A
CODE

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005, Phone : 011-47623456

Time : 3 hrs **MM : 198**

MOCK TEST - 3

for JEE (Advanced) – 2022

Paper - 1

ANSWERS

PHYSICS	CHEMISTRY	MATHEMATICS
1. (B)	19. (A)	37. (C)
2. (D)	20. (C)	38. (C)
3. (B)	21. (B)	39. (C)
4. (D)	22. (C)	40. (B)
5. (A)	23. (D)	41. (C)
6. (B)	24. (A)	42. (C)
7. (A, B, C)	25. (B)	43. (A, B, C, D)
8. (A, D)	26. (A)	44. (A, B, D)
9. (A, B, D)	27. (B, C, D)	45. (B, C)
10. (A, C)	28. (A, B, C, D)	46. (A, C)
11. (A, B)	29. (A, B, C)	47. (B, C)
12. (B, D)	30. (A, D)	48. (A, B)
13. (12.00)	31. (78.00)	49. (04.00)
14. (27.00)	32. (02.00)	50. (16.00)
15. (90.00)	33. (25.00)	51. (1023.00)
16. (24.00)	34. (17.00)	52. (72.00)
17. (00.00)	35. (12.00)	53. (04.00)
18. (01.00)	36. (24.00)	54. (02.00)

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MOCK TEST - 3

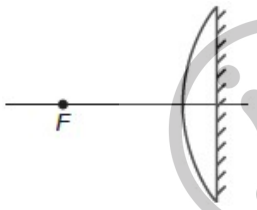
for JEE (Advanced) – 2022

Paper - I

ANSWERS & SOLUTIONS

PART – I : PHYSICS

1. Answer (B)



Object at focus ⇒ Image at ∞ ⇒ beam is incident normally on mirror

⇒ Rays retrace their path ⇒ Image is at *F*.

2. Answer (D)

$$\frac{\varepsilon R_1}{R_1+R_2}=\frac{\varepsilon C_2}{C_1+C_2}$$

$R_1C_1 + R_1C_2 = R_1C_2 + R_2C_2$

or $R_1C_1 = R_2C_2$

3. Answer (B)

At time *t*,

$$l' = (l - vt) = \frac{\lambda}{4}$$

∴ Fundamental frequency $f_0 = \frac{C}{4l}$

$$f_0 = \frac{C}{4(l-vt)}$$

$$\therefore \quad \frac{df}{dt}=\frac{-C}{4(l-vt)^2}(-v)=\frac{CV}{4l^2}$$

4. Answer (D)

$$\phi' \propto \frac{4}{3}\pi a^3 r$$

$$\phi \propto a^3 r$$

$$\therefore \quad \frac{\phi'}{\phi}=\frac{4}{3}\pi$$

$$\therefore \quad f' = \frac{4}{3}\pi\phi$$

5. Answer (A)

$$2\mu t = \left(n+\frac{1}{2}\right)\lambda$$

$$2\mu t = \frac{\lambda}{2}$$

6. Answer (B)

$$E_1 = \frac{hc}{\lambda}$$

$$E_2 = \frac{h^2}{\lambda^2 2M}$$

$$\Rightarrow \quad E_1 = xE_2$$

7. Answer (A, B, C)

$F = -8x + 2x^3$

$$F = -\frac{dU}{dx} = -8x + 2x^3$$

$$\Rightarrow \quad \int_0^U dU = -\int_0^x (-8x+2x^3)\,dx$$

$$\Rightarrow \quad U = -\left(-4x^2+\frac{x^4}{2}\right)$$

$$U_x = 4x^2 - \frac{x^4}{2} \quad \text{... PE} = f(x)$$

$$4x^2 - \frac{x^4}{2} = 3.5 \Rightarrow 8x^2 - x^4 = 7$$

If $x^2 = t \Rightarrow 8t - t^2 = 7$

$\Rightarrow t^2 - 8t + 7 = 0$

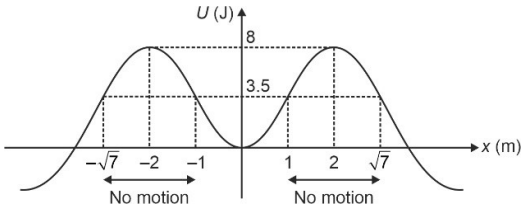
$\Rightarrow (t-7)(t-1) = 0$

$\Rightarrow t = 1, 7 \Rightarrow x^2 = 1, 7$

Let us draw the graph of PE(U) as a.

Function of x

$$U_x = 4x^2 - \frac{x^4}{2}$$

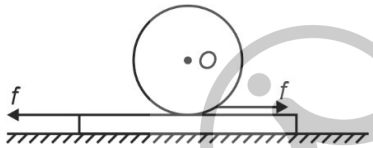


For motion KE > 0 $\Rightarrow$ PE < 3.5 J

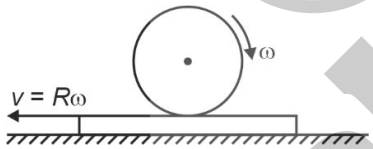
Particle cannot perform motion only in the region from $x = -\sqrt{7}$ to $x = -1$, and $x = 1$ to $x = +\sqrt{7}$.

8. Answer (A, D)

$\vec{L}_i = \vec{L}_f$



$$\Rightarrow \quad \frac{mR^2}{2}\omega_0 = mvR + \frac{mR^2}{2}\omega$$



$$\Rightarrow \quad \frac{mR^2}{2}\omega_0 = \frac{7}{2}mvR$$

$$\Rightarrow \quad v = \frac{R\omega_0}{3}$$

$$W_T = \Delta KE = K_f - K_i = -\frac{1}{6}mR^2\omega_0^2$$

Angular impulse = Change in angular momentum

9. Answer (A, B, D)

By first law of thermodynamics,

$Q = W + \Delta U$

$\Rightarrow \quad 2Q = \Delta U \qquad \ldots(i)$

$$\Rightarrow \quad 2nC(T_B - T_A) = n\frac{5R}{2}(T_B - T_A)$$

$$\therefore \quad C = \frac{5R}{4}$$

By equation (i),

$$Q = \frac{1}{2}\Delta U = \frac{1}{2}\left[\frac{5}{2}(6P_0V_0 - 4P_0V_0)\right] = \frac{5}{2}P_0V_0$$

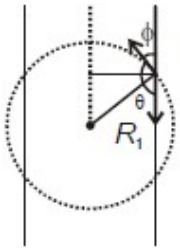
Since $\Delta U = -2W$, therefore, temperature goes on increasing from A to B.

10. Answer (A, C)

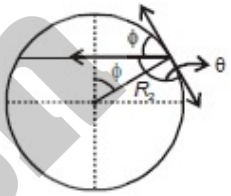
Clearly contact angle is

$\theta = \pi - \phi$ and $\cos\phi = \frac{R_0}{R_1}$

$$\Rightarrow \quad \theta = \pi - \cos^{-1}\left(\frac{R_0}{R_1}\right)$$



Now, in spherical container,

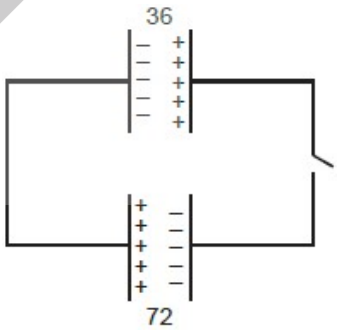


$R_2\cos\phi = h$

$$\Rightarrow \quad R_2 \cdot \frac{R_0}{R_1} = h$$

11. Answer (A, B)

12. Answer (B, D)



Charge stored on capacitor 3 μF = 36 μC

Charge stored on capacitor 6 μF = 72 μC

After connected the plates

$$\frac{-36 + Q}{3} = \frac{72 - Q}{6}$$

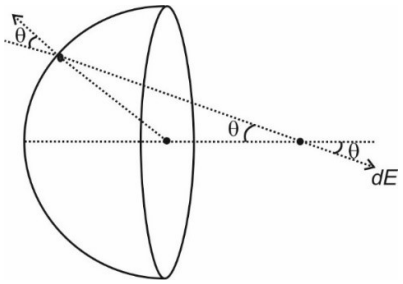
i.e., $-72 + 2Q = 72 - Q$

$3Q = 2 \times 72$

$Q = 48$

$$\therefore \quad V_{3\,\mu F} = \frac{48 - 36}{3} = 4 \text{ V}$$

13. Answer (12.00)



$E_p = \int dE\cos\theta$

$= \int \frac{k(\sigma ds)}{r^2}\cos\theta$

$= \frac{\sigma}{4\pi\varepsilon_0}\int \frac{ds\cos\theta}{r^2}$

$= \frac{\sigma}{4\pi\varepsilon_0}\left[2\pi\left(1-\frac{1}{\sqrt{2}}\right)\right]$

$= \frac{\sigma}{4\varepsilon_0}\left(2-\sqrt{2}\right)$

14. Answer (27.00)

Process is polytropic

$C = C_v - \frac{R}{m-1}$

$\frac{R}{2} = \frac{3}{2}R - \frac{R}{m-1} \Rightarrow m = 2$

$PV^2 = C$

$450 \times V_0^2 = P(2V_0)^2$

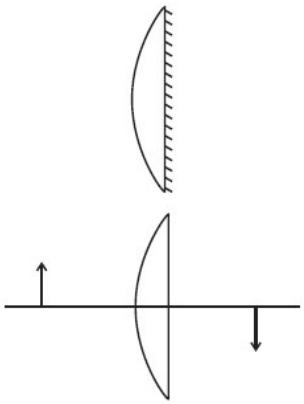
$P = 10$ kPa

$\frac{PV}{T} = \frac{P_0V_0}{T_0}$

$\frac{10 \times 2V_0}{T} = \frac{40 \times V_0}{600}$

$\Rightarrow \quad T = 300$ K $= 27°$C

15. Answer (90.00)



$\frac{1}{f_{eq}} = \frac{-2}{f_2} + \frac{1}{f_M} \qquad f_M = \infty$

$\Rightarrow \quad -\frac{1}{30} = -\frac{2}{f_2}$

$\Rightarrow \quad f_L = 60$ cm

$m = -\frac{1}{2} = \frac{v}{u}$

$u = -2v$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_L} = \frac{1}{60}$

$v = 90$ cm

16. Answer (24.00)

$600 = \frac{3}{2L}\sqrt{\frac{T}{\mu}} = \frac{3}{2L}\sqrt{\frac{T}{\rho A}}$

$(400)^2L^2\rho A = T$

$16 \times 10^4 \times 0.36 \times 8000 \times 10^{-5} = T$

$36 \times 16 \times 8 = T = 192k$ (Newton)

$\therefore \quad k = 24$

17. Answer (00.00)

Let c be acceleration of wedge C.

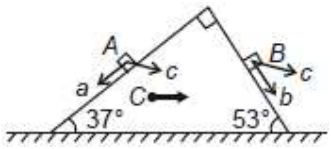
a be acceleration of block A w.r.t. wedge C.

b be acceleration of block B w.r.t. wedge C.

Applying Newtons law in horizontal direction of system of $A + B + C$.

$mc + m(c - a\cos37°) + m(c + b\cos53°) = 0$ …(i)

Applying Newton's law to block A and B along the incline gives



In case of A,

$mg\sin37° = m(a - c\cos37°)$ …(ii)

In case of B,

$mg\sin53° = m(b + c\cos53°)$ …(iii)

Solving (i), (ii), and (iii), we get $c = 0$

$a_c = 0$

18. Answer (01.00)

$V_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = 5.1$ m/s

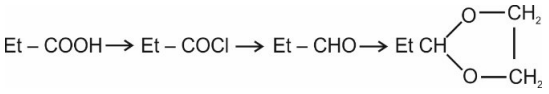
$\therefore \quad 5.1 = 10 - 0.5 \times 9.8t = t = 1$ s

PART – II : CHEMISTRY

19. Answer (A)

S_N1 reaction proceeds through the formation of carbocation intermediate.

20. Answer (C)



21. Answer (B)

22. Answer (C)

$$d = \frac{Z\ M}{N_A\ a^3}$$

$d_{BCC} = d_{FCC}$

$$\frac{2\times M_1}{N_A (a_1)^3} = \frac{4\times M_2}{N_A (a_2)^3}$$

$$\Rightarrow \left(\frac{a_1}{a_2}\right)^3 = \frac{1}{2}\frac{M_1}{M_2}$$

If $a_1 = a_2$ then $M_1 = 2M_2$

If $a_1 = \frac{1}{2}a_2$ then $M_2 = 4M_1$

23. Answer (D)

(i) $Ni(CO)_4 = 28 + 2 \times 4 = 36$

(ii) $[Ag^{I}(NH_3)_2]Cl = 46 + 2 \times 2 = 50$

(iii) $K_4[Fe^{II}(CN)_6] = 24 + 2 \times 6 = 36$

(iv) $K_2[Ni^{II}(CN)_4] = 26 + 2 \times 4 = 34$

(v) $K_2[Pt^{IV}Cl_6] = 74 + 2 \times 6 = 86$

(vi) $K_3[Cr^{III}(C_2O_4)_3] = 21 + 2 \times 6 = 33$

24. Answer (A)

Order of energy $d_{x^2-y^2} > d_{xy} > d_{z^2} > \left|\begin{matrix} d_{yz} \\ d_{xz} \end{matrix}\right|$

25. Answer (B)

ESR takes place on more reactive ring.

26. Answer (A)

Wittig reaction takes place.

27. Answer (B, C, D)

$XeOF_4$, XeO_2F_2 and XeO_3 are the products.

28. Answer (A, B, C, D)

$\because$ dG = VdP – SdT

dH = TdS – VdP

29. Answer (A, B, C)

Since the reaction (i) involves NH_3 as one of the products, (P) will be NH_4Cl. In reaction (ii), (Q) will be NH_4HCO_3. In reaction (iii), (R) will be $NaHCO_3$ and finally in reaction (iv), (S) will be CO_2.

(i) $\underset{(P)}{NH_4Cl} + NaOH \xrightarrow{\Delta} NH_3 + NaCl + H_2O$

(ii) $NH_3 + CO_2 + H_2O \longrightarrow \underset{(Q)}{NH_4HCO_3}$

(iii) $\underset{(Q)}{NH_4HCO_3} + NaCl \longrightarrow \underset{(R)}{NaHCO_3} + NH_4Cl$

(iv) $\underset{(R)}{2NaHCO_3} \xrightarrow{\Delta} Na_2CO_3 + H_2O + \underset{(S)}{CO_2}$

30. Answer (A, D)

$$\ln(P_2/P_1) = \frac{\Delta H_{vap}}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

$X(\ell) \rightarrow X(g)$

$\Delta H_{vap} = 10.3 - 2 = 8.3$ kJ

$\Delta S_{vap} = 20.75$ J

Boiling point = $\frac{\Delta H_{vap}}{\Delta S_{vap}} = \frac{8.3\times10^3}{20.75} = 400$ K

$Y(\ell) \rightarrow Y(g)$

$\Delta H_{vap} = 17.96 - 10.30 = 7.66$ kJ/mol

$\Delta S_{vap} = 20$ J/mol K

Boiling point = $\frac{\Delta H_{vap}}{\Delta S_{vap}} = 383$ K

Y is more volatile than X

For ideal solution $P_{Total} = P_X + P_Y$

$P_X = P_X^o X_X$, $P_Y = P_Y^o X_Y$

$X_X = 0.5$ $X_Y = 0.5$

P_x^o at 27°C

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

$$\ln\left(\frac{1}{P_x^o}\right) = \frac{8.3\times10^3}{8.3}\times\frac{100}{300\times400} = \frac{5}{6}$$

$\frac{1}{P_x^o} = e^{5/6} = 2.3 \Rightarrow P_x^o = \frac{1}{2.3} = 0.435$ atm

For P_y^o at 27°C

$\ln\left(1/P_y^o\right) = \frac{\Delta H}{R}\left(\frac{T_2 - T_1}{T_1 T_2}\right)$

$\ln(1/P_y^o) = \frac{7.66 \times 10^3 \times 83}{8.3 \times 300 \times 383} = \frac{2}{3}$

$P_y^o = \frac{1}{2} = 0.5$ atm

$P = P_X^o X_X + P_Y^o X_Y$

= (0.435 × 0.5) + (0.5 × 0.5)

= 0.47 atm

31. Answer (78.00)

$\Delta T_f = i K_f m$

$0.062 = i \times 1.86 \times 0.01$

$\Rightarrow$ i = 3.33

$\alpha = \frac{i-1}{n-1} = \frac{2.33}{4-1} = 0.776 \simeq 78\%$

32. Answer (02.00)

d, l dibromo derivatives will be resolved.

33. Answer (25.00)

For pure water, difference between T_f and T_b = 100°C

For given solution =105°C

So $\Delta T_f + \Delta T_b = 105 - 100 = 5$°C

$\Delta T_f + \Delta T_b = K_b m + K_f m$

$5°C = m(K_b + K_f)$

$m = \frac{5}{0.512 + 1.86} = 2.11$ mol/kg

Mass in 500 g = $\frac{2.11 \times 120}{2} = 126.5$ g

34. Answer (17.00)

Statement (iii) and (vii) are incorrect because half life of zero order reaction is dependent on concentration.

35. Answer (12.00)

A = PCl_3, B = AgCl

C $\Rightarrow$ $SnCl_4$

t = 1, y = 4, z = 3

t.y.z = 12

36. Answer (24.00)

	Oxidation Number
Siderite $FeCO_3$	2
Iron Pyrite FeS_2	2
Haematite Fe_2O_3	3
Malachite $CuCO_3.Cu(OH)_2$	2
Cuprite Cu_2O	1
Copper glance Cu_2S	1
Sphalerite ZnS	2
Calamine $ZnCO_3$	2
Zincite ZnO	2

PART – III : MATHEMATICS

37. Answer (C)

$(1 + a)\cos\theta\cos(2\theta - b) = (1 + a\cos2\theta)\cos(\theta - b)$

$\Rightarrow$ $2\cos\theta\cos(2\theta - b) + 2a\cos\theta\cos(2\theta - b)$

$= 2\cos(\theta - b) + 2a\cos2\theta\cos(\theta - b)$

$\Rightarrow$ $\cos(3\theta - b) - \cos(\theta - b)$

$= a\{\cos(\theta + b) - \cos(\theta - b)\}$

$\Rightarrow$ $\sin\theta\{\sin(2\theta - b) - a\sin b\} = 0$

Now, $\sin(2\theta - b) = a\sin b$ is meaningful only when $|a\sin b| \le 1$.

38. Answer (C)

Let α, β, γ be the roots.

$\therefore$ $\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = -p$

$\alpha\beta\gamma = -q$

$\Rightarrow$ 2 of them are positive and one is negative and negative root is numerically greater than the other two positive roots.

Let $\gamma < 0 < \alpha \le \beta$, where $|\alpha| \le |\beta| < |\gamma|$

$\Rightarrow$ $q - p\alpha < 0$

And $3q - 2p\alpha = -3\alpha\beta\gamma + 2\alpha(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= -\alpha(\beta - \gamma)(\gamma - \alpha) \ge 0$

$\Rightarrow$ $\alpha \in \left(\frac{q}{p}, \frac{3q}{2p}\right]$

39. Answer (C)

$f(x) = f'(x) \times f''(x)$ is satisfied by only the polynomial of degree 4.

Since $f(x) = 0$ satisfies x = 1, 2, 3 only. It is clear one of the root is twice repeated.

$\Rightarrow$ $f'(1)\, f'(2)\, f'(3) = 0$

40. Answer (B)

$$I=\int\frac{\left(\frac{2}{x^2}+\frac{1}{x\sqrt{x}}\right)dx}{\left(1+\frac{1}{\sqrt{x}}+\frac{1}{x}\right)^2}$$

Put $1+\frac{1}{\sqrt{x}}+\frac{1}{x}=t$;

$$-\left[\frac{1}{2x\sqrt{x}}+\frac{1}{x^2}\right]dx=dt=-2\int\frac{dt}{t^2}=\frac{2}{t}+c$$

$$\Rightarrow\quad I=\frac{2x}{x+\sqrt{x}+1}+c$$

41. Answer (C)

$2x^3y\,dy + (1-y^2)(x^2y^2+y^2-1)dx = 0$

$$\frac{2y}{(1-y^2)^2}\frac{dy}{dx}+\frac{y^2}{1-y^2}\frac{1}{x}=\frac{1}{x^3}$$

Put $\frac{y^2}{1-y^2}=u$

$$\Rightarrow\quad \frac{2y}{(1-y^2)^2}\frac{dy}{dx}=\frac{du}{dx}$$

$$\Rightarrow\quad \frac{du}{dx}+\frac{u}{x}=\frac{1}{x^3}$$

$$u\cdot x=\int\frac{1}{x^2}dx+c\ \Rightarrow\ x^2y^2=(cx-1)(1-y^2)$$

42. Answer (C)

Let $\frac{x^2}{a_n^2}+\frac{y^2}{b_n^2}=1$, $a_n > b_n$ be the ellipse with eccentricity = e_n

$$\Rightarrow\quad b_n^2=a_n^2(1-e_n^2) \qquad \ldots(i)$$

According to question $b_n = b_{n-1}$ …(ii)

And $a_{n-1} = a_n e_n$ …(iii)

Again for ellipse E_{n-1},

$$a_{n-1}^2=b_{n-1}^2(1-e_{n-1}^2) \qquad \ldots(iv)$$

Solving, we have

$$a_n^2e_n^2=a_n^2(1-e_n^2)(1-e_{n-1}^2)$$

$$\Rightarrow\quad e_n^2=(1-e_n^2)(1-e_{n-1}^2) \qquad \ldots(v)$$

$\because$ Eccentricity e_n is independent of

$$n \Rightarrow e_n = e_{n-1} = e$$

$$\Rightarrow\quad e^2 = (1-e^2)^2 \Rightarrow e=\frac{\sqrt{5}-1}{2}$$

43. Answer (A, B, C, D)

$$f(x+1)=\frac{f(x)-5}{f(x)-3} \qquad \ldots(i)$$

$f(x) \times f(x+1) - 3f(x+1) = f(x) - 5$

$$\Rightarrow\quad f(x)=\frac{3f(x+1)-5}{f(x+1)-1}$$

Replacing x by $(x-1)$, we have

$$f(x-1)=\frac{3f(x)-5}{f(x)-1} \qquad \ldots(ii)$$

Using (i), we have

$$f(x+2)=\frac{f(x+1)-5}{f(x+1)-3}$$

$$=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}=\frac{2f(x)-5}{f(x)-2} \qquad \ldots(iii)$$

Using (ii), we get

$$f(x-2)=\frac{3f(x-1)-5}{f(x-1)-1}$$

$$=\frac{3\left(\frac{3f(x)-5}{f(x)-1}\right)-5}{\frac{3f(x)-5}{f(x)-1}-1}$$

$$=\frac{2f(x)-5}{f(x)-2} \qquad \ldots(iv)$$

Using (iii) and (iv), we have

$f(x+2) = f(x-2)$

$\Rightarrow$ $f(x+4) = f(x)$

$\Rightarrow$ $f(x)$ is periodic with period 4.

44. Answer (A, B, D)

For some $\alpha\in(0,1)$,

$$|f'(\alpha)|=\left|\frac{f(1)-f(0)}{1-0}\right|\le|f(1)|+|f(0)|\le 2$$

Also, $F(\alpha)=(f(\alpha))^2+(f'(\alpha))^2\le 1+4=5$.

Similarly, $F(\beta)\le 5$ for some $\beta\in(-1,0)$. As $F(0)=6$, so there must be a point of local maxima for $F(x)$ in $(-1, 1)$ and at the point of maxima, $F(c)\ge 6$, $F'(c)=0$ and $F''(c)\le 0$.

45. Answer (B, C)

(B) $\frac{df(x)}{dx} - f(x) > 0$

$\Rightarrow \frac{e^{-x}df(x)}{dx} - e^{-x}f(x) > 0 \qquad (\because e^{-x} > 0)$

$\Rightarrow \frac{d}{dx}(e^{-x}f(x)) > 0$

$\Rightarrow e^{-x}f(x)$ is an increasing function.

(C) $e^{-x}f(x) > e^{-1}f(1) = 0$

$\Rightarrow e^{-x}f(x) > 0 \Rightarrow f(x) > 0$ for all $x > 1$

46. Answer (A, C)

$I = \frac{f^2(1)}{\int_0^1 (f(x))^3 dx}$

Now, $f^2(1) = \int_0^1 f^2(1)\,dx > \int_0^1 f^2(x)\,dx$ as $f(x)$ is increasing in [0, 1].

$\Rightarrow \int_0^1 f^2(1)\,dx > \int_0^1 f^3(x)\,dx$

$\Rightarrow I > 1$

47. Answer (B, C)

Since, $2\Delta = bc\sin A = ca\sin B = ab\sin C$

$\Rightarrow$ Given expression

$= \sum\sqrt{a^2b^2 - (ab\sin C)^2} = \sum ab\cos C$

$= ab\cos C + bc\cos A + ca\cos B$

$= \frac{a^2 + b^2 + c^2}{2}$

48. Answer (A, B)

$|z_1z_2| = \left|\frac{c}{a}\right|$ and $|z_1 + z_2| = \left|-\frac{b}{a}\right| = 1$

So, $(z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = 1 \Rightarrow 2 + \bar{z}_1z_2 + z_1\bar{z}_2 = 1$

$\Rightarrow 2 + \frac{z_2}{z_1} + \frac{z_1}{z_2} = 1 \Rightarrow \frac{(z_1 + z_2)^2}{z_1z_2} = 1$

$\Rightarrow \frac{b^2}{a^2} = \frac{c}{a} \Rightarrow b^2 = ac$

Now, $z_2 = z_1e^{i\theta} \Rightarrow |z_1 + z_2| = |z_1||1 + e^{i\theta}| = 1$

$= 2\cos\frac{\theta}{2}\left|\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right|$

$|z_1 + z_2| = 2\cos\frac{\theta}{2} = 1 \Rightarrow \frac{\theta}{2} = \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$

Now, $PQ = |z_2 - z_1| = |z_1||e^{i\theta} - 1| = \left|2\sin\frac{\theta}{2}\right|$

We know, $\theta = \frac{2\pi}{3}$

$\therefore \quad PQ = |z_2 - z_1| = \sqrt{3}$

49. Answer (04.00)

Let $\log_6(\cos x) = t$

$\sin^{-1}(|t^2 - 1|) + \cos^{-1}(|3t^2 - 7|) = \frac{\pi}{2}$

Possible only if $|t^2 - 1| = |3t^2 - 7| \le 1$

Possible $t = \pm\sqrt{2}$

$\Rightarrow \log_6(\cos x) = \pm\sqrt{2} \Rightarrow \cos x = 6^{\sqrt{2}}$ or $6^{-\sqrt{2}}$

$\Rightarrow \cos x = \frac{1}{6^{\sqrt{2}}}$

If $x \in [0, 4\pi]$, total 4 possible solutions.

50. Answer (16.00)

Let $|X| = PQ = \frac{|4x - 3y|}{5}$

$|Y| = PR = \frac{|3x + 4y|}{5}$

$\Rightarrow$ Given equation is $[|X|] + [|Y|] = 3$

Symmetric about X-axis and Y-axis.

For $X, Y \ge 0$

$\Rightarrow$ Area = 4 sq. units

$\Rightarrow$ Total area enclosed = $4 \times 4 = 16$ sq. units.

51. Answer (1023.00)

$(AB) \cdot (AB) = A(BA)B = A^3B^2$

$(AB)(AB)(AB) = A^2B^2AB = A^7B^3$

So, $(AB)^n = A^{2^n - 1}B^n$

So, $k = 2^{10} - 1 = 1023$

52. Answer (72.00)

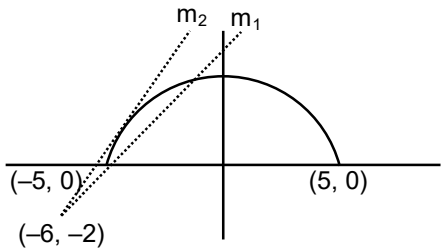
Total 3 digit natural number divisible by 3 = 300

3 digit natural number in which digits not repeated

$= {}^3C_2 \times 2 \times 2! + {}^3C_3 \times 3! \times 3 + {}^3C_1\,{}^3C_1 \times 2 \times 2!$

$+ {}^3C_1\,{}^3C_1\,{}^3C_1\,3! = 228$

53. Answer (04.00)



$M_1 = \frac{2}{1} = 2$

For the maximum value of slope in an extreme case that line should be the tangent to the semicircle with positive slope.

Tangent through (–6, –2)

$y + 2 = m(x + 6)$

$y = mx + (6m - 2)$

For tangent $(6m - 2)^2 = 25m^2 + 25$

$11m^2 - 24m - 21 = 0$

$m = \frac{12 + \sqrt{375}}{11}$

$m \in \left[2, \frac{12 + \sqrt{375}}{11}\right)$

So, $a = 2, b = \frac{12 + \sqrt{375}}{11}$

$[a + b] = 4$

54. Answer (02.00)

$\vec{\alpha} + \vec{\gamma} \times \vec{\beta} = \vec{\alpha} \times \vec{\gamma}$

$\vec{\alpha} \cdot \vec{\alpha} + \left[\vec{\alpha}\ \vec{\gamma}\ \vec{\beta}\right] = 0$

Also, $\vec{\beta} \cdot \vec{\alpha} - \left[\vec{\beta}\ \vec{\alpha}\ \vec{\gamma}\right] = 0$

and $|\vec{\alpha}| = -|\vec{\beta}| \cos\theta$

Also, $\vec{\gamma} \cdot \vec{\alpha} = 0 \Rightarrow \vec{\alpha}$ is perpendicular to $\vec{\gamma}$

$\vec{\alpha} = (\vec{\alpha} + \vec{\beta}) \times \vec{\gamma}$

$|\vec{\alpha}| = |\vec{\alpha} + \vec{\beta}| \cdot |\vec{\gamma}|$

$|\vec{\alpha}|^2 = |\vec{\beta}|^2 |\vec{\gamma}|^2 (1 - \cos^2\theta)$

$\Rightarrow |\vec{\gamma}| = \cot\theta$

$\Rightarrow |\vec{\gamma}|_{max} = \sqrt{3}$

$\Rightarrow |\vec{\gamma}|_{min} = 1$

So, A.M. of $|\vec{\gamma}|^2_{max}$ and $|\vec{\gamma}|^2_{min} = 2$

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B
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Time : 3 hrs **MM : 198**

MOCK TEST - 3

for JEE (Advanced) – 2022

Paper - 2

ANSWERS

PHYSICS	CHEMISTRY	MATHEMATICS
1. (1)	19. (9)	37. (5)
2. (2)	20. (9)	38. (0)
3. (5)	21. (8)	39. (2)
4. (3)	22. (2)	40. (1)
5. (1)	23. (8)	41. (6)
6. (2)	24. (4)	42. (3)
7. (A, B, C)	25. (A, C)	43. (A, B, C, D)
8. (B, C)	26. (B)	44. (A, B, C, D)
9. (A, B, C, D)	27. (A, B, C, D)	45. (A, B, D)
10. (A, D)	28. (A, B)	46. (A, D)
11. (B, C, D)	29. (A, B)	47. (A, C)
12. (A, C, D)	30. (B)	48. (A, B, D)
13. (06.00)	31. (70.00)	49. (469.00)
14. (04.00)	32. (02.00)	50. (10.00)
15. (28.00)	33. (10.00)	51. (07.00)
16. (02.35)	34. (80.00)	52. (2250.00)
17. (30.00)	35. (10.00)	53. (02.00)
18. (04.00)	36. (24.00)	54. (21.00)

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MOCK TEST - 3

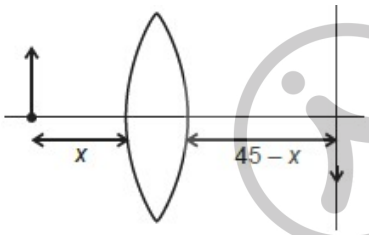
for JEE (Advanced) – 2022

Paper - 2

ANSWERS & SOLUTIONS

PART – I : PHYSICS

1. Answer (1)



$h_{I_1} \times h_{I_2} = h_0^2$

$4h_{I_1}^2 = h_0^2$

$\Rightarrow \quad \frac{h_{I_1}}{h_0} = \frac{-1}{2}$

$\frac{-1}{2} = \frac{(45 - x)}{-x}$

$\Rightarrow \quad x = 90 - 2x$

$x = 30$ cm

$\frac{1}{30} + \frac{1}{15} = \frac{1}{f}$

$\Rightarrow \quad f = 10$ cm

2. Answer (2)

$B = 2 \times \left[\frac{\mu_0 i}{4\pi a} + \frac{\mu_0 i}{4\pi a}\sin 45°\right]$

$B = \left[\frac{\mu_0 i}{4\pi a}\left(1 + \frac{1}{\sqrt{2}}\right)\right] \times 2$

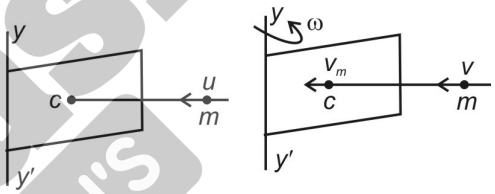
$B = \frac{2\mu_0 i}{4\pi a}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$

$B = \frac{2\mu_0 i}{4\pi a}\frac{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}{\sqrt{2}\left(\sqrt{2}-1\right)} = \frac{2\mu_0 i}{4\pi a.(2-\sqrt{2})}$

$x = 2$

3. Answer (5)

The plate is free to rotate about vertical axis *yy′*



Let V, V_{cm} and ω be the velocity of particle, velocity of centre of mass of plate and angular velocity of plate just after collision.

∴ From conservation of angular momentum about vertical axis passing through *O* is

$$mu\frac{a}{2} = mv\frac{a}{2} + \frac{ma^2}{3}\omega \qquad \ldots(i)$$

Since the collision is elastic, the equation of coefficient of restitution is

$$e = \frac{V_{cm} - V}{u} = 1 \qquad \ldots(ii)$$

But $V_{cm} = \frac{a\omega}{2}$ …(iii)

Solving equation (i), (ii) and (iii) we get

$\omega = \frac{12}{7}\frac{u}{a} = 5$ rad/s

4. Answer (3)

The concrete and iron section are subjected to the same strain. If σ_c and σ_i denote compressive stress, A_c and A_i cross-sectional area and Y_c and Y_t Young's modulus respectively, $\frac{\sigma_c}{Y_i} = \frac{\sigma_i}{Y_A}$

Let x fraction of load P be shared be concrete

$$\frac{xP}{A_cY_c}=\frac{P-xP}{Y_iA_i}$$

$$x=\frac{A_cY_c}{A_iY_i+A_cY_c}$$

Given $A_i=\frac{A_T}{4}, Y_c=\frac{Y_i}{10}\quad A_c=\frac{3A_r}{4}$

$$x=\frac{\frac{3A_T}{4}\times Y_c}{\frac{A_T}{4}10Y_c+\frac{3A_T}{4}Y_c}=\frac{3}{13}$$

Load on concrete $=\frac{3}{13}\times 1300\,\text{N}=3\times10^2\text{N}$

5. Answer (1)

Let the mass of rod is m and acceleration is $2a$ downward. If T is tension in string 2, then.

For ball, $2T-1.8\,mg=1.8\,mg$

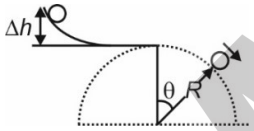
For rod, $mg-T=m\,(2a)$

$$\Rightarrow\quad a=\frac{0.29}{5.8}=\frac{g}{29}$$

The relative acceleration of ball to rod is $3a$, the time required to cross rod is

$$T=\sqrt{\frac{2l}{a_{\text{rel}}}}=\sqrt{\frac{2l}{3a}}\cong 1.4\text{ s}$$

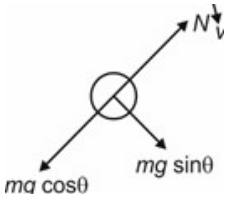
6. Answer (2)



$$\Delta h=\frac{R}{4}+R(1-\cos\theta)$$

$$\frac{1}{2}mv^2=\frac{mgR}{4}(1+4)(1-\cos\theta)$$

$$\therefore\quad \frac{mv^2}{R}=\frac{mg}{2}(5-4\cos\theta)$$



$$mg\cos\theta-N=\frac{mv^2}{R}$$

$$mg\cos\theta=\frac{mg}{2}(5-4\cos\theta)$$

$$\cos\theta=\frac{5}{6}$$

7. Answer (A, B, C)

$$a=4-\frac{1}{3}x$$

$$\Rightarrow\quad \int v\,dv=\int\left(4-\frac{1}{3}x\right)dx$$

$$\Rightarrow\quad v^2=2\times\left[4x-\frac{x^2}{6}\right]$$

$\Rightarrow$ $v=0$ at $x=24$ m, v (at $x=6$)

$$=\sqrt{2\times(24-6)}=6\text{ m/s}$$

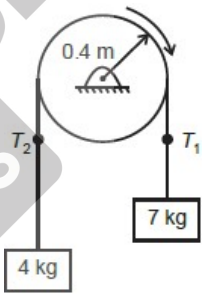
8. Answer (B, C)

$$Y_0=\frac{Dt}{d}(\mu_2-\mu_1)=6.67\text{ m}$$

$$\Delta x_0=t(\mu_2-\mu_1)=\frac{10}{6}\times10^{-6}$$

$\Delta\phi=\pi\Rightarrow I_0=0$

9. Answer (A, B, C, D)



Since pulley rotates cw, $T_1>T_2$

$$K_p=\frac{1}{2}\times I\omega^2=\frac{1}{2}\times\frac{\left(6R^2\right)}{2\omega^2}=1.5v^2$$

$$K_{\text{block}}=\frac{1}{2}\times4\times v^2=2v^2$$

$a_{cm}\downarrow$

$\Rightarrow$ $7g+4g+6g-F_{\text{hinge}}=17a_{cm}>0$

$\Rightarrow$ $F_{\text{hinge}}<17\,g$

10. Answer (A, D)

$$B=\frac{\mu_0NI}{2R}$$

$\vec{\mu}=n\pi r^2I$

$\therefore$ Torque initially

$\Rightarrow$ $\tau=\vec{\mu}\times\vec{B}$

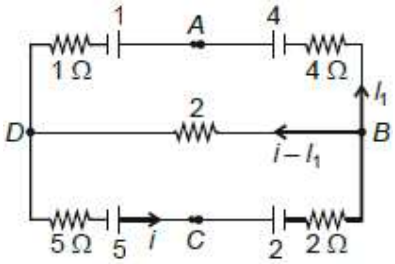
$$\Rightarrow\quad \tau=n\pi r^2I\cdot\frac{\mu_0NI}{2R}=\frac{\mu_0\pi r^2nN\cdot I^2}{2R}$$

Work done ΔW = |change in PE|

$\Rightarrow \Delta W = \left|\vec{\mu}\cdot\vec{B}_{|\text{initially}|} - \vec{\mu}\cdot\vec{B}_{|\text{finally}|}\right|$

$\Rightarrow \Delta W = \dfrac{\mu_0 \pi r^2 nN \cdot I^2}{2R}$

11. Answer (B, C, D)



$-5i + 5 - 2 - 2i - 4I_1 + 4 - 1 - I_1 = 0$

$\Rightarrow 6 = 7i + 5I_1$ …(i)

and $-5i + 5 - 2 - 2i - 2i + 2I_1 = 0$

$\Rightarrow 3 = 9i - 2I_1$ …(ii)

From eq. (i) and (ii).

$7i + 5I_1 = 18i - 4I_1$

$\Rightarrow I_1 = \dfrac{11}{9}i$

And $i = \dfrac{27}{59}$ and $I_1 = \dfrac{33}{59}$

$\therefore$ Current through 2 Ω is $\dfrac{6}{59}$ from *D* to *B*.

$V_G = -5i + 5 = 5\left(1 - \dfrac{27}{59}\right) = \dfrac{160}{59}$ volt

$V_H = 2 + 2i = \left(1 + \dfrac{27}{59}\right) = \dfrac{172}{59}$

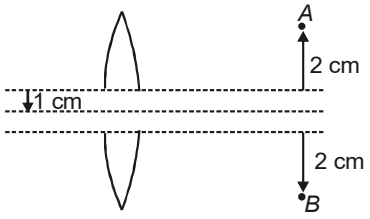
12 Answer (A, C, D)

(A) Due to induction charge on outer surface is $Q + q$

(C) Charge flows till potential of *A* and *B* will be same

(D) All charge will be on the outer surface of *A*

13. Answer (06.00)



$\dfrac{1}{v} - \dfrac{1}{-15} = \dfrac{1}{10}$

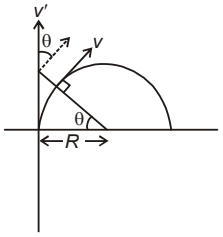
$\dfrac{1}{v} = \dfrac{1}{10} - \dfrac{1}{15} = \dfrac{3-2}{30}$

v = 30 cm

$m = \dfrac{v}{u} = \dfrac{30}{-15} = -2$

AB = 6 cm

14. Answer (04.00)



$\dfrac{v'\cos\theta}{R\sec\theta} = \dfrac{v}{R}$

$v' = v\sec^2\theta$

15. Answer (28.00)

Δm = 0.026 amu

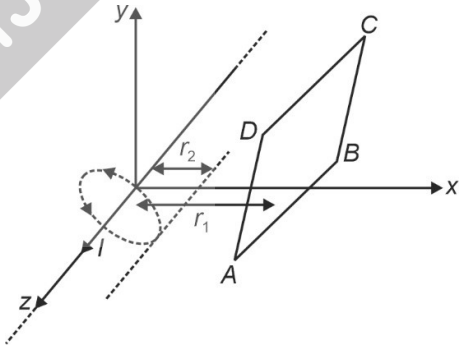
$\Delta E = 0.026 \times 931 \times 1.6 \times 10^{-13}$ J

$= 3.87 \times 10^{-12}$ J

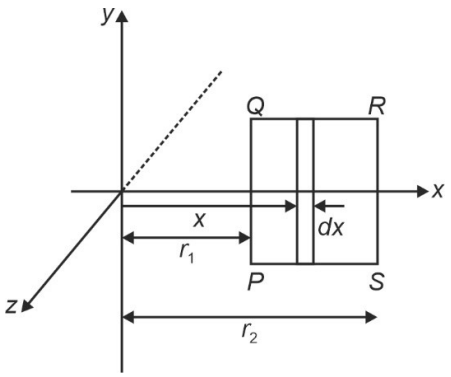
$t = \left(\dfrac{10^{50}}{3}\right)\left(\dfrac{3.87 \times 10^{-12}}{10^{10}}\right)$ s

$= 1.29 \times 10^{28}$ s

16. Answer (02.35)



Since magnetic field lines are in cylindrical form have flux through *ABCD* will be same as flux through *PQRS*.



$$d\phi = \frac{\mu_0 I}{2\pi x}\,\ell\,dx$$

$$\phi = \int d\phi$$

$$= \frac{\mu_0 I \ell}{2\pi}\ln\frac{r_2}{r_1}$$

$$r_2 = \sqrt{(\ell\sin 74)^2 + (\ell + \ell\cos 74)^2}$$

$$= \ell\sqrt{2\left(1+\frac{7}{25}\right)}$$

$$= \frac{8\ell}{5}$$

$$= \frac{\mu_0 I \ell}{2\pi}\ln\frac{8}{5}$$

$$= \frac{\mu_0 I \ell}{2\pi}\ln 1.6$$

$$= \frac{(\bar{\mu}_0)(10)\,\bar{\ell}}{2\pi}\times 0.47 = \frac{\mu_0 \ell}{\pi}\times 0.47\times 5$$

$$= 2.35\,\frac{\mu_0 \ell}{\pi}$$

17. Answer (30.00)

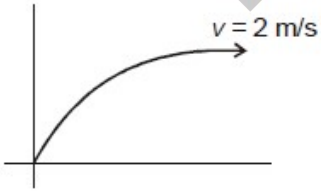
Apply Kirchhoff's law,

$R = 3CE$

$= 3 \times 2 \times 5$

$= 30\ \mu C$

18. Answer (04.00)



$$t = \frac{2\sqrt{3}}{E_0}$$

$$T = \frac{2\pi m}{aB}$$

$$T = \left(\frac{2\pi}{B_0}\right)$$

$$Z_m = 2R,\ R = \left(\frac{mu}{aB}\right)$$

PART – II : CHEMISTRY

19. Answer (9)

Concentration of drop

$$= \frac{\text{Mole}}{\text{Volume in mL}}\times 1000$$

$$= \frac{3\times 10^{-6}}{0.05}\times 1000 = 0.06\ \text{mol L}^{-1}$$

$$\text{Rate of disappearance} = \frac{\text{conc. change}}{\text{time}}$$

$$1\times 10^{7} = \frac{0.06}{\text{time}}$$

$$\boxed{t = 6\times 10^{-9}}$$

20. Answer (9)

We know that the isotonic solutions have the same molar concentration (i.e. moles/L)

Let the molecular weight of A be M.

$$\text{Mole of A} = \frac{1.72}{M}$$

$$\therefore\ \text{Molar conc. of A (mole/litre)} = \frac{1.72}{M}\times\frac{1000}{100}$$

$$= \frac{17.2}{M}$$

$$\text{Molar conc. of sucrose} = \frac{3.42}{342}\times\frac{1000}{100} = 0.1$$

$$\frac{17.2}{M} = 0.1\ ;\ M = 172$$

21. Answer (8)

$$PV = \frac{w}{m}RT$$

$$\frac{(766-16)}{760}\times\frac{27.96}{1000} = \frac{0.1015}{m}\times 0.0821\times 288\ ;$$

$$m = 86.797 = 87$$

$$\%\ C = \frac{12}{44}\times\frac{0.6098}{0.4020}\times 100 = 41.37\%$$

$$\%H = \frac{2}{18}\times\frac{0.2080}{0.4020}\times 100 = 5.749\%$$

$$\%\ N = \frac{1.4NV}{W} = \frac{1.4\times 0.5\times 23.3}{1.01} = 16.07\%$$

$$\%S = \frac{32}{233} = \frac{0.2772}{0.1033}\times 100 = 36.854\%$$

Number of (C) atoms

$$= \frac{\%}{100}\times\frac{Mw}{Aw} = \frac{41.37}{100}\times\frac{87}{12} = 3$$

Number of (H) atoms

$$= \frac{\%}{100} \times \frac{Mw}{Aw} = \frac{5.749}{100} \times \frac{87}{1} = 5$$

Number of (N) atoms

$$= \frac{\%}{100} \times \frac{Mw}{Aw} = \frac{16.07}{100} \times \frac{87}{14} = 1$$

Molecular formula = C_3H_5NS = Empirical formula

22. Answer (2)

Mer and Fac

23. Answer (8)

$$CH_3 - CH = CH - CH_3 \xrightarrow[(ii)\,OH^-/\Delta]{(i)\,O_3/Zn} CH_3 - CH = CH - CHO$$

24. Answer (4)

Let the moles of CO(g) = x and CO_2(g) = y moles

$\therefore$ Total number of moles of CO(g) and CO_2(g) = x + y

Initially, for n mole, ideal gas equation

PV = nRT

$1 \times V = (x + y) R \times 300$ …(i)

According to the question

Moles of O_2(g) = x + y

Now, $2CO(g) + O_2(g) \longrightarrow 2CO_2(g)$

Total number of moles after the completion of the reaction

= remaining moles of O_2(g) + moles of CO_2(g)
already present and produced

$$= (x+y) - \frac{x}{2} + y + x = \frac{3x}{2} + 2y$$

Now, PV = nRT

$$2.70 \times V = \left(\frac{3x}{2} + y\right) \times R \times 450 \quad \text{…(ii)}$$

Solving equation (i) and (ii),

y = 1.5x

% of CO in the original mixture

$$= \frac{x}{x+y} \times 100 = \frac{x}{x+1.5x} = \frac{1}{2.5} \times 100 = 40\%$$

25. Answer (A, C)

It is the part of Victor meyer's test for 1° alcohol.

26. Answer (B)

$$R - \underset{\underset{O}{\|}}{C} - Cl + H_2 \xrightarrow{Pd/BaSO_4} RCHO$$

$$R - C \equiv C - R + H_2 \xrightarrow{Pd/BaSO_4} \text{Cis-alkene}$$

27. Answer (A, B, C, D)

Fact based.

28. Answer (A, B)

(A), (B) Correct statements.

(C) Graphite, sp^2 hybridisation bond length 1.42 Å, diamond sp^3 hybridisation bond length 1.54 Å

(D) Diamond, more dense (3.51 g/ml) than graphite (2.25 g/ml)

29. Answer (A, B)

(A) Two hydroxyl groups lie in different planes (have open book like structure)

(B) It turns blue litmus red because of acidic character which is then bleached by bleaching action of H_2O_2.

30. Answer (B)

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{n} \log \frac{[Anode]}{[Cathode]}$$

(A) $$E_{cell} = E^\circ - \frac{0.059}{n} \log\left(\frac{Ag^+_{anode}}{Ag^+_{cathode}}\right)$$

$$E_{cell} = \frac{-0.059}{1} \log_{10}\left[\frac{10^{-9}}{10^{-3}}\right] = -0.059 \times (-6)$$

= 0.354 V

(B) $E^o_{cell} = 0.8$ V

$$E_{cell} = 0.8 - \frac{0.059}{1} \log\left(\frac{10^{-12}}{10^{-2}}\right) = 0.8 + 0.059(10)$$

= 1.39 V

(C) $E^o_{cell} = 0$

$$E_{cell} = 0 - \frac{0.059}{1} \log\left(\frac{10^{-3}}{10^{-6}}\right) = -0.177 \text{ V}$$

(D) $$E_{cell} = -0.34 - \frac{0.059}{2} \log\left(\frac{10^{-18}}{10^{-2}}\right) = 0.132 \text{ V}$$

31. Answer (70.00)

NaOH present in the solution

$$= \frac{2}{40} \times 1000 = 50 \text{ millimoles} = 50 \text{ milli eq.}$$

Na_2CO_3 present in the solution

$$= \frac{2.1}{106} \times 1000 = 18.8 \text{ millimoles} = 2 \times 18.8 \text{ milli eq.}$$

In presence of phenolphthalein at the end point NaOH is completely neutralized but Na_2CO_3 is half neutralized (up to $NaHCO_3$)

∴ Milli eq. of NaOH used for neutralized = 50

Milli eq. of Na_2CO_3 neutralized = 19.8

∴ Milli eq. of HCl used for neutralization

= 50 + 19.8 = 69.8

69.8 milli eq. of HCl will be present in 69.8 ml of 1 N HCl solution

32. Answer (02.00)

meq of $KMnO_4$ = meq of oxalate

33. Answer (10.00)

$\pi = i \times C \times R \times T$

T = 300 K, R = 0.082

i = 2

$$C = \frac{4.92}{0.082 \times 300 \times 2} = 0.1$$

$$0.1 = \frac{\text{Mol of salt}}{\text{Volume}}$$

Molar mass of salt = 18 g

$$d = \frac{1 \times 18}{6 \times 10^{23} \times a^3}$$

$$\Rightarrow \quad a^3 = \frac{18}{6 \times 10^{23} \times 30} = 10^{-24}\,cm^3$$

$a = 10^{-8}$ cm

a = 0.1 nm

34. Answer (80.00)

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

$$\frac{1}{n} = \tan\theta = \frac{1}{2},\ n = 2$$

P = 4, n = 2 log k = 2

$k = 10^2$

$$\frac{x}{m} = 100(4)^{\frac{1}{2}} = 200$$

x = 200 × 2 = M

$\Rightarrow$ M = 400 g

$$\Rightarrow \quad \frac{M}{5} = 80$$

35. Answer (10.00)

$A = PCl_3$, $B = S_2Cl_2$, $C = SO_2$

$$P_4 + 8SOCl_2 \longrightarrow \underset{(A)}{4PCl_3} + \underset{(C)}{4SO_2} + \underset{(B)}{2S_2Cl_2}$$

36. Answer (24.00)

Velocity at which fraction of molecule is maximum called most probable velocity.

$$V_{mp} = \sqrt{\frac{2RT}{M}} = 300$$

$$\Rightarrow \quad \frac{(2)(8.314)(T)\times 1000}{90} = 300 \times 300$$

$$\Rightarrow \quad T = \frac{300 \times 90 \times 500}{2 \times 8.314 \times 1000}$$

$$= \left(\frac{27}{2 \times 8.314}\right) \times 300 = \frac{27 \times 300}{2R}$$

$$K.E. = \left(\frac{3}{2}\right)(n)RT$$

$$= \left(\frac{3}{2}\right)\left(\frac{360}{90}\right)(R)\frac{27 \times 300}{2 \times 8.314}$$

= 3 × 27 × 300

= 81 × 300

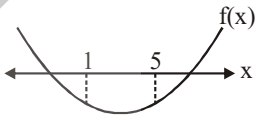
= 24300 joule

= 24.300 kJ

And hence answer will be 24.

PART – III : MATHEMATICS

37. Answer (5)



We have,

$$1 + \sum_{r=1}^{10}\left(3^r \cdot {}^{10}C_r + r \cdot {}^{10}C_r\right)$$

$$= 1 + \sum_{r=1}^{10} 3^r \cdot {}^{10}C_r + 10\sum_{r=1}^{10} {}^{9}C_{r-1}$$

$= 1 + 4^{10} - 1 + 10 \cdot 2^9$

$= 4^{10} + 5.2^{10}$

$= 2^{10}(4^5 + 5)$

$= 2^{10}(\alpha \cdot 4^5 + \beta)$

So, $\alpha = 1$ and $\beta = 5$

Now, $f(1) < 0$ and $f(5) < 0$

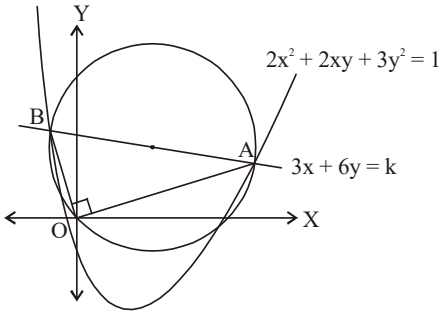
So, $f(1) < 0 \Rightarrow -k^2 < 0 \Rightarrow k \neq 0$

and $f(5) < 0 \Rightarrow 16 - k^2 < 0 \Rightarrow k^2 - 16 > 0$

$\Rightarrow \quad k \in (-\infty, -4) \cup (4, \infty)$

Hence, smallest positive integral value of $k = 5$.

38. Answer (0)



We have,

$$\frac{3x+6y}{k} = 1 \qquad \ldots(i)$$

$$2x^2 + 2xy + 3y^2 - 1 = 0 \quad \ldots(ii)$$

Now, homogenising (ii) with the help of (i), we get

$$\Rightarrow \quad 2x^2 + 2xy + 3y^2 - \left(\frac{3x+6y}{k}\right)^2 = 0$$

$$\Rightarrow \quad k^2(2x^2 + 3x + 3y^2) - (3x + 6y)^2 = 0$$

Now, coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \quad (2k^2 - 9) + (3k^2 - 36) = 0$$

$$\Rightarrow \quad 5k^2 = 45$$

$$\Rightarrow \quad k^2 = 9$$

$$\Rightarrow \quad k = 3 \text{ or } -3$$

39. Answer (2)

$$\vec{r}_1 + 2\vec{r}_2 = (p\vec{a} + q\vec{b} + \vec{c}) + 2(\vec{a} + p\vec{b} + q\vec{c})$$

$$\vec{r}_1 + 2\vec{r}_2 = (p+2)\vec{a} + (q+2p)\vec{b} + (1+2q)\vec{c}$$

$$2\vec{r}_1 + \vec{r}_2 = (2p+1)\vec{a} + (2q+p)\vec{b} + (2+q)\vec{c}$$

$$\frac{p+2}{2p+1} = \frac{q+2p}{2q+p} = \frac{1+2q}{2+q}$$

$$= \frac{p+q+2p+2q+3}{p+q+2p+2q+3} = 1$$

$$\Rightarrow \quad p = 1 \text{ and } q = 1$$

40. Answer (1)

We have,

$$\left((\vec{a}\times\vec{b})\times\vec{a}\right)\cdot\left((\vec{b}\times\vec{a})\times\vec{b}\right)$$

$$= \left((\vec{a}\cdot\vec{a})\vec{b} - (\vec{b}\cdot\vec{a})\vec{a}\right)\cdot\left((\vec{b}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{b})\vec{b}\right)$$

$$= \left(\vec{a}^2\,\vec{b}^2\right)\vec{a}\cdot\vec{b} - \vec{a}^2\,\vec{b}^2\left(\vec{a}\cdot\vec{b}\right) - \left(\vec{a}\cdot\vec{b}\right)\vec{a}^2\,\vec{b}^2 + \left(\vec{a}\cdot\vec{b}\right)^3$$

$$= -\left[\left(\vec{a}\cdot\vec{b}\right)\vec{a}^2\,\vec{b}^2 - \left(\vec{a}\cdot\vec{b}\right)^3\right]$$

$$= -\left(\vec{a}\cdot\vec{b}\right)\left[\vec{a}^2\,\vec{b}^2 - \left(\vec{a}\cdot\vec{b}\right)^2\right]$$

$$= -(\vec{a}\cdot\vec{b})\left|\vec{a}\times\vec{b}\right|^2$$

41. Answer (6)

We have,

$$S_n = (1 + 2T_n)(1 - T_n) \quad \ldots(i)$$

Putting n = 1, in equation (i), we get

$$S_1 = (1 + 2T_1)(1 - T_1) = T_1$$

$$\Rightarrow \quad 1 + T_1 - 2T_1^2 = T_1 \;\Rightarrow\; T_1 = \frac{1}{\sqrt{2}} \text{ (As } T_1 > 0)$$

Now, putting $n = 2$ in equation (i), we get

$$S_2 = (1 + 2T_2)(1 - T_2)$$

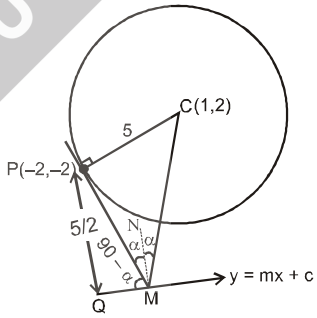
$$\Rightarrow \quad T_1 + T_2 = 1 + T_2 - 2T_2^2 \;\Rightarrow\; 2T_2^2 = 1 - \frac{1}{\sqrt{2}}$$

$$\therefore \quad T_2^2 = \frac{\sqrt{2}-1}{2\sqrt{2}} = \frac{\sqrt{4}-\sqrt{2}}{4} \equiv \frac{\sqrt{a}-\sqrt{b}}{4} \text{ (Given)}$$

So, $a = 4$ and $b = 2$

Hence, $(a + b) = 4 + 2 = 6$

42. Answer (3)



Let the equation of required straight line be $y = mx + c$.

$$\Rightarrow \quad \frac{5}{2} = \frac{|-2+2m-c|}{\sqrt{1+m^2}} \qquad \ldots(i)$$

For ΔPCM,

$$\frac{PC}{PM} = \tan 2\alpha$$

$$\Rightarrow \quad PM = 5\cot 2\alpha \qquad \ldots(ii)$$

For ΔPQM,

$$\frac{5}{2} = PM \sin(90 - \alpha)$$

$$\Rightarrow \quad \frac{5}{2} = \frac{5\cos 2\alpha}{\sin 2\alpha}\cos\alpha$$

On solving, we get

$\alpha = 30°$

Equation of tangent at $P(-2, -2)$ is

$3x + 4y + 14 = 0$

$\tan 60° = \left|\dfrac{m + 3/4}{1 - 3m/4}\right|$

$\Rightarrow \quad \sqrt{3} = \dfrac{m + 3/4}{1 - 3m/4} \Rightarrow m = \dfrac{4\sqrt{3} - 3}{4 + 3\sqrt{3}}$

Now, on substituting value of m in equation (i), we get

$c = \dfrac{11 + 2\sqrt{3}}{4 + 3\sqrt{3}}$ or $\dfrac{-39 + 2\sqrt{3}}{4 + 3\sqrt{3}}$

But c should be (–ve).

So, equation of line

$y = \dfrac{(4\sqrt{3} - 3)}{4 + 3\sqrt{3}}x + \left(\dfrac{-39 + 2\sqrt{3}}{4 + 3\sqrt{3}}\right)$

i.e., $(4\sqrt{3} - 3)x - (4 + 3\sqrt{3})y - (39 - 2\sqrt{3}) = 0$

Hence, $a = 3$, $b = 4$ and $c = 39$

Hence, $\dfrac{c}{a^2 + b} = 3$

43. Answer (A, B, C, D)

$a, b = ar, c = ar^2$ are in G.P., where $r \neq \pm 1$

$a + b + c = bx \Rightarrow 1 + r + r^2 = rx \quad \ldots(i)$

$\Rightarrow \quad r^2 + (1 - x)r + 1 = 0$

$\Delta \geq 0 \Rightarrow (1 - x)^2 - 4 \geq 0$

$(x - 1)^2 - 2^2 \geq 0, (x - 3)(x + 1) \geq 0$

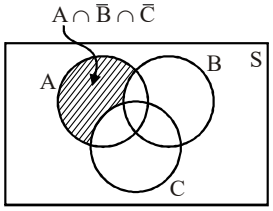
$x \in (-\infty, -1] \cup [3, \infty)$

If $x = -1$, then $r = -1 \Rightarrow x = -1$ (rejected)

and $x = 3$, then $r = 1 \Rightarrow x = 3$ (rejected)

$\therefore \quad x \in (-\infty, -1) \cup (3, \infty)$

44. Answer (A, B, C, D)



$P((A \cap \bar{B}) / \bar{C}) = \dfrac{P(A \cap \bar{B} \cap \bar{C})}{P(\bar{C})} =$

$\dfrac{P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)}{1 - P(C)} \quad \ldots(i)$

(MM), (TT), (AA), H, E, I, C, S

A : $\boxed{MM}$, T, T, A, A, H, E, I, C, S

$P(A) = \dfrac{\frac{(10)!}{2!\,2!}}{\frac{(11)!}{2!\,2!\,2!}} = \dfrac{2}{11}$

$= P(B) = P(C) \Rightarrow$ (A) is correct

$A \cap B$: $\boxed{MM}$, $\boxed{TT}$, A, A, M, E, I, C, S

$P(A \cap B) = \dfrac{\frac{9!}{2!}}{\frac{(11)!}{2!\ 2!\ 2!}} = \dfrac{2}{55}$

$= P(A \cap C) = P(B \cap C) \Rightarrow$ (B) is correct

$A \cap B \cap C$: $\boxed{MM}$, $\boxed{TT}$, $\boxed{AA}$, H, E, I, C, S

$P(A \cap B \cap C) = \dfrac{8!}{\frac{(11)!}{2!\,2!\,2!}} = \dfrac{4}{495} \Rightarrow$ (C) is correct

From (i),

$P((A \cap \bar{B}) / \bar{C}) = \dfrac{\frac{2}{11} - \frac{2}{55} - \frac{2}{55} + \frac{4}{495}}{1 - \frac{2}{11}}$

$= \dfrac{58}{405} \Rightarrow$ (D) is correct

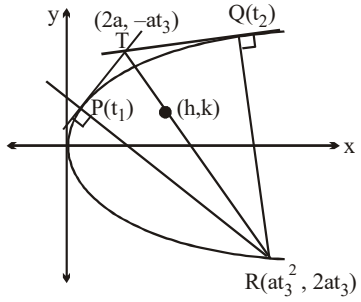
45. Answer (A, B, D)

We have,

$2h = t_3^2 + 2 \quad \ldots(i)$

$2k = t_3 \quad \ldots(ii)$

$\therefore \quad 2h = 4k^2 + 2$



Here, $a = 1$, $t_1t_2 = 2$, $t_1 + t_2 + t_3 = 0$

$\therefore \quad 2y^2 = x - 1$

$y^2 = \dfrac{1}{2}(x - 1)$ (Parabola)

Now interpret.

46. Answer (A, D)

Equation of chord joining $\left(ct_1, \frac{c}{t_1}\right)$ and $\left(ct_2, \frac{c}{t_2}\right)$ is

$x + t_1 t_2 y = (t_1 + t_2)c$ …(i)

$\because$ (i) is parallel to $y = x$

$\Rightarrow t_1 t_2 = -1$

Now, equation of circle will be

$$(x - ct_1)(x - ct_2) + \left(y - \frac{c}{t_1}\right)\left(y - \frac{c}{t_2}\right) = 0$$

$\Rightarrow x^2 + y^2 - cx(t_1 + t_2) + yc(t_1 + t_2) - 2c^2 = 0$

$\Rightarrow x^2 + y^2 - 2c^2 - c(t_1 + t_2)(x - y) = 0$ …(ii)

Equation (ii) is of the form $S + \lambda L = 0$, where S is $x^2 + y^2 - 2c^2 = 0$ and L is $x - y = 0$

Solving S and L, we get (c, c) and $(-c, -c)$

$\therefore$ (ii) will always pass through (c, c) and $(-c, -c)$

47. Answer (A, C)

L.H.L. at $x = 0$,

$$\lim_{x\to 0^-} g(f(x)) = \lim_{x\to 0^-} g\left(\frac{x}{\tan x}\right)$$

Let $\frac{x}{\tan x} = t$

$\therefore \lim_{x\to 0^-} g(f(x)) = \lim_{t\to 1^-} g(t) = \lim_{t\to 1^-} (t+4) = 5$

R.H.L. at $x = 0$,

$$\lim_{x\to 0^+} g(f(x)) = \lim_{x\to 0^+} g\left(\frac{\ln(1+2x)}{x}\right)$$

As, $\frac{\ln(1+2x)}{x} = \frac{2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} .....}{x}$

$$= 2 - 2x + \frac{8}{3}x^2 .....$$

Taking $\frac{\ln(1+2x)}{x} = t$, we get

$$\lim_{x\to 0^+} g(f(x)) = \lim_{t\to 2^-} g(t) = \lim_{t\to 2^-} (t^2 - 5t + 11)$$

$= 4 - 10 + 11$

$= 5$

48. Answer (A, B, D)

$$\Delta = \begin{vmatrix} x^2 & x+1 & x-2 \\ 2x^2 & 3x & 3x-3 \\ x^2 & 2x-1 & 2x-1 \end{vmatrix} + \begin{vmatrix} x & x+1 & x-2 \\ 3x-1 & 3x & 3x-3 \\ 2x+3 & 2x-1 & 2x-1 \end{vmatrix}$$

$$= \begin{vmatrix} 2x^2 & 3x & 3x-3 \\ 2x^2 & 3x & 3x-3 \\ x^2 & 2x-1 & 2x-1 \end{vmatrix} + \begin{vmatrix} 2 & 3 & x-2 \\ 2 & 3 & 3x-3 \\ 4 & 0 & 2x-1 \end{vmatrix}$$

$[R_1 \to R_1 + R_3]$ $[C_1 \to C_1 - C_3; C_2 \to C_2 - C_3]$

$$= 0 + \begin{vmatrix} 2 & 3 & x \\ 2 & 3 & 3x \\ 4 & 0 & 2x \end{vmatrix} + \begin{vmatrix} 2 & 3 & -2 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$$

$$= x\begin{vmatrix} 2 & 3 & 1 \\ 2 & 3 & 3 \\ 4 & 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 3 & -2 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$$

which is of the form $xA + B$, where

$$A = \begin{vmatrix} 2 & 3 & 1 \\ 2 & 3 & 3 \\ 4 & 0 & 2 \end{vmatrix} \text{ and } B = \begin{vmatrix} 2 & 3 & -2 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$$

49. Answer (469.00)

$T_1 = a^{(\log_3 7)(\log_3 7)} = 27^{\log_3 7} = 3^{3\log_3 7} = 343$

$T_2 = b^{(\log_7 11)(\log_7 11)} = 49^{\log_7 11} = 7^{2\log_7 11} = 121$

$T_3 = c^{(\log_{11} 25)(\log_{11} 25)}$

$= (\sqrt{11})^{\log_{11} 25} = 11^{\frac{1}{2}\log_{11} 25} = 5$

$\therefore$ Sum = $T_1 + T_2 + T_3 = 343 + 121 + 5 = 469$

50. Answer (10.00)

Given, $\sin A \sin B \sin C = \frac{2}{3}$;

$\cos A \cos B \cos C = \frac{1}{9}$;

$\tan A \tan B \tan C = \frac{2}{3} \cdot \frac{9}{1} = 6$

$\therefore$ $\tan A + \tan B + \tan C = 6$

$\left[\text{As, in } \Delta ABC, \prod \tan A = \prod \tan A\right]$

Hence the cubic is

$$x^2 - 6x^2 + \left(\sum \tan A \tan B\right)x^2 - 6 = 0 \quad \text{…(i)}$$

Now, $\sum \tan A \tan B$

$$= \frac{\sin A \sin B \cos C + \sin B \sin C \cos A + \sin C \sin A \cos B}{\cos A \cos B \cos C} \text{ ...(ii)}$$

Now, $A + B + C = \pi$

$\cos(A + B + C) = -1$

$\cos(A + B)\cos C - \sin(A + B)\sin C = -1$

$(\cos A\cos B - \sin A\sin B)\cos C$

$-\sin C(\sin A\cos B + \cos A\sin B) = -1$

$1 + \cos A\cos B\cos C$

$= \sin A\sin B\cos C + \sin B\sin C\cos A$

$+ \sin C\sin A\cos B$ …(iii)

Substituting in (ii), we get

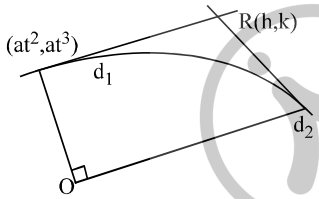
$$\sum \tan A\tan B = \frac{1+\cos A\cos B\cos C}{\cos A\cos B\cos C}$$

$$= \frac{1+\frac{1}{9}}{\frac{1}{9}} = \frac{10}{9}\cdot\frac{9}{1} = 10$$

Hence, the cubic is $x^3 - 6x^2 + 10x - 6 = 0$

Clearly, the sum of the products of the roots taken two at a time is 10.

51. Answer (07.00)



$x = at^2$; $y = at^3$

$$\frac{dx}{dt} = 2at;\ \frac{dy}{dt} = 3at^2$$

$$\frac{dy}{dx} = \frac{3t}{2}$$

$$y - at^3 = \frac{3t}{2}(x - at^2) \quad \text{…(i)}$$

$2k - 2at^3 = 3th - 3at^3$

$at^3 - 3th + 2k = 0$

$t_1t_2t_3 = -\frac{2k}{a}$ (put $t_1t_2 = -1$); hence $t_3 = \frac{2k}{a}$

Now, t_3 must satisfy the equation (i) which gives the required locus.

52. Answer (2250.00)

We have,

$$F(x) + F\left(x+\frac{1}{2}\right) = 3 \quad \text{…(i)}$$

Replace x by $x+\frac{1}{2}$ in (i), we get

$$F\left(x+\frac{1}{2}\right) + F(x+1) = 3 \quad \text{…(ii)}$$

$\therefore$ From (i) and (ii), we get

$F(x) = F(x+1)$ …(iii)

$\Rightarrow$ $F(x)$ is periodic function.

Now, consider

$$I = \int_0^{1500} F(x)dx$$

$$= 1500\int_0^1 F(x)dx$$

$$= 1500\left[\int_0^{\frac{1}{2}} F(x)dx + \int_{\frac{1}{2}}^1 F(x)dx\right]$$

[Using property of periodic function]

Put $x = y + \frac{1}{2}$ in 2nd integral, we get

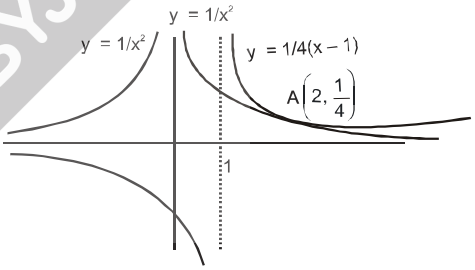
$$I = 1500\left[\int_0^{\frac{1}{2}} F(x)\,dx + \int_0^{\frac{1}{2}} F\left(y+\frac{1}{2}\right)dy\right]$$

$$= 1500\int_0^{\frac{1}{2}}\left(F(x) + F\left(x+\frac{1}{2}\right)\right)dx$$

$$= 1500\int_0^{\frac{1}{2}} 3\,dx \text{ [Using (i)]}$$

Hence, $I = 1500(3)\left(\frac{1}{2}\right) = 750\times 3 = 2250$

53. Answer (02.00)



$y_1 = \frac{1}{x^2}$ and $y_2 = \frac{1}{4(x-1)}$

These curves are touching at $x = 2$.

$$A = \int_2^a\left(-\frac{1}{x^2} + \frac{1}{4(x-1)}\right)dx = \frac{1}{x} + \frac{1}{4}\ln(x-1)\Big|_2^a$$

or $\left(\frac{1}{a} + \frac{1}{4}\ln(a-1) - \frac{1}{2}\right) = \frac{1}{a}$

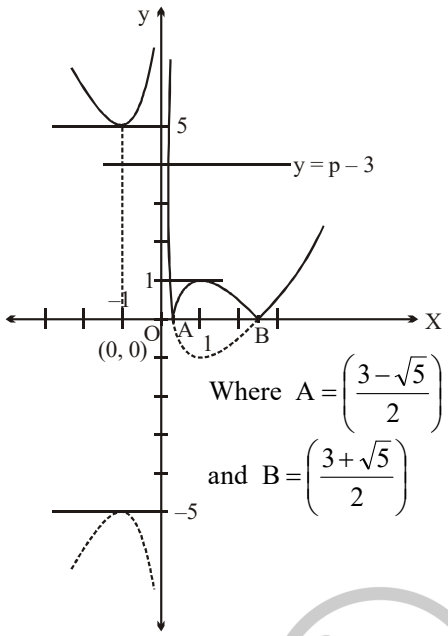
or $\ln(a-1) - 2 = 0 \Rightarrow a = 1 + e^2$

Also, $A = \int_b^2\left(-\frac{1}{x^2} + \frac{1}{4(x-1)}\right)dx = 1 - \frac{1}{b}$

or $\frac{1}{2} - \frac{1}{4}\ln(b-1) - \frac{1}{b} = 1 - \frac{1}{b}$

or $b = 1 + 1/e^2$

54. Answer (21.00)



Where $A=\left(\frac{3-\sqrt{5}}{2}\right)$

and $B=\left(\frac{3+\sqrt{5}}{2}\right)$

Consider $y = x+\frac{1}{x}-3$

$\Rightarrow \quad \frac{dy}{dx}=1-\frac{1}{x^2}=0$

$\therefore \quad \frac{dy}{dx}=0 \Rightarrow x=1 \text{ or } -1$

As $x \to 0^+$, $y \to \infty$ and $x \to 0^-$, $y \to -\infty$

Also, roots of $x+\frac{1}{x}-3=0 \Rightarrow x^2-3x+1=0$

$$x=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

For two distinct solutions either $p-3=0 \Rightarrow p=3$

or $1 < p-3 < 5$

$4 < p < 8$

Hence, $p \in \{3\} \cup (4, 8)$

$p = \{3, 5, 6, 7\} \Rightarrow$ Sum = 21

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