A

CODE

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005. Ph.011-47623456

MOCK TEST-2

for JEE (Advanced) – 2022

Paper - I

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(B, C, D)	19.	(B, D)	37.	(A, B)
2.	(A, C, D)	20.	(D)	38.	(A, D)
3.	(A, D)	21.	(A, B, D)	39.	(B, C, D)
4.	(A, D)	22.	(A, C)	40.	(A, B, C, D)
5.	(A, B, D)	23.	(B, D)	41.	(A, B, C, D)
6.	(B, D)	24.	(A, C)	42.	(A, C, D)
7.	(04)	25.	(01)	43.	(60)
8.	(01)	26.	(08)	44.	(89)
9.	(90)	27.	(02)	45.	(54)
10.	(02)	28.	(01)	46.	(19)
11.	(24)	29.	(02)	47.	(76)
12.	(25)	30.	(05)	48.	(05)
13.	(22)	31.	(06)	49.	(12)
14.	(03)	32.	(05)	50.	(25)
15.	(C)	33.	(C)	51.	(A)
16.	(C)	34.	(C)	52.	(B)
17.	(A)	35.	(C)	53.	(B)
18.	(B)	36.	(D)	54.	(A)

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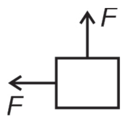
Paper - I

ANSWER & SOLUTIONS

PART - I : PHYSICS

1. Answer (B, C, D)



$F\sqrt{2} = \mu mg \;\Rightarrow\; F = \dfrac{\mu mg}{\sqrt{2}}$

$a_x = a_y = \dfrac{\mu g}{\sqrt{2}}$

$v^2 = u^2 - 2aS$

$\Rightarrow\; (0)^2 = (30)^2 - (2)\left(\dfrac{0.6\times 10}{\sqrt{2}}\right)S$

$\Rightarrow\; S_{\min} = 75\sqrt{2}$ m

2. Answer (A, C, D)

$\dfrac{dv}{dt} = 6$ m/s^2 for 1st 2 sec and $\dfrac{dv}{dt} = 3$ m/s^2 for last 4 sec

3. Answer (A, D)

$\dfrac{Kq_1}{R} + \dfrac{Kq_2}{3R} = 2$ V, $\dfrac{Kq_1}{3R} + \dfrac{Kq_2}{3R} = V$, $\dfrac{q_1}{q_2} = 1$

After earthing $\dfrac{Kq'}{3R} + \dfrac{Kq_2}{3R} = 0$, $\dfrac{q_1}{q_2} = -1$

4. Answer (A, D)

$dI = \dfrac{I}{2\pi R}h$

$\vec{B}_1 + \vec{B}_2 = 0$ (inside tube)

$\Rightarrow\; |\vec{B}_1| = \dfrac{(\mu_0)dI}{2\pi r} = \dfrac{(\mu_0)}{2\pi r}\dfrac{Ih}{2\pi R}$

$|\vec{B}_1| = \dfrac{\mu_0 Ih}{4\pi^2 Rr} \;\Rightarrow\; \vec{B}_A = \dfrac{\mu_0 Ih}{4\sqrt{2}\pi^2 R^2}$

$|\vec{B}_B| = \dfrac{\mu_0 Ih}{8\pi^2 R^2}$

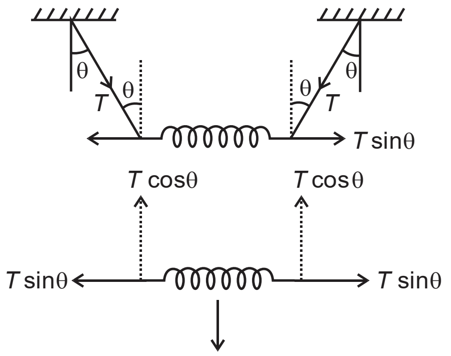
5. Answer (A, B, D)

$\vec{L}_i = \vec{L}_f$ about bump

$\Rightarrow\; MR\omega_0(R-h) + \dfrac{2}{5}MR^2\omega_0 = \dfrac{7}{5}MR^2\omega$

$\Rightarrow\; \omega = \omega_0\left(1 - \dfrac{5h}{7R}\right)$

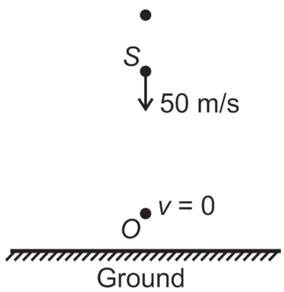
6. Answer (B, D)



$T\sin\theta = kx$

$2T\cos\theta = W\;\;\dfrac{\tan\theta}{2} = \dfrac{kx}{W}\quad x = \dfrac{W\tan\theta}{2k}$

7. Answer (04)



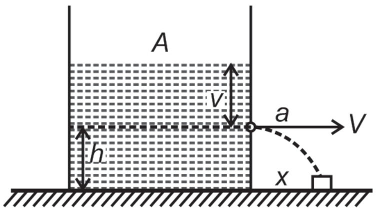
After 5 s, speed of detector = 50 – 10 × 5 = 0 and that of source = 0 + 10 × 5 = 50 m/s

and the source has fallen a distance = $\frac{1}{2}\times 10\times(5)^2$ = 125 m and the detector has risen

a distance = $50\times 5-\frac{1}{2}\times 10(5)^2$ = 125 m

So, $f' = 130\left(\frac{300-0}{300-50}\right) = 156$ Hz

8. Answer (01)



Velocity of efflux $v=\sqrt{2gy}$

Range $x=\sqrt{2gy}\times\sqrt{\frac{2h}{g}}$

The velocity of the block must be $\left(\frac{dx}{dt}\right)$

$$\therefore \quad V_b = \frac{dx}{dt} = \sqrt{\frac{2h}{g}}\times\sqrt{2g}\times\frac{1}{2\sqrt{y}}\frac{dy}{dt}$$

$$V_b = \frac{\sqrt{h}}{\sqrt{y}}\cdot\frac{dy}{dt} \qquad \ldots(i)$$

Using equation of continuity

$$\frac{Ady}{dt} = a\sqrt{2gy} \qquad \ldots(ii)$$

Equation (i) and (ii)

$$V_b = \sqrt{\frac{h}{y}}\times\frac{a}{A}\sqrt{2gy}$$

$$V_b = \sqrt{2gh}\times\frac{a}{A}$$

$$= 20\times\frac{1}{20} = 1\ \text{ms}^{-1}$$

9. Answer (90)

$$v_1 = -3\hat{i} - g\frac{\sqrt{3}}{5}\hat{j}$$

$$v_2 = 4\hat{i} - g\frac{\sqrt{3}}{5}\hat{j}$$

$$\vec{v_1}.\vec{v_2} = -12+\frac{g^2\times 3}{25}$$

$$\Rightarrow \quad \vec{v_1}.\vec{v_2} = 0$$

$\therefore \quad \theta = 90°$

10. Answer (02)

$$\left.\begin{array}{l}\text{Since } \quad X = 16 \text{ m} \\ \text{and } \quad Y = -8 \text{ m}\end{array}\right\}$$

$\Rightarrow \quad \theta = 45$

$\Rightarrow \quad t_0 = 2$ s

11. Answer (24)

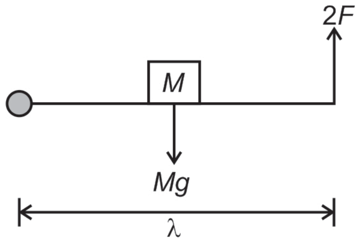
$$\frac{V^2}{R}t = 15\times S\times(100-T)$$

$$\frac{8V^2}{5R}\times t = m\times S\times(100-T)$$

$$\frac{8}{5}\times 15 = m$$

m = 24 kg

12. Answer (25)



For equilibrium

$$\left|Mg\frac{\lambda}{2}\right| = |2F|1$$

$$F = \frac{mg}{4} = \frac{100}{4} = 25 \text{ N}$$

13. Answer (22)

$$E = \frac{3}{2}kT = \frac{3}{2}\times 1.44\times 10^{-23}\times 300$$

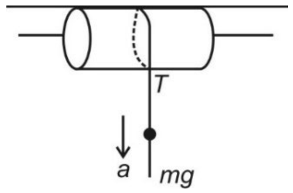
$$\lambda = \frac{h}{\sqrt{2m_n k}}$$

$$= \frac{6.6\times 10^{-34}}{\sqrt{2\times 1.69\times 10^{-27}\times 1.44\times 10^{-23}\times\frac{3}{2}\times 300}}$$

$$= \frac{22}{156}\times 10^{-9}\text{m}$$

14. Answer (03)

Suppose that the tension in the string is T.



Then, $m_0g - T = m_0a$

$\therefore \quad T = m_0(g - a)$ where a = acceleration

Further,

α = angular acceleration and $T.r$ = moment of force acting on cylinder $\tau = I\alpha$

$a = r\alpha$

or, $T = \frac{I\alpha}{r} = \left(\frac{mr^2}{2}\right)\left(\frac{\alpha}{r}\right) = \frac{mr\alpha}{2} = \frac{ma}{2}$

$a = \frac{2T}{m} = \frac{2m_0(g-a)}{m}$

Solving for a, we get

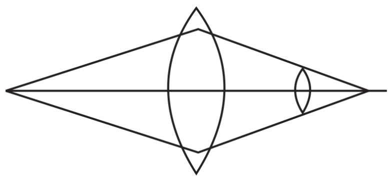
$a = \left(\frac{2m_0g}{m + 2m_0}\right) = \frac{g}{3}$

$Xg = \frac{g}{3}$

$X = \frac{1}{3}$ or $\frac{1}{X} = 3$

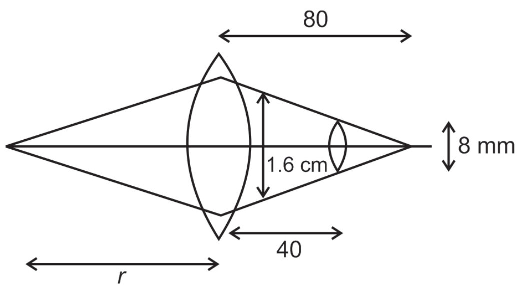
15. Answer (C)
16. Answer (C)
17. Answer (A)
18. Answer (B)

Solution for Q. Nos. 17 and 18



All light coming from lens will fall on sphere.

Intensity of light = $\frac{P}{4\pi r^2}$



Power incident on lens = $\frac{P}{4\pi r^2} \times \frac{\pi d^2}{4}$

Power exiting the lens = $\frac{P}{4\pi r^2} \times \frac{\pi d^2}{4} \times 0.80$

Let n be the number of photoelectron/s then

$$\frac{P}{4\pi r^2} \times \frac{\pi d^2}{4} \times 0.80 \times 10^{-4} = n\frac{hc}{\lambda}$$

$$\frac{Pd^2}{16r^2} \times 0.80 \times 10^{-4} = n \times 5 \times 1.6 \times 10^{-10}\ \text{V}$$

$$n = \frac{3.2 \times (1.6)^2 \times 10^{-4} \times 0.8}{16 \times (8.0)^2 \times 8 \times 10^{-19}}$$

$$= \frac{0.4 \times 10^{15} \times 0.8}{40000}$$

$n = 8 \times 10^9$

Current = $1.6 \times 10^{-19} \times 8 \times 10^9$

PART - II : CHEMISTRY

19. Answer (B, D)

$K = Ae^{-E_a/RT}$; A = PZ, $K = PZe^{-\frac{E_a}{RT}}$

20. Answer (D)

Millimoles of $AgNO_3$ = 0.1 × 300 = 30

Millimoles of ionic bromide = 0.2 × 50 = 10

The number of moles of $AgNO_3$ used is three times that of ionic bromide. Therefore, the formula of ionic bromide is ZBr_3

21. Answer (A, B, D)

Factual

22. Answer (A, C)

Factual

23. Answer (B, D)

Q = I × t = 96500 C = IF

Cathode :

$$2H_2O + 2e^- \longrightarrow H_2 + 2OH^-$$

Anode :

$$H_2O \longrightarrow \frac{1}{2}O_2 + 2H^+ + 2e^-$$

2 mol electron produce = $\left(1 + \frac{1}{2}\right)$ mol of gas.

1 mol electron produce = 0.75 mol of gas.

24. Answer (A, C)

The reaction which has higher value of activation energy has lesser rate of reaction.

25. Answer (01)

Mass of ethanol needed 1.7 × 46 = 78.2

Volume of ethanol measured out

$$\frac{\text{Mass of ethanol}}{\text{Density of ethanol}} = \frac{78.2}{0.784} \cong 100 \text{ ml}$$

26. Answer (08)

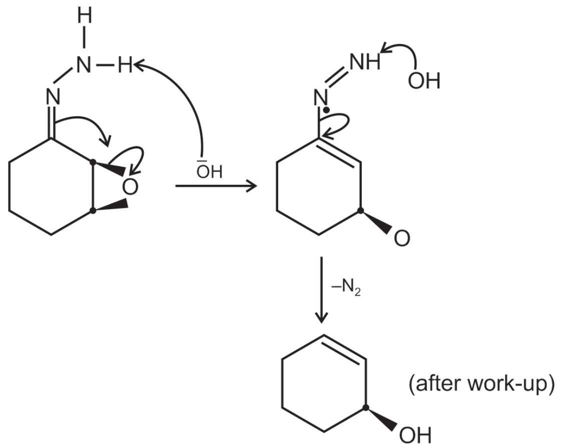
$$P_T = P_{gas} + P_{H_2O(V)}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ for gas}$$

27. Answer (02)

CH_4 and O_2 are colourless and odourless gases.

28. Answer (01)



29. Answer (02)

Two reactive intermediates

30. Answer (05)

$\pi = iCRT$

$$\begin{array}{ccccc} MgCl_2 & \rightarrow & Mg^{+2} & + & 2Cl^{-1} \\ 1-\alpha & & \alpha & & 2\alpha \end{array}$$

$i = (1+2\alpha) = (1+2\alpha)$

$\Rightarrow$ 4.8 = (1 + 2α) (0.1) (0.08) (300)

$$\Rightarrow \quad (1+2\alpha) = \frac{(4.8)}{(0.1)(0.08)(300)}$$

$$= \frac{4.8}{3 \times 0.08}$$

$$= \frac{480}{30 \times 8} = 2$$

$\Rightarrow$ $(1+2\alpha) = 2$

2α = 1

$\Rightarrow$ α = 0.5

$\Rightarrow$ 50% dissociation

31. Answer (06)

Total geometrical isomers = 15

Total optical isomers = 30

Total stereoisomer = 15 + $\frac{30}{2}$ = 30

$$\therefore \quad \frac{x}{5} = \frac{30}{5} = 6$$

32. Answer (05)

$$\frac{\Delta P}{P_s} = i\,\frac{n_{solute}}{n_{solvent}}$$

$$\frac{\Delta P}{P_s} = i\,\frac{m_{solute}}{\frac{1000}{18}}$$

$$\frac{18.5 - 18.496}{18.496} = i\,\frac{10^{-2}}{\frac{1000}{18}}$$

i = 1.2

α = i – 1

α = 0.2

$$K_b = \frac{C\alpha^2}{1-\alpha} = \frac{0.01 \times (0.2)^2}{0.8} = 5 \times 10^{-4}$$

33. Answer (C)

Equilibrium achieved at t_3, as conc. become constant.

34. Answer (C)

Concentrations should be constant at equilibrium.

35. Answer (C)

High oxidizing power of Fluorine is due to less bond energy and high hydration energy. As oxidizing power is the combination values of bond energy, electron affinity and hydration energy, their sum is most negative for F_2.

36. Answer (D)

Iodine is weaker oxidizing agent than other halogens. The free energy change of the following reaction is positive.

$$I_2 + H_2O \rightarrow 2H^+ + 2I^- + \frac{1}{2}O_2,\ \Delta G^\circ = +105 \text{ kJ/mol}$$

PART - III : MATHEMATICS

37. Answer (A, B)

$$\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}.dx = Ax + B\ell n\,|9e^{2x} - 4| + C$$

Put $4e^x + 6e^{-x} = P\left(9e^x - 4e^{-x}\right) + Q(9e^x + 4e^{-x})$

$\Rightarrow$ $4 = 9P + 9Q$ and $6 = 4Q - 4P$

Comparing, $P = -\dfrac{19}{36}$, $Q = \dfrac{35}{36}$

$$I = -\frac{19}{36}\int dx + \frac{35}{36}\int \frac{9e^x + 4e^{-x}}{9e^x - 4e^{-x}}dx$$

$$= -\frac{19}{36}.x + \frac{35}{36}\ell n\left|\left(9e^x - 4e^{-x}\right)\right| + C$$

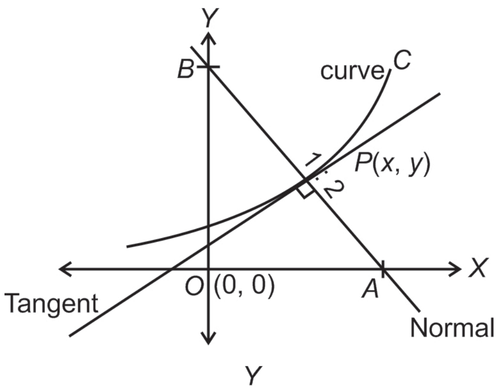
$$= -\frac{19}{36}x + \frac{35}{36}\ell n\left|\left(9e^{2x} - 4\right)\right| - \frac{35}{36}x + C$$

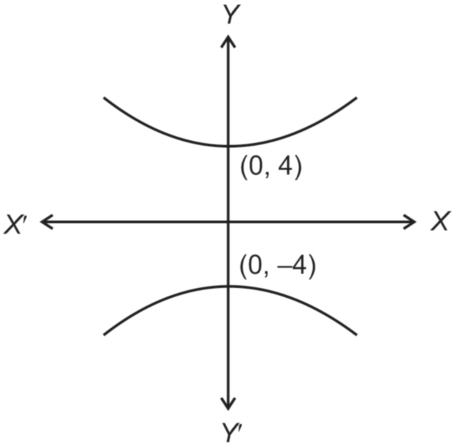
$$= \frac{35}{36}\ell n\left|\left(9e^{2x} - 4\right)\right| - \frac{54}{36}x + C$$

$$= \frac{35}{36}\ell n\left|\left(9e^{2x} - 4\right)\right| - \frac{3}{2}x + C$$

So, $A = -\dfrac{3}{2}$, $B = \dfrac{35}{36}$, $C \in R$

38. Answer (A, D)





The equation of normal at

$P(x, y)$ is $(Y - y) = \dfrac{-1}{\frac{dy}{dx}}(X - x)$

$\therefore$ $A\left(x + y\dfrac{dy}{dx}, 0\right)$ and $B\left(0, y + \dfrac{x}{\frac{dy}{dx}}\right)$

Now $\dfrac{1\left(x + y\frac{dy}{dx}\right) + 2(0)}{1+2} = x \Rightarrow x + y\dfrac{dy}{dx} = 3x$

$\therefore$ $y\dfrac{dy}{dx} = 2x$ ....(1)

$\Rightarrow$ $\int y\,dy = \int 2x\,dx \Rightarrow \dfrac{y^2}{2} = x^2 + C$

Also (0, 4) satisfy it, so $C = 8$

$\therefore$ $y^2 = 2x^2 + 16$ (equation of curve) which represent a hyperbola.

Also $\left.\dfrac{dy}{dx}\right]_{(4,4\sqrt{3})} = \dfrac{2(4)}{4\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

$\therefore$ The equation of tangent at $(4, 4\sqrt{3})$ is

$y - 4\sqrt{3} = \dfrac{2}{\sqrt{3}}(x - 4) \Rightarrow 2x - \sqrt{3}y + 4 = 0$

39. Answer (B, C, D)

It may be observed that

$$[\vec{U}\ \vec{V}\ \vec{W}] = \begin{vmatrix} 2 & 3 & -6 \\ 6 & 2 & 3 \\ 3 & -6 & -2 \end{vmatrix} = 343 \neq 0$$

$\Rightarrow$ $\vec{U}, \vec{V}, \vec{W}$ are non-coplanar hence linearly independent.

Further $\vec{U} \times \vec{V} = \vec{W}$ and $\vec{V} \times \vec{W} = \vec{U}$

$\Rightarrow$ They form a right handed triplet of mutually perpendicular vectors and of course!

$(\vec{U} \times \vec{V}) \times \vec{W} = \vec{0} = \vec{U} \times (\vec{V} \times \vec{W})$

40. Answer (A, B, C, D)

$AB = 0$

$\therefore$ $|AB| = 0 \Rightarrow |A|\,|B| = 0$

$\therefore$ $\det A \neq 0$

$\therefore$ A^{-1} exist

$\therefore$ $A^{-1}(AB) = A^{-1}(0) = 0$

$IB = 0$

$B = 0 \Rightarrow B$ must be null matrix.

41. Answer (A, B, C, D)

(A) Clearly,

S = sin130° + cos130° = (sin130° – sin40°)

= 2cos85° sin45° > 0.

(B) Given,

$\text{cosec}^2 30° + k(\cos 36° - \sin 18°) = 5$

$\Rightarrow \quad 4 + k\left[\left(\frac{\sqrt{5}+1}{4}\right) - \left(\frac{\sqrt{5}-1}{4}\right)\right] = 5$

$\Rightarrow \quad 4 + \frac{k}{2} = 5 \Rightarrow \frac{k}{2} = 1 \Rightarrow k = 2.$

(C) Let $E = 27\tan^2\theta + 3\cot^2\theta$

$= \left(3\sqrt{3}\tan\theta - \sqrt{3}\cot\theta\right)^2 + 18$

$\therefore \quad E_{\min} = 18$

(D) Let $P(x) = x^2 + 2ax + b$.

If $P(x) > 0 \ \forall\ x \in R$

then $D < 0$

$\Rightarrow \quad 4a^2 - 4b < 0 \quad \Rightarrow a^2 < b$

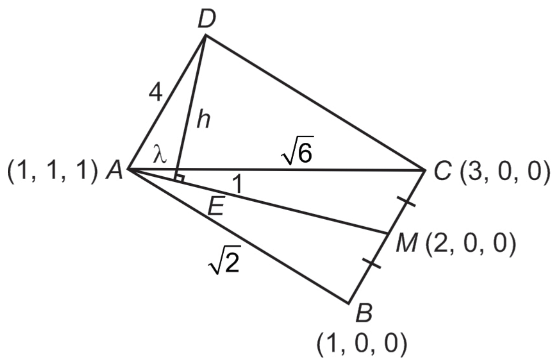
Alternate:

Let $f(x) = x^2 + 2ax + b$

$\therefore\ D = 4a^2 - 4b = 4(a^2 - b) < 0$

So, $f(x) > 0\ \forall\ x \in R$ (As, leading coefficient equals 1.)

42. Answer (A, C, D)



Given $V = \frac{2\sqrt{2}}{3}$

Now $\frac{1}{3}\cdot\frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix}\right| h = \frac{2\sqrt{2}}{3}$

$\Rightarrow \quad h\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix}\right| = 4\sqrt{2}$

[Note ABC is a right triangle $\rightarrow$ Area = $\frac{1}{2}(2)\left(\sqrt{2}\right) = \sqrt{2}$]

$h\left|\hat{i}(-1+1) + 2(\hat{j} - \hat{k})\right| = 4\sqrt{2}$

$\Rightarrow \quad h\left|\hat{j} - \hat{k}\right| = 2\sqrt{2} \quad \Rightarrow h = 2$

$\Rightarrow$ (A) and (D)

Let E divides AM in the ratio $\lambda : 1$

Hence $E : \left(\frac{2\lambda+1}{\lambda+1}, \frac{1}{\lambda+1}, \frac{1}{\lambda+1}\right)$

Now, $(AE)^2 + (DE)^2 = (AD)^2$

$\left(\frac{2\lambda+1}{\lambda+1} - 1\right)^2 + \left(1 - \frac{1}{\lambda+1}\right)^2 + \left(1 - \frac{1}{\lambda+1}\right)^2 + 4 = 16$

$\left(\frac{\lambda}{\lambda+1}\right)^2 + 2\left(\frac{\lambda}{\lambda+1}\right)^2 = 12$

$\Rightarrow \quad \left(\frac{\lambda}{\lambda+1}\right)^2 = 4 \quad \Rightarrow \quad \frac{\lambda}{\lambda+1} = 2 \text{ or } -2$

$\therefore$ These are two positions for E which are (–1, 3, 3) and (3, – 1, – 1)]

43. Answer (60)

$U = 2x^4 - 30x^2 + 8x + 10$

Given $x = 2 + \sqrt{3}$

$\Rightarrow \quad (x-2)^2 = 3$

$\Rightarrow \quad x^2 - 4x + 1 = 0$

$\underbrace{2x^2(x^2 - 4x + 1)}_{\text{zero}} + 8x^3 - 32x^2 + 8x + 10$

$\Rightarrow \quad \underbrace{8x(x^2 - 4x + 1)}_{\text{zero}} + 10 = 10$

$\Rightarrow \quad \boxed{U = 10}$

$V = 2x^2 + 2xy - 7x - 3y + p$

$a = 2;\ b = 0;\ h = 1;\ g = -\frac{7}{2}$

$f = -\frac{3}{2};\ c = p$

$\therefore \quad (0) + 2\left(-\frac{3}{2}\right)\left(-\frac{7}{2}\right)(1) - 2\cdot\left(\frac{9}{4}\right) - 0 - p = 0$

$\frac{21}{2} - \frac{9}{2} = p$

$\therefore \quad p = \dfrac{12}{2} = 6$

$\Rightarrow \quad \boxed{V = 6}$

Hence $(UV) = 10\,(6) = 60$.

44. Answer (89)

$$\left(1+i\sqrt{3}\right)^n = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^n$$

$$= 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$$

$$f\left(\left(1+i\sqrt{3}\right)^n\right) = \text{real part of } z = 2^n\cos\frac{n\pi}{3}$$

$$\therefore \quad \sum_{n=1}^{6a}\log_2\left|2^n\cos\frac{n\pi}{3}\right| = \sum_{n=1}^{6a}\left(n + \log_2\left|\cos\frac{n\pi}{3}\right|\right)$$

$$= \frac{6a(6a+1)}{2} + \underbrace{(-1-1+0-1-1+0)}_{6a \text{ such terms}}$$

$$= 3a(6a+1) - 4a = 18a^2 - a$$

45. Answer (54)

Let $\alpha_1 = A$, $\beta_1 = AR$, $\alpha_2 = AR^2$, $\beta_2 = AR^3$

we have $\alpha_1 + \beta_1 = 6$

$\Rightarrow A(1+R) = 6$ ...(i)

$\alpha_1\beta_1 = p \Rightarrow A^2R = p$ ...(ii)

Also $\alpha_2 + \beta_2 = 54$

$\Rightarrow \quad AR^2(1+R) = 54$ ...(iii)

$\alpha_2\beta_2 = q \Rightarrow A^2R^5 = q$ ...(iv)

Now, on dividing equation (iii) by equation (i), we get

$$\frac{AR^2(1+R)}{A(1+R)} = \frac{54}{6} = 9 \Rightarrow R^2 = 9$$

$\therefore \quad R = 3$ (As it is an increasing G.P.)

$\therefore \quad$ On putting $R = 3$ in equation (i), we get

$$A = \frac{6}{4} = \frac{3}{2}$$

$\therefore \quad p = A^2R = \dfrac{9}{4}\times 3 = \dfrac{27}{4}$ and

$$q = A^2R^5 = \frac{9}{4}\times 243 = \frac{2187}{4}$$

Hence $\dfrac{1}{10}(q-p) = \dfrac{2187-27}{4\times 10} = \dfrac{2160}{40} = 54$

46. Answer (19)

Equation of family of curves passing through intersection of C_1 and C_2 is

$-\lambda x^2 + 4y^2 - 2xy - 9x + 3 + \mu(2x^2 + 3y^2 - 4xy + 3x - 1) = 0$ ...(i)

It will give the joint equation of pair of lines passing through origin,

If coefficient of $x = 0$ & Constant $= 0$

$\Rightarrow \quad \mu = 3$

Put $\mu = 3$ in equation (i), we get

$-\lambda x^2 + 4y^2 - 2xy + 6x^2 + 9y^2 - 12xy = 0$

It will subtend 90° at origin if coefficient of x^2 + coefficient of $y^2 = 0 \Rightarrow \lambda = 19$

47. Answer (76)

L : $3^4 - [{}^3C_1(2^4 - 2) + {}^3C_2] = 36$

M : If $x > 0$, sgn $(x) = 1$

$f(x) = 0 + \dfrac{\pi}{2} = \dfrac{\pi}{2}$

for $x = 0$, $f(x)$ is not defined

$\therefore \quad$ For $x < 0$, $f(x) = \pi - \dfrac{\pi}{2} = \dfrac{\pi}{2}$

$\therefore \quad M = 1$

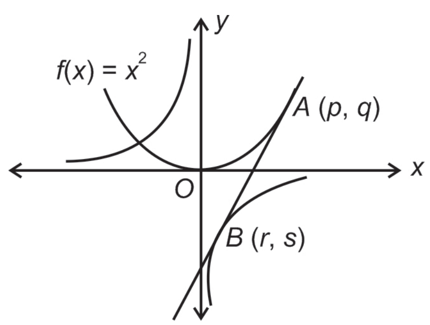
N : Coefficient of t^5 = coefficient of t^2 in $(1+t^2)^5$ × coefficient of t^3 in $(1+t^3)^8$

$= 5 \times 8 = 40$

Hence $L = 36$; $M = 1$ and $N = 40$

$\Rightarrow \quad LM + N = 36 + 40 = 76$

48. Answer (05)



$y = x^2$ and $y = -\dfrac{8}{x}$; $\quad q = p^2$ and $s = -\dfrac{8}{r}$ ...(1)

Equating $\dfrac{dy}{dx}$ at A and B, we get

$2p = \dfrac{8}{r^2}$ ....(1) $\Rightarrow$ $pr^2 = 4$

Now $m_{AB} = \dfrac{q-s}{p-r}$ $\Rightarrow$ $2p = \dfrac{p^2 + \frac{8}{r}}{p-r}$

$\Rightarrow$ $p^2 = 2pr + \dfrac{8}{r}$ $\Rightarrow$ $p^2 = \dfrac{16}{r}$

$\Rightarrow$ $\dfrac{16}{r^4} = \dfrac{16}{r}$ $\Rightarrow$ $r = 1$ $(r \neq 0)$ $\Rightarrow$ $p = 4$

$\therefore$ $r = 1$

Hence $p + r = 5$

49. Answer (12)

Single element subsets $^7C_1$ (1 cannot be taken)

Two element subsets $^7C_2$ (2 cannot be taken)

Three element subsets $^7C_3$ (3 cannot be taken)

Four element subsets $^7C_4$ (4 cannot be taken)

Five element subsets $^7C_5$ (5 cannot be taken)

Six element subsets $^7C_6$ (6 cannot be taken)

Seven element subsets $^7C_7$ (7 cannot be taken)

$\therefore$ Total number of non-empty subsets are $^7C_1 + {}^7C_2 + {}^7C_3 + ...+ {}^7C_7 = 127$.

50. Answer (25)

Let $f(x, y) = x^2 - 16xy - 11y^2 - 12x + 6y + 21$

and $g(x, y) = 9x^2 - 16y^2 - 18x - 32y - 151$

$\dfrac{\partial f}{\partial x} = 0$ $\Rightarrow$ $2x - 16y - 12 = 0$ …(i)

$\dfrac{\partial f}{\partial y} = 0$ $\Rightarrow$ $-16x - 22y + 6 = 0$ …(ii)

Solving (i) & (ii) we get $C_1\left(\dfrac{6}{5}, \dfrac{-3}{5}\right)$

Where C_1 is the centre of 1st hyperbola

Similarly $C_2 = (1, -1)$

given that $C_1C_2 = d$

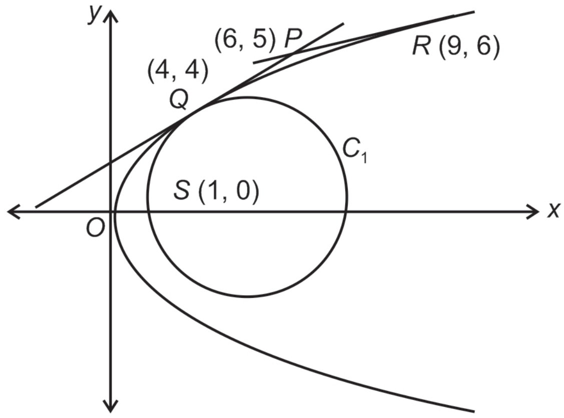
$\Rightarrow$ $\dfrac{1}{25} + \dfrac{4}{25} = d^2 = \dfrac{1}{5}$

$\therefore$ $125d^2 = 25$

51. Answer (A)

52. Answer (B)

Solution for Q. Nos. 51 & 52



Equation of tangent of slope m to $y^2 = 4x$ is

$y = mx + \dfrac{1}{m}$ ...(1)

(i) As (1) passes through $P(6, 5)$, so

$5 = 6m + \dfrac{1}{m}$

$\Rightarrow$ $6m^2 - 5m + 1 = 0$

$\Rightarrow$ $m = \dfrac{1}{2}$ or $m = \dfrac{1}{3}$

Points of contact are $\left(\dfrac{1}{m_1^2}, \dfrac{2}{m_1}\right)$ and $\left(\dfrac{1}{m_2^2}, \dfrac{2}{m_2}\right)$

Hence $R(4, 4)$ and $Q(9, 6)$

Area of $\Delta PQR = \dfrac{1}{2}\begin{vmatrix} 6 & 5 & 1 \\ 4 & 4 & 1 \\ 9 & 6 & 1 \end{vmatrix} = \dfrac{1}{2} \Rightarrow$ (A)

(ii) $y = \dfrac{1}{2}x + 2$ $\Rightarrow$ $x - 2y + 4 = 0$ …(2)

and $y = \dfrac{1}{3}x + 3$ $\Rightarrow$ $x - 3y + 9 = 0$

Now equation of circle C_2 touching $x - 3y + 9 = 0$ at (9, 6), is

$(x - 9)^2 + (y - 6)^2 + \lambda(x - 3y + 9) = 0$

As above circle passes through (1, 0), so

$64 + 36 + 10\lambda = 0$ $\Rightarrow$ $\lambda = -10$

Circle C_2 is

$x^2 + y^2 - 28x + 18y + 27 = 0$ ...(3)

Radius of C_2 is

$r_2^2 = 196 + 81 - 27 = 277 - 27$

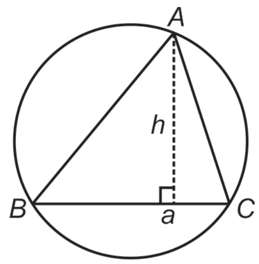
$= 250$

$\Rightarrow \quad r_2 = 5\sqrt{10} \Rightarrow$ (B)

53. Answer (B)

54. Answer (A)

Solution for Q. Nos. 53 & 54



Area of ΔABC

We have, $\Delta = \frac{1}{2}ah = 12$

$\Rightarrow ah = 24 \Rightarrow h = \frac{24}{a} = \frac{24}{2R\sin A}$

$\Rightarrow h = \frac{24}{2\times 6\times \sin A} \Rightarrow h = 2\operatorname{cosec} A$

So, $y = f(x) = 2\operatorname{cosec}x$

(ii) We have,

$g(x) = f(\sin^{-1}x) = 2\operatorname{cosec}(\sin^{-1}x) = \frac{2}{x}$,

$g'(x) = \frac{-2}{x^2} \Rightarrow g'\left(\frac{4}{5}\right) = \frac{-2.25}{16} = \frac{-25}{8}$

(iii) We have,

$h(x) = \sec^{-1}\left(\frac{1}{2}2\operatorname{cosec} x\right)$

$= \sec^{-1}(\operatorname{cosec} x) = \frac{\pi}{2} - x$

Now, $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} \frac{e^{2h(x)} - 2e^{\left(\frac{\pi}{2}-x\right)} + \sin x}{h(x)\cos x}$

$= \lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} \frac{e^{2\left(\frac{\pi}{2}-x\right)} - 2e^{\left(\frac{\pi}{2}-x\right)} + \sin x}{\left(\frac{\pi}{2}-x\right)\cos x}$

Put, $x = \frac{\pi}{2} - h$, we get

$\lim\limits_{h\to 0} \frac{e^{2h} - 2e^{h} + 1 - (1-\cos h)}{h\cdot\sin h}$

$= \lim\limits_{h\to 0} \frac{\frac{\left(e^h - 1\right)^2}{h^2} - \frac{(1-\cos h)}{h^2}}{\frac{\sin h}{h}}$

$= 1 - \frac{1}{2} = \frac{1}{2}$

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B
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MOCK TEST-2

for JEE (Advanced) – 2022

Paper – 2

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(A, B, C, D)	19.	(A, C)	37.	(A, D)
2.	(D)	20.	(B, C)	38.	(B, C)
3.	(A, C)	21.	(A)	39.	(A, D)
4.	(B, D)	22.	(A, B, C, D)	40.	(A, C)
5.	(B, C)	23.	(A, B, C, D)	41.	(A, C)
6.	(B, C)	24.	(A, C)	42.	(B, C)
7.	(03)	25.	(02)	43.	(00)
8.	(05)	26.	(09)	44.	(00)
9.	(07)	27.	(06)	45.	(06)
10.	(07)	28.	(04)	46.	(03)
11.	(10)	29.	(07)	47.	(01)
12.	(04)	30.	(02)	48.	(04)
13.	(30)	31.	(05)	49.	(12)
14.	(03)	32.	(12)	50.	(01)
15.	(A)	33.	(C)	51.	(B)
16.	(C)	34.	(B)	52.	(C)
17.	(D)	35.	(A)	53.	(A)
18.	(A)	36.	(D)	54.	(C)

B
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MOCK TEST-2
for JEE (Advanced) – 2022
Paper - 2
ANSWER & SOLUTIONS

PART - I : PHYSICS

1. Answer (A, B, C, D)

2. Answer (D)

$$e_A : e_D : e_C = 1 : \frac{1}{2} : \frac{1}{4}$$

$$e_A T_A^4 = e_B T_B^4 = e_C T_C^4$$

$$\Rightarrow \quad T_A^4 = \frac{T_B^4}{2} = \frac{T_C^4}{4}$$

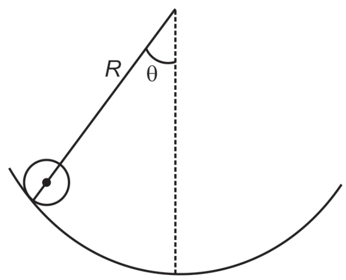
and $\frac{1}{\lambda_A^4} = \frac{1}{2\lambda_B^4} = \frac{1}{4\lambda_C^4}$

3. Answer (A, C)

Use symmetry $R_{eq}^{AB} = 2\ \Omega$

$$I_0 = \frac{2}{3} \Rightarrow I = \frac{1}{6}\text{ A}$$

4. Answer (B, D)



$$a = \frac{g\sin\theta}{1+\frac{k^2}{r^2}}$$

$$a = \frac{g}{\left(1+\frac{k^2}{r^2}\right)}\frac{x}{(R-r)}$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{g}{(R-r)\left(1+\frac{k^2}{r^2}\right)}},\ T = \frac{2k}{\omega} = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

5. Answer (B, C)

6. Answer (B, C)

Look at the given figure carefully to find that two plates of the capacitor marked as *C* are directly connected to the terminals of the battery.

Potential difference across the marked capacitor = 20 V

$\therefore$ Charge on it = 20 μC

Similarly, $Q_1 = C_1V = 1 \times 40 = 40\ \mu$C

7. Answer (03)

$$F = 2T\sin\theta$$

$$T = \frac{F}{2\sin\theta}$$

$$T = \frac{314\times 180}{2\times 3\times \pi} = 3000$$

8. Answer (05)

$V_{cm} = 1$ m/s

In the C-frame, $\frac{1}{2}kx^2 = \frac{1}{2}\times 0.1v_1^2 + \frac{1}{2}\times 0.3\times v_2^2$

$120 \times 0.4^2 = 0.1v_1^2 + 0.3v_2^2$ …(i)

$0.1v_1 + 0.3v_2 = 0$ …(ii)

$\Rightarrow \quad v_1 = -3v_2$

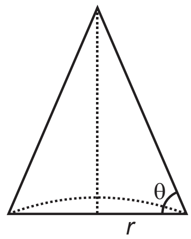
$$120 \times \frac{16}{100} = \frac{1}{10} \times 9v_2^2 + \frac{3}{10}v_2^2$$

$$12 \times 16 = 12v_2^2$$

$v_2 = 4$ m/s

$\therefore$ Velocity of 300 g cart in ground frame $= 5$ m/s

9. Answer (07)



$$V = \frac{1}{3}\pi r^2 h$$

$$\mu_s = \tan\theta = \frac{h}{r}$$

$$r = \frac{h}{\tan\theta} = \frac{h}{\mu_s}$$

So, $V = \frac{1}{3}\pi\left(\frac{h}{\mu_s}\right)^2 h$

$$\Rightarrow \quad h = \left(\frac{3V\mu_s^2}{\pi}\right)^{1/3} = 7\,\text{cm}$$

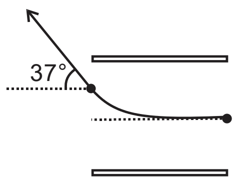
10. Answer (07)

$\Rightarrow$ 19 m s × 9 = ms + mL

$\Rightarrow$ 170 ms = mL

$\Rightarrow$ L = 7.14 × 10^2 kJ/kg

11. Answer (10)



$$\tan 37^\circ = \frac{V_y}{V_x} = \frac{V_y}{V_0}$$

$$V_y = a_y t = \frac{qE_y}{m} \times \frac{l}{V_0}$$

$$E_y = \frac{V}{d} = \frac{iR}{d} = \frac{\varepsilon R}{(R+r)d}$$

$$\Rightarrow \quad \frac{3}{4}V_0 = V_y = \frac{ql}{mv_0} \times \frac{\varepsilon R}{(R+r)d}$$

$$\frac{3}{4}V_0 = \frac{16}{91} \times 10^{12} \times \frac{3 \times R}{(R+2)10^{-3}} \times 0.182$$

$2.5R + 5 = 3R$

$5 = 0.5R$; $R = 10\ \Omega$

12. Answer (04)

$$4.896 = 13.6Z^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$13.6 = 13.6\frac{Z^2}{n^2}$$

$Z = n$

$$\frac{4.896}{13.6} = 1 - \frac{n^2}{(n+1)^2} = \frac{n}{n+1} = \frac{4}{5}$$

$n = 4$

13. Answer (30)

$$m_1 m_2 = \frac{f}{f-x} \times \frac{f'}{\frac{xf}{x-f} - d + f'}$$

$$= \frac{ff'}{-xf - d(f-x) + f'(f-x)}$$

$-xf + dx - xf' = 0$

$f + f' = d = 30$ cm

14. Answer (03)

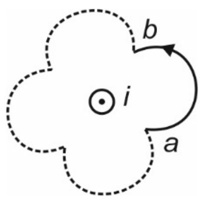
$$\int F dt\ h = I\omega$$

$$\int F dt\ h = mv$$

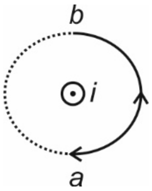
From above relations $h = \frac{2}{3}R$

Height from ground = $\frac{5}{3}R$

15. Answer (A)



A : If we consider four such identical paths with similar orientation forming a closed path then for all that path $\int \vec{B} \cdot \vec{dl} = \mu_0 i$. By symmetry, $\int \vec{B} \cdot \vec{dl}$ along the given path is $\frac{\mu_0 i}{4}$.

B : 

Two such given paths form a closed surface around the conductor

$$\therefore \quad \int_a^b \vec{B}\cdot\vec{dl} = \frac{\mu_0 i}{2}$$

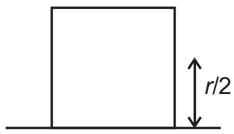
C : From Ampere's law, as the given path is closed around the conductor, $\oint \vec{B}\cdot\vec{dl} = \mu_0 i$

D : $\int_a^b \vec{B}\cdot\vec{dl}$ is same in magnitude with each of the two conductors but reverse in sign.

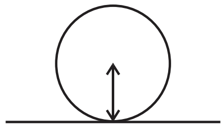
Hence net value is zero.

16. Answer (C)

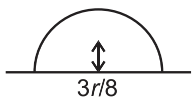
(P) $\frac{dT}{dt} = \frac{CA}{ms}(T - T_0) \; = \; \frac{dT}{dt} \propto \frac{\text{surface area}}{\text{volume}}$

(Q) 

$$\frac{\text{P}r^3}{12}\left(r^2 + r^2\right) = \frac{\text{P}r^5}{12}$$



$$\frac{2}{5}r^2 \times \rho \times \frac{4}{3}\pi r^3 = \frac{8\pi\rho r^5}{15}$$



$$\frac{2}{5} \times r^2 \times \rho \times \frac{2}{3}\pi^3 = \frac{4\pi\rho r^5}{15}$$

(R) For cone $C_{(cm)} = \frac{r}{4}$

17. Answer (D)

18. Answer (A)

For L = 1 H, X_L = 1000 Ω

And C = 1 μF, X_C = 1000 Ω

So, Z = 1000 Ω

i_{max} = 2 A

Reading of V_1 = 1414 V

Reading of V_2 = 1414 V

Reading of V_3 = 1414 V

Reading of V_5 = 1414 V

Reading of V_4 = 0

When L = 2 H $\quad X_L$ = 2000 Ω and

When C = 0.5 μF $\quad X_C$ = 2000 Ω

PART - II : CHEMISTRY

19. Answer (A, C)

Factual

20. Answer (B, C)

Reaction sequence is as follows

(1) X is soluble in H_2O and C_2H_5OH

(2) $(\text{X}) \xrightarrow{\text{Heat}} \underset{\text{Brown gas}}{(\text{Y})} + \text{Grey residue}$

(3) $(\text{X}) \xrightarrow{NH_4OH} \text{Ammonical solution of } (\text{X})$

$\xrightarrow{CH_3CHO} \text{Silver mirror}$

(4) (X) is reduced by a ferrous salt.

(5) $(\text{X})_{(aq.)} + K_2CrO_4 \longrightarrow \text{Brick red ppt.}$

Observation of set (3) indicates that (X) is a salt of silver. Step (2) shows that (X) may contain NO_3^- ions. Hence, (X) is $AgNO_3$

(1) $AgNO_3 \xrightarrow{H_2O} AgNO_{3(aq.)}$

$AgNO_3 \xrightarrow{C_2H_5OH} AgNO_3\,(C_2H_5OH)$

(2) $\underset{(\text{X})}{2AgNO_3} \xrightarrow{\Delta} 2Ag + \underset{(\text{Y})}{2NO_2} + O_2$

(3) $AgNO_3 + NH_4OH \longrightarrow AgOH + NH_4NO_3$

$AgOH + 2NH_4OH$

$\longrightarrow \underset{\text{Soluble}}{Ag(NH_3)_2^+} + 2H_2O + OH^-$

$2Ag(NH_3)_2^+ + CH_3CHO + H_2O$

$\longrightarrow \underset{\text{Silver mirror}}{2Ag\downarrow} + CH_3COOH + 2NH_4^+$

(4) $AgNO_3 + Fe^{2+} \longrightarrow Ag\downarrow + Fe^{3+} + NO_3^-$

(5) $2AgNO_3 + K_2CrO_4 \longrightarrow \underset{\text{Brick red ppt.}}{Ag_2CrO_4} + 2KNO_3$

21. Answer (A)

22. Answer (A, B, C, D)

(A) HCl because of its high bond enthalpy.

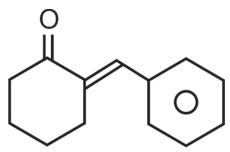
(B) $KClO_3 \longrightarrow KCl + O_2$.

(C) Correct order, F^- because of high electron density on account of small size easily donate the electron pair, is strongest base.

(D) It is used as bleaching agent (as it acts as strong oxidising agent).

23. Answer (A, B, C, D)

24. Answer (A, C)



25. Answer (02)

Use POAC with 95% yield.

26. Answer (09)

Since the volume of a gas at STP depends only on the mass, the volume of the dissolved gas (reduced to STP) is proportional to the partial pressure of the gas.

Partial pressure of hydrogen = (0.70) (5.0 atm) = 3.50 atm

Solubility of H_2 at 20°C and 1 atm

$$= \left(\frac{1.00 \text{ atm}}{3.50 \text{ atm}}\right)(31.5 \text{ mL / L}) = 9.0 \text{ mL(STP) / L}$$

27. Answer (06)

DU in x = 6, y = 1

28. Answer (04)

x = 12

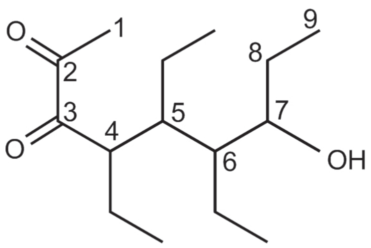
y = 12

z = 6

w = 6

29. Answer (07)

Substituent ethyl groups are marked with circles:



30. Answer (02)

Let the mole of propane (C_3H_8) = n_1

Moles of methane CH_4 = n_2

We know that PV = nRT

$320 \times V = (n_1 + n_2)$ RT ...(i)

After combustion

$$\underset{n_1}{C_3H_8} + 5O_2 \rightarrow \underset{3n_1}{3CO_2} + 4H_2O,$$

$$\underset{n_2}{CH_4} + 2O_2 \rightarrow \underset{n_2}{CO_2} + 2H_2O$$

Total moles of CO_2 formed = $(3n_1 + n_2)$

Once again we have PV = nRT

$448 \times V = (3n_1 + n_2)RT$ ...(ii)

Dividing equation (ii) by equation (i), we have

$$\frac{448}{320} = \frac{(3n_1 + n_2)}{(n_1 + n_2)} \Rightarrow \frac{n_1}{n_2} = 0.25 \quad n_1 = 0.25n_2$$

$$\therefore \frac{n_1}{n_1 + n_2} = \frac{0.25n_2}{0.25n_2 + n_2} = 0.2$$

31. Answer (05)

$n_f = 2$ for $Ca(COO^-)_2$

$$\Rightarrow \frac{W}{128} \times 2 \times \frac{1}{10} = \frac{8 \times 0.1}{1000}$$

$$\Rightarrow W = 0.512$$

$$\% \text{ impurity} = \frac{0.54 - 0.512}{0.54} \times 100 \cong 5\%$$

32. Answer (12)

X	Y	Z	W
4	1	5	2

X = 4: α-Naphthylamine is soluble in aqueous HCl solution.

Y = 1: Benzoic acid is soluble in aqueous $NaHCO_3$ solution

Z = 5: Naphthalene insoluble in aqueous NaOH solution.

W = 2: Salicylaldehyde has lower b.p. than p-hydroxy benzaldehyde.

Sum of numbers corresponding to X, Y, Z and W = 4 + 1 + 5 + 2 = 12

33. Answer (C)

34. Answer (B)

35. Answer (A)

36. Answer (D)

PART - III : MATHEMATICS

37. Answer (A, D)

$AP = \sqrt{(1-\cos\alpha)^2 + \sin^2\alpha} = 2\left|\sin\frac{\alpha}{2}\right| = 2\sin\frac{\alpha}{2}$

$\left(\because \ \sin\frac{\alpha}{2} > 0\right)$

Similarly, $AQ = 2\sin\frac{\beta}{2}$ and $AR = 2\sin\frac{\gamma}{2}$

Now AP, AQ, AR are in G.P. $\Rightarrow$ $\sin\frac{\alpha}{2}$, $\sin\frac{\beta}{2}$, $\sin\frac{\gamma}{2}$ are in G.P.

$\therefore \quad \dfrac{\sin\frac{\alpha}{2} + \sin\frac{\gamma}{2}}{2} \geq \sin\frac{\beta}{2}$

$\Rightarrow \quad \sin\frac{\alpha+\gamma}{4}\cos\frac{\alpha-\gamma}{4} \geq \sin\frac{\beta}{2}$

Also, $\sin\frac{\alpha}{2}\sin\frac{\gamma}{2} \leq \sin\frac{\beta}{2}$.

38. Answer (B, C)

Any tangent to $xy = 4\sin^2\theta$ is $y = mx \pm 4\sin\theta\sqrt{-m}$.

If it is normal to any circle of given family it will pass through the centre of the circle i.e. (1, 1)

$1 = m \pm 4\sin\theta\sqrt{-m}$

$\Rightarrow \quad m^2 + 2[8\sin^2\theta - 1]m + 1 = 0$

For non-real roots $D < 0$

$\Rightarrow \quad \sin^2\theta < \frac{1}{4}$ or $-\frac{1}{2} < \sin\theta < \frac{1}{2}$.

$\therefore \quad \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right) - \{\pi\}$

39. Answer (A, D)

If two vertices of the triangle are on the same side of the square, then the third vertex would be a height < 1 above the side and hence in the interior of the square which is contradiction. So there must be one vertex of the triangle on each of three sides of the square.

40. Answer (A, C)

$2\log(y - 1) - \log x - \log(y - 2) = 0$ is defined when $y > 2$ and $x > 0$

Given equation can be written as

$\log\dfrac{(y-1)^2}{(y-2)} = \log x$

$\Rightarrow \quad \dfrac{(y-1)^2}{(y-2)} = x$

let $f(t) = \dfrac{(t-1)^2}{t-2} \quad t > 2$

$f'(t) = \dfrac{(t-2)\cdot 2(t-1) - (t-1)^2\cdot 1}{(t-2)^2}$

$= \dfrac{(t-1)(2t-4-t+1)}{(t-2)^2} = \dfrac{(t-1)(t-3)}{(t-2)^2} > 0$

$\Rightarrow \quad t < 1$ or $t > 3$

$f(t)$ is increasing in $(3, \infty)$ decreasing in $(2, 3)$

$f(t)_{\min}$ at $t = 3$

so $f(3) = 4$

$\Rightarrow \quad x \geq 4$ i.e. domain is $[4, \infty)$ range is $(2, \infty)$

41. Answer (A, C)

$$f(x)\begin{cases} 0, & 1 \leq x < 2 \\ \log_e 2, & 2 \leq x < 3 \\ \log_e x, & 3 \leq x < 4 \end{cases}$$

Clearly $f(x)$ is continuous and differentiable everywhere except possibly at $x = 2, 3$

As $\lim\limits_{x\to 2^-} f(x) = 0, \lim\limits_{x\to 2^+} f(x) = \log_e 2$

$f(x)$ is not continuous at $x = 2$

$\lim\limits_{x\to 3^-} f(x) = \log_e 2, \lim\limits_{x\to 3^+} f(x) = \log_e 3$

$\Rightarrow \quad f(x)$ is not continuous at $x = 3$

Hence $f(x)$ is not derivable at $x = 2, 3$

42. Answer (B, C)

$f'(x) = 2x - a$

At (2, 4)

$f'(x) = 4 - a$

Equation of normal at (2, 4) is

$(y - 4) = -\dfrac{1}{(4-a)}(x-2)$.

Let point of intersection with x and y axis be A and B respectively then

$$A \equiv (-4a + 18, 0),\ B \equiv \left(0,\ \frac{4a-18}{a-4}\right)$$

Hence $a > \frac{9}{2}$ as

$\therefore$ Area of triangle $= \frac{1}{2}(4a-18)\frac{(4a-18)}{(a-4)} = 2$

$\Rightarrow$ $(4a - 17)(a - 5) = 0$

$\Rightarrow$ a = 5 or $\frac{17}{4}$

43. Answer (00)

$$S_n = \sum_{k=0}^{n} (-4)^k \ {}^{n+k}C_{2k}$$

$$S_{n+1} = \sum_{k=0}^{n+1} (-4)^k \ {}^{n+k+1}C_{2k}$$

$$S_{n-1} = \sum_{k=0}^{n-1} (-4)^k \ {}^{n-1+k}C_{2k}$$

Consider ${}^{n+1}C_r + {}^{n-1}C_r - 2\,{}^nC_r$ = coefficient of x^r in $\{(1 + x)^{n+1} + (1 + x)^{n-1} - 2(1 + x)^n\}$

= coefficient of x^r in $(1 + x)^{n-1}\{(1 + x)^2 + 1 - 2(1 + x)\}$

= coefficient of x^r in $(1 + x)^{n-1}\{x^2 + 2x + 2 - 2x - 2\}$

= coefficient of x^{r-2} in $(1 + x)^{n-1} = {}^{n-1}C_{r-2}$

Put $n \to n + k$ and $r \to 2k$

$\Rightarrow$ ${}^{n+k+1}C_{2k} + {}^{n+k-1}C_{2k} - 2\,{}^{n+k}C_{2k}$

$= {}^{n+k-1}C_{2(k-1)}$.

Multiplying by $(-4)^k$ and applying summation on both sides

$$\sum_{k=0}^{n+1}(-4)^k\ {}^{n+k+1}C_{2k} + \sum_{k=0}^{n-1}(-4)^k\ {}^{n+k-1}C_{2k}$$

$$-2\sum_{k=0}^{n}(-4)^k\ {}^{n+k}C_{2k} = \sum_{k=1}^{n+1}(-4)^k\ {}^{n+k-1}C_{2(k-1)}$$

(Note that the limits of summation are different for each term)

$\Rightarrow$ $S_{n+1} + S_{n-1} - 2S_n = \sum_{k=1}^{n+1}(-4)^k\ {}^{n+k-1}C_{2(k-1)}$

Let $k - 1 = t$

$$\sum_{t=0}^{n}(-4)^t(-4)^1\ {}^{n+t}C_{2t} = -4S_n$$

$S_{n+1} + 2S_n + S_{n-1} = 0$

Put $n = 2010$

44. Answer (00)

$$\Delta = \begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

45. Answer (06)

Let $S(0, -2)$ and S' be foci of the ellipse.

The slope of $AS' = \frac{1}{3}$ and $AS' = 3\sqrt{10}$.

So, the coordinates of S' will be (10, 4).

And centre is mid-point of S and S'.

46. Answer (03)

$$2\sin^2 x + \frac{2\sin x\cos x}{2} = n$$

$$1 - \cos 2x + \frac{\sin 2x}{2} = n$$

$\sin 2x - 2\cos 2x = 2n - 2$

$$-\sqrt{5} \le 2n - 2 \le \sqrt{5}$$

$$\frac{-\sqrt{5}}{2} \le n - 1 \le \frac{\sqrt{5}}{2}$$

$$1 - \frac{\sqrt{5}}{2} \le n \le 1 + \frac{\sqrt{5}}{2}$$

47. Answer (01)

$$\frac{2f(x)\cdot f'(x)}{\sqrt{1-(f(x))^4}} - 2x \ge 0 \Rightarrow \frac{d}{dx}\left(\sin^{-1}(f(x))^2 - x^2\right) \ge 0$$

Let $g(x) = \sin^{-1}((f(x))^2) - x^2$ is a non-decreasing function.

$$\Rightarrow \lim_{x \to x_1^+} g(x) \le \lim_{x \to x_2^-} g(x) \Rightarrow \frac{\pi}{2} - x_1^2 \le \frac{\pi}{6} - x_2^2$$

$$\Rightarrow x_1^2 - x_2^2 \ge \frac{\pi}{3} \Rightarrow \left[x_1^2 - x_2^2\right] \ge 1$$

48. Answer (04)

Equation of the line L is $x + y - 1 + \lambda z = 0$, $x - y - 2 + \mu(y + z - 3) = 0$.

As line passes through (1, 1, 1) so the value of λ will be -1 and $\mu = -2$

49. Answer (12)

$f(x) \geq 0$

$$\Rightarrow \left(x^2+\frac{p}{2}x\right)^2+\left(3-\frac{p^2+q^2}{4}\right)x^2+\left(\frac{qx}{2}+1\right)^2 \geq 0$$

which holds only when $3-\dfrac{p^2+q^2}{4} \geq 0$

$\Rightarrow p^2 + q^2 \leq 12$

50. Answer (01)

We have $k = 1 - \dfrac{2}{\alpha+1}-\alpha^2$

gives $k = -1$ only.

51. Answer (B)

(P) It can be reduced to

$$\int_0^1 \sqrt{\frac{1+x}{1-x}}dx = \int_0^1 \frac{1+x}{\sqrt{1-x^2}}dx$$

$$= \left[\sin^{-1}x - \sqrt{1-x^2}\right]_0^1$$

$$= \frac{\pi}{2}-(-1) = \frac{\pi}{2}+1$$

(Q) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{\sqrt{1-\frac{1}{n^2}}}+\frac{1}{\sqrt{1-\left(\frac{2}{n}\right)^2}}+\cdots+\frac{1}{\sqrt{1-\left(\frac{n-1}{n}\right)^2}}\right]$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n-1}\frac{1}{\sqrt{1-\left(\frac{r}{n}\right)^2}}$$

Replace $\dfrac{r}{n}$ by x and $\dfrac{1}{n}dx$ also when $r = 1$,

$x = 0$ when $r = n - 1$, $x = 1$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1-x^2}}dx = \left(\sin^{-1}x\right)_0^1$$

$$= \sin^{-1}1 - \sin^{-1}0 = \frac{\pi}{2}$$

(R) $\displaystyle A = \lim_{n\to\infty}\left[\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{n}{n}\right]^{1/n}$

$$\log A = \lim_{n\to\infty}\left[\log\frac{1}{n}+\log\frac{1}{n}+\log\frac{3}{n}+\cdots+\log 1\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)$$

Put $\dfrac{r}{n} = x$ $\quad \dfrac{1}{n} = dx$ and limits 0 to 1

$$\log A = \int_0^1 \log x\,dx = \left(x\log x\right)_0^1 - \int_0^1 x\cdot\frac{1}{x}$$

$$= \left(x\log x - x\right)_0^1 = -1$$

$\Rightarrow A = e^{-1}$

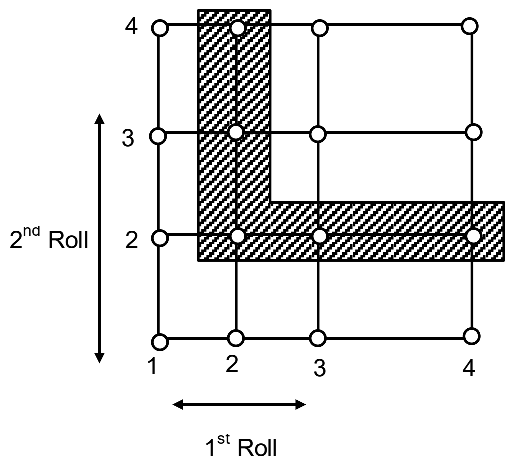
(S) $f(x)$ = R.P of $e^{\cos x}.e^{i\sin x}$ = R.P of $e^{\cos x + i\sin x}$

$= e^{e^{ix}}$ = R.P of $\left[1+e^{ix}+\dfrac{e^{2ix}}{2!}+\cdots\right]$

$f(x) = 1 + \cos x + \dfrac{1}{2!}\cos 2x+\dfrac{1}{3!}\cos 3x+\cdots$

$$\int_0^{2\pi} f(x)dx = \left[x\right]_0^{2\pi}+0+0+\cdots = 2\pi$$

52. Answer (C)



Let us make grid for 1st and 2nd rolling of dice.

The set S is the shaded region consisting of 5 elements.

The set R = {max(X, Y) = m}

shares with S two elements. If m = 3 or m = 4, one element and no element if m = 1.

As if m = 3

max(A, B) = 3

$\Rightarrow$ Either (2, 3) or (3, 2)

If m = 4

Max(A, B) = 4

Either (4, 2) or (2, 4)

If m = 2

Max(A, B) = 2

$\Rightarrow$ (2, 2)

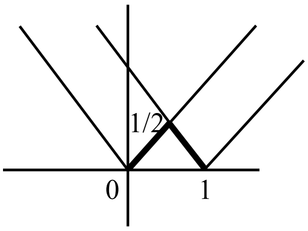
If m = 1

Max(A, B) = 1

No pair exist.

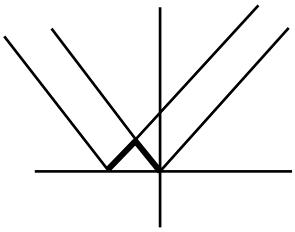
53. Answer (A)

(P)

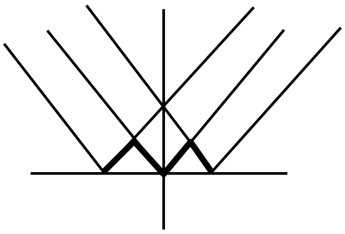


$A = \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$

(Q) $A = \frac{1}{4}$

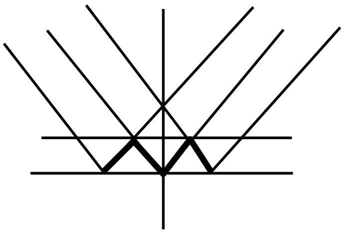


(R)



$A = \frac{1}{2}$

(S)



$A = \frac{3}{4}$

54. Answer (C)

(P) $\because\ n! \approx n^n e^{-n}$

$$\lim_{n\to\infty}\left(\frac{3n!}{n^{3n}}\right)^{1/n} \approx \lim_{n\to\infty}\left(3n^{-2n}e^{-n}\right)^{1/n} = 0$$

(Q) $(n!)^3 \approx n^{3n}e^{-3n}$

$$\therefore\ \lim_{n\to\infty}\left(\frac{(n!)^3}{n^{3n}e^{-n}}\right)^{1/n} \approx \lim_{n\to\infty}\left(e^{-2n}\right)^{1/n} = e^{-2}$$

(R) $(n!)^2 \approx n^{2n}e^{-2n}$

$$\therefore\ \lim_{n\to\infty}\left(\frac{(n!)^2}{n^{2n}}\right)^{1/n} \approx \lim_{n\to\infty}\left(e^{-2n}\right)^{1/n} = e^{-2}$$

(S) $(2n!) \approx (2n)^{2n}e^{-2n}$

$$\therefore\ \lim_{n\to\infty}\left(\frac{n^{2n}}{(2n)!}\right)^{1/n} \approx \lim_{n\to\infty}\left(\frac{1}{2^{2n}e^{-2n}}\right)^{1/n} = \frac{e^2}{4}$$

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