NCERT Solutions For Class 10 Maths Chapter 12- Areas Related to Circles

Exercise: 12.1

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has a circumference equal to the sum of the circumferences of the two circles.

Solution:

The radius of the 1st circle = 19 cm (given)
∴ Circumference of the 1st circle = $2\pi\times19 = 38\pi$ cm
The radius of the 2nd circle = 9 cm (given)
∴ Circumference of the 2nd circle = $2\pi\times9 = 18\pi$ cm
So,
The sum of the circumference of two circles = $38\pi+18\pi = 56\pi$ cm
Now, let the radius of the 3rd circle = R
∴ The circumference of the 3rd circle = $2\pi R$
It is given that sum of the circumference of two circles = circumference of the 3rd circle
Hence, $56\pi = 2\pi R$
Or, R = 28 cm.

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Solution:

Radius of 1st circle = 8 cm (given)
∴ Area of 1st circle = $\pi(8)^2 = 64\pi$
Radius of 2nd circle = 6 cm (given)
∴ Area of 2nd circle = $\pi(6)^2 = 36\pi$
So,
The sum of 1st and 2nd circle will be = $64\pi+36\pi = 100\pi$
Now, assume that the radius of 3rd circle = R
∴ Area of the circle 3rd circle = πR^2
It is given that the area of the circle 3rd circle = Area of 1st circle + Area of 2nd circle
Or, $\pi R^2 = 100\pi cm^2$
$\Rightarrow R^2 = 100cm^2$
So, R = 10cm

3. Fig. 12.3 depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold

score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Solution:

The radius of 1st circle, r_1 = 21/2 cm (as diameter D is given as 21 cm)

So, area of gold region = $\pi r_1^2 = \pi(10.5)^2 = 346.5$ cm^2

Now, it is given that each of the other bands is 10.5 cm wide,

So, the radius of 2nd circle, r_2 = 10.5cm+10.5cm = 21 cm

Thus,

∴ Area of red region = Area of 2nd circle − Area of gold region = $(\pi r_2^2 - 346.5)$ cm^2

= $(\pi(21)^2 - 346.5)$ cm^2

= 1386 − 346.5

= 1039.5 cm^2

Similarly,

The radius of 3rd circle, r_3 = 21 cm+10.5 cm = 31.5 cm

The radius of 4th circle, r_4 = 31.5 cm+10.5 cm = 42 cm

The Radius of 5th circle, r_5 = 42 cm+10.5 cm = 52.5 cm

For the area of nth region,

A = Area of circle n - Area of circle (n-1)

∴ Area of blue region (n=3) = Area of third circle - Area of second circle

= $\pi(31.5)^2$ - 1386 cm^2

= 3118.5 - 1386 cm^2

= 1732.5 cm^2

∴ Area of black region (n=4) = Area of fourth circle - Area of third circle

= $\pi(42)^2$ - 1386 cm^2

= 5544 - 3118.5 cm^2

= 2425.5 cm^2

∴ Area of white region (n=5) = Area of fifth circle - Area of fourth circle
= $\pi(52.5)^2$ - 5544 cm^2
= 8662.5 - 5544 cm^2
= 3118.5 cm^2

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Solution:
The radius of car's wheel = 80/2 = 40 cm (as D = 80 cm)
So, the circumference of wheels = $2\pi r$ = 80 π cm
Now, in one revolution, the distance covered = circumference of the wheel = 80 π cm
It is given that the distance covered by the car in 1 hr = 66km
Converting km into cm we get,
Distance covered by the car in 1hr = (66×10^5) cm
In 10 minutes, the distance covered will be = $(66\times10^5\times10)/60$ = 1100000 cm/s
∴ Distance covered by car = 11×10^5 cm

Now, the no. of revolutions of the wheels = (Distance covered by the car/Circumference of the wheels)
=$(11\times10^5)/80\pi$ = 4375.

5. Tick the correct Solution: in the following and justify your choice : If the perimeter and the area of a circle are numerically equal, then the radius of the circle is
(A) 2 units
(B) π units
(C) 4 units
(D) 7 units

Solution:
Since the perimeter of the circle = area of the circle,
$2\pi r = \pi r^2$
Or, r = 2
So, option (A) is correct i.e. the radius of the circle is 2 units.

NCERT Solutions For Class 10 Maths Chapter 12- Areas Related to Circles

Exercise: 12.2

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Solution:
It is given that the angle of the sector is 60°
We know that the area of sector = $(\theta/360°)\times\pi r^2$
∴ Area of the sector with angle 60° = $(60°/360°)\times\pi r^2$ cm^2
= $(36/6)\pi$ cm^2
= $6\times22/7$ cm^2 = $132/7$ cm^2

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution:
Circumference of the circle, C = 22 cm (given)
It should be noted that a quadrant of a circle is a sector which is making an angle of 90°.
Let the radius of the circle = r
As $C = 2\pi r = 22$,
$R = 22/2\pi$ cm = $7/2$ cm
∴ Area of the quadrant = $(\theta/360°)\times\pi r^2$
Here, $\theta = 90°$
So, $A = (90°/360°)\times\pi r^2$ cm^2
= $(49/16)\pi$ cm^2
= $77/8$ cm^2 = 9.6 cm^2

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:
Length of minute hand = radius of the clock (circle)
∴ Radius (r) of the circle = 14 cm (given)
Angle swept by minute hand in 60 minutes = 360°
So, the angle swept by the minute hand in 5 minutes = $360°\times5/60 = 30°$
We know,
Area of a sector = $(\theta/360°)\times\pi r^2$
Now, area of the sector making an angle of 30° = $(30°/360°)\times\pi r^2$ cm^2
= $(1/12)\times\pi 14^2$

= (49/3)×(22/7) cm^2

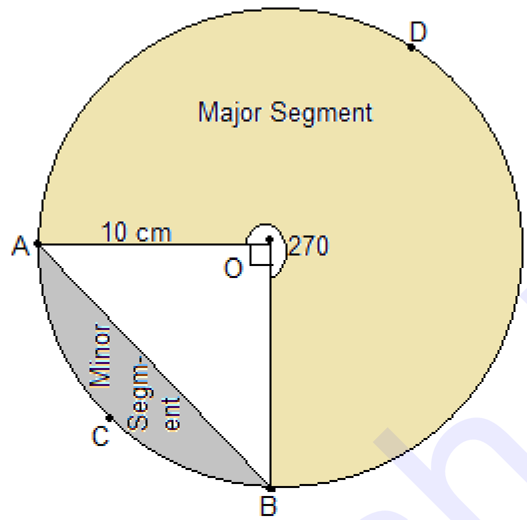
= 154/3 cm^2

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment

(ii) major sector. (Use π = 3.14)

Solution:



Here AB be the chord which is subtending an angle 90° at the center O.

It is given that the radius (r) of the circle = 10 cm

(i) Area of minor sector = (90/360°)×πr^2

= (¼)×(22/7)×10^2

Or, Area of minor sector = 78.5 cm^2

Also, area of ΔAOB = ½×OB×OA

Here, OB and OA are the radii of the circle i.e. = 10 cm

So, area of ΔAOB = ½×10×10

= 50 cm^2

Now, area of minor segment = area of minor sector - area of ΔAOB

= 78.5 - 50

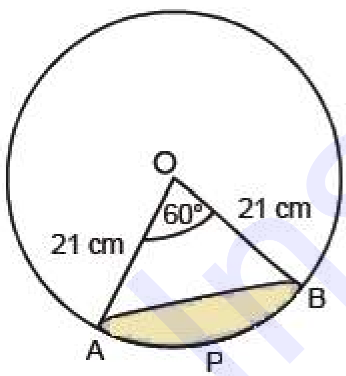
= 28.5 cm^2

(ii) Area of major sector = Area of circle - Area of minor sector

= $(3.14\times10^2)-78.5$
= $235.5\ cm^2$

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc
(ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord

Solution:



Given,
Radius = 21 cm
$\theta = 60°$
(i) Length of an arc = $\theta/360°\times$Circumference$(2\pi r)$
∴ Length of an arc AB = $(60°/360°)\times2\times(22/7)\times21$
= $(1/6)\times2\times(22/7)\times21$
Or Arc AB Length = 22cm

(ii) It is given that the angle subtend by the arc = 60°
So, area of the sector making an angle of 60° = $(60°/360°)\times\pi\ r^2\ cm^2$
= $441/6\times22/7\ cm^2$
Or, the area of the sector formed by the arc APB is $231\ cm^2$

(iii) Area of segment APB = Area of sector OAPB - Area of ΔOAB
Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔOAB is an equilateral triangle. So, its area will be $\sqrt{3}/4\times a^2$ sq. Units.

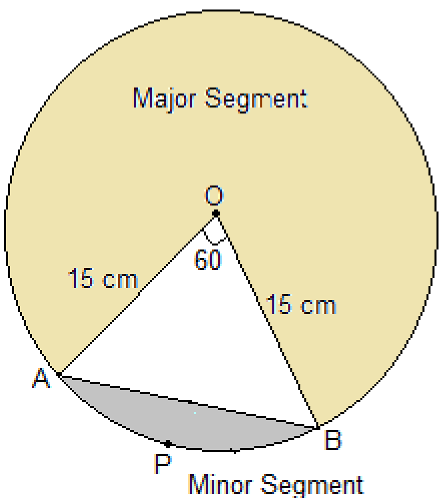
Area of segment APB = $231-(\sqrt{3}/4)\times(OA)^2$

= 231-(√3/4)×21^2

Or, Area of segment APB = [231-(441×√3)/4] cm^2

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:



Given,

Radius = 15 cm

θ = 60°

So,

Area of sector OAPB = (60°/360°)×πr^2 cm^2

= 225/6 πcm^2

Now, ΔAOB is equilateral as two sides are the radii of the circle and hence equal and one angle is 60°

So, Area of ΔAOB = (√3/4) ×a^2

Or, (√3/4) ×15^2

∴ Area of ΔAOB = 97.31 cm^2

Now, area of minor segment APB = Area of OAPB - Area of ΔAOB

Or, area of minor segment APB = ((225/6)π - 97.31) cm^2 = 20.43 cm^2

And,

Area of major segment = Area of circle - Area of segment APB

Or, area of major segment = $(\pi\times15^2) - 20.4 = 686.06\ cm^2$

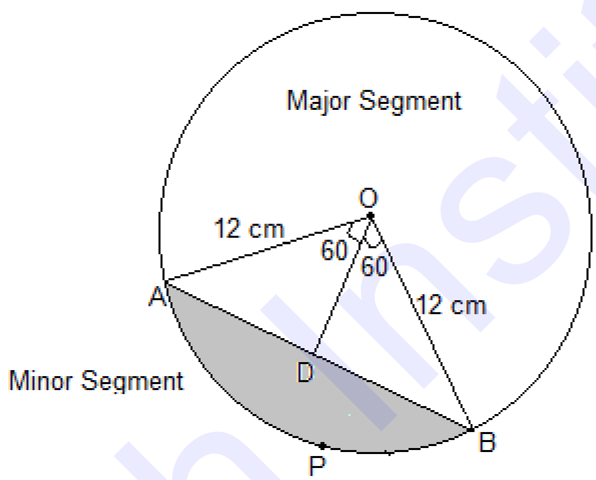
7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:

Radius, r = 12 cm

Now, draw a perpendicular OD on chord AB and it will bisect chord AB.

So, AD = DB



Now, the area of the minor sector = $(\theta/360°)\times\pi r^2$

= $(120/360)\times(22/7)\times12^2$

= $150.72\ cm^2$

Consider the ΔAOB,

∠ OAB = 180°-(90°+60°) = 30°

Now, cos 30° = AD/OA

⇒√3/2 = AD/12

Or, AD = 6√3 cm

We know OD bisects AB. So,

AB = 2×AD = 12√3 cm

Now, sin 30° = OD/OA

Or, ½ = OD/12

∴ OD = 6 cm

So, the area of ΔAOB = ½ × base × height

Here, base = AB = 12√3 and

Height = OD = 6

So, area of ΔAOB = $\frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3}$ cm = 62.28 cm^2

∴ Area of the corresponding Minor segment = Area of the Minor sector - Area of ΔAOB
= 150.72 cm^2- 62.28 cm^2 = 88.44 cm^2

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find
(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Fig. 12.11

Solution:
As the horse is tied at one end of a square field, it will graze only a quarter (i.e. sector with θ = 90°) of the field with radius 5 m.
Here, the length of rope will be the radius of the circle i.e. r = 5 m
It is also known that the side of square field = 15 m

(i) Area of circle = πr^2 = 22/7 × 5^2 = 78.5 m^2
Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle) = 78.5/4 = 19.625 m^2

(ii) If the rope is increased to 10 m,
Area of circle will be = πr^2 =22/7×10^2 = 314 m^2
Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle)
= 314/4 = 78.5 m^2
∴ Increase in the grazing area = 78.5 m^2 - 19.625 m^2 = 58.875 m^2

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 12.12. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

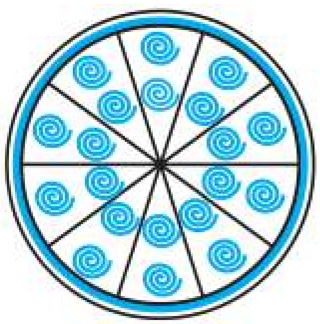


Fig. 12.12

Solution:

Diameter (D) = 35 mm

Total number of diameters to be considered= 5

Now, the total length of 5 diameters that would be required = 35×5 = 175

Circumference of the circle = $2\pi r$

Or, C = πD = 22/7×35 = 110

Area of the circle = πr^2

Or, A = $(22/7)\times(35/2)^2$ = 1925/2 mm^2

(i) Total length of silver wire required = Circumference of the circle + Length of 5 diameter

= 110+175 = 185 mm

(ii) Total Number of sectors in the brooch = 10

So, the area of each sector = total area of the circle/number of sectors

∴ Area of each sector = (1925/2)×1/10 = 385/4 mm^2

10. An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Fig. 12.13

Solution:

The radius (r) of the umbrella when flat = 45 cm

So, the area of the circle (A) = πr^2 = $(22/7)\times(45)^2$ =6364.29 cm^2

Total number of ribs (n) = 8

∴ The area between the two consecutive ribs of the umbrella = A/n

⇒ 6364.29/8 cm^2

Or, The area between the two consecutive ribs of the umbrella = 795.5 cm^2

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:

Given,

Radius (r) = 25 cm

Sector angle (θ) = 115°

Since there are 2 blades,

The total area of the sector made by wiper = $2\times(\theta/360°)\times\pi\, r^2$

$= 2\times(115/360)\times(22/7)\times25^2$

$= 2\times158125/252\ cm^2$

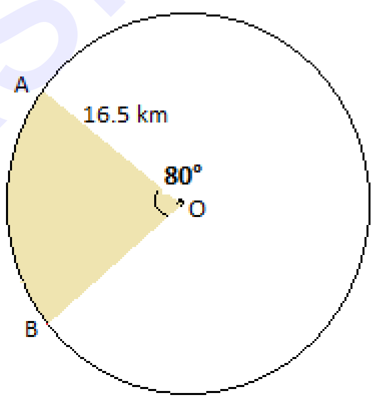
$= 158125/126 = 1254.96\ cm^2$

12. To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.

(Use π = 3.14)

Solution:

Let O bet the position of Lighthouse.



Here the radius will be the distance over which light spreads.

Given, radius (r) = 16.5 km

Sector angle (θ) = 80°

Now, the total area of the sea over which the ships are warned = Area made by the sector

Or, Area of sector = $(\theta/360°)\times\pi r^2$

= $(80°/360°)\times\pi r^2$ km^2

= 189.97 km^2

13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use √3 = 1.7)

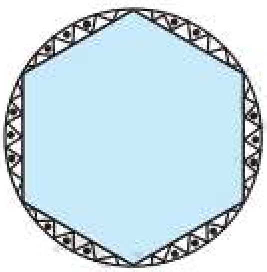


Fig. 12.14

Solution:

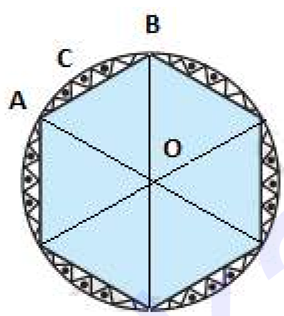


Fig. 12.14

Total number of equal designs = 6

∠AOB= 360°/6 = 60°

Radius of the cover = 28 cm

Cost of making design = ₹ 0.35 per cm^2

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔAOB is an equilateral triangle. So, its area will be $(\sqrt{3}/4)\times a^2$ sq. units

Here, a = OA

∴ Area of equilateral $\Delta AOB = (\sqrt{3}/4)\times 28^2 = 333.2\ cm^2$

Area of sector ACB = $(60°/360°)\times\pi r^2\ cm^2$

= $410.66\ cm^2$

So, area of a single design = area of sector ACB - area of ΔAOB

= $410.66\ cm^2 - 333.2\ cm^2 = 77.46\ cm^2$

∴ Area of 6 designs = $6\times 77.46\ cm^2 = 464.76\ cm^2$

So, total cost of making design = $464.76\ cm^2 \times$Rs.0.35 per cm^2

= Rs. 162.66

14. Tick the correct solution in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $p/180 \times 2\pi R$

(B) $p/180 \times \pi R^2$

(C) $p/360 \times 2\pi R$

(D) $p/720 \times 2\pi R^2$

Solution:

The area of a sector = $(\theta/360°)\times\pi r^2$

Given, $\theta = p$

So, area of sector = $p/360\times\pi R^2$

Multiplying and dividing by 2 simultaneously,

= $(p/360)\times 2/2\times\pi R^2$

= $(2p/720)\times 2\pi R^2$

So, option (D) is correct.

Exercise: 12.3

1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

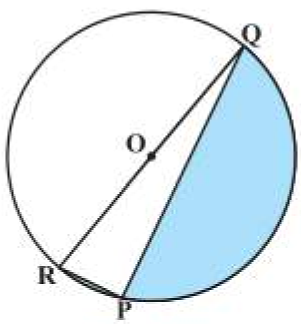


Fig. 12.19

Solution:

Here, ∠P is in the semi-circle

and so, ∠P = 90°

So, it can be concluded that QR is hypotenuse of the circle and is equal to the diameter of the circle.

∴ QR = D

Using Pythagorean theorem,

$QR^2 = PR^2+PQ^2$

Or, $QR^2 = 7^2+24^2$

⇒ QR= 25 cm = Diameter

Hence, the radius of the circle =

25/2 cm Now, the area of the

semicircle = $(\pi R^2)/2$ =

$(22/7)\times(25/2)\times(25/2)/2$ cm^2

= 13750/56 cm^2 = 245.54 cm^2

Also, area of the ΔPQR = ½×PR×PQ

=(½)×7×24 cm^2

= 84 cm^2

Hence, the area of the shaded region = 245.54 cm^2-84 cm^2

= 161.54 cm^2

2. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.

Solution:

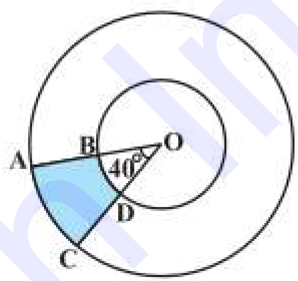


Fig. 12.20

Given,

Angle made by sector = 40°,

Radius the inner circle = r = 7 cm, and

Radius of the outer circle = R = 14 cm

We know,

Area of the sector = (θ/360°)×πr^2

So, Area of OAC = (40°/360°)×πr^2 cm^2

= 68.44 cm^2

Area of the sector OBD = (40°/360°)×πr^2 cm^2

= $(1/9)\times(22/7)\times 7^2 = 17.11\ cm^2$

Now, area of the shaded region ABDC = Area of OAC - Area of the OBD

= $68.44\ cm^2 - 17.11\ cm^2 = 51.33\ cm^2$

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

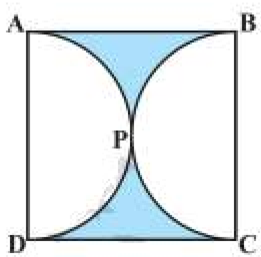


Fig. 12.21

Solution:

Side of the square ABCD (as given) = 14 cm

So, Area of ABCD = a^2

= $14\times 14\ cm^2 = 196\ cm^2$

We know that the side of the square = diameter of the circle = 14 cm

So, side of the square = diameter of the semicircle = 14 cm

∴ Radius of the semicircle = 7 cm

Now, area of the semicircle = $(\pi R^2)/2$

= $(22/7\times 7\times 7)/2\ cm^2$ =

= $77\ cm^2$

∴ Area of two semicircles = $2\times 77\ cm^2 = 154\ cm^2$

Hence, area of the shaded region = Area of the Square - Area of two semicircles

= $196\ cm^2 - 154\ cm^2$

$= 42\ cm^2$

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

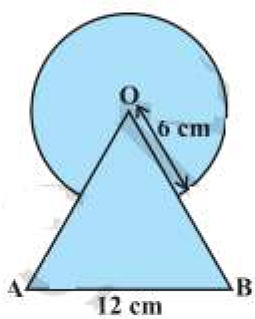


Fig. 12.22

Solution:

It is given that OAB is an equilateral triangle having each angle as 60°

Area of the sector is common in both.

Radius of the circle = 6 cm.

Side of the triangle = 12 cm.

Area of the equilateral triangle = $(\sqrt{3}/4)\ (OA)^2 = (\sqrt{3}/40 \times 12^2 = 36\sqrt{3}\ cm^2$

Area of the circle = $\pi R^2 = (22/7) \times 6^2 = 792/7\ cm^2$

Area of the sector making angle 60° = $(60°/360°) \times \pi r^2\ cm^2$

$= (1/6) \times (22/7) \times 6^2\ cm^2 = 132/7\ cm^2$

Area of the shaded region = Area of the equilateral triangle + Area of the circle - Area of the sector

$= 36\sqrt{3}\ cm^2 + 792/7\ cm^2 - 132/7\ cm^2$

$= (36\sqrt{3} + 660/7)\ cm^2$

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

Solution:

Side of the square = 4 cm

Radius of the circle = 1 cm

Four quadrant of a circle are cut from corner and one circle of radius are cut from middle.

Area of square = $(\text{side})^2 = 4^2 = 16\ cm^2$

Area of the quadrant = $(\pi R^2)/4\ cm^2 = (22/7)\times(1^2)/4 = 11/14\ cm^2$

∴ Total area of the 4 quadrants = $4 \times (11/14)\ cm^2 = 22/7\ cm^2$

Area of the circle = $\pi R^2\ cm^2 = (22/7\times1^2) = 22/7\ cm^2$

Area of the shaded region = Area of square - (Area of the 4 quadrants + Area of the circle)

$= 16\ cm^2 - (22/7)\ cm^2 + (22/7)\ cm^2$

$= 68/7\ cm^2$

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design.

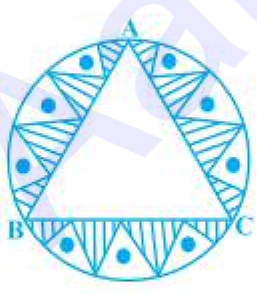


Fig. 12.24

Solution:

Radius of the circle = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.

⇒ BD = AB/2

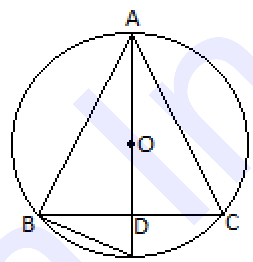
Since, AD is the median of the triangle

∴ AO = Radius of the circle = (2/3) AD

⇒ (2/3)AD = 32 cm

⇒ AD = 48 cm

In ΔADB,



By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

⇒ $AB^2 = 48^2 + (AB/2)^2$

⇒ $AB^2 = 2304 + AB^2/4$

⇒ $3/4\ (AB^2) = 2304$

⇒ $AB^2 = 3072$

⇒ AB= 32√3 cm

Area of ΔADB = $\sqrt{3}/4 \times (32\sqrt{3})^2\ cm^2 = 768\sqrt{3}\ cm^2$

Area of circle = πR^2 = (22/7)×32×32 = 22528/7 cm^2

Area of the design = Area of circle - Area of ΔADB

$= (22528/7 - 768\sqrt{3})\ cm^2$

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

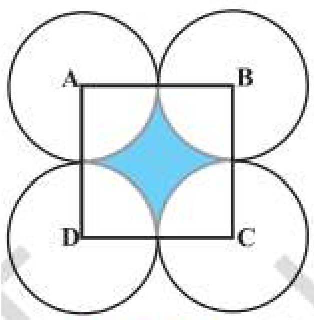


Fig. 12.25

Solution:

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

∴ Radius of the circles = 14/2 cm = 7 cm

Area of the square ABCD = $14^2 = 196\ cm^2$

Area of the quadrant = $(\pi R^2)/4\ cm^2 = (22/7) \times 7^2/4\ cm^2$

$= 77/2\ cm^2$

Total area of the quadrant = $4 \times 77/2\ cm^2 = 154 cm^2$

Area of the shaded region = Area of the square ABCD - Area of the quadrant

$= 196\ cm^2 - 154\ cm^2$

$= 42\ cm^2$

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inner edge

(ii) the area of the track.

Fig. 12.26

Solution:

Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m

DE = CF = 60 m

Radius of inner semicircle, r = OD = O'C

= 60/2 m = 30 m

Radius of outer semicircle, R = OA = O'B

= 30+10 m = 40 m

Also, AB = CD = EF = GH = 106 m

Distance around the track along its inner edge = CD+EF+2×(Circumference of inner semicircle)

= 106+106+(2×πr) m = 212+(2×22/7×30) m

= 212+1320/7 m = 2804/7 m

Area of the track = Area of ABCD + Area EFGH + 2 × (area of outer semicircle) - 2 × (area of inner semicircle)

= $(AB \times CD)+(EF \times GH)+2\times(\pi r^2/2) - 2\times(\pi R^2/2)\ m^2$

= $(106\times10)+(106\times10)+2\times\pi/2(r^2-R^2)\ m^2$

= $2120+22/7\times70\times10\ m^2$

= $4320\ m^2$

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

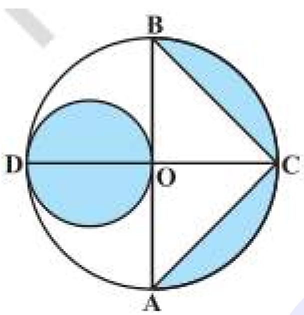


Fig. 12.27

Solution:

Radius of larger circle, R = 7 cm

Radius of smaller circle, r = 7/2 cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of ΔBCA = 1/2 × AB × OC = (½)×7×14 = 49 cm^2

Area of larger circle = $\pi R^2 = (22/7)\times7^2 = 154\ cm^2$

Area of larger semicircle = $154/2\ cm^2 = 77\ cm^2$

Area of smaller circle = πr^2 = (22/7)×(7/2)×(7/2) = 77/2 cm^2

Area of the shaded region = Area of larger circle - Area of triangle - Area of larger semicircle + Area of smaller circle

Area of the shaded region = (154-49-77+77/2) cm^2

= 133/2 cm^2 = 66.5 cm^2

10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use π = 3.14 and √3 = 1.73205)

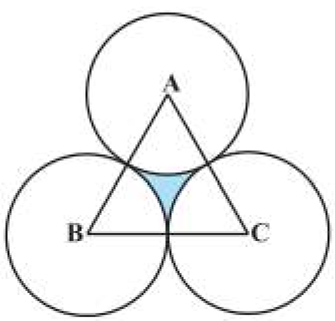


Fig. 12.28

Solution:

ABC is an equilateral triangle.

∴ ∠ A = ∠ B = ∠ C = 60°

There are three sectors each making 60°.

Area of ΔABC = 17320.5 cm^2

⇒ √3/4 ×$(side)^2$ = 17320.5

⇒ $(side)^2$ =17320.5×4/1.73205

⇒ $(side)^2$ = 4×10^4

⇒ side = 200 cm

Radius of the circles = 200/2 cm = 100 cm

Area of the sector = $(60°/360°)\times\pi r^2$ cm^2

= $1/6\times3.14\times(100)^2$ cm^2

= $15700/3 cm^2$

Area of 3 sectors = $3\times15700/3 = 15700$ cm^2

Thus, area of the shaded region = Area of equilateral triangle ABC - Area of 3 sectors

= $17320.5-15700$ cm^2 = 1620.5 cm^2

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

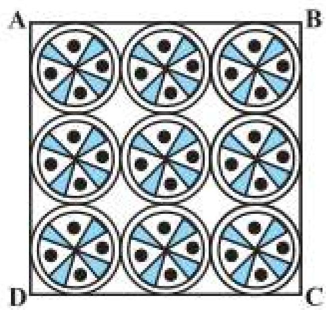


Fig. 12.29

Solution:

Number of circular designs = 9

Radius of the circular design = 7 cm

There are three circles in one side of square handkerchief.

∴ Side of the square = 3×diameter of circle = 3×14 = 42 cm

Area of the square = 42×42 cm^2 = 1764 cm^2

Area of the circle = $\pi r^2 = (22/7)\times7\times7 = 154$ cm^2

Total area of the design = $9\times154 = 1386$ cm^2

Area of the remaining portion of the handkerchief = Area of the square - Total area of the design = 1764 - 1386 = 378 cm^2

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB,

(ii) shaded region.

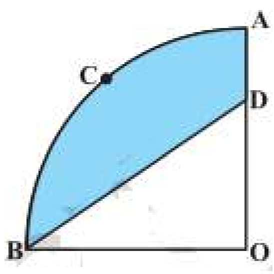


Fig. 12.30

Solution:

Radius of the quadrant = 3.5 cm = 7/2 cm

(i) Area of quadrant OACB = $(\pi R^2)/4$ cm^2

= $(22/7)\times(7/2)\times(7/2)/4$ cm^2

= 77/8 cm^2

(ii) Area of triangle BOD = $(½)\times(7/2)\times2$ cm^2

= 7/2 cm^2

Area of shaded region = Area of quadrant - Area of triangle BOD

= (77/8)-(7/2) cm^2 = 49/8 cm^2

= 6.125 cm^2

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

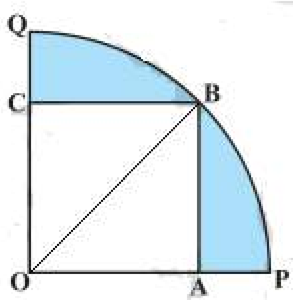


Fig. 12.31

Solution:

Side of square = OA = AB = 20 cm

Radius of the quadrant = OB

OAB is right angled triangle

By Pythagoras theorem in ΔOAB,

$OB^2 = AB^2+OA^2$

⇒ $OB^2 = 20^2 +20^2$

⇒ $OB^2 = 400+400$

⇒ $OB^2 = 800$

⇒ OB= $20\sqrt{2}$ cm

Area of the quadrant = $(\pi R^2)/4$ cm^2 = $(3.14/4)\times(20\sqrt{2})^2$ cm^2 = 628cm^2

Area of the square = 20×20 = 400 cm^2

Area of the shaded region = Area of the quadrant - Area of the square

= 628-400 cm^2 = 228cm^2

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠AOB = 30°, find the area of the shaded region.

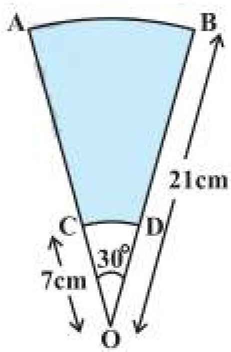


Fig. 12.32

Solution:

Radius of the larger circle, R = 21 cm

Radius of the smaller circle, r = 7 cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = $(30°/360°)\times\pi R^2$ cm^2

$= (1/12)\times(22/7)\times 21^2$ cm^2

$= 231/2 cm^2$

Area of the smaller circle = $(30°/360°)\times\pi r^2$ cm^2

$= 1/12\times 22/7\times 7^2$ cm^2

$=77/6$ cm^2

Area of the shaded region = $(231/2) - (77/6)$ cm^2

$= 616/6$ $cm^2 = 308/3 cm^2$

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

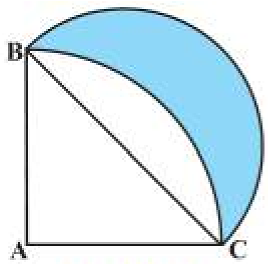


Fig. 12.33

Solution:

Radius of the quadrant ABC of circle = 14 cm

AB = AC = 14 cm

BC is diameter of semicircle.

ABC is right angled triangle.

By Pythagoras theorem in ΔABC,

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 14^2 + 14^2$

$\Rightarrow BC = 14\sqrt{2}$ cm

Radius of semicircle = $14\sqrt{2}/2$ cm = $7\sqrt{2}$ cm

Area of ΔABC =(½)×14×14 = 98 cm^2

Area of quadrant = (¼)×(22/7)×(14×14) = 154 cm^2

Area of the semicircle = (½)×(22/7)×$7\sqrt{2}\times7\sqrt{2}$ = 154 cm^2

Area of the shaded region =Area of the semicircle + Area of ΔABC - Area of quadrant

= 154 +98-154 cm^2 = 98cm^2

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

Solution:

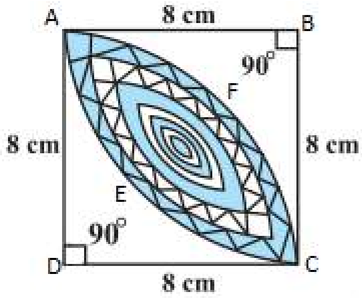


Fig. 12.34

AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC = (½)×8×8 = 32 cm^2

Area of quadrant AECB = Area of quadrant AFCD = (¼)×22/7×8^2

= 352/7 cm^2

Area of shaded region = (Area of quadrant AECB - Area of ΔABC) = (Area of quadrant AFCD - Area of ΔADC)

= (352/7 -32)+(352/7- 32) cm^2

= 2×(352/7-32) cm^2

= 256/7 cm^2