Test Booklet Code

43

Date: 25/11/2023

Corporate Office: **Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456**

Answers & Solutions

Time : 120 Minute

Max. Marks : 216

for

National Standard Examination in Astronomy (NSEA) 2023

INSTRUCTIONS TO CANDIDATES

(1) There are 60 questions in this paper.

(2) Question paper has two parts. In **Part A1** (Q. No. 1 to 48) each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

Q. No. 22 (a) (●) (c) (d)

In **Part A2** (Q. No. 49 to 60) each question has four alternatives, out of which **any number of alternative (s)** (1, 2, 3 or 4) may be correct. You have to choose ALL correct alternative(s) and fill the appropriate bubble(s), as shown.

Q. No. 54 (a) (●) (c) (●)

(3) For **Part A1**, each correct answer carries **3 marks** whereas 1 mark will be deducted for each wrong answer. In **Part A2**, you get **6 marks** if all the correct alternatives are marked. No negative marks in this part.

[Attempt All Sixty Questions]

A - 1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBBLE THE CORRECT OPTION

1. Standing waves have been produced in a 51 cm long, open end organ pipe with just one node within the pipe. The wavelength of the sound wave, which forms a standing wave in the same organ pipe such that there are three nodes within the pipe, is

   (a) 68 cm (b) 51 cm

   (c) 34 cm (d) 20.4 cm

Answer (c)

Sol. For the first information,

$$L = \frac{\lambda}{2} \quad \text{...(i)}$$

For the second information,

$$L = \frac{3\lambda'}{2} \quad \text{...(ii)}$$

$$\Rightarrow \quad \lambda' = \frac{2}{3} \times L = 34 \text{ cm}$$

2. For the air-glass interface if the Brewster angle (polarizing angle) is ϕ_P, then the critical angle (i_c) for the glass-air interface is expressed as

   (a) $\sin^{-1}(\tan \phi_P)$ (b) $\cos^{-1}(\tan \phi_P)$

   (c) $\sin^{-1}(\cot \phi_P)$ (d) $\cos^{-1}(\cot \phi_P)$

Answer (c)

Sol. For Brewster angle

$\tan\phi_P = \mu$

$\Rightarrow$ Critical angle $i_c = \sin^{-1}\left[\frac{1}{\mu}\right]$

$= \sin^{-1}[\cot \phi_P]$

3. A gas in equilibrium attains a state of equipartition, wherein every active degree of freedom of every particle has on the average the same energy. In particular, the average translational kinetic energy will be the same for every particle; it will be equal to $\frac{3}{2}kT$ where, k is the Boltzmann constant and T is the temperature of the gas in kelvin. Consider air at 300 kelvin. If the root mean square velocity of nitrogen molecules is v_{rms} m/s, the magnitude of the mean velocity and the root mean square velocity of oxygen molecules are respectively

   (a) $\left(0, \frac{v_{rms}}{2\sqrt{3}}\right)$ (b) $(0, 0.94v_{rms})$

   (c) $\left(\frac{3}{2}v_{rms}, 1.07v_{rms}\right)$ (d) $(0, 0.875v_{rms})$

Answer (b)

Sol. N_2 : $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{28}}$

O_2 : $v'_{rms} = \sqrt{\frac{3RT}{M'}} = \sqrt{\frac{3RT}{32}} = \sqrt{\frac{28}{32}}v_{rms} = 0.94v_{rms}$

Mean velocity = 0 [Due to random motion]

4. Formation of the iron peak elements ${}^{56}_{26}Fe$, ${}^{56}_{27}Co$ and ${}^{56}_{28}Ni$ by nuclear fusion marks the end of energy production in a star. If the mass of the iron core is greater than the Chandrasekhar mass (1.4 solar masses) the core cannot be supported by the degeneracy pressure of the electrons and will collapse to form a neutron star. Matter is crushed to nuclear densities with protons combining with electrons as $p^+ + e^- \rightarrow n + v_e$. Neutron stars whose masses are not too high are supported by neutron degeneracy pressure against gravitational collapse. The number of neutrinos released, when a neutron star of 1.4 solar masses forms, is approximately

(a) 7.77×10^{56} (b) 1.74×10^{57}

(c) 8.68×10^{56} (d) 8.38×10^{57}

Answer (c)

Sol. Number of proton or neutron $= \frac{2\times10^{30}}{1.67\times10^{-27}}$

Neutrino = Number of proton

Averagely 27 p out of 56 will be protons and hence equal number of neutrino

$\nu = \frac{27}{56}\times\frac{2\times10^{30}\times1.4}{1.67\times10^{-27}}$

$\approx 0.61 \times 10^{57} \times 1.4$

$\approx 6.1 \times 10^{56} \times 1.4$

$\approx 8 \times 10^{56}$

5. The Seebeck coefficient of a material characterizes the voltage built up when a small temperature gradient is set up across it. It is defined by the relation $S = -\frac{V_{left} - V_{right}}{T_{left} - T_{right}}$. Only the relative Seebeck coefficients of materials may be determined since, connecting a voltmeter introduces other voltages. The relative Seebeck coefficients of some elemental materials in μV/K are: (A) Aluminium : 3.5 (B) Carbon : 3 (C) Sodium : –2 (D) Platinum : 0. The absolute Seebeck coefficient of Platinum is –5 μV at room temperature. The combination of two metals which will give maximum voltage difference when used to make a thermocouple is

(a) (A, D) (b) (C, D)

(c) (A, C) (d) (B, C)

Answer (c)

Sol. Seebeck coefficient for couples

For Aluminium – Platinum = 3.5

For Sodium – Platinum = 2

For Aluminium – Sodium = 3.5 – (–2) = 5.5

For Carbon – Sodium = 3 – (–2) = 5

∴ Maximum voltage will be given by Aluminium – Sodium.

6. The mean power received per unit area just outside Earth's atmosphere from the Sun known as the solar constant, is 1.362 kilowatt per square meter (kW/m^2). For receiving this much power per unit area at Earth, the Sun, if it is radiating like a black body, must have a surface temperature of

(a) 5349 K (b) 5779 K

(c) 5709 K (d) 5479 K

Answer (b)

Sol. Solar constant = $\frac{P}{4\pi R^2}$

R : Separation between Earth & Sun

$$\Rightarrow \frac{\sigma A T^4}{4\pi R^2} = 1.362 \times 1000$$

A : Area of Sun

$$\Rightarrow \frac{5.67\times10^{-8}\times4\pi\times(6.96\times10^8)^2 T^4}{4\times(1.5\times10^{11})^2\times\pi} = 1362$$

$$\Rightarrow T \simeq 5779 \text{ K}$$

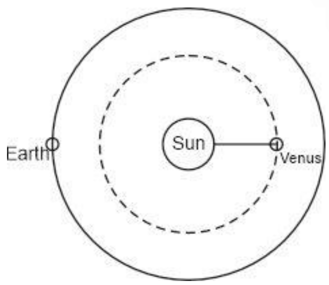
7. Venus has an equatorial radius of 6052 km. The semi-major axis of its orbit around the Sun is 0.72 AU. The smallest diameter of the primary mirror/lens (objective) of an optical telescope to be used to just resolve the Venus disc during the transit of Venus is approximately

(a) 200 cm (b) 20 cm

(c) 10 cm (d) 0.25 cm

Answer (c)

Sol.



$\therefore$ D = 1 AU + 0.72 AU = 1.72 AU

$\therefore$ $\frac{r}{D} = \frac{1.22\lambda}{a}$, a = Diameter of lens

$$\Rightarrow a = \frac{1.22\lambda D}{r}$$

$$= \frac{1.22\times500\times10^{-9}\times1.72\times1.5\times10^{11}}{6052\times10^3}$$

$$\simeq 3 \text{ cm}$$

So, closest answer is 10.

8. The Earth's atmosphere is transparent across the most of the optical region and ________ region of the electromagnetic spectrum. Choose the correct option to fill the blank.

(a) Ultraviolet
(b) X-ray
(c) Some portions of the radio wave
(d) Gamma-ray

Answer (c)

Sol. Earth's atmosphere is transparent to visible & some part of radio wave region.

9. It is given that the pupil of the human eye is about 5 mm (in diameter). How many times more light-gathering power does a telescope with a primary mirror of diameter about 20 cm (8 inches) have than the human eye?

(a) 4 times
(b) 16 times
(c) 40 times
(d) 1600 times

Answer (d)

Sol. Light gathering power $\propto$ Area

$$\Rightarrow \quad \text{Ratio} = \left[\frac{20 \text{ cm}}{5 \text{ mm}}\right]^2 = 1600$$

10. An astronomer observes a spectral emission line from a galaxy at a wavelength of 600 nm. This same spectral line is measured in the laboratory using a stationary source and is seen to have wavelength 500 nm. The speed of the galaxy toward or away from Earth (in units of the speed of light c) is equal to

(a) 0.2 c moving away from Earth
(b) 0.2 c moving toward Earth
(c) 0.9 c moving away from Earth
(d) 1.0 c moving toward Earth

Answer (a)

Sol. $\Delta\lambda$ = +100 nm

$\therefore$ Galaxy is moving away from earth

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

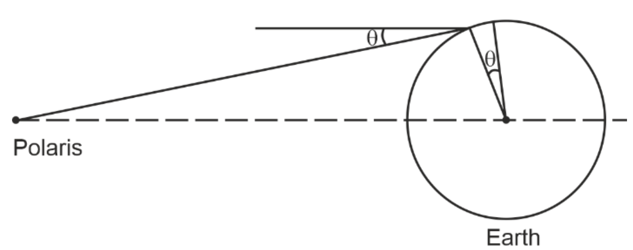
$$\therefore \quad v = c \times \frac{100}{500} = 0.2 \text{ c}$$

11. An astronomer observes that Polaris is 40 degrees above her northern horizon. What can you say about the longitude of her location from her observations?

(a) Nothing
(b) That it is 40 degrees north
(c) That it is 50 degrees south
(d) That it is 40 degrees east

Answer (a)

Sol.



θ = 40° above the horizontal

$\Rightarrow$ Latitude would be 40° north

But longitude cannot be determined.

So, correct option is (a).

12. The spectrum of a cloud of cool gas seen against a bright background black body would show
    (a) Bright lines (emission spectrum) against a continuum spectral emission background
    (b) A continuous spectrum
    (c) Dark lines (absorption lines) against a continuum emission background
    (d) Either bright or dark lines, depending on distance, against a continuum emission background

Answer (c)

Sol. A bright black body emits all wavelengths.

And, cloud of cool gas absorbs some wavelengths.

Therefore, the spectrum will have dark lines against continuous emission background.

$\therefore$ Correct option is (c).

13. If a star has a parallax of one-eighth of a second of arc, its distance from the earth is
    (a) 2,06,265 astronomical units
    (b) Eight light years
    (c) One-eighth parsec
    (d) Eight parsec

Answer (d)

Sol. Here, $d = \frac{1}{p}$, where, d is distance measured in parsec and p is measured in arc-seconds.

$$\therefore \quad d = \frac{1}{(1/8)} = 8 \text{ parsec}$$

14. The elemental composition of the Sun, by mass, is about
    (a) 50% metals, 50% hydrogen
    (b) 71% hydrogen, 27% helium, 2% others
    (c) 75% helium, 20% hydrogen, 5% others
    (d) 75% carbon, 25% helium

Answer (b)

Sol. Elemental composition of sun by mass is

Hydrogen 70%

Helium 27%

Rest is traces of other elements.

15. Which of the following planets has essentially no atmosphere?
    (a) Venus
    (b) Mercury
    (c) Jupiter
    (d) Mars

Answer (b)

Sol. Mercury is the only planet which has no atmosphere.

16. Sunspots are

(a) Relatively cool compared to the photosphere
(b) Related to convection cells
(c) Related to the Sun's electric field
(d) Cyclonic storms similar to Jupiter's great red spot

Answer (a)

Sol. Sunspots are cooler than the surface of sun.

17. A planet moves fastest in its orbit

(a) When it is in opposition
(b) When it is closest to the Sun
(c) The greater its mass
(d) When it is farthest from the Sun

Answer (b)

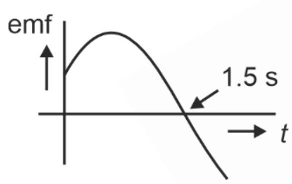
Sol. Angular momentum of planet remains constant.

$\therefore \quad L = mvr = \text{Constant}$

$\Rightarrow \quad vr = \text{Constant}$

$\Rightarrow \quad v$ is maximum when r is minimum.

18. A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the east-west (x-z) plane. A uniform magnetic field of strength B = 0.1 T is set up across the plane of the coil in the north-south direction. The coil is rotating about a vertical axis $(\vec{\omega} \,||\, \hat{k})$ through its center at the rate of one rotation per 4 second. At time t = 0, the plane of the coil makes an angle θ = +45° with the east-west (x-z) plane. The correct schematic diagram giving the emf induced in the coil is

(a)


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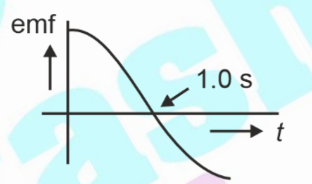
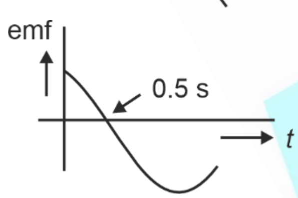
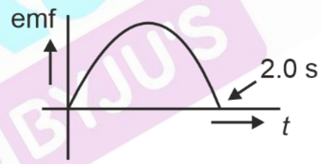
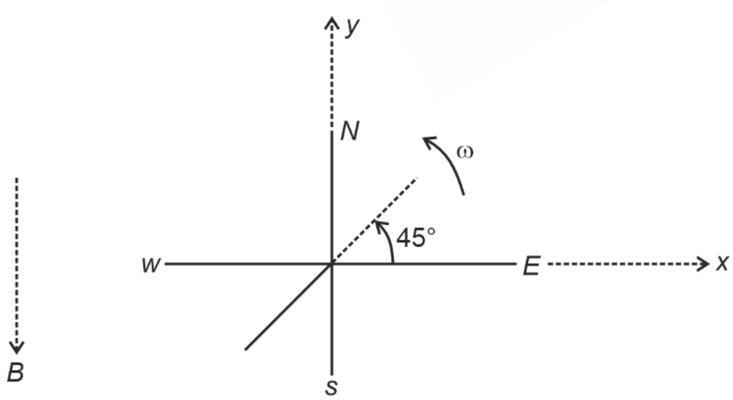


(c)


(d)


Answer (c)

Sol.



$$\Rightarrow \quad \phi = B \times a^2 \times \cos\left[\omega t + \frac{3\pi}{4}\right]$$

$$\Rightarrow \quad \varepsilon = \frac{-d\phi}{dt} = Ba^2\omega \sin\left[\omega t + \frac{3\pi}{4}\right] = Ba^2\omega \sin\left[\frac{\pi t}{2} + \frac{3\pi}{4}\right]$$

19. In 1912 Henrietta Leavitt making observations of Cepheid variable stars in the Magellanic clouds discovered the *period luminosity relation* - fainter stars have shorter periods. The relation between the apparent magnitude m (the measured magnitude of a star situated at its actual distance D pc) and its absolute magnitude M (its magnitude if located at a distance of 10 pc) is given by the magnitude distance relation $m - M = 5 \log D - 5$. Here D is the distance in parsec. The distance from Milky Way to the Andromeda galaxy is 765 kpc. The observed period luminosity relation determined by observations in the K band for Cepheid variables in the Andromeda galaxy is $m_K = -3.26 (\log P - 1)$ 18.73 with slope –3.26 and zero-point 18.73. For Cepheids in the Milky Way $M_K = -3.26 (\log P - 1) + c$, where c is approximately

(a) – 5.69
(b) 5.67
(c) +18.73
(d) – 10.67

Answer (c)

Sol. Distance of Andromeda galaxy = 100 × Distance of milky way

$$\therefore \quad m = M + 5\log\left(\frac{d}{10}\right)$$

$$\therefore \quad m_1 - m_2 = 5\log 100 = 10$$

$$\frac{b_1}{b_2} = (100)^{\frac{m_2 - m_1}{5}} = \frac{1}{(100)^2}$$

$$b \propto \frac{L}{d^2}$$

$$\therefore \quad \frac{L_1}{L_2} = \frac{b_1}{b_2}\left(\frac{d_1}{d_2}\right)^2 = \frac{1}{(100)^2} \times (100)^2 = 1$$

$\therefore$ Luminosity of both the cases is equal

$\therefore$ $c = 18.73$

20. A Red Giant star of mass $M_{Red\ Giant}$ and a White Dwarf star of mass $M_{white\ Dwarf}$ form a binary system and are rotating in circular orbits around their common centre of mass with a period of one year. The White Dwarf is orbiting just within the photosphere of the Red Giant star. The Red Giant has a radius of 3.6 AU. An estimate for the mass of the Red Giant star in solar mass units is (note that $m_{White\ Dwarf} << M_{Red\ Giant}$)

(a) 188
(b) 94
(c) 47
(d) 13

Answer (c)

Sol. As $m_{\text{White Drawf}} << M_{\text{Red Giant}}$

And $r \approx R_{\text{red Giant}}$

$$T = \frac{2\pi R_{\text{Red Giant}}^{3/2}}{\sqrt{G M_{\text{Red Giant}}}} = \frac{2\pi d_{\text{Sun}} (3.6)^{3/2}}{\sqrt{GM_{\text{Sun}}}\sqrt{N}} = 1 \text{ year}$$

We know $\dfrac{2\pi d_{\text{Sun}}}{\sqrt{GM_{\text{Sun}}}} = 1\,\text{year}$ (d_{Sun} is distance of sun from earth)

$$\therefore \quad (3.6)^{3/2} = \sqrt{N}$$

$N = 46.65 \approx 47$

21. In spectroscopy doublet lines are closely spaced spectral lines that arise from transitions from a common fundamental state to states which differ only in their total angular momentum value. If a source of light is moving away from us the light from the source will appear shifted to the longer wavelength (red) side of the spectrum. The shift is quantified by the redshift $z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$, where λ is the observed wavelength of the spectral line when the emitting source is at rest with respect to the observer and Δλ is the observed shift in the wavelength. Hubble observed that spectral lines of galaxies not very close to us are redshifted. A galaxy is showing a redshift of z = 0.005. The observed wavelength separation, in nanometre, of the sodium doublet at 589 and 589.6 nm and of the potassium doublet at 766.5 and 769.9 nm in the spectrum of the galaxy are respectively

(a) 0.603, 3.417
(b) 0.003, 0.017
(c) 38.33, 29.45
(d) 38.33, 38.5

Answer (a)

Sol. For sodium

$\Delta\lambda_1 = z\lambda_1 = 0.005 \times 589 = 2.945$ nm

$\therefore \quad \lambda_1' = \lambda_1 + \Delta\lambda_1 = 591.945$ nm

$\Delta\lambda_2 = z\lambda_2 = 0.005 \times 589.6 = 2.948$ nm

$\lambda_2' = 592.548$ nm

$\therefore \quad \lambda_2' - \lambda_1' = 0.603$ nm

For Potassium

$\Delta\lambda_1 = 0.005 \times 766.5 = 3.8325$ nm

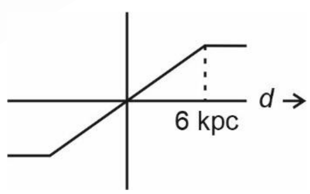
$\therefore \quad \lambda_1' = 770.3325$ nm

$\Delta\lambda_2 = 0.005 \times 769.9 = 3.8495$ nm

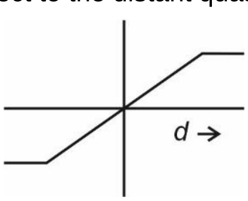
$\lambda_2' = 773.7495$ nm

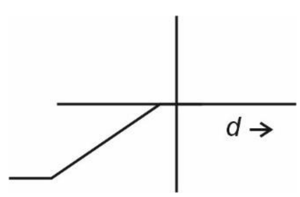
$\therefore \quad \lambda_2' - \lambda_1' = 3.417$ nm

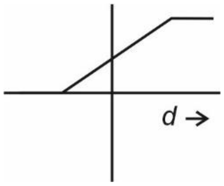
22. The rotation curve of a spiral galaxy gives the 'instantaneous' local mean tangential velocity in the plane of the galaxy of stars/gas clouds in the galaxy lying along a line through the centre of the galaxy, with respect to the distant quasars as observed from the centre of the galaxy. The following is a schematic diagram of the rotation curve of a particular spiral galaxy.

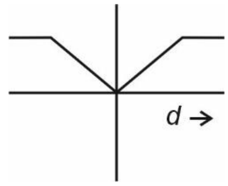


As measured from a star orbiting at a distance r = 8 kilo parsec from the centre, which one of the following diagrams gives a schematic representation of the mean tangential velocity $v(d)$ with which objects at various distances d from the star, along the line joining the star to the centre of the galaxy, will appear to move with respect to the distant quasars?

(a)


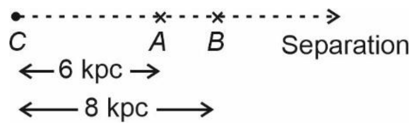
(b)


(c) 

(d) 

Answer (b)

Sol. Let C : centre of the galaxy



Now, all points to the right of A have the same velocity (let us say v_0)

$\Rightarrow$ With respect to B, all the points with position (–2 kpc, ∞) will have zero velocity. Also, the whole graph would be shifted downward by v_0, when seen with respect to B.

23. The relation R on Z the set of integers, defined by $(x, y) \in R$ if and only if $xy \neq 0$ is

(a) Reflexive, symmetric and transitive

(b) Reflexive, symmetric but not transitive

(c) Symmetric, transitive but not reflexive

(d) Transitive, reflexive but not symmetric

Answer (c)

Sol. Reflexive: No, because (x, x) is not always in R.

Symmetric: Yes, because if (x, y) is in R, then (y, x) is also in R.

Transitive: Yes, because if (x, y) and (y, z) are in R, then (x, z) is also in R.

So the given relation is symmetric, transitive but not reflexive.

24. A is a 3 × 3 non-singular matrix such that $A^4 = 4A$. What is the determinant of A?

(a) 0 (b) 4

(c) 16 (d) 64

Answer (b)

Sol. $A^4 = 4A$

$\Rightarrow |A|^4 = 4^3|A|$

$\Rightarrow |A|^3 = 4^3$

$\Rightarrow |A| = 4$

25. The number of real values of θ that satisfy the equation $\sin^2\theta - 5\sin\theta + 6 = 0$ is

(a) 0 (b) 1

(c) 2 (d) 3

Answer (a)

Sol. $\sin^2\theta - 5\sin\theta + 6 = 0$

$(\sin\theta - 2)(\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = 2$ or $\sin\theta = 3$

not possible not possible

$\therefore$ zero solution

26. If z is a complex number such that $\frac{z-2}{z+2}$ = i, the value of $|z|$ is

(a) 0
(b) 1
(c) 2
(d) 3

Answer (c)

Sol. $\frac{z-2}{z+2} = i$

$\Rightarrow$ $z - 2 = iz + 2i$

$\Rightarrow$ $z(1 - i) = 2 + 2i$

$\Rightarrow$ $z = \frac{2+2i}{1-i}$

$$|z| = \frac{|2+2i|}{|1-i|} = \frac{\sqrt{4+4}}{\sqrt{2}} = 2$$

27. The number of positive divisors of 202300 is

(a) 8
(b) 18
(c) 26
(d) 54

Answer (d)

Sol. N = 202300

= 2023 × 100

= 2023 × 2^2 × 5^2

= $2^2 \times 5^2 \times 7^1 \times 17^2$

Number of factors = (2 + 1) (2 + 1) (1 + 1) (2 + 1)

= 54

Option (d) is correct.

28. For what values of a and b is the following function continuous at x = 2?

$$f(x) = \begin{cases} ax^2 + bx + 1 & \text{if } x \geq 2 \\ 3 & \text{if } x = 2 \\ bx^2 - ax - 11 & \text{if } x < 2 \end{cases}$$

(a) a = 1, b = 9
(b) a = 2, b = 12
(c) a = –1, b = 3
(d) a = –3, b = –3

Answer (c)

Sol. $f(x) = \begin{cases} ax^2 + bx + 1 & \text{if } x \geq 2 \\ 3 & \text{if } x = 2 \\ bx^2 - ax - 11 & \text{if } x < 2 \end{cases}$

$$\lim_{x \to 2^-} f(x) = 4a + 2b + 1$$

$$\lim_{x \to 2^+} f(x) = 4b - 2a - 11$$

$4a + 2b + 1 = 3$

$\Rightarrow$ $4a + 2b = 2$ …(1)

$4b - 2a - 11 = 3$

$\Rightarrow$ $4b - 2a = 14$ …(2)

Solving (1) and (2),

$10b = 30$

$b = 3$

$a = -1$

29. The value of the integral $\int_0^{\pi} |\cos 2x|\, dx$ is

(a) 0 (b) 1

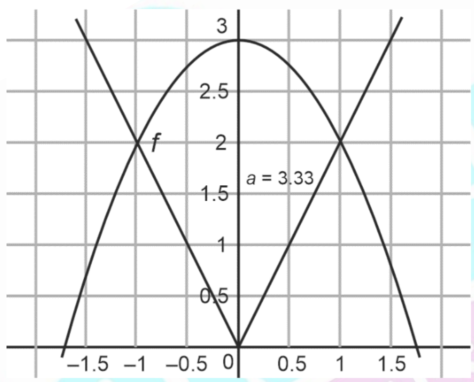
(c) 2 (d) 3

Answer (c)

Sol. $\int_0^{\pi} |\cos 2x|\, dx = \int_0^{\pi/4} \cos 2x dx + \int_{\pi/4}^{3\pi/4} -\cos 2x dx + \int_{3\pi/4}^{\pi} \cos 2x dx$

$$\left[\frac{\sin 2x}{2}\right]_0^{\pi/4} - \left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4} + \left[\frac{\sin 2x}{2}\right]_{3\pi/4}^{\pi} = \frac{1}{2} - [-1] + \frac{1}{2} = 2$$

30. The area bounded by $y = 3 - x^2$ and $y = 2|x|$ is



(a) $\frac{5}{3}$ (b) $\frac{8}{3}$

(c) $\frac{10}{3}$ (d) $\frac{16}{3}$

Answer (c)

Sol. Area $= 2\int_0^1 (3 - x^2 - 2x)dx = 2\left[3x - \frac{x^3}{3} - x^2\right]_0^1 = 2\left[3 - \frac{1}{3} - 1\right] = \frac{10}{3}$ sq. unit

31. Two unbiased dice are thrown simultaneously. What is the probability that both the dice show prime numbers?

(a) $\frac{7}{36}$ (b) $\frac{1}{12}$

(c) $\frac{1}{6}$ (d) $\frac{1}{4}$

Answer (d)

Sol. Prime numbers = {2, 3, 5}

$$P(E) = \frac{3 \times 3}{6 \times 6} = \frac{1}{4}$$

32. The differential equation $\frac{dy}{dx}=\frac{y}{x}$ represents

(a) A family of concurrent straight lines
(b) A family of straight lines passing through (1, 1)
(c) A family of straight lines parallel to X-axis
(d) A family of straight lines parallel to Y-axis

Answer (a)

Sol. $\frac{dy}{dx}=\frac{y}{x}$

$\int\frac{dy}{y}=\int\frac{dx}{x}$

log y = log x + log c

$y = xc$

$\Rightarrow$ a family of lines passing through (0, 0)

33. The volume of a magical right circular cylinder of fixed height changes at a constant rate. Initially, the base radius was 3 units and it changed to 6 units in 3 seconds. What is the radius of the base at time t?

(a) $3\sqrt{\pi t+1}$
(b) $3\sqrt{t+1}$
(c) $3\sqrt{t+\pi}$
(d) $3\sqrt{\pi t+\pi}$

Answer (b)

Sol. $v=\pi r^2h$, h is fixed

$\Rightarrow \frac{dv}{dt}=\left(2\pi r\frac{dr}{dt}\right)h=a$, a is a constant

$\Rightarrow rdr=\frac{a}{2\pi h}dt$

$\Rightarrow \frac{r^2}{2}=\frac{a}{2\pi h}t+c \quad \ldots \text{(i)}$

Now, at t = 0, r = 3

$\Rightarrow \frac{9}{2}=c$

at t = 3, r = 6

$18=\frac{3a}{2\pi h}+\frac{9}{2}$

$\Rightarrow \frac{a}{2\pi h}=\frac{1}{3}\left(18-\frac{9}{2}\right)=\frac{9}{2}$

$\Rightarrow$ using (i)

$\frac{r^2}{2}=\frac{9t}{2}+\frac{9}{2}$

$\Rightarrow r^2=9t+9$

$\Rightarrow r=3\sqrt{t+1}$

34. The Lane Emden equation $\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+y^n=0$, is the governing equation for a crude model for a star. Here the star is treated as a sphere of gas with a polytropic equation of state. The differential equation admits exact solutions for polytropic index n = 0, 1 and 5 as $y(x)=1-\frac{x^2}{6}$, $\frac{\sin x}{x}$ and $\frac{1}{\sqrt{1+\frac{x^2}{3}}}$. The value of $x=x_0$ is such that $y(x_0)=0$ defines the surface of the star. The quantity $R=\frac{x_0^3}{3\left|-x^2\frac{dy}{dx}\right|_{x_0}}$ gives $\frac{\rho_{central}}{\rho_{average}}$ the ratio of the central density to the average density for the model star. $\left|-x^2\frac{dy}{dx}\right|_{x_0}$ means the value of $-x^2\frac{dy}{dx}$ evaluated at x_0. The value of R for n = 0, 1 are

(a) $1, \frac{\pi}{3}$

(b) $\sqrt{6}, \frac{\pi}{3}$

(c) $\propto, \frac{\pi^2}{3}$

(d) $1, \frac{\pi^2}{3}$

Answer (d)

Sol. For n = 0, $y(x)=1-\frac{x^2}{6}$

$$y(x_0)=0 \Rightarrow x_0=\pm\sqrt{6}$$

$$\frac{dy}{dx}=\frac{-x}{3}, \left.\frac{dy}{dx}\right)_{x=\sqrt{6}}=\frac{-\sqrt{6}}{3}$$

$$R=\frac{(\sqrt{6})^3}{3\left|(-6)\left(\frac{-\sqrt{6}}{3}\right)\right|}=\left|\frac{6\sqrt{6}}{6\sqrt{6}}\right|=1$$

For n = 1, $y(x)=\frac{\sin x}{x}, y(x_0)=0 \Rightarrow x_0=\pi$

$$\frac{dy}{dx}=\frac{x\cos x-\sin x}{x^2} \Rightarrow \left.\frac{dy}{dx}\right)_{x_0=\pi}=\frac{\pi\cos\pi-\sin\pi}{\pi^2}=\frac{-1}{\pi}$$

$$R=\frac{\pi^3}{3\left|-\pi^2\times\frac{1}{\pi}\right|}=\frac{\pi^2}{3}$$

35. The logistic map iteratively maps x_{n+1} to x_n (n = 0, 1, 2, … … … … …) through the equation $x_{n+1}=\lambda x_n(1-x_n)$. Consider the two different initial values $x_0=0.25$ and $x_0'=x_0+0.01=0.26$. For $\lambda=3.25$, the difference between the second iterates $(x_2'-x_2)$ is approximately

(a) 0.01

(b) –0.01

(c) 0.02

(d) 0.001

Answer (b)

Sol. $x_1 = \left(\frac{13}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{39}{64}$

$$x_2 = \left(\frac{13}{4}\right)\left(\frac{39}{64}\right)\left(1-\frac{39}{64}\right) = \left(\frac{13}{4}\right)\left(\frac{39}{64}\right)\left(\frac{25}{64}\right) = \frac{(169\times 3)100}{(64)^2\times 16}$$

$$x_1' = \left(\frac{13}{4}\right)\left(\frac{13}{50}\right)\left(1-\frac{13}{50}\right) = \left(\frac{13}{4}\right)\left(\frac{13}{50}\right)\left(\frac{37}{50}\right) = \frac{37\times 169}{10000}$$

$$x_2' = \frac{13}{4}\left(\frac{37\times 169}{10000}\right)\left(1-\frac{37\times 169}{10000}\right)$$

$$x_2' - x_2 = \frac{-169\times 300}{16\times 64^2} + \frac{13^3\times 37}{10^8\times 4}(10000 - 37\times 169)$$

$$= \frac{3747\times 13^3\times 37}{10^8\times 4} - \frac{300\times 169}{16\times 64^2} = 0.76147 - 0.77362 = -0.01$$

36. In a class of 40 students 25 have opted Hindi as second language (event *A*). 35 students got first class (event *B*) in the examinations of which 20 had taken Hindi as second language. It is found that a student picked at random has secured first class. The probability *P*(*A*/*B*) that this student has taken Hindi as second language is

(a) $\frac{4}{7}$

(b) $\frac{5}{8}$

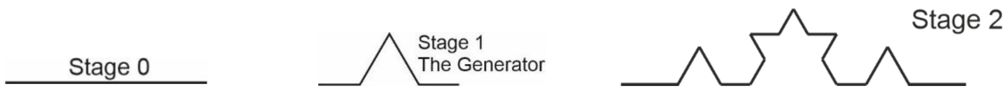
(c) $\frac{7}{8}$

(d) $\frac{1}{2}$

Answer (a)

Sol. $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{\left(\frac{35}{40}\right)\times\left(\frac{20}{35}\right)}{\frac{35}{40}} = \frac{20}{35} = \frac{4}{7}$

37. When a line segment/square/cube of side length *L* is measured using a line segment/square/cube of side length *l*, the number *N* of line segments/squares/cubes covering the original line segment/square/cube is given by $\left(\frac{L}{l}\right), \left(\frac{L}{l}\right)^2, \left(\frac{L}{l}\right)^3$ respectively. Define the dimension of an object as $D = [-\log(N)/\log(l)]$ as $l \to 0$. The $n = 0$, $n = 1$ and $n = 2$, steps of iteration of the process of obtaining the fractal curve called a Koch curve, of stretching and bending the middle one third of a line segment (of length *L*) into a triangle shape of side length *L*/3 is shown in the figure; in the second ($n = 2$) iteration each of the 4 line segments of length *L*/3 will have their middle one third stretched and bend into triangle shapes of side $l = L/(3\times 3)$. After *n* iterations the total number *N* of line segments of length $l = L/3^n$ making up the figure will be 4^n. The dimension *D* of the Koch curve, obtained as $n \to \propto$ (*i.e.*, as $l \to 0$ is)



(a) $\frac{\log 4}{\log 3}$

(b) $\log\left(\frac{4}{3}\right)$

(c) 1

(d) 1.3

Answer (a)

Sol. $N = 4^n$ after n iteration

$$N = \frac{L}{l} = 4^n$$

$\Rightarrow$ $\log N = n\log 4$

$\log l = -n\log 3$

$$l = \frac{L}{3^n}$$

$$D = \left(-\frac{\log N}{\log 4}\right) = \lim_{n\to\infty} \frac{n\log 4}{n\log 3} = \frac{\log 4}{\log 3}$$

38. Consider the two sets $G = \{1, i, -i, -1\}$ and $G' = \{R_{\pi/2}, R_{\pi}, R_{3\pi/2}, R_{2\pi}\}$, where R_θ stands for rotation of a square by angle θ in the anticlockwise direction about an axis through its centre and perpendicular to its plane. If product of two elements of G is taken by complex multiplication and the product of two elements of G' are taken as $R_\theta R_\varphi = R_{\theta+\varphi}$, then, under which of the following mappings between G and G' do the product relations hold unchanged?

(a) $\begin{pmatrix} g_1 \to g_1' \\ g_2 \to g_2' \\ g_3 \to g_3' \\ g_4 \to g_4' \end{pmatrix}$

(b) $\begin{pmatrix} g_1 \to g_4' \\ g_2 \to g_1' \\ g_3 \to g_3' \\ g_4 \to g_2' \end{pmatrix}$

(c) $\begin{pmatrix} g_1 \to g_3' \\ g_2 \to g_4' \\ g_3 \to g_1' \\ g_4 \to g_2' \end{pmatrix}$

(d) $\begin{pmatrix} g_1 \to g_2' \\ g_2 \to g_3' \\ g_3 \to g_4' \\ g_4 \to g_1' \end{pmatrix}$

Answer (b)

Sol. Sets G can be mapped with G' as Euler representation such that multiplication relation holds

$1 = e^{i2\pi} \Rightarrow g_1 \to g_4'$

$-1 = e^{i\pi} \Rightarrow g_4 \to g_2'$

$i = e^{i\frac{\pi}{2}} \Rightarrow g_2 \to g_1'$

$-i = e^{i\frac{3}{2}\pi} \Rightarrow g_3 \to g_3'$

39. The objective of a reflecting telescope is a concave mirror of focal length 750 mm. To double the magnification, a concave lens of focal length 20 mm is placed near the focus of the primary mirror. What should be the position of the eyepiece focus from the concave lens?

(a) 10 mm away from mirror

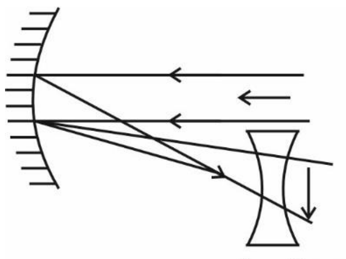
(b) 10 mm towards mirror

(c) 20 mm away from mirror

(d) 20 mm towards mirror

Answer (a)

Sol.



$m = 2$

$\Rightarrow \quad v = 2u$

$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\Rightarrow \quad \frac{1}{-u} + \frac{1}{2v} = \frac{1}{-2}$

$\Rightarrow \quad u = 1$ cm.

$\therefore \quad v = 2$ cm

$\Rightarrow$ The primary image is shifted by 10 mm away from the mirror.

And, focus of eyepiece lie at this position.

40. On the basis of Bohr Theory of atomic structure, the total energy required to remove both electrons from the helium atom in its ground state is 79 eV. How much energy is required to ionize helium *i.e.* to remove the first electron?

(a) 79.0 eV
(b) 54.4 eV
(c) 39.5 eV
(d) 24.6 eV

Answer (d)

Sol. Energy required to remove second electron $= 13.6 \times \frac{Z^2}{n^2}$ = 13.6 × 4 = 54.4 eV

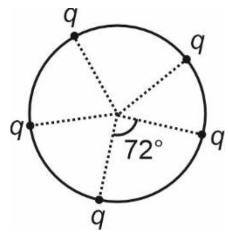
$\therefore$ Energy required to remove first electron = (79 – 54.4) eV = 24.6 eV

41. Five positive charges, each of magnitude q, are arranged symmetrically on the circumference of a circle of radius r. The magnitude of the electric field E at the center of the circle is

(a) Zero
(b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
(c) $\frac{1}{\pi\varepsilon_0}\frac{q}{r^2}$
(d) $\frac{5}{4\pi\varepsilon_0}\frac{q}{r^2}$

Answer (a)

Sol. Due to mutual repulsion, separation between charge will be same



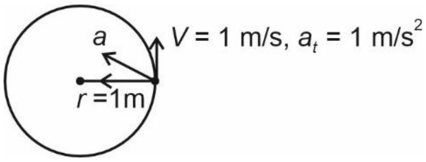
So net field at centre will be zero.

42. A particle is constrained to move along a circle of radius R = 1.0 m. At a certain instant of time the speed of the particle is 1.0 m/s and the speed is increasing at the rate of 1.0 m/s^2. The angle (in radian) between the velocity and the acceleration vectors of the particle is

(a) Zero
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{2}$

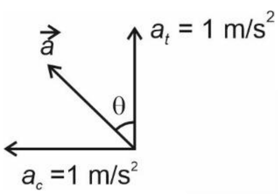
Answer (c)

Sol.



$$a_c = \frac{v^2}{r} = \frac{(l)^2}{l} = 1 \text{ m/s}^2$$

Net acceleration (a):



$$\tan\theta = \frac{ac}{at} = 1$$

$$\theta = \frac{\pi}{4}$$

43. A certain amount of helium gas is expanded adiabatically to one and half times of its initial volume. If the initial temperature is 27°C, the final temperature of the gas will be

(a) +3.6°C
(b) –5.8°C
(c) –10.9°C
(d) –44.1°C

Answer (d)

Sol. $TV^{\gamma-1}$ = constant

Here, He is monoatomic, so, $\gamma = \frac{5}{3}$

$V_1 = V$

$$V_2 = \frac{3}{2}V$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{2}{3}\right)^{\frac{5}{3}-1}$$

$$T_2 = 300\left(\frac{2}{3}\right)^{\frac{2}{3}}$$

T_2 = 228.94 K

T_2 = –44.057°C = –44.1°C

44. Absorbing a neutron, ${}^{235}_{92}U$ undergoes fission producing two fragments in the intermediate atomic mass range namely ${}^{89}_{36}Kr$ and ${}^{144}_{56}Ba$ along with three neutrons and energy. By What factor is the size of the so produced ${}^{144}_{56}Ba$ nucleus smaller than that of the ${}^{235}_{92}U$ nucleus?

(a) 0.61
(b) 0.72
(c) 0.78
(d) 0.85

Answer (d)

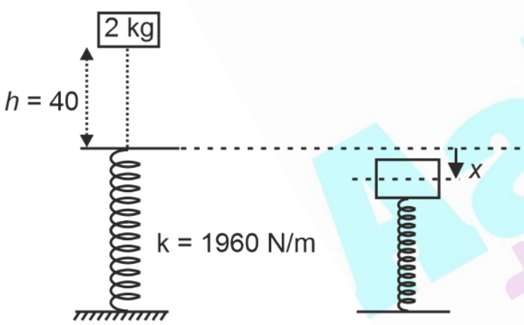
Sol. $\frac{\gamma_{Ba}}{\gamma Ur} = \left(\frac{144}{135}\right)^{1/3} = 0.85$

45. A small wooden block of mass 2 kg is dropped from a height h = 40 cm on a vertical spring of force constant k = 1960 N/m. The maximum compression of the spring is

(a) 0.1 cm
(b) 1.0 cm
(c) 10.0 cm
(d) 23.0 cm

Answer (c)

Sol.



From energy conservation,

$$mgh = -mgx + \frac{1}{2}kx^2$$

$$2\times 9.8\left(\frac{40}{100}\right) = -2\times 9.8x + \frac{1}{2}\times 1960x^2$$

$$4\times\frac{9.8}{5} = -2\times 9.8x + \frac{1}{2}\times 1960x^2$$

$$\frac{4}{5} = -2x + \frac{1}{2}\times 200x^2$$

$$4 = -10x + 500x^2$$

$$2 = -5x + 250x^2$$

$250x^2 – 5x – 2 = 0$

$250x^2 - 25x + 20x - 2 = 0$

$25x\,(10x - 1) + 2(10x - 1) = 0$

$x = \frac{1}{10}\text{m}$, $x = -\frac{2}{25}\text{m}$, $x = 10$ cm, $x = -8$ cm

From given as $x = 10$ cm.

46. The intensity level of a particular sound source is increased by 10 dB, the corresponding change in the amplitude of the sound wave is

(a) $\sqrt{10}$ times

(b) $10\sqrt{10}$ times

(c) 10 times

(d) 100 times

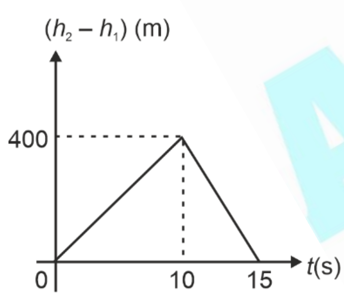
Answer (a)

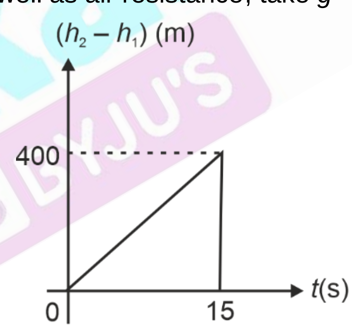
Sol. Intensity, $L = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}(10) = 10$

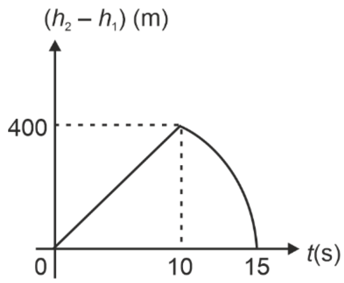
We know, Intensity $\propto$ (Amplitude)^2

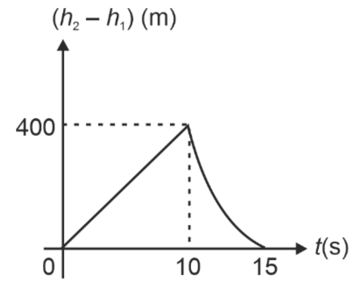
Amplitude change = $\sqrt{10}$

47. Two stones of equal mass are thrown vertically up simultaneously from the edge of a cliff 400 m high with initial speeds of v_1 = 10 m/s and v_2 = 50 m/s respectively. Which of the following graphs best represents the time variation of relative position [height $(h_2 - h_1)$] of the second stone with respect to the first? (Assume that the stones are not obstructed at the cliff while going down and do not rebound after hitting the ground. Also ignore the small horizontal separation between the stones as well as air resistance, take g = 10 m/s^2)

(a)


(b)


(c)


(d)


Answer (c)

Sol. The first stone touches the ground at t = 10 s

Before that $(h_2 - h_1) = (v_2 - v_1)t$

$(h_2 - h_1) = 40t$

After t = 10 s till 16.18 s. Stone 2 is moving down with acceleration g.

48. A crow is sitting on a standard electric power line. The crow is not affected by the potential drop across its feet because

(a) It has non-conducting pads at the bottom of its feet

(b) The potential drop is too small for any significant current to flow through its body

(c) The crow is carefully sitting on the neutral line

(d) None of the above

Answer (b)

Sol. The potential drop is too small across feet.

A - 2

ANY NUMBER OF OPTIONS [4, 3, 2 OR 1] MAY BE CORRECT
MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED AND NO INCORRECT

49. Consider the electromagnetic radiation emitted by two stars. The hotter star will

(a) Emit more radiation at all wavelengths

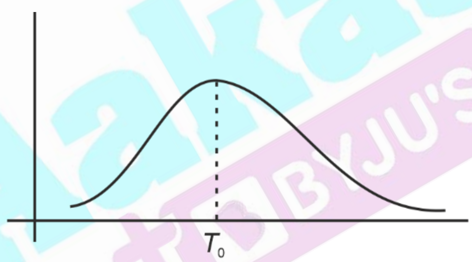
(b) Have a higher frequency of peak emission in its spectrum

(c) Radiate energy at more than one wavelength

(d) Exhibit a continuous spectrum

Answer (a, b, c, d)

Sol. We know that graph of $\frac{dE}{d\lambda}$ vs λ is



Also, $\lambda_0 T_0 = b$

$\Rightarrow$ We can say that the hotter star will

1. Have a lower $\lambda_0 \Rightarrow$ higher f_0

2. Radiate energy at more than one wavelength.

3. Exhibit a continuous spectrum.

4. Emit more radiation at all wavelengths (Since graph always remains above)

50. Because of the precession of the axis of rotation of Earth

(a) The polaris is not always our 'pole star'

(b) The average length of a sidereal day changes slowly with time

(c) The declinations of the stars change slowly with time

(d) The Vernal Equinox moves with respect to the stars

Answer (a, b, c, d)

Sol. Due to precession of axis of rotation of earth,

(i) The polaris position keep shifting continuously, so it is not always pole star.

(ii) Length of sidereal day changes slowly

(iii) Declination changes slowly

(iv) Vernal Equinox changes with time

So correct options are (a, b, c, d).

51. A planet orbits the Sun at a distance of 4.0 AU from the Sun. The orbital speed (v_0) and the time period (T) of the planet are approximately

(a) v_0 = 7.92 km/s

(b) v_0 = 14.93 km/s

(c) T = 4 years

(d) T = 8 years

Answer (b, d)

Sol. By Kepler's Law

$$\left[\frac{T}{1\,\text{year}}\right]^2 = \left[\frac{4\text{ AU}}{1\text{ AU}}\right]^3 \Rightarrow T = 8 \text{ years}$$

Also, $v = \sqrt{\frac{GM_{sun}}{4\text{ AU}}} = \sqrt{\frac{6.67\times10^{-11}\times2\times10^{30}}{4\times1.5\times10^{11}}}$ m/s = 1.49×10^4 m/s

52. The rotation period of a spherical moon about its own axis is equal to its period of revolution round its parent planet. Which of the following statements is/are true?

(a) If the orbital and spin angular momentum vectors of the moon are parallel to each other then, the moon will always show the same portion of its surface to the planet

(b) If the spin angular momentum vector of the moon lies in its orbital plane, then every portion of the surface of the moon may be seen from the planet

(c) If the orbital and spin angular momentum vectors of the moon are antiparallel with respect to each other then, viewed from the planet, the moon will appear to be non-rotating

(d) If the spin angular momentum vector of the moon lies in its orbital plane, then, viewed from the planet, the moon will appear to be rotating with a higher angular speed

Answer (a, b, c, d)

Sol. The orbital and spin angular momentum vectors of moon are parallel and their time period are equal.

Therefore, the same face of moon appears always to the earth.

53. The true statement(s) for the functions $f(x) = \begin{cases} \frac{\sin|x|}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is/are

(a) $\lim_{x\to0} f(x)$ exists

(b) $\lim_{x\to0} f(x)$ does not exist

(c) f is differentiable at $x = 0$

(d) f has removable discontinuity at $x = 0$

Answer (a, d)

Sol. $f(x)=\begin{cases}\dfrac{\sin|x|}{|x|} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$

$\lim\limits_{x\to 0^-}\dfrac{-\sin x}{-x}=1$

$\lim\limits_{x\to 0^+}\dfrac{\sin x}{x}=1$

$\because$ LHL = RHL $\Rightarrow \lim\limits_{x\to 0} f(x)$ exist

But LHL = RHL $\neq f(0) \Rightarrow f(x)$ is not continuous.

Also $f(x)$ has removable type of discontinuity.

54. Consider the differential equation $x\dfrac{dy}{dx}+3y=x^3$. Which of the following is/are true?

(a) $3x^3$ is an integrating factor of this equation

(b) $y=\dfrac{x^3}{6}+\dfrac{2023}{x^3}$ is a solution of this equation

(c) $y=\dfrac{x^3}{6}-\dfrac{8}{2023x^3}$ is a solution of this equation

(d) The equation is a homogeneous differential equation

Answer (a, b, c)

Sol. $x\dfrac{dy}{dx}+3y=x^3$

$\dfrac{dy}{dx}+3\left(\dfrac{y}{x}\right)=x^2$

$\text{I.F.}=e^{\int\frac{3}{x}dx}=e^{3\ln x}=e^{\ln x^3}=x^3$

I.F. can be $3x^3$

$\Rightarrow\ y(3x^3)=\int(3x^3)x^2dx$

$\Rightarrow\ 3yx^3=\dfrac{3x^6}{6}+c$

$\Rightarrow\ y=\dfrac{x^3}{6}+\dfrac{c}{x^3}$

55. Consider the LPP (Linear Programming Problem) Max $Z = x + y$ subject to the constraints $x + 3y \geq 6$, $y \leq 3$, $x + y < 4$, $x, y \geq 0$ Which of the following is/are true?

(a) $x = 1, y = 2$ is a feasible solution of the LPP

(b) $x = 2, y = 2$ is an optimal solution of the LPP

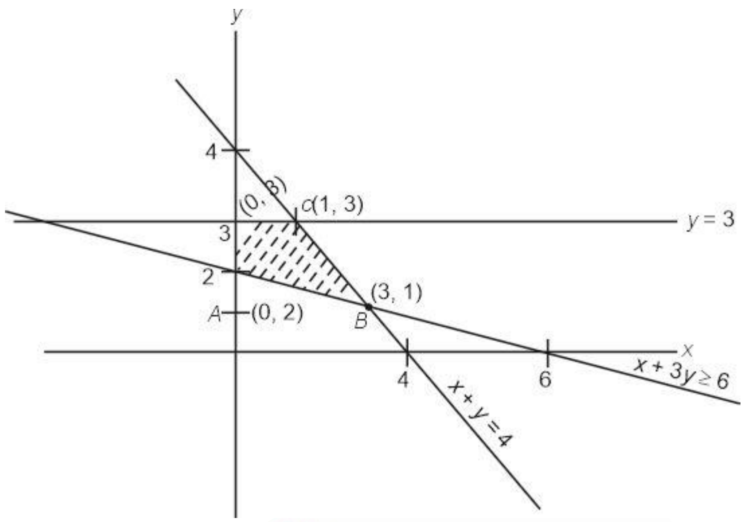
(c) The only optimal solutions of the LPP are $x = 1, y = 3$ and $x = 3, y = 1$

(d) $x = 3, y = 1$ is an optimal solution of the LPP

Answer (a, b, d)

Sol. max $z = x + y$ s.t

$x + 3y \geq 6$, $y \leq 3$, $x + y \leq 4$, $x, y \geq 0$



(a) (1, 2) lie in feasible region.

(b) Z(A) = 2, Z(B) = 4, Z(C) = 4, Z(D) = 3

Z(2, 2) = 4 which is equal to optional value

(c) (2, 2) is also optional.

(d) (3, 1) is an optional

56. For vectors A, B & C in three-dimensional Euclidean space, let $AB = A \times B$ stand for the cross product, $[A, B] = AB - BA$ for the commutator of A & B and $\{A, B, C\} = (AB)C - A(BC)$ for the associator of A, B & C. Then which of the following relations hold true?

(a) $A^2 = 0$ for $A \neq 0$

(b) $[A, B] = 0$ for $A \neq B$ and $A, B \neq 0$

(c) $\{A, B, C\} = 0$ for $A \neq B \neq C$ and $A, B, C \neq 0$

(d) $A(BC) + B(CA) + C(AB) = 0$ for $A \neq B \neq C$ and $A, B, C \neq 0$

Answer (a, d)

Sol. $AB = A \times B$, $[A, B] = AB - BA = A \times B - B \times A$

$[A, B, C] = (AB)C - A(BC) = ((A \times B)) \times C - A \times (B \times C)$

(a) $A^2 = A \times A = 0$ for $A \neq 0$, true

(b) $[A, B] = 0 = A \times B = B \times A$

$\Rightarrow 2(A \times B) = 0$

$\Rightarrow A \times B = 0$

$\Rightarrow A, B$ parallel or $A, B = 0$

(c) $[A\ B\ C] = 0 \Rightarrow (A \times B) \times C = A \times (B \times C)$

$\Rightarrow (C \cdot A)B - (C \cdot B)A = (A \cdot C)B - (A \cdot B)C = (B \cdot C)A = (A \cdot B)C$

(d) $A \times (B \times C) + B \times (C \times A) + C \times (A \times B) = (A \cdot C)B - (A \cdot B)C + (A \cdot B)C - (B \cdot C)A + (C \cdot B)A - (C \cdot A)B = 0$, true

$\Rightarrow$ (a, d) are correct answers.

57. An artificial satellite is moving in an orbit around the Earth with orbital period of 8 hours, from west to east, with its orbit making an angle of 30° with equator. An observer on the equator will see it

(a) Again after 6 hour, moving from east to west

(b) Again after about 11 hours, moving from west to east

(c) Going up to about 46.8° from the equatorial plane

(d) Going up to about 43.2° from the equatorial plane

Answer (b, d)

Sol. For rough estimation,

$\omega_{rel} = \omega_1 - \omega_2$

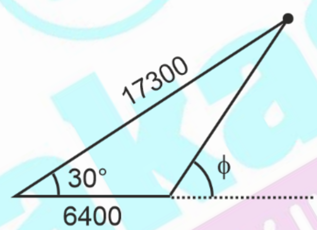
$= \frac{2\pi}{8} - \frac{2\pi}{24}$

$= 2\pi \times \left(\frac{1}{12}\right)$

$\therefore \quad \Delta T_{rel} = \frac{2\pi}{\omega_{rel}} \simeq 12$ hours

And, $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\Rightarrow \quad r \simeq 17300$ km



$\tan\phi = \frac{17300\sin 30°}{17300\cos 30° - 6400}$

$\Rightarrow \quad \phi \simeq 43.2°$ up

Correct option is (b, d).

58. A particle of mass '*m*' is moving under the influence of a central force; the correct statement(s) is/are

(a) The motion always remains constrained to be in a plane

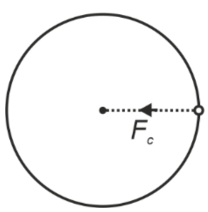
(b) The trajectory will always be elliptical

(c) The total angular momentum remains constant

(d) The torque about the force center is always zero

Answer (a, c, d)

Sol.



For the given condition, motion of particle will be in circle or ellipse.

F_c (central force) only depends on separation between centre

So, particle will move in plane, in circular or elliptical path depends on nature of central force.

Torque of centre (τ) = $rF_c \sin\theta = 0$

59. When a raw poori round (kneaded wheat dough rolled into thin rounds) is slipped into hot oil and the hot oil is swished immediately over it repeatedly, the poori puffs up because

 (a) The dough has comparatively low heat conductivity

 (b) Hot oil causes a crust to form

 (c) The density of steam is low

 (d) The matrix of proteins in the dough formed during kneading helps trap gases

Answer (a, b, c, d)

Sol. During kneading wheat, air trapped inside which expand on getting heat and try to move up because of low density but again trapped in layer and it puffs up.

Due to low conductivity of heat, one layer heat quickly than other and puffs up.

60. Two positively charged conducting spheres *A* and *B* with +*Q* charge on each, are placed on the *x*-axis at (*a*, 0) and (–*a*, 0). A very small metallic ball with charge +*q* is gently placed at the origin (midway between the two spheres). The small ball with charge +*q*, in the middle, is

 (a) In stable equilibrium if constrained to move only along *x*-axis

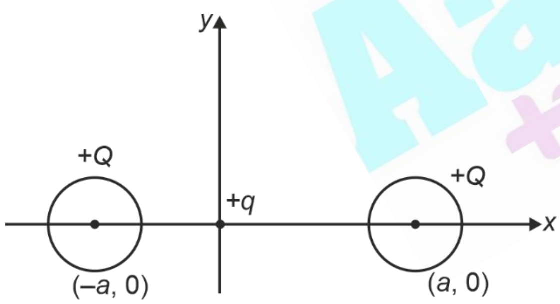
 (b) In unstable equilibrium if constrained to move only perpendicular to the *x*-axis

 (c) Always in unstable equilibrium

 (d) Always in neutral equilibrium

Answer (a, b)

Sol.



Case-I: If small ball will displaced along *x*-axis, it will always tend to move at point of minimum potential energy that is origin.

Case-II : If small ball displaced along *y*-axis, it will move far away because of repulsion force and will stay at point of minimum potential energy that is infinite.

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