Aakash
Medical | IIT-JEE | Foundations
Corporate Office : *AESL, 3rd Floor, Incuspaze Campus-2, Plot No. 13, Sector-18, Udyog Vihar, Gurugram, Haryana - 122015*

Previous Year RMO Questions

1. Let N be the set of all positive integers and $S = \{(a, b, c, d) \in N^4 : a^2 + b^2 + c^2 = d^2\}$. Find the largest positive integer m such that m divides $abcd$ for all $(a, b, c, d) \in S$. **[2023]**
2. Let ω be a semicircle with AB as the bounding diameter and let CD be a variable chord of the semicircle of constant length such that C, D lie in the interior of the arc AB. Let E be a point on the diameter AB such that CE and DE are equally inclined to the line AB. Prove that
   (a) The measure of $\angle CED$ is a constant
   (b) The circumcircle of triangle CED passes through a fixed point. **[2023]**
3. For any natural number n, expressed in base 10, let $s(n)$ denote the sum of all its digits. Find all natural numbers m and n such that $m < n$ and $(s(n))^2 = m$ and $(s(m))^2 = n$. **[2023]**
4. Let Ω_1, Ω_2 be two intersecting circles with centres O_1, O_2 respectively. Let l be a line that intersects Ω_1 at points A, C and Ω_2 at points B, D such that A, B, C, D are collinear in that order. Let the perpendicular bisector of segment AB intersect Ω_1 at points P, Q; and the perpendicular bisector of segment CD intersect Ω_2 at points R, S such that P, R are on the same side l. Prove that the midpoints of PR, QS and O_1O_2 are collinear. **[2023]**
5. Let $n > k > 1$ be positive integers. Determine all positive real numbers $a_1, a_2, \ldots, a_n$ which satisfy
$$\sum_{i=1}^{n} \sqrt{\frac{ka_i^k}{(k-1)a_i^k + 1}} = \sum_{i=1}^{n} a_i = n$$ **[2023]**
6. Consider a set of 16 points arranged in a 4 × 4 square grid formation. Prove that if any 7 of these points are coloured blue, then there exists an isosceles right-angled triangle whose vertices are all blue. **[2023]**
7. Let $n > 1$ be a positive integer. Call a rearrangement $a_1, a_2, \ldots, a_n$ of 1, 2, …, n nice if for every $k = 2, 3, \ldots, n$, we have that $a_1 + a_2 + \ldots + a_k$ is not divisible by k.
   (a) If $n > 1$ is odd, prove that there is no nice rearrangement of 1, 2, …, n.
   (b) If n is even, find a nice rearrangement of 1, 2, …, n. **[2024]**
8. For a positive integer n, let $R(n)$ be the sum of the remainders when n is divided by 1, 2, …, n. For example $R(4) = 0 + 0 + 1 + 0 = 1$, $R(7) = 0 + 1 + 1 + 3 + 2 + 1 + 0 = 8$. Find all positive integers n such that $R(n) = n - 1$.

9. Let ABC be an acute triangle with $AB = AC$. Let D be the point on BC such that AD is perpendicular to BC. Let O, H, G be the circumcenter, orthocenter and centroid of triangle ABC respectively. Suppose that $2 \cdot OD = 23 \cdot HD$. Prove that G lies on the incircle of triangle ABC. **[2024]**

10. Let a_1, a_2, a_3, a_4 be real numbers such that $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1$. Show that there exist i, j with $1 \le i < j \le 4$, such that $\left(a_i - a_j\right)^2 \le \frac{1}{5}$. **[2024]**

11. Let $ABCD$ be a cyclic quadrilateral such that AB is parallel to CD. Let O be the circumcenter of $ABCD$ and L be the point on AD such that OL is perpendicular to AD. Prove that $OB \cdot (AB + CD) = OL \cdot (AC + BD)$. **[2024]**

12. Let $n \ge 2$ be a positive integer. Call a sequence $a_1, a_2, \ldots, a_k$ of integers an n-chain if $1 = a_1 < a_2 < \ldots < a_k = n$ and a_i divides a_{i+1} for all i, $1 \le i \le k - 1$. Let $f(n)$ be the number of n-chains where $n \ge 2$. For example, $f(4) = 2$ corresponding to the 4-chain {1, 4} and {1, 2, 4}.
Prove that $f(2^m \cdot 3) = 2^{m-1}(m + 2)$ for every positive integer m. **[2024]**

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