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## **MOCK TEST PAPER**

for

# JEE (Main)-2023

### **General Instructions:**

- 1. Duration of Test is 3 hrs.
- 2. The Test booklet consists of 90 questions. The maximum marks are 300.
- There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part has two sections.
  - (i) **Section-I**: This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.
  - (ii) **Section-II**: This section contains 10 questions. In Section II, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

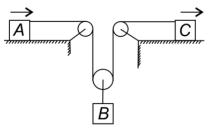
### PART - A: PHYSICS

### **SECTION - I**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

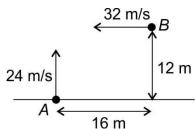
- 1. A particle moves in the x, y plane according to the law, x = kt and  $y = kt(1-\alpha t)$ , where k and  $\alpha$  are constants and t is time. Time instant when angle between acceleration and velocity is  $45^{\circ}$ , is
  - (1)  $\frac{2}{\alpha}$
- (2)  $\frac{1}{\alpha}$
- (3)  $\frac{1}{2\alpha}$
- (4)  $\frac{4}{\alpha}$

2. In the shown system the block A moves towards right with velocity v = 2t m/s and block C moves towards right with constant velocity 4 m/s (time t is in seconds). Velocity of block 'B' at t = 4 sec, is

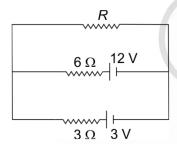


- (1) Zero
- (2) 2 m/s
- (3) 4 m/s
- (4) 3 m/s

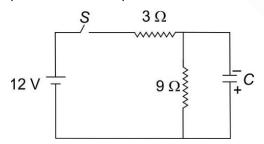
3. Two particles A and B are projected with velocities as indicated in vertical plane. Distance of the particles from point of projection of particle A where both particles collide, is  $[g = 10 \text{ m/s}^2]$ 



- (1) 9.00 m
- (2) 8.00 m
- (3) 10.75 m
- (4) 15.00 m
- 4. For the circuit shown in figure, value of resistance R is adjusted so that power delivered to resistor, R is maximum and is equal to  $P_0$ . Value of  $P_0$ , is

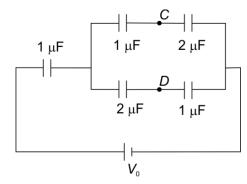


- (1) 6.0 W
- (2) 3.0 W
- (3) 9.0 W
- (4) 4.5 W
- 5. For the *RC* circuit as shown, capacitor '*C*' is charged to 4 V with polarity as shown. Switch '*S*' is closed at t = 0. Time,  $t_0$  at which potential across capacitor becomes 6 V is

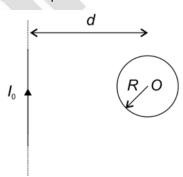


- (1) 4C In(2)
- (2)  $4C \ln \left(\frac{13}{6}\right)$
- $(3) 2C \ln(3)$
- (4)  $\frac{9C}{4} \ln \left( \frac{13}{3} \right)$

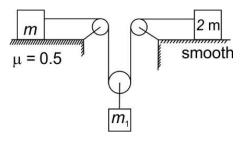
6. For the circuit shown, magnitude of potential difference across *C* and *D* is



- (1)  $\frac{4V_0}{7}$
- (2)  $\frac{2V_0}{7}$
- (3)  $\frac{3V_0}{7}$
- (4)  $\frac{V_0}{7}$
- 7. Current,  $I_0$  flows in long straight conductor as shown. If magnetic field at center of circular loop in the same plane is zero, then current in the circular loop is



- (1)  $2I_0$
- (2)  $I_0 \frac{d}{R}$
- $(3) \ \frac{\pi I_0 R}{2d}$
- $(4) \quad \frac{I_0 R}{\pi d}$
- 8. Value of mass  $m_1$  is chosen so that mass 'm' is just in equilibrium. Acceleration of mass ' $m_1$ ' is



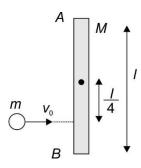
(1)  $\frac{g}{8}$ 

(2)  $\frac{g}{2}$ 

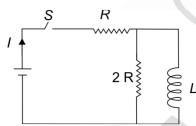
(3)  $\frac{g}{4}$ 

(4)  $\frac{3g}{4}$ 

9. A metre stick lies on a frictionless horizontal plane. A small body of mass 'm' moving with velocity,  $v_0$  collides elastically with the stick as shown. If mass 'm' comes to rest after collision, then velocity of end 'A' just after the collision is



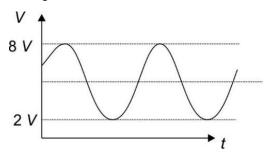
- (1)  $\frac{2v_0}{7}$
- (2)  $\frac{v_0}{7}$
- (3)  $\frac{3v_0}{7}$
- (4)  $\frac{4v_0}{7}$
- 10. For the *L-R* circuit as shown, ratio of currents, I at t=0 and  $t=\infty$  after closing the switch is



(1) 2

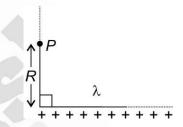
(2) 3

- (3)  $\frac{1}{2}$
- (4)  $\frac{1}{3}$
- A sinusoidally varying source voltage is given as a function of time as shown. RMS value of voltage is

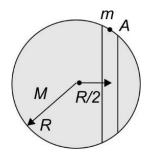


- (1)  $\sqrt{\frac{59}{2}}V$
- (2)  $\frac{7}{\sqrt{2}}$
- (3) 4V
- (4) 6V

- 12. Two particles A and B are performing SHM with amplitude,  $A_0$  and time period T about the same mean position. At t=0, A is at mean position and B is at distance  $\frac{A_0}{2}$  from mean position and is going towards mean position. At what time they will be at maximum separation? (At t=0, direction of velocities of A and B are same)
  - (1)  $\frac{T}{12}$
- $(2) \quad \frac{7}{8}$
- (3)  $\frac{T}{30}$
- (4)  $\frac{T}{24}$
- 13. Electric field at point 'P' due to long rod having uniform charge density,  $\lambda$  as shown is



- (1)  $\frac{\lambda}{4\pi\epsilon_0 R}$
- (2)  $\frac{\lambda}{2\sqrt{2}\pi\epsilon_0 R}$
- (3)  $\frac{\lambda}{2\pi\epsilon_0 R}$
- $(4) \quad \frac{\lambda}{\sqrt{2}\pi\varepsilon_0 R}$
- 14. A point mass 'm' is released from rest at point A along the smooth tunnel made at distance  $\frac{R}{2}$  from center of the earth as shown. Velocity of point mass once it reaches the center of tunnel is

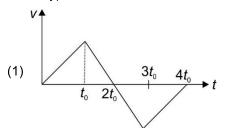


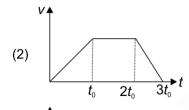
- (1)  $\sqrt{\frac{3GM}{2R}}$
- (2)  $\sqrt{\frac{GM}{4R}}$
- (3)  $\sqrt{\frac{3GM}{8R}}$
- $(4) \quad \sqrt{\frac{3GM}{4R}}$

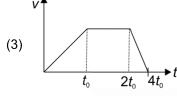
- 15. Molar heat capacity of the diatomic gas undergoing the process  $PV^3$  = constant is
  - (1) 2R
- (2) 3R

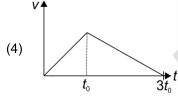
(3) R

- (4) 4R
- 16. A solid ball of density half that of water falls freely under gravity from a height of 29.4 m and then enters water. *v-t* graph of solid ball is shown by the curve, (neglect friction and viscosity).

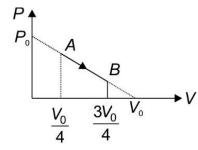






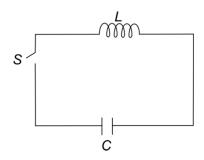


17. A diatomic gas undergoes the process  $A \rightarrow B$  as shown in P-V diagram. Volume of gas when it changes from endothermic to exothermic, is

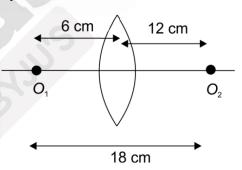


- (1)  $\frac{3V_0}{8}$
- (2)  $\frac{5V_0}{8}$
- (3)  $\frac{5V_0}{9}$
- $(4) \frac{7V_0}{12}$

18. Switch 'S' is closed at t = 0 and initial charge on capacitor is  $Q_0$ . At what time energy stored in capacitor is 3 times that of energy stored in inductor?



- $(1) \ \frac{\pi}{6}\sqrt{LC}$
- $(2) \quad \frac{\pi}{3}\sqrt{LC}$
- $(3) \ \frac{\pi}{2} \sqrt{LC}$
- $(4) \quad \frac{\pi}{4}\sqrt{LC}$
- 19. Two objects O<sub>1</sub> and O<sub>2</sub> are placed in front of a thin lens as shown. What should be the focal length of lens so that images of both the objects are formed on the same location?

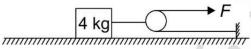


- (1) 7 cm
- (2) 9 cm
- (3) 8 cm
- (4) 10 cm
- 20. In an electromagnetic wave, if  $\bar{E}$  and  $\bar{B}$  represent electric and magnetic field respectively, then
  - (1)  $\bar{E}$  and  $\bar{B}$  oscillate in same phase
  - (2)  $\bar{E}$  and  $\bar{B}$  oscillate in opposite phase
  - (3)  $\bar{E}$  and  $\bar{B}$  oscillate perpendicular to each other and  $\bar{E}$  leads  $\bar{B}$  by phase of  $\frac{\pi}{2}$
  - (4)  $\bar{E}$  and  $\bar{B}$  oscillate parallel to each other

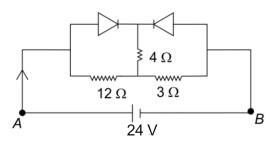
#### **SECTION - II**

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

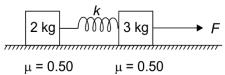
21. If coefficient of friction between all the surfaces is 0.50, then force, F (in N) required to move the block of mass 4 kg is  $[g = 10 \text{ m/s}^2]$ 



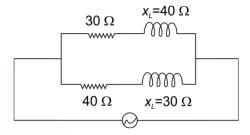
- 22. Fundamental frequency of an open organ pipe is  $f_0$ . If it is closed at one end, fundamental frequency becomes  $nf_0$ . Value of n is \_\_\_\_\_\_. (ignore end correction)
- 23. In a hydrogen atom, electron jumps from 4<sup>th</sup> excited state to 2<sup>nd</sup> excited state. Wavelength of photon emitted is  $\frac{9n}{16R}$ . Value of n is [R: Rydberg constant]
- 24. In a photoelectric experiment stopping potential changes from  $V_0$  to  $3V_0$  when frequency of incident radiation is changed from  $v_0$  to  $2v_0$ . If work function of metal is  $\frac{hv_0}{n}$  then find the value of n.
- 25. In the circuit with ideal diodes as shown, current (in A) through battery is



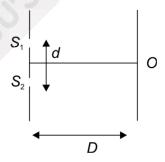
26. For the system shown in figure, initially spring is unstretched. Find minimum force F (in N) required to just move the block of mass 2 kg. [ $g = 10 \text{ m/s}^2$ ]



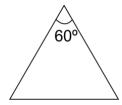
27. For the AC circuit as shown, power factor of circuit is  $\frac{1}{\sqrt{n}}$ . Find n.



28. In a standard YDSE two identical slits  $S_1$  and  $S_2$  are illuminated by light of wavelength of  $\lambda$ . If light wave through slit  $S_1$  leads  $S_2$  by phase of  $\frac{\pi}{3}$  at point O then nearest minima to O is at distance of  $\frac{\lambda D}{nd}$ . Find n.



- 29. If force (F), area (A) and time (T) are fundamental units, then dimension of mass, is  $[F^{x}A^{y}T^{z}]$ , then value of x + y + z, is
- 30. In an equilateral prism minimum deviation is obtained when angle of incidence is  $\theta = 45^{\circ}$ . Value of  $\frac{d\theta}{dn}$  is  $\frac{1}{\sqrt{x}}$ , where n is refractive index of glass prism. Find x.



### **PART - B: CHEMISTRY**

### SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

- 31. Which one of the following halides will not undergo oxidation with dimethyl sulfoxide?
  - (1) C<sub>6</sub>H<sub>5</sub> CH– CH<sub>3</sub>
  - (2)  $C_6H_5 C(CH_3)_2 Br$
  - (3) C<sub>6</sub>H<sub>5</sub> CH– C<sub>6</sub>H<sub>5</sub> I Br
  - (4)  $C_6H_5 CO CH_2Br$
- 32. The rate of a chemical reaction is found to be 0.92 mol L<sup>-1</sup> min<sup>-1</sup> at 15 min and 0.23 mol L<sup>-1</sup> min<sup>-1</sup> at 45 min from the start of the reaction. What is the order of reaction if half-life of the reaction is 15 min?
  - (1) Zero order
- (2) Second order
- (3) Third order
- (4) First order
- 33. Positronium is a species consisting of an electron bound to a positron. What would be the radius of its first excited state?
  - (1) 2.12 Å
- (2) 1.06 Å
- (3) 0.53 Å
- (4) 4.24 Å
- 34. Gallium dichloride exists as
  - (1) GaCl<sub>2</sub>
  - (2) Ga[GaCl<sub>4</sub>]
  - (3) Ga<sub>2</sub>[GaCl<sub>6</sub>]
  - (4) [GaCl<sub>2</sub>]<sub>2</sub>
- 35. Oxidising power of perhalate ions (into halate ion) follows the order:
  - (1)  $BrO_4^- > IO_4^- > CIO_4^-$
  - (2)  $CIO_4^- > BrO_4^- > IO_4^-$
  - (3)  $IO_4^- > BrO_4^- > CIO_4^-$
  - (4)  $IO_4^- > CIO_4^- > BrO_4^-$

36. 
$$CH_{2} CN CN C_{2}H_{5}ONa C_{2}H_{5}OH$$

$$(Z) \leftarrow \frac{H_{3}O^{+}}{\Delta}(Y) \leftarrow \frac{H_{2}O}{\Delta}(X)$$

A sequence of reaction is given above.

Which of the following statements is correct about the products obtained?

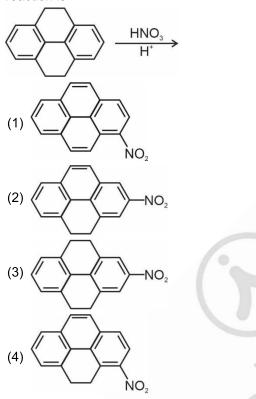
(1) (X) is 
$$CH_2 - CH = NH$$

$$CH_2 - CH = NH$$

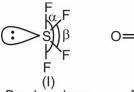
$$OH$$

- 37. The  $K_f$  and  $K_b$  values of a solvent are 5.0 K kg mol<sup>-1</sup> and 14.0 K kg mol<sup>-1</sup> respectively. The boiling point of pure solvent is 84.0°C and its freezing point is (–2.0°C). What is the boiling point of a solution of solute (X) if it freezes at (–2.75°C)?
  - (1) 87.9°C
- (2) 87.0°C
- (3) 86.1°C
- (4) 85.2°C
- 38. Silver acetate is a slightly soluble salt of weak acid (K<sub>a</sub> = 1.75 × 10<sup>-5</sup>). At 20°C, 100 g of water dissolves 1.04 g of crystalline silver acetate. The density of saturated solution of silver acetate at 20°C is 1.01 g/cc. The solubility product constant for silver acetate at 20°C is (Atomic mass of Ag is 108 u and neglect hydrolysis of acetate ion)
  - $(1) 3.87 \times 10^{-3}$
- (2)  $6.022 \times 10^{-3}$
- $(3) 2.84 \times 10^{-4}$
- (4) 5.35 × 10<sup>-5</sup>

- 39. The correct order of increasing X O Xbond angle is (X:H, F or CI)
  - (1)  $H_2O > Cl_2O > F_2O$
  - (2)  $Cl_2O > H_2O > F_2O$
  - (3)  $F_2O > Cl_2O > H_2O$
  - (4)  $F_2O > H_2O > Cl_2O$
- 40. The major product obtained in the following reaction is



41. For the following geometry which of the given inequality is correct?



(II) Bond angles :

Bond angles : Bond angles : 
$$F_{ax} - S - F_{ax} = \alpha \qquad F_{ax} - S - F_{ax} = \gamma$$
$$F_{eq} - S - F_{eq} = \beta \qquad F_{eq} - S - F_{eq} = \delta$$

$$F_{ax} - S - F_{ax} = \gamma$$
  
 $F_{ax} - S - F_{ax} = \delta$ 

- (1)  $\alpha = \gamma$  and  $\beta = \delta$
- (2)  $\alpha > \gamma$  and  $\beta < \delta$
- (3)  $\alpha > \gamma$  and  $\beta > \delta$
- (4)  $\alpha < \gamma$  and  $\beta > \delta$
- 42. The overall reaction for the electrolytic production of aluminium by means of the Hall-Heroult process may be represented as

$$Al_2O_3(s) + 3C(s) \longrightarrow 2Al(s) + 3CO(g)$$

At 1000°C, the standard free energy change for the process is -594 kJ mol-1. The minimum voltage required to produce one mole aluminium at this temperature is

- (1) 1.02 V
- (2) 2.05 V
- (3) 3.08 V
- (4) 4.03 V

43. 
$$(X) \leftarrow \frac{H_2O}{Boil} - H_4P_2O_7 \xrightarrow{Strong heating} (Y)$$

In the sequence of reactions given above, compounds (X) and (Y) are respectively

- (1) P<sub>2</sub>O<sub>5</sub>, H<sub>3</sub>PO<sub>4</sub>
- (2) H<sub>3</sub>PO<sub>4</sub>, H<sub>3</sub>PO<sub>2</sub>
- (3) P<sub>2</sub>O<sub>5</sub>, H<sub>3</sub>PO<sub>3</sub>
- (4) H<sub>3</sub>PO<sub>4</sub>, P<sub>2</sub>O<sub>5</sub>
- 44. For a second order reaction, the ratio of t<sub>3/4</sub> to t<sub>1/2</sub> is
  - (1) 2:1
- (2) 3:2
- (3) 3:1
- (4) 4:1
- 45. Treatment of D-fructose with concentrated HCI mainly gives product P. The degree of unsaturation of product P is \_\_\_
  - (1) 2

(2) 1

(3) 4

- (4) 3
- 46. The iodide content of a solution was determined by titration with Ce(SO<sub>4</sub>)<sub>2</sub> in presence of HCl in which I- is converted to ICI. A 250 mL sample of the solution required 20 mL of 0.05 N Ce4+ solution. What is the iodide concentration in the original solution in g/L? (Atomic mass of I is 127 u)
  - (1) 5.08 g/L
- (2) 0.854 g/L
- (3) 0.254 g/L
- (4) 3.26 g/L
- 47. Consider the following sequential reaction starting with excess of (A):

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

Where  $k_1 = 2 \times 10^{-3}$  mol  $L^{-1}s^{-1}$ ;  $k_2 = 3 \times 10^{-3}$  s<sup>-1</sup>. Calculate the maximum concentration of B obtained during the reaction.

- (1)  $2.0 \text{ mol } L^{-1}$
- (2)  $1.5 \text{ mol L}^{-1}$
- (3)  $0.5 \text{ mol } L^{-1}$
- (4)  $0.67 \text{ mol L}^{-1}$
- 48. Identify the product formed on heating (alone) the following allyl vinyl ether.

$$CH_2 = CH - CH_2 - O - CH = CH_2 \xrightarrow{\Delta}$$

- (1)  $CH_3 CH = CH \ddot{C} CH_3$
- (2)  $CH_2 = CH CH_2OH + HC \equiv CH$
- (3)  $CH_2 = CH CH_2 CH_2 C H$
- (4)  $CH_3 CH_2 CH = CH C -$

49. Identify the major product (X) formed in the following reaction.

$$O = CH = CH - C - CH_3 \xrightarrow{H^+} (X)$$

$$O = CH = CH - C - CH_3 \xrightarrow{H^+} (X)$$

$$O = CH_3 \xrightarrow{CH_3}$$

$$O = CH_3 \xrightarrow{CH_3$$

The major product (P) formed in the above reaction would be

### **SECTION - II**

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

- 51. A 2 g sample of impure anhydrous oxalic acid is added to 100 mL of 0.1 M acidified Ba(MnO<sub>4</sub>)<sub>2</sub>. Excess of Ba(MnO<sub>4</sub>)<sub>2</sub> was completely reduced by 60 mL of 1 M FeSO<sub>4</sub>. Calculate the percentage purity of the anhydrous oxalic acid sample.
- 52. An aromatic compound contains 69.4% carbon, 5.8% hydrogen. A sample of 0.303 g of this compound was analysed for nitrogen by Kjeldahl's method. The NH<sub>3</sub> evolved was absorbed in 50 mL of 0.05 M H<sub>2</sub>SO<sub>4</sub>. The excess of acid required 25 mL of 0.1 M NaOH for neutralisation. Find out the percentage of oxygen in the aromatic compound. (Report answer to the nearest integer)
- 53. The solubility of CaF<sub>2</sub> in water at 25°C is  $1.56 \times 10^{-3}$  g per 100 mL. The solubility product of CaF<sub>2</sub> at 25°C is x ×  $10^{-12}$ . The value of x is (Report answer to the nearest integer). Molar mass of CaF<sub>2</sub> is 78 g mol<sup>-1</sup>.
- 54. When excess of KCN is added to aqueous solution of  $CuSO_4$ , a co-ordination compound  $K_x[Cu(CN)_y]$  is formed. What is the value of (x + y)?
- 55. The total number of stereoisomers possible for 2,5-dibromohex-3-ene is \_\_\_\_.
- 56. The half-cell potentials of half cell A<sup>(x+n)+</sup>, A<sup>x+</sup>| Pt were found to be as follows % of reduced form 75 60
  Half cell potential (V) 0.1066 0.115
  Determine the value of 'n' (Round off to the nearest integer) assuming reduction takes place. Use log2 = 0.3.
- 57. 0.044 mol of solid NaOH is added to 250 mL of 0.1 M NiCl<sub>2</sub> solution. Calculate the approximate pH of the final solution, [K<sub>sp</sub> (Ni(OH)<sub>2</sub>) = 1.6 × 10<sup>-14</sup> M<sup>3</sup>] rounded off to the nearest integer. Given log3 = 0.48.

- 58. Consider the following compounds.
  - (A) Phenol
  - (B) Propyne
  - (C) Benzene sulphonic acid
  - (D) Benzoic acid
  - (E) Cyclopentadiene
  - (F) Ethane
  - (G) Picric acid
  - (H) 4-Nitro Cyclohexanol

If (X) is the total number of compounds which evolve  $H_2$  gas with Na metal and (Y) is the

- total number of compounds which evolve  $CO_2$  gas with NaHCO<sub>3</sub>. Then find the value of (X-Y).
- 59. If equivalent volume of  $N_2$  in a given reaction under STP conditions is x liter per equivalent  $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g), \text{ then the value of } \\ \frac{3x}{5.6} \text{ will be } \_\_\_\_ . \text{ (Report the answer to the nearest integer)}$
- 60. Find the number of angles less than 120° in  $PF_5$ .

### **PART - C: MATHEMATICS**

#### **SECTION - I**

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

- 61. If two real numbers a and b are randomly chosen from the interval (0, 1), then the probability that the equation  $x^2 \sqrt{a}x + b = 0$  has real roots is
  - $(1) \frac{1}{4}$
- (2)  $\frac{1}{8}$
- (3)  $\frac{5}{15}$
- (4)  $\frac{3}{16}$
- 62. Let  $S = \{1, 2, 3, ..., 99, 100\}$ . Then, the number of ordered triplet (x, y, z) such that x,  $y, z \in S$ , x < z and y < z is equal to
  - (1)  $3 \cdot {}^{100}C_2$
- (2)  $^{100}C_2 + ^{101}C_3$
- (3)  ${}^{101}C_3 + {}^{100}C_3$
- (4)  $2 \cdot {}^{101}C_3$
- 63. Sum of 36 terms of the series  $\frac{1}{9\sqrt{11}+11\sqrt{9}}$

$$+\frac{1}{11\sqrt{13}+13\sqrt{11}}+\frac{1}{13\sqrt{15}+15\sqrt{13}}+\dots$$
 is

equal to

- (1)  $\frac{2}{3}$
- $(2) \frac{1}{6}$
- (3)  $\frac{2}{9}$

 $(4) \frac{2}{3}$ 

- 64. The value of  $\frac{\sin\frac{4\pi}{9}}{\sin\frac{\pi}{9}} \frac{\sqrt{3}}{2\sin\frac{4\pi}{9}}$  is equal to
  - (1) 2

(2) 3

(3) 4

- (4) 1
- 65. Let  $a, b, c \in I^+$ , such that the roots of the three quadratic equations  $x^2 2ax + b = 0$ ,  $x^2 2bx + c = 0$ ,  $x^2 2cx + a = 0$  are all positive integers, then the maximum value of the product abc is equal to
  - (1) 1

(2) 2

(3) 4

- (4) 6
- 66. Let  $f: Q \{0, 1\} \to Q$  be a function such that  $x^2 f(x) + f\left(\frac{x-1}{x}\right) = 2x^2$  for all rational numbers  $x \neq 0$ , 1. Then the value of f(2) is
  - (1)  $\frac{3}{4}$
  - (2)  $\frac{7}{4}$
  - (3)  $\frac{11}{4}$
  - (4)  $\frac{13}{4}$

- 67. The locus of point of intersection of tangents of the parabola  $y^2 = 4(x + 1)$  and  $y^2 = 8(x + 2)$  which are perpendicular to each other is
  - (1) x + 7 = 0
  - (2) x y = 4
  - (3) x + y = 3
  - (4) x + 3 = 0
- 68. In a triangle ABC, coordinates of A are (1, 2) and the equations of medians through B and C are x + y = 5 and x = 4 respectively. The coordinates of C and B are respectively
  - (1) (3, 3), (7, 2)
  - (2) (-4, 3), (7, -2)
  - (3) (-4, -3), (7, 2)
  - (4) (4, 3), (7, -2)
- 69. The equation of circle having the pair of lines  $x^2 + 2xy + 3x + 6y = 0$  as its normal and having size just sufficient to contain the circle x(x-4) + y(y-3) = 0 is
  - (1)  $x^2 + y^2 + 6x 3y 35 = 0$
  - (2)  $x^2 + y^2 6x + 3y 35 = 0$
  - (3)  $x^2 + y^2 + 6x 3y 45 = 0$
  - (4)  $x^2 + y^2 6x + 3y 45 = 0$
- 70. From a point 'O' on circle  $x^2 + y^2 = r^2$ , tangents *OP* and *OQ* are drawn to ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then the locus of mid-point of chord *PQ* is

(1) 
$$r^2 \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} \right) = x^2 + y^2$$

(2) 
$$r^2 \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = x^2 + y^2$$

(3) 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2(x^2 + y^2)$$

(4) 
$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = r^2(x^2 + y^2)$$

- 71. Let  $f(x) = g(x) \frac{e^{1/x} e^{-1/x}}{e^{1/x} + e^{-1/x}}$  and  $x \ne 0$ , where g(x) is a continuous function. Then  $\lim_{x \to 0} f(x)$ 
  - exists if
  - (1) g(x) is any polynomial
  - (2) g(x) = x + 4
  - (3)  $g(x) = x^2$
  - (4)  $g(x) = 2 + 3x + 4x^2$
- 72. Let f(x), f'(x) and f''(x) are all positive for all  $x \in [0, 7]$ . If  $f^{-1}(x)$  exists, then  $2f^{-1}(1) + f^{-1}\left(\frac{3}{2}\right) +$

$$3f^{-1}\left(\frac{32}{5}\right)$$
 is

- (1) Always negative
- (2) Always positive
- (3) Non-positive
- (4) Nothing can be said
- 73. The value of  $\lim_{x\to 0^+} \left(\frac{(1+\{x\})^{\frac{1}{\{x\}}}}{e}\right)^{\frac{1}{\{x\}}}$ , where  $\{x\}$

represents fractional part of x, is equal to

- (1)  $\frac{2}{e}$
- (2) √e

- (3)  $\frac{e}{2}$
- $(4) \quad \frac{1}{\sqrt{e}}$
- 74. Let  $g(x) = \begin{cases} \frac{x^2 + x \tan x x \tan 2x}{ax + \tan x \tan 3x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

If g'(0) = b, then the value of  $\frac{b}{a}$  where

- $a \cdot b \neq 0$  is
- (1)  $\frac{7}{52}$
- (2)  $\frac{9}{52}$
- (3)  $\frac{7}{51}$
- (4)  $\frac{9}{51}$

- 75. The angle between the tangents at any point P and the line joining the origin O to the point P, of the curve  $\ln(x^2 + y^2) = 3\tan^{-1}\left(\frac{y}{x}\right)$  is  $\theta$ , then  $\tan\theta$  is equal to
  - (1) √3
- (2)  $\frac{1}{\sqrt{3}}$
- (3)  $\frac{1}{3}$
- (4)  $\frac{2}{3}$
- 76. If  $|z-1|+|z+3| \le 8$ , where z=x+iy, where  $x, y \in R$  and  $i=\sqrt{-1}$ , then range of the values of |z-4| is
  - (1) [0, 7]
- (2) [1, 8]
- (3) [2, 12]
- (4) [1, 9]
- 77. The value of  $\int \frac{e^x dx}{(\sin e^x + e^{-x} \cos e^x)^2}$  is equal

to

- (1)  $\tan (e^x \tan^{-1} e^x) + C$
- (2)  $\tan (e^{-x} \tan^{-1} e^{-x}) + C$
- (3)  $\tan (e^x + \tan^{-1} e^x) + C$
- (4)  $\tan (e^x \tan^{-1} e^{-x}) + C$
- 78. The value of  $\int_{1/e}^{\tan x} \frac{t}{1+t^2} dt + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$  is equal to  $\left(\text{where } x \in \left(0, \frac{\pi}{2}\right)\right)$ 
  - (1) Zero
  - (2) 1
  - (3) 2e
  - (4)  $\frac{2}{e}$
- 79. If the area of the region bounded by the curves  $y=\frac{1}{x}, y=\frac{1}{2x-1}, x=2$  and x=a; where a>2 equal to  $\ln\frac{4}{\sqrt{5}}$ , then one possible value of a is
  - (1) 5
  - (2) 6
  - (3) 7
  - (4) 8

- 80. Let f(x) be a continuous function which takes positive values for  $x \ge 0$  and satisfy  $\int_0^x f(t)dt = x\sqrt{f(x)} \quad \text{with} \quad f(1) = \frac{1}{2}. \text{ Then the }$  value of  $f(2\sqrt{2}+2)$  is equal to
  - $(1) \frac{1}{4}$
  - (2)  $\frac{1}{7}$
  - (3)  $\frac{1}{9}$
  - (4)  $\frac{1}{18}$

#### **SECTION - II**

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

- 81.  $\vec{b}$  and  $\vec{c}$  are non collinear vectors. If  $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a} \cdot \vec{b}) \vec{b} = (4 2x \sin y) \vec{b} + (x^2 1) \vec{c}$  and  $(\vec{c} \cdot \vec{c}) \vec{a} = \vec{c}$ , then the value of  $x^{\sin y} + (4 \sin y)^x$  is
- 82. The distance from the point (3, 4, 5) to the point where the line  $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$  meets the plane x + y + z = 17 is equal to
- 83. If the distance between line 3(x+3)= 2(y-5) = z+1 and a line passing through (1, -2, 0) and having direction ratios (2, 3, 6) is K units, then  $K^2$  is equal to
- 84. If z = x + iy, where  $x, y \in R$  and  $i = \sqrt{-1}$  satisfies the relation |2z + 5| = |6z 9|, then  $|z|^2 = a \operatorname{Re}(z) + b$  where  $a, b \in R$ . The value of (a 4b) is

- 85. Let a matrix  $a = [a_{ij}]_{n \times n}$  be such that  $a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases} \text{ then in inverse of } A, \text{ each}$  diagonal elements is equal to  $\frac{p-n}{n-q}$ . The value of  $(p+q)^3$  is
- 86. The coefficient of  $x^{18}$  in  $(1 x)^{15}$   $(1 + x + x^2 + ... + x^8)^{14}$  is equal to
- 87. The number of solutions of the equation  $\cos(\pi\sqrt{x-4})\cdot\cos(\pi\sqrt{x})=1 \text{ is equal to}$
- 88. In a  $\triangle ABC$ , the maximum value of  $\frac{\sum a \cos^2 \frac{\lambda}{2}}{a+b+c}$  is equal to  $\lambda$ , then 16 $\lambda$  is equal to

- 89. Let f(x) be a thrice differentiable function satisfying  $f(x + y) = f(x y) + y \{f(x + y) + f'(x y)\}$ , where f(0) = 0, f(0) = 1 and f(1) = 2, then f(5) is equal to
- 90. Let f(x) be cubic polynomial which has local maximum at x = -1 and f(x) has local minimum at x = 1. If f(-1) = 10 and f(3) = -22, then the distance between its two horizontal tangents is

