. Answer (A$\to$Q); (B$\to$R); (C$\to$ Q); (D$\to$P, Q, R, S, T)

(A)



Solving, $y = \dfrac{2k^2 - k}{3} > 0$ and $x = \dfrac{4k - 2k^2}{3} > 0$

Hence $k \in \left(\dfrac{1}{2}, 2\right)$

(B) Given, $g(x) = \ln(\cos^{-1}x)$

As $0 \le \cos^{-1}x \le \pi \ \forall\ x \in [-1, 1]$

So, Domain of $g(x) = [-1, 1)$

Hence, number of integers are two (i.e., – 1 and 0).

(C) Clearly, domain of expression = $\{-1, 1\}$. As $x > 0$ (Given)

So, $x = 1$

Hence, the value of expression $\dfrac{\left(1+\sin^{-1}x\right)^{2020}\left(1+\cos^{-1}x\right)^{2021}\left(1+\tan^{-1}x\right)^{2022}}{\left(1+\operatorname{cosec}^{-1}x\right)^{2020}\left(1+\sec^{-1}x\right)^{2021}\left(1+\cot^{-1}x\right)^{2022}}$ = 1.

(D) $f(x) = \dfrac{x+a}{x+b}$ ......(1)

$f^{-1}(x) = \dfrac{a-bx}{x-1}$ ......(2)

Given $f(x) = f^{-1}(x)$

$\Rightarrow \dfrac{x+a}{x+b} = \dfrac{a-bx}{x-1}$

$\Rightarrow (1+b)x^2 + (b^2-1)x - a(1+b) = 0 \ \forall\ x \in D_f$

Hence, $b = -1$ and $a \in R$.

Aliter: According to the given condition,

$x = f(f(x)) = \dfrac{f(x)+a}{f(x)+b} \Rightarrow x = \dfrac{\frac{x+a}{x+b}+a}{\frac{x+a}{x+b}+b}$

$\Rightarrow \dfrac{x}{1} = \dfrac{(x+a)+a(x+b)}{(x+a)+b(x+b)}$

$\Rightarrow \dfrac{x}{1} = \dfrac{(1+a)x + a(1+b)}{(1+b)x + \left(a+b^2\right)}$

$\Rightarrow (1+b)x^2 + (a+b^2)x = (1+a)x + a(1+b)$

$\Rightarrow (1+b)x^2 + (b^2-1)x - a(1+b) = 0 \ \forall\ x \in D_f$

Hence, $b = -1$ and $a \in R$.

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B
CODE

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Time : 3 hrs

MOCK TEST - 1

MM : 264

for JEE (Advanced) – 2022

Paper – 2

ANSWERS

PHYSICS	CHEMISTRY	MATHEMATICS
1. (8)	21. (6)	41. (6)
2. (2)	22. (3)	42. (5)
3. (3)	23. (4)	43. (3)
4. (5)	24. (6)	44. (5)
5. (5)	25. (9)	45. (0)
6. (4)	26. (3)	46. (3)
7. (6)	27. (8)	47. (1)
8. (2)	28. (5)	48. (8)
9. (A, B)	29. (A, B, C)	49. (C, D)
10. (A, D)	30. (A, B)	50. (A, B)
11. (A, B)	31. (A, B, C, D)	51. (B, D)
12. (C, D)	32. (B, C)	52. (A, B, D)
13. (A, C)	33. (A, B, D)	53. (B, D)
14. (B, C)	34. (A, B, C)	54. (B, D)
15. (B, D)	35. (A, B, D)	55. (B, C)
16. (A)	36. (A, C)	56. (B, D)
17. (B, C)	37. (A)	57. (A)
18. (A)	38. (A)	58. (A)
19. (A, B)	39. (C)	59. (A)
20. (C)	40. (B)	60. (B)

B
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MOCK TEST - I

for JEE (Advanced) – 2022

Paper - 2

MM : 264

ANSWER & SOLUTIONS

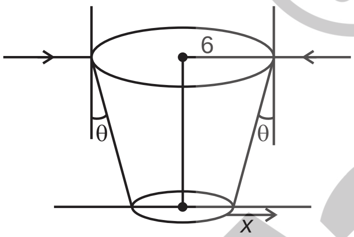
PART – I : PHYSICS

1. Answer (8)

$F = 24 \times 10^{-21}$ N

$\therefore \quad n = 8$

2. Answer (2)



$1\sin 90° = 1.25 \sin\theta$

$\theta = 53°$

$\frac{x}{3} = \tan\theta$

$\Rightarrow \quad x = 4$

$r = 6 - 4 = 2$ m

3. Answer (3)

$\Delta PE = \Delta KE$

$\Rightarrow \quad mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}\right)mR^2\omega^2$

Also $v_{cm} = r\omega = \frac{R}{2}\omega$

$v_{max} = \left(R + \frac{R}{2}\right)\omega = \frac{3R}{2}\omega$

$= \left(\frac{3}{\cancel{2}}\right)(2)\sqrt{\frac{10gh}{13}}$

$= 3$ m/s

4. Answer (5)

Case-1 :

'i' is very small $\Rightarrow R_V$ is very high

Case-2 :

V across ammeters is V, small $\Rightarrow R_A$ is very low

$\Rightarrow$ From Case-1, we can say that

$V_{cell} = 100$ volt

From Case-2, we can say

$25 \times 10^{-3} = V_{cell} - ir$

$= 10 - 2.5\,r$

$r = 40\ \Omega$

5. Answer (5)

$f_A = \frac{340 - 10}{340 - 10 + 10} \times 85 = \frac{330}{4}$ Hz

$f_B = \frac{340 + 10}{340 + 10 + 10} \times 85 = \frac{330}{4}$ Hz

$f_{beat} = f_B - f_A = 5$ Hz

6. Answer (4)

$\vec{p} = q\vec{l} = 4\left[\vec{r_2} - \vec{r_i}\right]$

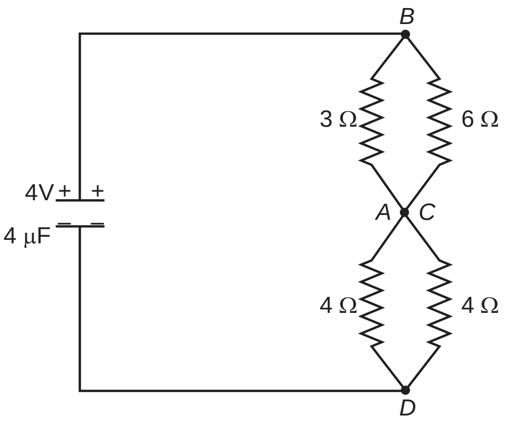
$= 4 \times 10^{-3}\left[\left(-1.2\hat{i} + 1.1\hat{j}\right) - \left(-1.4\hat{i} - 1.3\hat{j}\right)\right]$

$= \left[-26\hat{i} + 2.4\hat{j}\right] \times 10^{-3} \quad \vec{\tau} = \vec{p} \times \vec{E}$

$= 4 \times 10^{-3}\left[-2.6\hat{i} + 2.4\hat{j}\right] \times \left[2500\hat{i} - 5000\hat{j}\right]$

$= \left[5.2k - 2.4k\right] = 28$ N-m k

7. Answer (6)



Potential difference across the capacitor is

$$V_B - O = \frac{q_0}{C} = \frac{16}{4} = 4 \text{ volt}$$

E.M.F. of the battery = 24 V, $Y = 24$

Time constant

$\tau = RC$

$= (4\ \Omega)\ (4\ \mu F)$

$= 16\ \mu s$

Equation of discharge

$$q = q_0 e^{-t/\tau}$$

$$4\ \mu C = 16\ \mu C e^{\left(-\frac{X}{16}\right)}$$

$X = 32 \ln 2$

8. Answer (2)

$$\frac{dN}{dt} = \frac{P_0 \times \lambda}{hc} \times \eta$$

$$= 0.1 \times \frac{6.63 \times 10^{-6} \times 300 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} = 10^{12} \text{ sec}^{-1}$$

$E = h(f - f_T)$

$P = 2.2 \times 10^{-4}$ W

9. Answer (A, B)

$2T\sin 37 = 2\ mg$

$$\cancel{2} \times T_1 \times \frac{3}{5} = \cancel{2}\ mg$$

$$\Rightarrow \quad T_1 = \frac{5\ mg}{3}$$

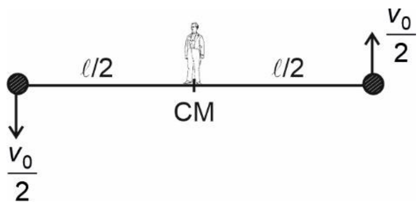
$T_2 = T_1 \cos 37$

$$= \frac{5\ mg}{3} \times \frac{4}{5} = \frac{4\ mg}{3}$$

10. Answer (A, D)

May be in uniform circular motion

11. Answer (A, B)



$$v_{cm} = \frac{v_0}{2}, \omega = \frac{v_0}{\ell}$$

$$T = \frac{2\pi\ell}{v_0}, \frac{T}{4} = \frac{\pi\ell}{2v_0}$$

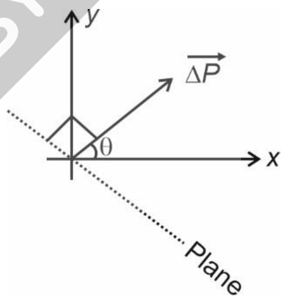
$$\vec{S}_1 = \vec{S}_{1,cm} + \vec{S}_{cm} = \left(\frac{\ell}{2} + \frac{\pi\ell}{4}\right)\hat{j} + \frac{\ell}{2}\hat{i}$$

$$\vec{S}_2 = \left(\frac{\pi\ell}{4} - \frac{\ell}{2}\right)\hat{j} + \frac{\ell}{2}\hat{i}$$

12. Answer (C, D)

The wave speed depends on properties of the medium, not on how you generate the wave. For a string $v = \sqrt{T_S/\mu}$. Increasing the tension or decreasing the linear density (lighter string) will increase the wave speed.

13. Answer (A, C)



$$\overrightarrow{\Delta v} = \overrightarrow{v_2} - \overrightarrow{v_1} = (\hat{i} - \hat{j}) - (-2\hat{i} - 3\hat{j})$$

$$= \left(3\hat{i} + 2\hat{j}\right)$$

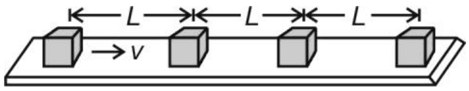
$$\Rightarrow \quad \overrightarrow{\Delta P} = \left(3\hat{i} + 2\hat{j}\right)$$

$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$

14. Answer (B, C)

Force of upthrust will be there on mass *m* shown in given figure, so *A* weighs less than 2 kg. Balance will show sum of load of beaker and reaction of upthrust so it reads more than 5 kg.

15. Answer (B, D)



Since collision is perfectly inelastic so all the blocks will stick together one by one and move in a form of combined mass.

Time required to cover a distance 'L' by first block = $\frac{L}{v}$

Now first and second block will stick together and move with $\frac{v}{2}$ velocity (by applying conservation of momentum) and combined system will take time $\frac{L}{v/2}=\frac{2L}{v}$ to reach up to block third.

Now these three blocks will move with velocity $\frac{v}{3}$ and combined system will take time $\frac{L}{v/3}=\frac{3L}{v}$ to reach up to the block fourth.

So, total time =

$$\frac{L}{v}+\frac{2L}{v}+\frac{3L}{v}+...\frac{(n-1)L}{v}=\frac{n(n-1)L}{2v}$$

and velocity of combined system having n blocks as $\frac{v}{n}$.

16. Answer (A)

$r = 1.5 \times 10^8$ km, $T = 1$ year

$= 3.14 \times 10^7$ s, $m = 6 \times 10^{24}$ kg

Linear velocity, $v = r\omega$

$$=\frac{1.5\times10^{11}\times2\pi}{3.14\times10^{7}}=3\times10^{4}\text{ m/s}$$

Work-energy theorem,

$$W=K_p-K_I=0-\frac{1}{2}mv^2$$

$$=-\frac{6}{2}\times10^{24}\times9\times10^{8}$$

$$=-\frac{54}{2}\times10^{32}\text{ J}$$

$$=-\frac{54}{x}\times10^{32}\text{ J}\Rightarrow x=2$$

17. Answer (B, C)

From basic photoelectric equation, we can get work function.

18. Answer (A)

Number of electrons emitted

$$=\frac{J}{e}=\frac{4.8\times10^{-3}}{e}=3\times10^{16}$$

Efficiency $\eta=\frac{\text{no. of electrons emitted}}{\text{no. of photons incident}}$

$$=\frac{JE_1}{e}=\frac{3\times10^{16}(hc)}{3000\times10^{-10}}=0.0198$$

19. Answer (A, B)

Potential gradient,

$$K=\frac{V_{AB}}{L}=\frac{I\times R}{L}=\frac{4\text{ volt}}{(15+10)\text{ohm}}\times\frac{10\text{ ohm}}{50\text{cm}}$$

$$=\frac{40}{25\times50}\text{volt/cm}$$

$$\therefore\ E_2=KI=\frac{40}{25\times50}\times31.25=1\text{ volt}$$

20. Answer (C)

When key K_2 is open and K_1 is closed : In this case R_1 is short circuited and e.m.f E_2 is balanced against potentiometer

i.e., $E_2=\left[\frac{4}{10}\times\frac{10}{50}\right]\times1=\frac{1\times10\times50}{4x10}=12.5$ cm

PART – II : CHEMISTRY

21. Answer (6)

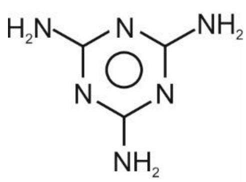
For a first order reaction

$$t=\frac{2.303}{k}\log\frac{[A]_0}{[A]_t}$$

$$=\frac{2.303}{7.67\times10^{-2}}\log\frac{1.20}{0.75}$$

$$=\frac{2.303\times0.2}{7.67\times10^{-2}}\simeq6\text{ sec}$$

22. Answer (3)



(Melamine)

Three lone pairs of electrons are involved in resonance.

23. Answer (4)

Moles of (X) = $\frac{12\times10^{-3}}{80}=1.5\times10^{-4}$

Moles of H_2 = $\frac{10.08}{22400}$ = 4.5×10^{-4}

Thus (X) has three double bonds. It is

$CH_2 = CH - CH = CH - CH = CH_2$

24. Answer (6)

Compounds other than, Salicylaldehyde and Pentan-2-one give positive Fehling solution test.

25. Answer (9)

Mass of mixture of Al and Al_2O_3 = 0.742 g

$2Al + 2NaOH + 2H_2O \rightarrow 2NaAlO_2 + 3H_2$

$Al_2O_3 + 2NaOH \rightarrow 2NaAlO_2 + H_2O$

Volume of H_2 gas at STP = 840 mL

No. of moles of $H_2 = \frac{840}{22400} = 0.0375$

No. of moles of Al = $\frac{2}{3}\times$ No. of moles of H_2

= $\frac{2}{3}\times 0.0375 = 0.025$

Mass of aluminium = 0.025 × 27 = 0.675 g

% of Al_2O_3 in the mixture = $\left(\frac{0.742-0.675}{0.742}\right)^{100}$

= 9%

26. Answer (3)

Glucose, Mannose and Fructose give the same osazone with excess of phenyl hydrazine.

27. Answer (8)

$$\underset{Z=100}{A}\xrightarrow{-e}\underset{Z_1=101}{B}\xrightarrow{-\alpha}\underset{Z_2=97}{C}$$

$$\frac{Z_1+Z_2}{25}=\frac{101+97}{25}=\frac{198}{25}\simeq 8$$

28. Answer (5)

$A \longrightarrow B$

$[A]_t = a - b\,t$

Pressure , p of gas A is given by

p = (a – b t) RT

$$R = -\frac{dp}{dt} = bRT = 0.4\times\frac{1}{12}\times 300 = 10 \text{ atm sec}^{-1}$$

29. Answer (A, B, C)

(A) For an ideal monoatomic gas, $\gamma = \frac{5}{3}$

Adiabatic equation for expansion of ideal monoatomic gas

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{T_2}{T_1}\right)^{3/2}$$

(B) At low pressures attractive forces dominate and 'b' is negligible

$$\therefore\ Z = 1 - \frac{a}{V_m bRT}$$

(C) A gas can be liquified below critical temperature just by increasing pressure

(D) van der Waal's constant 'a' is a measure of intermolecular forces of attraction

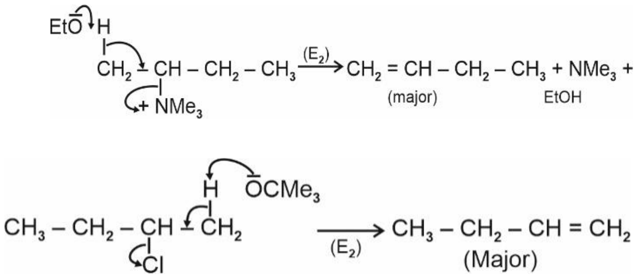
30. Answer (A, B)

Both nitrate ion $\left(NO_3^-\right)$ and bromide ion (Br^-) give brown fumes with conc. H_2SO_4

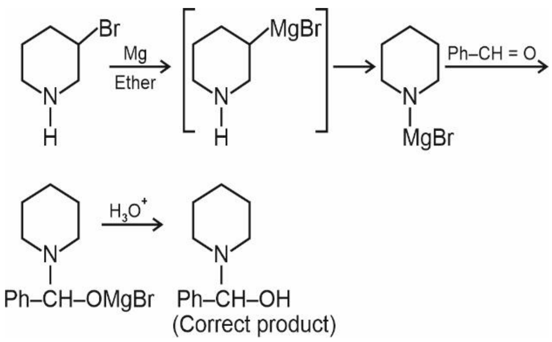
31. Answer (A, B, C, D)

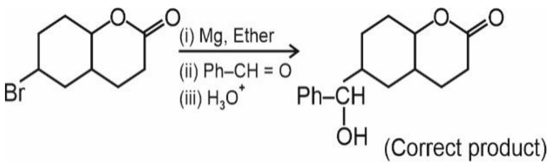
By all the above processes coagulation can be done.

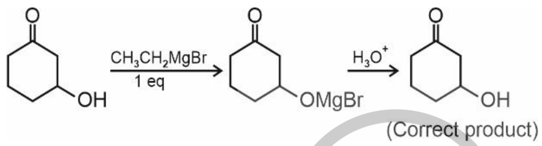
32. Answer (B, C)



33. Answer (A, B, D)

(A) 

(B) 

(D) 

34. Answer (A, B, C)

Sulphonic acid group is stronger than carboxylic acid group.

35. Answer (A, B, D)

Gel has liquid dispersed phase and solid dispersion medium

36. Answer (A, C)

$$\text{Lactose} \xrightarrow{H_3O^+} \text{Glucose + Galactose}$$

37. Answer (A)

$$\Delta H = -1575 - \frac{3}{2}\times 241.8 + 2021$$

= 83.3 kJ/mol

$$\therefore \Delta H \text{ (per kg)} = 83.3\times\frac{1000}{172}$$

= 484.3 kJ

38. Answer (A)

$$\Delta S = 130.5 + \frac{3}{2}\times 188.6 - 194$$

= 219.4 J/K

39. Answer (C)

$$\underset{(A)}{(NH_4)_2SO_4\cdot FeSO_4\cdot 6H_2O} + aq \rightarrow 2NH_4^+ + \overset{2+}{Fe} + 2SO_4^{2-}$$

$$NH_4^+ + 2K_2HgI_4 + 4KOH \rightarrow \underset{\text{Brown}}{H_2N-Hg-O-Hg-I}\downarrow + 7KI + K^+ + 3H_2O$$

$$3Fe^{2+} + 2K_3[Fe(CN)_6] \rightarrow \underset{\text{(Blue ppt.)}}{Fe_3[Fe(CN)_6]_2} + 6K^+$$

$$BaCl_2 + SO_4^{2-} \rightarrow \underset{\text{(White)}}{BaSO_4}\downarrow 2Cl^-$$

40. Answer (B)

$$FeSO_4 + NO + 5H_2O \rightarrow \underset{(D)}{[Fe(H_2O)_5NO]\,SO_4}$$

PART – III : MATHEMATICS

41. Answer (6)

$$S = \sum_{r=1}^{50}\tan^{-1}\left(\frac{1+r+r^2-(1-r+r^2)}{1+(1+r+r^2)(1-r+r^2)}\right)$$

$$= \sum_{r=1}^{50}\tan^{-1}(1+r+r^2) - \tan^{-1}(1-r+r^2)$$

$$= \tan^{-1}(1+50+50^2) - \tan^{-1}1 = \tan^{-1}\left(\frac{2550}{2552}\right)$$

$\Rightarrow k = 6.$

42. Answer (5)

If there is a subset with 6 elements then it has 15 pairs, each with sum at least 1 + 2 = 3 and at most 8 + 9 = 17. There are only 15 numbers at least 3 and at most 17, so each must be realised.

But the only pair with sum 3 is 1, 2 and with sum 17 is 8, 9 and then 1 + 9 = 8 + 2, so six elements is impossible.

Also {1, 2, 3, 5, 8} has five elements and all pair with a different sum.

43. Answer (3)

The circle through points of intersections of three distinct tangents to parabola, taken pairwise always passes through the focus of the parabola.

44. Answer (5)

$f(0) = 0$; $x = 0$, $y = 0$

$f(x^3) = xf(x^2)$; $y = 0$

$f(y^3) = yf(y^2)$

$f(x^3) + f(y^3) = xf(x^2) + yf(y^2) = f(x^3 + y^3)$

$\Rightarrow f(x) + f(y) = f(x + y)$

$$f'(x) = \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(h)-f(0)}{h} = f'(0) = 5 \Rightarrow f'(x) = 5.$$

45. Answer (0)

$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})=(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{b})-\vec{a}\cdot\vec{c}$

$(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})=(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{c})-\vec{a}\cdot\vec{b}$

$(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})=(\vec{a}\cdot\vec{b})(\vec{a}\cdot\vec{c})-\vec{b}\cdot\vec{c}$

given that $|\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=3\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$

$\Rightarrow \lambda = (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})+(\vec{b}\cdot\vec{c})(\vec{c}\cdot\vec{a})+(\vec{c}\cdot\vec{a})(\vec{a}\cdot\vec{b})$

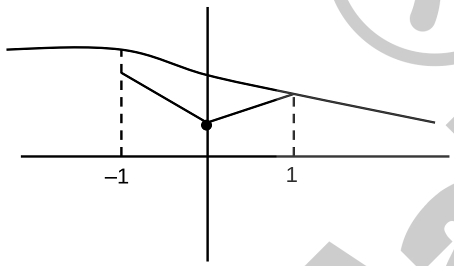
$-\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{c}-\vec{c}\cdot\vec{a}$

$\Rightarrow \lambda = (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})+(\vec{b}\cdot\vec{c})(\vec{c}\cdot\vec{a})+(\vec{c}\cdot\vec{a})(\vec{a}\cdot\vec{b})$

$\Rightarrow \lambda \le 0$ (since $x + y + z = 0$, $xy + yz + xz \le 0$)

$\lambda_{max} = 0$ only when $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{a}\cdot\vec{c}=0$

46. Answer (3)



$f(1^+) = f(1) = \dfrac{\pi}{4}$ and $f(1^-) = \dfrac{\pi}{4}$

also, $f(-1^+) = \dfrac{\pi}{4}$, $f(-1^-) = f(-1) = \dfrac{3\pi}{4}$

$\Rightarrow$ $f(x)$ is continuous at $x = 0, 1$ but discontinuous at $x = -1$.

domain of $f'(x)$ does not contains $x = -1, 0, 1$.

47. Answer (1)

$\sin\sin^{-1}[x] = [x]$ if $-1\le [x] \le 1$

i.e. $-1 \le x < 2$

$\cos^{-1}\cos x = x$ if $0 \le x \le \pi$

$\Rightarrow$ Given equation becomes $[x] + x = 1$ where $0 \le x < 2$

$[x] = 1 - x \qquad 0 \le x < 2$

No solution except $x = 1$.

Number of solutions is one.

48. Answer (8)

$(Y+2\sqrt{2})^2 = 4(X+1)+4$

Let chord is $Y = mX + c$

$Y^2+4\sqrt{2}Y\left(\dfrac{Y-mX}{c}\right)-4\times\left(\dfrac{Y-mX}{c}\right)=0$

Coefficient of X^2 + coefficient of Y^2 = 0

$1+\dfrac{4\sqrt{2}}{c}+\dfrac{4m}{c}=0$

$c + 4\sqrt{2} + 4m = 0$

$Y = mX - 4\sqrt{2} - 4m$

$(Y + 4\sqrt{2}) - m(X - 4) = 0$

$X = 4$, $Y = -4\sqrt{2}$

$x = 5$, $y = 2\sqrt{2}-4\sqrt{2}$.

49. Answer (C, D)

b is H.M. of a and c

$b < \dfrac{a+c}{2}$

$b - a < c - b$

$a - b > b - c$

$\dfrac{1}{a-b}-\dfrac{1}{b-c}<0$

also $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P.

so $bc(1 - a)$, $ac(1 - b)$, $ab(1 - c)$ are in A.P.

50. Answer (A, B)

Take any four numbers

Ex: 5, 1, 3, 3

They can be arranged only in one way such that each number is not smaller than the preceding i.e. 1, 3, 3, 5.

Probability

$$= \frac{\left({}^6C_1 \cdot 1 + {}^6C_2 \cdot 3 + {}^6C_3 \cdot 3 + {}^6C_4 \cdot 1\right)}{6^4} = \frac{7}{72}$$

51. Answer (B, D)

Area A = area of $PQRS$

$= (b \sin\theta + a \cos\theta)(a \sin\theta + b \cos\theta)$

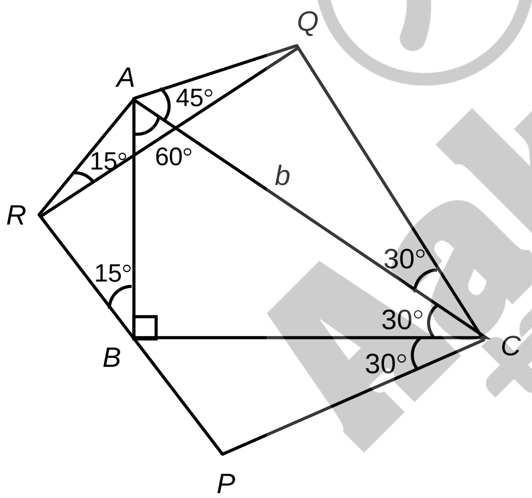
$= ab + (a^2 + b^2) \sin\theta \cos\theta$

$= ab + \frac{a^2 + b^2}{2} \sin 2\theta$

A is maximum when $\sin 2\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

$A_{\max} = ab + \frac{a^2 + b^2}{2} = \frac{1}{2}(a + b)^2$

52. Answer (A, B, D)



$AR = RB = \frac{b}{4\cos 15^\circ}$

$AQ = \frac{2b}{4\cos 15^\circ}$, $BP = \frac{\sqrt{3}b}{4\cos 15^\circ}$

$\Rightarrow RQ^2 = \frac{7b^2}{16\cos^2 15^\circ} = RP^2$

$\Rightarrow RP = RQ$ and $\left(\frac{RQ}{b}\right)^2 = \frac{7}{4\sqrt{3} + 8} < 1.$

Hence $RQ < b$ and quadrilateral $PCQR$ is a square hence $PQ = RC$.

53. Answer (B, D)

$$\lim_{x\to\infty} 4x\left(\frac{\pi}{4} - \tan^{-1}\frac{x+1}{x+2}\right) = \lim_{x\to\infty} 4x\left(\tan^{-1}\frac{1 - \frac{x+1}{x+2}}{1 + \frac{x+1}{x+2}}\right)$$

$$= \lim_{x\to\infty} 4x \frac{\tan^{-1}\left(\frac{1}{2x+3}\right)}{\left(\frac{1}{2x+3}\right)} \times \frac{1}{2x+3} = 2$$

$y^2 + 4y + 5 = 2$

$y = -1, -3$

54. Answer (B, D)

$$f(x) = \int_0^1 5 + (1 - t)\,dt + \int_1^x 5 - (1 - t)\,dt$$

$$\Rightarrow f(x) = \begin{cases} 5x + 1, & x \le 2 \\ \frac{x^2}{2} + 4x + 1, & x > 2 \end{cases}$$

55. Answer (B, C)

$$\left|z_1^2 - z_2^2\right| = \left|\bar{z}_1^2 + \bar{z}_2^2 - 2\bar{z}_1\bar{z}_2\right|$$

$$= \left|z_1^2 + z_2^2 - 2z_1z_2\right|$$

$|(z_1 - z_2)(z_1 + z_2)| = |z_1 - z_2||z_1 - z_2|$

$\Rightarrow |z_1 + z_2| = |z_1 - z_2|$

$$(z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$|z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + z_2\bar{z}_1 = |z_1|^2 + |z_2|^2 - z_1\bar{z}_2 - z_2\bar{z}_1$$

$\Rightarrow z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

$\Rightarrow \frac{z_1}{z_2} = -\frac{\bar{z}_1}{\bar{z}_2}$

$\Rightarrow \frac{z_1}{z_2}$ is purely imaginary and $|\arg z_1 - \arg z_2|$

$= \frac{\pi}{2}$

56. Answer (B, D)

$$g'(x) = \frac{(x - 1)\left(x^3 - 3x^2 + 5x + 1\right).e^x}{\left(x^2 + 1\right)^3}$$

Now, '$x^3 - 3x^2 + 5x + 1$' is strictly increasing and has a root in $(-1, 0)$

57. Answer (A)

SP.S′P = $a^2\sin^2\theta + b^2\cos^2\theta = a^2 + \cos^2\theta(b^2 - a^2)$

$\Rightarrow$ Maximum of SP · S′P = a^2 at $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

then point P is $(0, \pm 1)$.

58. Answer (A)

Curve X is $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$be = 1$

$a^2 + b^2 = 1$

$\Rightarrow \frac{a^2}{b} = 2$

$b^2 + 2b - 1 = 0$

$b = \sqrt{2} - 1$, $a^2 = 2\left(\sqrt{2} - 1\right)$

$$\frac{-x^2}{2\left(\sqrt{2}-1\right)} + \frac{y^2}{\left(\sqrt{2}-1\right)^2} = 1$$

59. Answer (A)

A = area of ΔORS = $\frac{1}{2}\sqrt{|\vec{a}|^2 + |\vec{c}|^2}\,\frac{|\vec{b}|}{2}\cos\theta$

$\sin\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

60. Answer (B)

$\frac{1}{2}\sqrt{|\vec{a}|^2 + |\vec{c}|^2} \times h = \frac{|\vec{a}\times\vec{c}|}{2}$

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