Date: 22 November, 2025

Question Paper Code

41

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Medical | IIT-JEE | Foundations

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Answers & Solutions

Time : 120 Minute

for

Max. Marks : 216

Indian Association of Physics Teachers
National Standard Examination in
ASTRONOMY (NSEA)-2025

INSTRUCTIONS TO CANDIDATES

(1) There are 60 questions in this paper. Attempt all the 60 Questions.

(2) Question paper has two parts. In **Part A-1** (Q. No. 1 to 48) each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

Q. No. 22 (a) (●) (c) (d)

In **Part A-2** (Q. No. 49 to 60) each question has four alternatives, out of which **any number of alternative (s)** (1, 2, 3 or 4) may be correct. You have to choose ALL correct alternative(s) and fill the appropriate bubble(s), as shown.

Q. No. 54 (a) (●) (c) (●)

(3) For **Part A-1**, each correct answer carries **3 marks** whereas 1 mark will be deducted for each wrong answer. In **Part A-2**, you get **6 marks** if all the correct alternatives are marked. No negative marks in this part.

Attempt All Sixty Questions

A - 1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBBLE THE CORRECT OPTION

1. In a hypothetical scenario the force experienced by a particle is given by $F = Ae^{\frac{px}{t^2}} + BV_t^2$, where $V_t = \sqrt{\frac{2mg}{\rho\eta S}}$ is the expression for terminal velocity. Where m is the mass, g is the acceleration due to gravity, ρ is the density of the medium, S is the surface area of the particle, and η is the coefficient of viscosity of the medium. The possible units of A, B and p are

(a) A: newton, B: newton kg m^{-1}, p: m s^2
(b) A: no units, B: newton kg m^{-1}, p: $m^{-1}s^2$
(c) A: newton, B: kg m^{-1} , p: no units
(d) A: newton, B: newton s^2 m^{-2}, p: $m^{-1}s^2$

Answer (d)

Sol. [A] will dimension of force.

and $[B] = \frac{MLT^{-2}}{L^2T^{-2}} = ML^{-1}$

$\frac{\rho x}{t^2}$ will be dimensionless.

So, $[p] = \frac{t^2}{x} = L^{-1}T^2$

2. Gravity on the surface of Ganymede, a satellite of Jupiter, is $\left(\frac{1}{7^{th}}\right)$ of that on the Earth, while the gravity on the surface of the Moon is $\left(\frac{1}{6^{th}}\right)$ of that on the Earth. Two identical pendulums are taken one on Ganymede and the other on the Moon. The two pendulums start oscillating together, after how many oscillations on Moon will they come again in the same phase with approximately 1% uncertainty?

(a) 24
(b) 25
(c) 26
(d) 27

Answer (d)

Sol. $\frac{GM_e}{R_e^2} = g_0$

$\frac{GM_G}{R_G^2} = g_G = \frac{g_0}{7}$

$\frac{GM_M}{R_M^2} = \frac{g_0}{6}$

$$T_G = 2\pi\sqrt{\frac{l\times 7}{g_0}} \Rightarrow W_G = \frac{1}{2\pi}\sqrt{\frac{g_0}{7l}}$$

$$T_M = 2\pi\sqrt{\frac{l\times 6}{g_0}} \Rightarrow W_M = \frac{1}{2\pi}\sqrt{\frac{g_0}{6l}}$$

$T_G = T_0\sqrt{7}$ and $T_M = T_0\sqrt{6}$

$$\Rightarrow \frac{T_G}{T_M} = \sqrt{\frac{7}{6}} \approx 1.08 = \frac{N_M}{N_G}$$

$\Rightarrow N_G = \frac{N_M}{1.08}$. If we try for different value of N_M = 27. N_G is closest integer.

3. Consider two telescopes having equal apertures of 400 mm. One has a focal ratio of $\frac{f}{5}$, the other has the focal ratio of $\frac{f}{10}$. What is the relation between their focal lengths?

(a) Focal length of the telescope with $\frac{f}{5}$ ratio is twice as compared to the focal length of telescope with $\frac{f}{10}$ ratio.

(b) Focal length of the telescope with $\frac{f}{10}$ ratio is twice as compared to the focal length of telescope with $\frac{f}{5}$ ratio.

(c) Focal length of the telescope with $\frac{f}{10}$ ratio is four times smaller as compared to the focal length of telescope with $\frac{f}{5}$ ratio

(d) Focal lengths of both telescopes are the same

Answer (a)

Sol. Focal ratio = $\frac{\text{Focal length } (f)}{\text{Apertur } (D)}$

So, for $F_{(R)_1} = \frac{f}{5}$

$$\frac{f}{5} = \frac{f_{(1)}}{D} \Rightarrow f_{(1)} = \frac{Df}{5}$$

And for $F_{(R)_2} = \frac{f}{10}$

$$\frac{f}{10}=\frac{f_{(2)}}{D}\Rightarrow f_{(2)}=\frac{Df}{10}$$

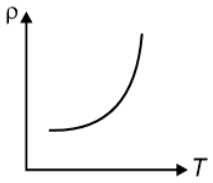
Clearly $f_{(1)}=2f_{(2)}$

4. Statement I: The radius vector of a planet sweeps equal area in equal time while revolving around the Sun.
Statement II: Gravitational force between the Sun and the planet is along the line joining the two.
(a) Both the Statements are true, and Statement II is correct reason of Statement I.
(b) Both the Statements are true, but Statement II is not the correct reason of Statement I.
(c) The Statement I is true and the Statement II is false.
(d) The Statement I is false and the Statement II is true.

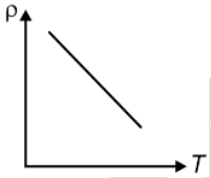
Answer (a)

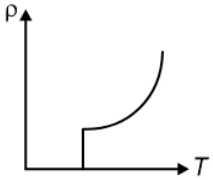
Sol. Gravitational force is central force so torque is zero and for that reason area velocity comes out to be a constant.

5. Which of the following graphs qualitatively depicts the variation in resistivity of a semiconductor with respect to temperature correctly?

(a) 

(b) 

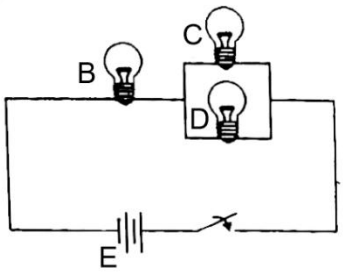
(c) 

(d) 

Answer (b)

Sol. For semiconductors the resistivity decreases rapidly as the temperature increases.

6. Three identical bulbs, B, C and D are connected in a circuit as shown below.
By connecting one more identical bulb

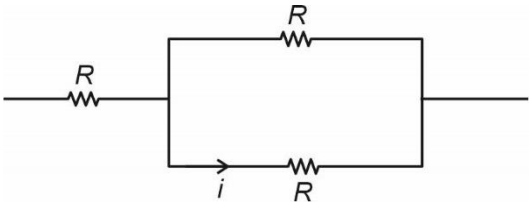


(a) in series with bulb B, the intensity of bulb B will decrease but that of bulb C and D will increase.
(b) in parallel with bulb B, the intensities of all the three bulbs B, C and D will decrease.
(c) in parallel with bulbs C and D, the intensities of all the three bulbs B, C and D will increase.
(d) in series with bulb C, the intensity of bulb B will decrease and that of bulb D will increase.

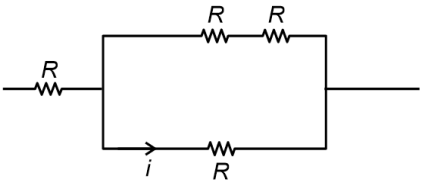
Answer (d)

Sol. (a) In series with B overall resistance increase so current in all decrease.
(b) Intensity of C and D will increase
(c) Intensity of C and D will decrease

(d)



$$i = \frac{V}{3R}$$

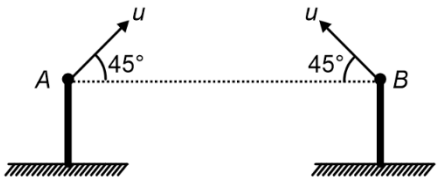


$$i' = \frac{2}{3}\left(\frac{3V}{5R}\right)$$

$$i' = \frac{2V}{5R}$$

$\Rightarrow \quad i' > i$

7. Two cannons are placed on 1000 m high towers at a horizontal distance of 400 m between them along x-axis. Ball A is fired at an angle of 45° to the +ve x-axis whereas Ball B is fired at an angle of 45° to the -ve x-axis. Initial velocity of projection given to each ball is u = 40m / s in magnitude. The point $P(x, y)$ at which the two balls collide is



(a) x = 200 m, y = – 45 m

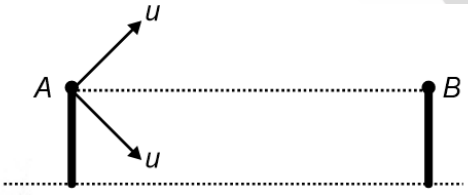
(b) x = 200 m, y = 0

(c) x = 100 m, y = – 200 m

(d) the two balls will not collide

Answer (a)

Sol. W.r.t. ball B,



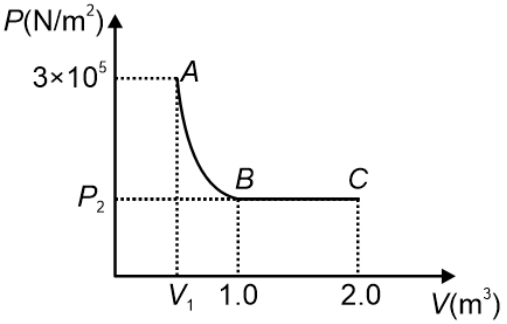
$$t = \frac{400}{40\sqrt{2}} = 5\sqrt{2} \text{ sec.}$$

$$\vec{s} = 20\sqrt{2}\cdot 5\sqrt{2}\hat{i} + (20\sqrt{2}\times 5\sqrt{2} - \frac{1}{2}\times 9.8\times 50)\hat{j}$$

$$= 200\,\hat{i} + (200 - 245)\hat{j}$$

$$= 200\,\hat{i} - 45\hat{j}$$

8. Sixty moles of Helium gas are initially at 28°C (at A). It undergoes an isothermal process from A to B and then an isobaric process from B to C. Total change in the internal energy of the gas in the complete process from A to C would be approximately,



(a) 206 kJ

(b) 450 kJ

(c) 431 kJ

(d) 225 kJ

Answer (d)

Sol. $P_2 = \frac{nRT_1}{1}$

$B \rightarrow C$, isobaric process

$T_C = 2T_B = 2T_1$

$\Delta U = nC_V\Delta T = 60\times\frac{3}{2}R\times 301 \simeq 225$ kJ

9. In an atom, the nucleus consists of 2 protons and 2 neutrons and one electron is revolving around the nucleus. According to Bohr's atomic model, the only visible wavelength corresponds to the transition

(a) 2^{nd} to 1^{st} orbit

(b) 4^{th} to 2^{nd} orbit

(c) 4^{th} to 3^{rd} orbit

(d) 5^{th} to 3^{rd} orbit

Answer (c)

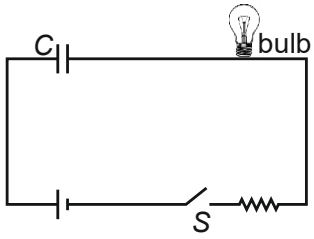
Sol. Visible range 400 nm to 750 nm

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = 1.1\times 10^7\times 4\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\lambda = \frac{16\times 9}{7\times 4\times 1.1\times 10^7} = 467 \text{ nm}$$

10. Select the statement describing correctly the functioning of the circuit, shown below.



(a) From the time the switch S is closed, the bulb will start glowing with intensity increasing uniformly to a maximum value and will turn off after some time.

(b) As soon as the switch S is closed, the bulb will glow with maximum intensity which will then slowly decrease to zero.

(c) From the time the switch S is closed, the bulb will start glowing with intensity increasing uniformly to a maximum value and will continue glowing thereafter.

(d) If distance between the parallel plates of the capacitor is increased, bulb intensity will attain maximum value in shorter time after the switch is closed.

Answer (b)

Sol. For RC circuit, $i = i.\left(e^{-t/RC}\right)$

Current is decreasing with time.

11. A mechanical spring deviates from Hooke's law as $F \propto -k(e^x - 1)$, where x is the strain. At what value of strain x, the force deviates from the one obeying Hooke's law with the same k and unit length, just by 1%?

(a) x = 1% (b) x = 2%

(c) x = 5% (d) x = 20%

Answer (a)

Sol. $F \propto -k(e^x - 1)$

$F = -k(e^x - 1)$

$dF = ke^x dx$

And for normal spring, $F' = -kx$

$dF' = -kdx$

$\frac{dF - dF'}{dF'} \times 100 = 1$

$\Rightarrow \quad \frac{kdx(e^x - 1)}{kdx} = \frac{1}{100}$

$$\Rightarrow \quad e^x = 1 + \frac{1}{100}$$

$$\Rightarrow \quad e^x = \frac{101}{100}$$

$$x = \log_e(1.01) = 0.01$$

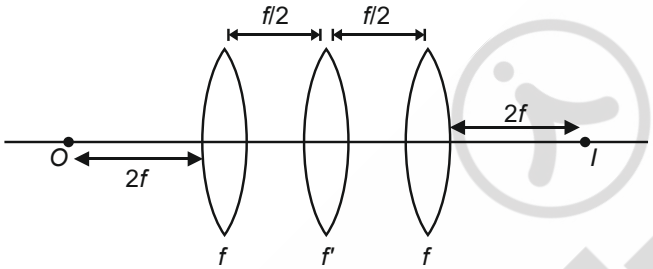
12. Three lenses with focal lengths f, f' and f respectively are kept with common principal axis in that order with successive separation of $\frac{f}{2}$ between each pair. If the point object is placed on the common principal axis to the left of 1st lens at $2f$ distance from it, what should be the value of focal length f' so as to form the final image at distance of $2f$ from 3rd lens on the right of it?

(a) $-\frac{4f}{3}$ (b) $+\frac{4f}{3}$

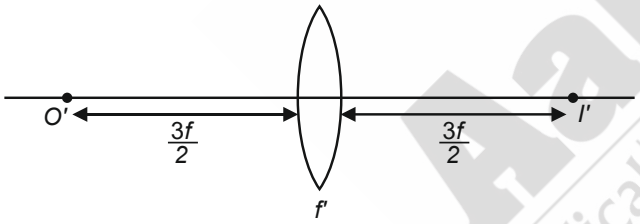
(c) $-\frac{3f}{4}$ (d) $-\frac{3f}{5}$

Answer (c)

Sol.



For middle lens



$\therefore$ Focal length of middle lens $f' = \frac{-3f}{4}$

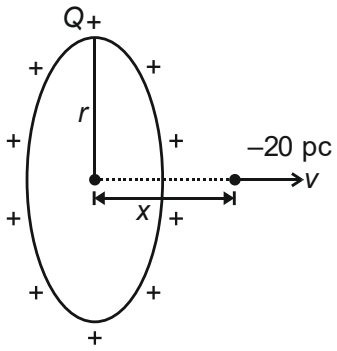
13. A negatively charged small ball with charge –20 pC and mass 1.00 mg is placed at the centre of a uniformly charged ring of radius 5.00 cm. The negatively charged ball is allowed to move only along the axis of the ring. The ball executes **SHM with frequency 1.00 kHz.** The charge on the ring is

(a) 27.4 mC (b) 0.55 mC

(c) 13.9 μC (d) 0.69 μC

Answer (a)

Sol.



Total energy of the point particle will remain conserved.

$$\frac{1}{2}mv^2 - \frac{KQ\times q}{(r^2+x^2)^{1/2}} = \text{constant}$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{KQq}{r\left(1+\frac{x^2}{r^2}\right)^{1/2}} = \text{constant}$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{KQq}{r}\left(1-\frac{x^2}{2r^2}\right) = \text{constant}$$

Differentiating both sides

$$\Rightarrow mva + \frac{KQq\times 2xv}{2r^3} = 0$$

$$\Rightarrow a = -\frac{KQqx}{mr^3}$$

$$\omega = \sqrt{\frac{KQq}{mr^3}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{KQq}{mr^3}} = 10^3$$

$$\Rightarrow \frac{1}{4\pi^2}\times\frac{9\times10^9\times Q\times20\times10^{-12}}{10^{-3}\times125\times10^{-6}} = 10^3$$

Q = 27.4 mC

14. Two hunters approach a prey from opposite directions. One hunter has a probability of $\frac{1}{3}$ for hitting the prey, the other has a probability of $\frac{2}{3}$ for hitting the prey. Both fires a burst of 3 shots each simultaneously. Assume that both the hunters fire one shot each at the same time simultaneously, followed by another shot each at the same time, followed by the third shot each at the same time, each round to shots have negligible time gap between them. The probability that the prey, in question, is hit

(a) after the three rounds is 1

(b) in the first round of shots is 1

(c) after the three rounds is $\frac{7}{9}$

(d) after the three rounds is $\frac{721}{729}$

Answer (d)

Sol. Probability that one hunter hit the prey is $\frac{1}{3}$

Probability that he doesn't hit the prey is $1-\frac{1}{3}=\frac{2}{3}$

Probability that other hunter hit the prey is $\frac{2}{3}$

Probability that other hunter doesn't hit the prey is $1-\frac{2}{3}=\frac{1}{3}$

P (prey is hit after three rounds) = 1 – (prey doesn't get hit in any of the round)

$$= 1-\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3$$

$$= 1-\frac{8}{3^6}$$

$$= \frac{721}{729}$$

15. A particle moving with constant velocity along X-axis has angular momentum p = 120 units about a reference point in XY plane. Which of the following statements is correct?

(a) If the reference point is shifted parallel to Y axis, angular momentum need not change necessarily.

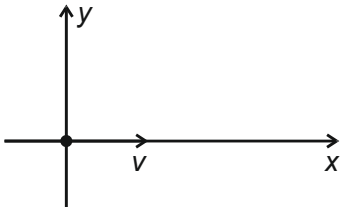
(b) If the reference point is taken to double the present distance from origin, instantaneous angular velocity become half the present value.

(c) It is always possible to increase instantaneous angular velocity to double the present value by shifting reference point parallel to X axis.

(d) If the reference point is shifted parallel to X axis, angular momentum will surely not change about it.

Answer (d)

Sol.



(a) If reference points is shifted parallel to *y*-axis, angular momentum would change.

(b) $\omega = \frac{v}{r}$ and if distance is doubled, it would not necessarily halve the angular velocity.

(c) If reference point is shifted parallel to *x* axis, it would not change perpendicular distance and hence angular momentum would remain same.

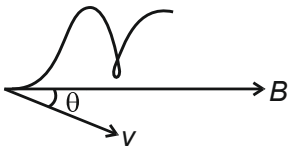
16. An electron moving with speed of 0.1% speed of light, enters a uniform magnetic field of strength 2.5 gauss at 60° angle. Calculate the number of helical turns it takes in moving 1 km distance.

(a) 20000 to 22000 (b) 22000 to 25000

(c) 30000 to 32000 (d) 45000 to 47000

Answer (d)

Sol.



v = 0.1% of c

$$r = \frac{mv\sin\theta}{qB} = \frac{\sqrt{3}mv}{2qB}$$

$$v\sin 60° = \frac{2\pi r}{T}$$

$$T = \frac{2\pi}{\frac{v\sqrt{3}}{2}} \times \frac{\sqrt{3}mv}{2qB} = \frac{2\pi mv}{qB}$$

$$v\cos 60° = \frac{1000}{t}$$

$$t = \frac{2000}{v}$$

No. of turns = $\frac{t}{T} = \frac{\frac{2000}{v} \times qB}{2\pi mv} \approx 46662$

17. If $a+\frac{1}{a}=\sqrt{3}$ then $a^6+\frac{1}{a^6}$ is

(a) 27 (b) 0

(c) –2 (d) 81

Answer (c)

Sol. $a+\frac{1}{a}=\sqrt{3}$

Cube both sides,

$a^3+\frac{1}{a^3}+3a\cdot\frac{1}{a}\left(a+\frac{1}{a}\right)=3\sqrt{3}$

$a^3+\frac{1}{a^3}+3\left(\sqrt{3}\right)=3\sqrt{3}$

$a^3+\frac{1}{a^3}=0$

Square both sides,

$\left(a^3+\frac{1}{a^3}\right)^2=0$

$a^6+\frac{1}{a^6}+2a^3\cdot\frac{1}{a^3}=0$

$a^6+\frac{1}{a^6}=-2$

18. If $\sin x+\cos x=\sqrt{2}$ then $\sin^4 x + \cos^4 x$ is

(a) 4 (b) –1

(c) 1 (d) $\frac{1}{2}$

Answer (d)

Sol. $\sin x+\cos x=\sqrt{2}$

Range of $a\sin x + b\cos x$:

$-\sqrt{a^2+b^2}\le a\sin x+b\cos x\le\sqrt{a^2+b^2}$

$-\sqrt{2}\le\sin x+\cos x\le\sqrt{2}$

Given that $\sin x+\cos x=\sqrt{2}$

So $\sin x + \cos x$ is taking its maximum value.

This is possible only when $\sin x=\cos x=\frac{1}{\sqrt{2}}$

$\sin^4 x+\cos^4 x=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

19. If a, b, c are in arithmetic progression and a^2, b^2, c^2 are in geometric progression, then the common ratio is

(a) 0 (b) 1

(c) 2 (d) 3

Answer (b)

Sol. If $a, b, c,$ in A.P. $\Rightarrow 2b = a + c$

If $a^2, b^2, c^2,$ in G.P. $\Rightarrow \left(b^2\right)^2 = a^2 \cdot c^2$

$\Rightarrow \quad b^2 = ac$ or $b^2 = -ac$

Case (A) If $b^2 = ac \Rightarrow a = b = c$ as a, b, c are in GP.

Case (B) If $b^2 = -ac \Rightarrow a^2 + b^2 + 6ac \Rightarrow$ no such ratio in given options.

20. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} =$

(a) $\frac{1}{3}$ (b) $\frac{11}{18}$

(c) $\frac{1}{2}$ (d) $\frac{4}{17}$

Answer (b)

Sol. $\frac{1}{n(n+3)} = \frac{1}{3n} - \frac{1}{3(n+3)} = \frac{1}{3}\left(\frac{1}{n} - \frac{1}{n+3}\right)$

$$S_N = \sum_{n=1}^{N} \frac{1}{3}\left(\frac{1}{n} - \frac{1}{n+3}\right) = \frac{1}{3}\sum_{n=1}^{N}\left(\frac{1}{n} - \frac{1}{n+3}\right)$$

$$= \frac{1}{3}\left[\left(1-\frac{1}{4}\right)+\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{3}-\frac{1}{6}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)\cdots\left(\frac{1}{N-1}-\frac{1}{N+2}\right)+\left(\frac{1}{N}-\frac{1}{N+3}\right)\right]$$

$$= \frac{1}{3}\left[1+\frac{1}{2}+\frac{1}{3}-\frac{1}{N+1}-\frac{1}{N+2}-\frac{1}{N+3}\right]$$

$$S = \lim_{N\to\infty} \frac{1}{3}\left[1+\frac{1}{2}+\frac{1}{3}-\frac{1}{N+1}-\frac{1}{N+2}-\frac{1}{N+3}\right]$$

$$= \frac{1}{3}\times\frac{11}{6} = \frac{11}{18}$$

21. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = \max\{x^2, 1\}$. Then

(a) f is differentiable everywhere

(b) f is continuous everywhere but not differentiable at $x = \pm 1$

(c) f is not continuous at $x = \pm 1$

(d) f is neither continuous nor differentiable

Answer (b)

Sol. $f(x)=\begin{cases}1, & -1\le x\le 1\\ x^2, & x<-1 \text{ or } x>1\end{cases}$

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-} 1=1$

$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^-} x^2=1$

$f(x) = 1$

continuous at $x = 1$

$\lim\limits_{x\to -1^-} f(x)=\lim\limits_{x\to -1^+} f(x)=f(-1)=1$

$\therefore$ Continuous at $x = -1$

$f'(x)=\begin{cases}0, & -1<x<1\\ 2x, & x<-1 \text{ or } x>1\end{cases}$

At $x = 1$

LHD = 0

RHD = 2

LHD $\neq$ RHD

$\therefore$ Non-differentiable at $x = 1$

At $x = -1$

LHD = $\lim\limits_{x\to -1^-} 2x=-2$

RHD = $\lim\limits_{x\to -1^+} 0=0$

LHD $\neq$ RHD

$\therefore$ Non-differentiable at $x = -1$

$\therefore$ f is continuous everywhere but non-differentiable at $x = \pm 1$

22. Let T_1 denote an equilateral triangle of side length a, T_2 be the triangle whose vertices are midpoints of sides of T_1, T_3 be the triangle whose vertices are midpoints of sides of T_2. We continue in similar manner to obtain the triangles T_4, T_5, …. For each natural number n, let P_n denote the perimeter of the triangle T_n. Then $\sum_{n=1}^{\infty} P_n =$

(a) $12a$ (b) $8a$

(c) $6a$ (d) $3a$

Answer (c)

Sol. Side length of triangle T_1 is a

Perimeter, $P_1 = 3a$

Side length of triangle T_2 is $\frac{a}{2}$

$\therefore\ P_2 = \dfrac{3a}{2}$

Similarly, Side length of triangle T_3 is $\dfrac{a}{4}$

$\therefore\ P_3 = \dfrac{3a}{4}$

$P_1, P_2, P_3.... = 3a, \dfrac{3a}{2}, \dfrac{3a}{4},...$

$\sum_{n=1}^{\infty} P_n = S = \dfrac{3a}{1-\frac{1}{2}} = \dfrac{3a}{1/2}$

$= 6a$

23. $\sqrt{i} + \sqrt[3]{i^2} = x + iy$, where $i = \sqrt{-1}$. $(x, y) = ?$

(a) (–0.293, 1.573) (b) (–0.293, 0.707)

(c) (–0.207, 1.573) (d) (1.207, 1.573)

Answer (d)

Sol. $\sqrt{i} + (i^2)^{1/3} = \sqrt{i} + (-1)^{1/3} = \left(e^{i\frac{\pi}{2}\cdot\frac{1}{2}}\right) + \left(e^{i\pi\cdot\frac{1}{3}}\right) = e^{i\frac{\pi}{4}} + e^{i\frac{\pi}{3}}$

$\Rightarrow\quad x = \cos\dfrac{\pi}{4} + \cos\dfrac{\pi}{3} \approx 1.207$

$y = \sin\dfrac{\pi}{4} + \sin\dfrac{\pi}{3} \approx 1.573$

24. A standard parabola $x^2 = 36y$ is approximated as an arc of a circle for small values of x. What will be the radius of that circle?

(a) 72 (b) 36

(c) 18 (d) 9

Answer (c)

Sol. Radius of curvature, $R = \dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}$

Curve is $x^2 = 36y$

$y = \frac{1}{36}x^2 \Rightarrow \frac{dy}{dx} = \frac{x}{18}$ and $\frac{d^2y}{dx^2} = \frac{1}{18}$

For small value of x : $x = 0$

$\left.\frac{dy}{dx}\right|_{x=0} = 0$

$\Rightarrow R = \frac{(1+0^2)^{3/2}}{1/18} = 18$

Option (c) is correct.

25. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\, dx =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) 0 (d) π

Answer (b)

Sol. $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ … (1)

$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$

$I = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$ …(2)

Adding (1) and (2)

$2I = \int_0^{\pi/2} dx$

$2I = \frac{\pi}{2} \Rightarrow \boxed{I = \frac{\pi}{4}}$

Option (b) is correct.

26. Let $f(x)$ be continuous on $[0, \pi]$ and $f(x) + f(\pi - x) = \pi$. Then $\int_0^{\pi} f(x)\, dx =$

(a) π^2 (b) $\frac{\pi}{2}$

(c) $\frac{\pi^2}{2}$ (d) $\frac{\pi^2}{4}$

Answer (c)

Sol. $f(x) + f(\pi - x) = \pi$

$$I = \int_0^{\pi} f(x)dx \quad \ldots (1)$$

$$I = \int_0^{\pi} f(\pi - x)dx \quad \ldots (2)$$

Adding (1) and (2)

$$2I = \int_0^{\pi} (f(x) + f(\pi - x))dx$$

$$2I = \int_0^{\pi} \pi dx$$

$$2I = \pi^2$$

$$\boxed{I = \frac{\pi^2}{2}}$$

Option (c) is correct.

27. $\lim\limits_{x \to 3} \dfrac{\sqrt{1 - \cos 2(x-3)}}{x-3}$

(a) Exists and it equals $\sqrt{2}$

(b) Exists and it equals $-\sqrt{2}$

(c) Does not exist because $(x - 1) \to 0$

(d) Does not exist because left hand limit is not equal to right hand limit

Answer (d)

Sol. $\lim\limits_{x \to 3} \dfrac{\sqrt{1 - \cos(2)(x-3)}}{x-3} = \dfrac{\sqrt{2\sin^2(x-3)}}{(x-3)}$

$$\lim_{x \to 3} \frac{\sqrt{2}|\sin(x-3)|}{x-3}$$

$$\lim_{x \to 3^+} \frac{\sqrt{2}\sin(x-3)}{x-3} = \sqrt{2}$$

$$\lim_{x \to 3^-} -\sqrt{2}\frac{\sin(x-3)}{(x-3)} = -\sqrt{2}$$

$\Rightarrow$ limit does not exist

as LHL $\neq$ KHL.

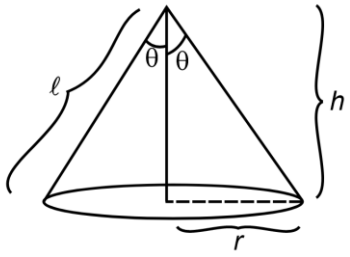
28. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{3}{4}\right)$. Water is poured into it at a constant rate of $5m^3$/min. Then, the rate (in m/min) at which the water surface moves along the slant surface at the instant when the depth of water in the tank is 10 m is

(a) $\frac{1}{9\pi}$ (b) $\frac{9}{192\pi}$

(c) $\frac{1875}{4\pi}$ (d) $\frac{4}{27\pi}$

Answer (a)

Sol.



$\tan\theta = \frac{3}{4}$

$\tan\theta = \frac{3}{4} = \frac{r}{h} \quad \Rightarrow r = \frac{3}{4}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\frac{9}{16}h^2 \cdot h$

$V = \frac{3}{16}\pi h^3$

$\frac{dv}{dt} = \frac{9}{16}\pi h^2 \frac{dh}{dt} = 5 \quad \left\{\because \frac{dv}{dt} = 5\right\}$

$\frac{dh}{dt} = \frac{80}{9\pi h^2} \qquad \left.\frac{dh}{dt}\right]_{h=10} = \frac{8}{90\pi}$

Now $\cos\theta = \frac{h}{\ell}$

$\ell = \frac{h}{\cos\theta}$

$\frac{1}{\cos\theta}\frac{dh}{dt} = \frac{d\ell}{dt}$

$\because \tan\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{4}{5}$

$$\frac{8}{90\pi\cos\theta} = \frac{d\ell}{dt}$$

$$\frac{8\times5}{90\pi\times4} = \frac{d\ell}{dt}$$

$$\frac{d\ell}{dt} = \frac{1}{9\pi}$$

29. How many people should there be in a group such that there is more than half the probability, that two people from the group have their birthdays with the same date, irrespective of in which months they were born?

(a) 3 or more (b) 4 or more

(c) 7 or more (d) 23 or more

Answer (c)

Sol. Let there are n-persons in a group

P(atleast 2 share birthday on same day) = 1 – *P*(all distinct day birthday)

$$= 1 - \frac{31.30...(31-n+1)}{31^n} > \frac{1}{2}$$

$\Rightarrow \approx n = 7$

30. *A* and *B* alternatively toss a coin. The one who gets a head first wins. If *A* starts the game, then what is the probability that *A* wins?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{3}{5}$

Answer (b)

Sol. *A* = *A* wins the match.

B = *B* wins the match.

$$P(A) = \frac{1}{2} \qquad P(B) = \frac{1}{2}$$

$$P(A \text{ wins}) = P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})(\bar{B})P(A)...$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + ...$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

31. Consider the expression $Z = 1 + 2^2 + (3^3)^3 + ((4^4)^4)^4 + \cdots$. From which term onwards, the total value of the expression Z exceeds 10^{1000}?

(a) 4 (b) 6

(c) 8 (d) 10

Answer (b)

Sol. $T_n = (n)^{(n)^{n-1}}$

$S_n = T_1 + T_2 + T_3 + \ldots T_N > 10^{1000}$

$T_5 = 5^{5^4} = 5^{625}$

$\log_{10} T_5 = 625 \log_{10} 5 \approx 437 < 1000$

$\log_{10} T_6 = \log\left(6^{6^5}\right) = 7776 \log 6 \approx 6050 > 1000$

$\Rightarrow T_6 > 10^{1000} \Rightarrow T_5 + T_6 > 10^{1000}$

$\therefore$ T_6 onwards the sum exceeds $(10)^{1000}$

32. Let $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$. Which of the following statements is not true?

(a) A is symmetric

(b) If $a + b + c = 0$ then $det\ A = 0$

(c) $det\ A = a^3 + b^3 + c^3 - 3abc$

(d) $det\ A = det\ A^T$

Answer (c)

Sol. $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

A is a symmetric matrix

$$\det(A) = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$|A| = 0$ if $a + b + c = 0$

$det(A) = det(A^T)$

$det(A) = -a^3 - b^3 - c^3 + 3abc$

$\therefore$ Option (c) is incorrect

33. It has been four hours since Regulus (10h 08m, +11°58′) has crossed the local meridian at Mumbai (19°2′11.11″*N,* 72°51′34.09″ *E*). Which of the following stars will be closest to the meridian now?

(a) Arcturus (14h 16m 50.74s, 19°02′.8″) (b) Sirius (6h 46m 15.1s, –16°44′.6″)

(c) Betelgeuse (5h 56m 31.86s, 19°02′.8″) (d) Spica (13h 26m 32.96s, –11°17′.44.7″)

Answer (a)

Sol. It's been 4 hr. Since Regulus crossed the Meridian.

So, current LST = 4 h + (RA) = 10 h + 08 m + 4 h = 14 h + 08 m

So, correct among the given is Arcturus.

34. A star has an apparent magnitude of 10, and an absolute magnitude of 10. How many parsecs away from the Earth is it?

(a) 100 (b) 10

(c) 1 (d) 0.1

Answer (b)

Sol. $m - M = \sum \log_{10}\left(\frac{d}{10pc}\right)$

So, $\left(\frac{d}{10pc}\right) = 1$

$\Rightarrow d = 10\ pc$

35. Which of the following statement or statements about stars are true?

(a) Among all one solar mass stars, the one with the largest radius is also the hottest.

(b) Type I supernovae are characterized by the absence of Hydrogen in the spectrum.

(c) Cooler stars show less absorption lines in their spectra.

(d) The stars are spectrally classified as O, B, A, F, G, K, M; with O type stars showing dense hydrogen line while M type stars having very less hydrogen lines.

Answer (b)

Sol. Type I supernova happens because of reaching Chandrashekhar limit.

36. As seen from Earth, angular separation between Proxima Centauri and Alpha Centauri is 2.2°. What is the physical separation between the two stars?

(a) 4.25 light years (b) 0.16 light years

(c) 2.2 light years (d) 0.33 light years

Answer (b)

Sol. $\theta = \frac{l}{r}$

$\Rightarrow \quad l = r\left(\frac{2.2 \times \pi}{180}\right)$

We know distance between Earth and alpha century is around 4.37 light year. So,

$l = \left(\frac{2.2 \times \pi}{180}\right) \times 4.37 \approx 0.167$ light year

37. Two stars in a binary system are separated by 3.0 AU and have mass ratio of 2 : 1. Their orbital period is 6.0 years. What are the masses of the stars in terms of solar mass?

(a) 2.0, 1.0 (b) 0.50, 0.25

(c) 2.25, 1.125 (d) 1.78, 0.88

Answer (b)

Sol. Let the masses be M_1 and M_2 then $T^2 = \frac{4\pi^2}{G(M_1 + M_2)} \cdot a^3$

$\Rightarrow$ We know that $T_0^2 = \frac{4\pi^2}{GM_\odot}(1\text{ Au})^3$

$\Rightarrow \quad \left(\frac{T}{T_0}\right)^2 = \frac{a^3 M_\odot}{(M_1 + M_2)(1\text{ Au})^3}$

$\Rightarrow \quad 36 = 27.\frac{M_\odot}{(M_1 + M_2)}$

$\Rightarrow \quad 4(M_1 + M_2) = 3M_\odot$

Also, $M_1 = 2M_2$

So, $M_2 = \frac{M_\odot}{4}$ and $M_1 = \frac{M_\odot}{2}$

$\Rightarrow \quad M_2 = 0.25\, M_\odot$ and $M_1 = 0.50 M_\odot$

38. A comet's closest approach to Sun is at 1 AU. What is the radial component of its velocity at this position?

(a) 0 km/s (b) 21.2 km/s

(c) 30 km/s (d) 42.4 km/s

Answer (a)

Sol. At closest approach. $\bar{r} \perp \bar{v}$.

39. A new space station orbits the sun every four and a half years. In a particular year, It is seen on the local meridian at 1:00 am of the 21st of June, 2020. It will be again seen on the local meridian from the Earth, approximately, on

(a) 21 June, 2024, at 1:00 am (b) 20 December, 2024, at 4:00 pm

(c) 3 October, 2021, 4:00 pm (d) 21 March, 2021, 9:00 pm

Answer (c)

Sol. $T_{(sf)} = 4.5$ years

So, in $T_{(sf)}$ it traverse 2π rad.

So, in 1 year = $\frac{360 \times 10}{45} = 80°$

So, in 1 year the station will be 80° ahead of earth (w.r.t. Sun).

Now to catch space station earth has to move 80° + extra distance travelled by space station.

So, it takes $\frac{12 \times 80}{360}$ = month.

$\Rightarrow$ 2 month + 20 days + 1 month

40. What is the advantage of an equatorial telescope mount over an alt-azimuth mount?

(a) Reduced vibrations and provides a more stable viewing platform.

(b) It allows tracking celestial objects using only one axis of motion.

(c) It eliminates the need for polar alignment before observing.

(d) It is easier to carry and transport due to its lightweight design.

Answer (b)

Sol. An equatorial mount is aligned with earth's rotation axis. So once aligned you track a star by rotating only the right ascension axis at a steady rate.

41. Which is the following statements is correct about constellations?

(a) Any star cannot belong to two constellations simultaneously.

(b) Only the bright stars which are imagined as some figure in the sky make constellations.

(c) Brightest star in any constellation has magnitude 1.

(d) There are only 12 constellations along the Ecliptic belt.

Answer (a)

Sol. Any star cannot belong to two constellations simultaneously.

42. What would be the speed of a comet, on a parabolic orbit around the sun, whose point of closest approach is 1 AU, when at a distance of 4.0 AU?

(a) 42.1 km/s (b) 29.8 km/s

(c) 21.1 km/s (d) 84.4 km/s

Answer (c)

Sol. For parabolic path

$$v = \sqrt{\frac{2GM}{r}}$$

$$= v = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{4 \times 1.5 \times 10^{11}}} = 21.087 \text{ km/s}$$

43. Globular clusters are typically found in the halo of Milky Way. In which of the following constellation, there are higher chances of seeing globular cluster?

(a) Orion (b) Sagittarius

(c) Ursa Minor (d) Virgo

Answer (d)

Sol. Globular clusters lie in the halo of the Milk way, and we can see them best out of the galactic plane.

The constellation far from milky way plane in the list is Virgo.

44. Ecliptic plane makes approximately 60° with the Milky Way plane. One point of intersection lies in the constellation of Sagittarius, in which constellation does the other intersection point lie?

(a) Aquarius (b) Libra

(c) Pieces (d) Gemini

Answer (d)

Sol. The opposite intersection is 180° away along the ecliptic so it is Gemini.

45. Which of the following places will have minimum duration between the two zero shadow days in a given calendar year?

(a) Manila (14° 36′ N, 120° 59′ E) (b) Monteiro (7° 53′ S, 37° 7′ W)

(c) Kansanshi (12° 6′ S, 26° 26′ W) (d) Barah (13° 42′ N, 30° 22′ E)

Answer (a)

Sol. Minimum duration between two zero shadow day occurs closet to equator. It will cross the point reaches a tropic and return to the point. So the points difference with the tropic will be minimum. So it is Manila (14° 36′).

46. An observer from Delhi, will see the Sun on the local meridian 365 times in the year 2025. A star, located on the celestial equator, will be seen how many times on the local meridian by the same observer in 2025?

(a) 364 (b) 365

(c) 366 (d) 367

Answer (c)

Sol. For a star in celestial equator, the number of culminations in a year = Number of sidereal day in the year = 366.

47. Considering the nuclear reactions that power the energy output of the stars, the correct statements is:

(a) the p-p chain is the dominant process that creates He in very massive stars.

(b) the fusion of H into He is an exothermic process.

(c) the CNO (Carbon-Nitrogen-Oxygen) cycle results in the creation of the elements like Si, S and P following the fusion of the lighter elements.

(d) the fusion process is replaced by the fission process in heavier stars.

Answer (b)

Sol. H $\rightarrow$ He fusion is exothermic.

48. Which of the following constellations is broken in two disjoint parts in the sky?

(a) Draco (b) Ursa

(c) Serpens (d) Eridanus

Answer (c)

Sol. A constellation broken into two disjoint parts = Serpens.

A – 2

ANY NUMBER OF OPTIONS (4, 3, 2 or 1) MAY BE CORRECT

MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED AND NO INCORRECT

49. Inside a cylindrical well, at the bottom and touching the wall, a red ball is thrown at an angle of 45° to the horizontal towards the diametrically opposite end of the wall, and it hits the wall after a time interval of $\frac{v\sqrt{2}}{g}$, where v is magnitude of velocity of the red ball and g is acceleration due to gravity. A black ball identical in shape and mass to the red ball is thrown vertically upwards from the bottom of the well with a kinetic energy half of that of the red ball. A green ball having a mass half of the red ball is thrown from the bottom of the well but diametrically opposite to the red ball with a kinetic energy half of that of the red ball. All the balls, if and when they hit the wall, undergo completely elastic collision.

(a) The red ball bounces off the bottom of the well for the first time when it hits the wall the third time

(b) The black ball will take double the time to hit the bottom of the well to that of the red ball

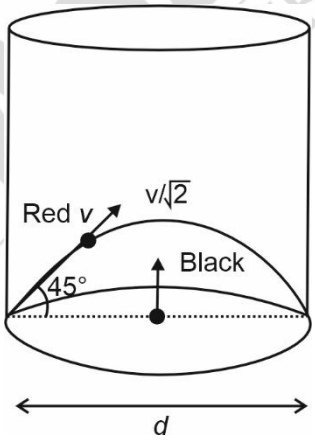
(c) The green ball hits the bottom of the well at the same spot as the red ball

(d) The green ball will take the same time as the red ball to hit the bottom of the well the first time

Answer (c, d)

Sol. For red ball

$v_x t = x$



$$\frac{v}{\sqrt{2}} \times \frac{v\sqrt{2}}{g} = d$$

$$d = \frac{v^2}{g}$$

as t = Time of flight

d is range as well

For green ball

$$kg = \frac{1}{2}kR$$

$$\frac{1}{2}\frac{m}{2}v_g^2 = \frac{1}{2}\frac{1}{2}mv^2$$

$$\boxed{v_g = v}$$

(a) The red ball collides wall and bottom for first time only

(b) Vertical component of red and black are same, so T is same.

(c) $\xrightarrow[m]{v/\sqrt{2}}$ $\xleftarrow[m/2]{v/\sqrt{2}}$

(d) For green and red speed are same.

50. Read the two statements, I and II, about a RLC series circuit driven by an AC voltage source using an inductor having internal resistance r. Assume that the maximum amplitude of ac signal is v_m and frequency is F:

I: When frequency F is equal to the resonance frequency of the circuit, potential difference across the series combination of L and C has a non-zero amplitude.

II: When frequency F is equal to the resonance frequency of the circuit, current in the circuit has amplitude less than v_m/R.

Select correct statement(s) from the following:

(a) If statement I is correct, statement II has to be correct.

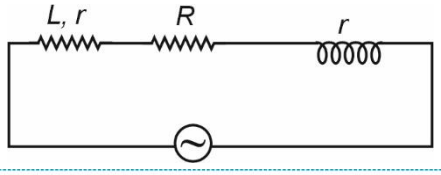
(b) If statement II is false, statement I cannot be false.

(c) Statement II is false for most of the RLC series circuits with ac voltage source.

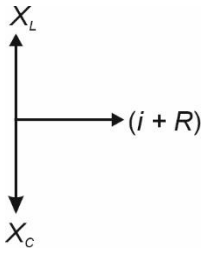
(d) Statement I is correct for most of the RLC series circuits with ac voltage source.

Answer (a, d)

Sol.



$$i = \frac{\varepsilon}{\sqrt{(R+r)^2 + (X_L - X_c)^2}}$$



$V_{L,C} = V_L + V_C + V_r$

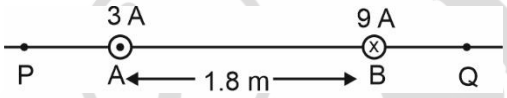
. but $V_L + V_C = 0$

Therefore $V_{L,C} = V_r$ non-zero

At resonance $i = \frac{V}{(R+r)} < \frac{V}{R}$

As generally resistance of inductor is non-zero.

51. Two current carrying wires A and B are held fixed, parallel to each other, at a distance of 1.8 m. A current of 3 A flows through wire A in a direction, coming out of the plane of paper and that through the wire B is 9 A going in to the plane of paper as shown.



Which of the following statement(s) is/are true with reference to the given situation?

(a) There can be a point on the left of wire A (PA side) where net magnetic field is zero.

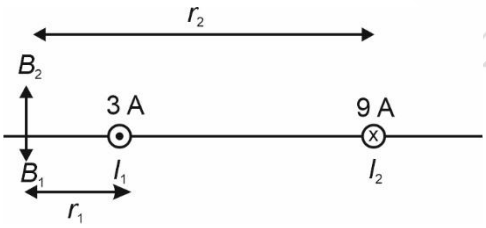
(b) There can be a point on the right of wire B (BQ side) where net magnetic field is zero.

(c) There can be a point between the wires A and B, closer to A, where magnetic field produced by the two wires will be in the same direction having equal magnitude.

(d) There can be a point Q on the right of wire B (BQ side) where net magnetic field is in the upward direction parallel to the plane of paper.

Answer (a, c)

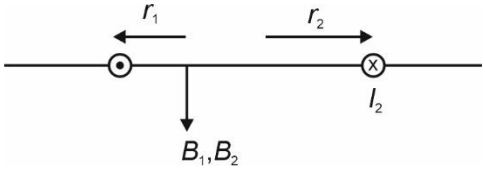
Sol.



(a) $B_1 = B_2$ $r_1 = r_1$ $B = 0$

(b) in right of B, $B_2 > B_1$ so $B_{net} \neq 0$

(c)



If $r_2 = 3r_1$

$B_1 = B_2$

(d) in right of $B_2 > B_1$, so always downward.

52. Two black bodies A and B are emitting in the approximate ratio 2 : 5. Which of the following statements may be correct from the given information?

(a) Body B is 20% hotter and 10% larger in diameter than body A.

(b) Body B is 50% hotter but half in diameter than body A.

(c) Body B is 10% hotter and 30% larger in diameter than body A.

(d) Body B has double the temperature of A and only 40% diameter of body A.

Answer (a, d)

Sol. : $p = \sigma e A T^4$

$$\frac{A_1 T_1^4}{A_2 T_2^4} = \frac{2}{5}$$

(a) $$\frac{AT^4}{(1.1)^2 (1.2)^4 AT^4} = 0.4$$

(b) $$\frac{AT^4}{\frac{A}{4}(1.5\,T)^4} = 0.8$$

(c) $$\frac{AT^4}{(1.3)^2 A(1.1T)^4} = 0.31$$

(d) $$\frac{AT^4}{(0.16\,A)(2\,T)^4} = 0.4$$

53. Let $\delta = \begin{vmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x^2 & 1 & x \end{vmatrix}$. Which of the following statements is not true?

(a) δ is an even function

(b) $\delta = 0$ for all real x

(c) δ is a polynomial of degree 6

(d) If $x = \sqrt[3]{1}$, $\delta = 0$ for all roots

Answer (a, b)

Sol. $\delta = \begin{vmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x^2 & 1 & x \end{vmatrix} = f(x)$ (let)

$= -x^3 - x^6 - 1 + 3x^3$

$= -\left(1+x+x^2\right)\left(1+x^2+x^4-x-x^2-x^3\right)$

$= -\left(1+x+x^2\right)\left(x^4-x^3-x+1\right)$

$= -(1+x+x^2)(x^3(x-1)-(x-1))$

$= -\left(1+x+x^2\right)\left(x^3-1\right)(x-1)$

$f(x)$ is not an even function as $f(x) \neq f(-x)$

$f(x) = 0$ for $x = 1$

$f(x)$ is of degree 6

54. Which of the following functions are periodic?

(a) $\sin x + \cos x$ (b) $\tan x$

(c) $x \sin x$ (d) $\sin x + \sin\sqrt{2}x$

Answer (a, b)

Sol. (a) $\sin(x+2\pi)+\sin(x+2\pi) = \sin x + \cos x \Rightarrow$ periodic

(b) $\tan(\pi + x) = \tan x \Rightarrow$ periodic

(c) $x \sin x = (x+T)\sin(x+T)$

Notice that: $f\left(\left(2n+\frac{1}{2}\right)\pi\right) = \left(2n+\frac{1}{2}\right)\pi \sin\left(2n\pi+\frac{\pi}{2}\right) = \left(2n+\frac{1}{2}\right)\pi$

Since for integer n, $\left(2n+\frac{1}{2}\right)\pi$ is unbounded, hence not periodic.

(d) $\sin x + \sin(\sqrt{2}x) \Rightarrow$ Since differentiation of periodic function is periodic, if for sake of contradiction, assume, $\cos x + \sqrt{2}\cos(\sqrt{2}x)$ is periodic.

$\Rightarrow$ Maximum value at zero and also at when $\cos x = 1$, $\cos(\sqrt{2}x) = 1$, $x \neq 0$

$\Rightarrow 2\pi K = \frac{2\pi m}{\sqrt{2}}$, for $m, n \in I \Rightarrow$ Contradiction.

$\Rightarrow$ Not periodic.

55. If $x+\frac{1}{x}=2$, then which of the following are true?

(a) $x^2+\frac{1}{x^2}=2$ (b) $x^3+\frac{1}{x^3}=2$

(c) $x=1$ (d) $x^4+\frac{1}{x^4}=2$

Answer (a, b, c, d)

Sol. $x+\frac{1}{x}=2 \Rightarrow x^2-2x+1=0,\ x\neq 0$

$\Rightarrow (x-1)^2=0$

$\Rightarrow x=1$

$x^2+\frac{1}{x^2}+2=4 \Rightarrow x^2+\frac{1}{x^2}=2$

$x^3+\frac{1}{x^3}=1+1=2,$

$x^4+\frac{1}{x^4}=2$

56. Which of the following series are not convergent?

(a) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)$ (b) $\sum_{n=1}^{\infty}\left(3+\left(\frac{2}{3}\right)^n\right)$

(c) $\sum_{n=1}^{\infty} n^{1/n}$ (d) $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

Answer (a,b,c)

Sol. (a) Harmonic sequence is unbounded hence not convergent.

(b) $\sum_{n=1}^{\infty}(3)$ diverges.

(d) $\sum_{n=1}^{\infty}\frac{(n+1)-n}{n(n+1)}=\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=1 \Rightarrow$ Convergent

(c) Since $\lim_{n\to\infty} n^{1/n}=1\neq 0 \Rightarrow$ Series diverges.

57. The Pole star

(a) can be seen at night from all locations on the Earth.

(b) will be visible during a solar eclipse from nearly the whole of the northern hemisphere.

(c) will be visible during a solar eclipse from the equator.

(d) can, in principle, be observed from India anytime during the day and night.

Answer (b,c,d)

Sol. Polar star is circumpolar for northern hemisphere. So principally it is visible all the time in day and night if sun's glare can be reduced. For equator it will be on horizon. So it will be visible during solar eclipse.

58 An observer measures the location of the Sun, moon, planets, and bright stars like Sirius, very diligently.

The correct conclusions that she may reach is/are

(a) Planets rise and set at the same time as per the sidereal clock but not as per the solar clock for all days of the year.

(b) The Sun rises and sets at the same time as per the solar clock and not as per the sidereal clock for all days of the year.

(c) The stars rise and set at the same time as per the sidereal clock but not as per the solar clock for all days of the year.

(d) The moon rises and sets at different times as per the sidereal clock and also as per the solar clock for all days of the month.

Answer (b, c, d)

Sol. Sun rises and sets approximately at the same time as per solar clock. Stars appears at same location at same side real times.

59. Which of the following stars is/are circumpolar in Warsaw (52°14′N 21°01′E)?

(a) α Cygni (16h 41m, +31°36′)

(b) β Böotis (15h 01m, +40°23′)

(c) θ Aurigae (5h 59m, +37°12′)

(d) γ Draconis (17h 56m, +51°26′)

Answer (b, d)

Sol. To be a circumpolar star in northern hemisphere

$\delta \geq 90° - \phi$

Here $90 - \phi = 90 - 52.23 = 37.77°$

So for any star $\delta \geq 37.77°$ is circumpolar in Warsaw.

For (a) α Cygni

$31{\cdot}6° < 37.77$

(b) β Bootis ($40°.23 > 37.77$)

(c) θ Aurigae ($37°.12 < 37.77$)

(d) γ Draconis ($51.43° > 37.77°$)

60. Star A rises half an hour before star B and it sets half an hour after star B from a particular location. Which of the following statement(s) is/are correct?

(a) The location is on equator.

(b) The location is in southern hemisphere and star A is more south than star B.

(c) The location is in northern hemisphere and star A is more north than star B.

(d) Both stars have the same right ascension.

Answer (b, c, d)

Sol. Al equator all stars rise and set 12 hr. apart.

For more south → declination more negative so it will stay above horizon for longer time.

For more north → declination more positive so it will stay above horizon for longer time.

Both star may have same right ascension.

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