Time : 3 hrs.

Questions & Answers

for

JEE (Advanced)-2024 (Paper-2)

Max. Marks: 180

PART-I : MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - 2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right) \text{ is}$$

(A) $\dfrac{7}{24}$
(B) $\dfrac{-7}{24}$

(C) $\dfrac{-5}{24}$
(D) $\dfrac{5}{24}$

Answer (B)

2. Let $S = \left\{(x,y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x \text{ and } 3y + \sqrt{8}x \leq 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

(A) $\dfrac{17}{2}$

(B) $\dfrac{17}{3}$

(C) $\dfrac{17}{4}$

(D) $\dfrac{17}{5}$

Answer (B)

3. Let $k \in \mathbb{R}$. If $\lim\limits_{x \to 0+} \left(\sin(\sin kx) + \cos x + x\right)^{\frac{2}{x}} = e^6$, then the value of k is

(A) 1

(B) 2

(C) 3

(D) 4

Answer (B)

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{\pi}{x^2}\right) & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

(A) $f(x) = 0$ has infinitely many solutions in the interval $\left[\dfrac{1}{10^{10}}, \infty\right)$.

(B) $f(x) = 0$ has no solutions in the interval $\left[\dfrac{1}{\pi}, \infty\right)$.

(C) The set of solutions of $f(x) = 0$ in the interval $\left(0, \dfrac{1}{10^{10}}\right)$ is finite.

(D) $f(x) = 0$ has more than 25 solutions in the interval $\left(\dfrac{1}{\pi^2}, \dfrac{1}{\pi}\right)$.

Answer (D)

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: + 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

5. Let S be the set of all $(\alpha, \beta) \in R \times R$ such that

$$\lim_{x \to \infty} \frac{\sin(x^2)(\log_e x)^\alpha \sin\left(\dfrac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^\beta} = 0$$

Then which of the following is (are) correct?

(A) $(-1, 3) \in S$ (B) $(-1, 1) \in S$

(C) $(1, -1) \in S$ (D) $(1, -2) \in S$

Answer (B, C)

6. A straight line drawn from the point $P(1,3,2)$, parallel to the line $\dfrac{x-2}{1} = \dfrac{y-4}{2} = \dfrac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R. Then which of the following statements is(are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are $(1,6,3)$

(C) The centroid of the triangle PQR is $\left(\dfrac{4}{3}, \dfrac{14}{3}, \dfrac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$

Answer (A, C)

7. Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If $O = (0,0)$ and $C_1 = (-4,0)$, then which of the following statements is (are) TRUE?

(A) The length of the line segment OA_1 is $4\sqrt{3}$

(B) The length of the line segment A_1B_1 is 16

(C) The orthocentre of the triangle $A_1B_1C_1$ is $(0, 0)$

(D) The orthocentre of the triangle $A_1B_1C_1$ is $(1, 0)$

Answer (A, C)

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

Full Marks	:	+4	If **ONLY** the correct integer is entered;
Zero Marks	:	0	In all other cases.

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, and $g : \mathbb{R} \to (0, \infty)$ be a function such that $g(x + y) = g(x)\, g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\dfrac{-3}{5}\right) = 12$ and $g\left(\dfrac{-1}{3}\right) = 2$, then the value of

$$\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0) \text{ is } \underline{\qquad}.$$

Answer (51)

9. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let W_i, G_i, and B_i denote the events that the ball drawn in the ith draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \dfrac{2}{5N}$ and the conditional probability $P(B_3 \mid W_1 \cap G_2) = \dfrac{2}{9}$, then N equals $\underline{\qquad}$.

Answer (11)

10. Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}}\frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}}\frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}.$$

Then the number of solutions of $f(x) = 0$ in $\mathbb{R}$ is $\underline{\qquad}$.

Answer (1)

11. Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β and γ, we have $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$, then the value of γ is $\underline{\qquad}$.

Answer (2)

12. A normal with slope $\dfrac{1}{\sqrt{6}}$ is drawn from the point $(0, -a)$ to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -a)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r : s = 1 : 16$, then the value of $24a$ is _____.

Answer (12)

13. Let the function $f : [1, \infty) \to \mathbb{R}$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1}2, & \text{if } t = 2n - 1, n \in \mathbb{N}, \\ \dfrac{(2n+1-t)}{2}f(2n-1) + \dfrac{(t-(2n-1))}{2}f(2n+1), & \text{if } 2n - 1 < t < 2n + 1, n \in \mathbb{N}. \end{cases}$$

Define $g(x) = \displaystyle\int_1^x f(t)\,dt, x \in (1, \infty)$. Let α denote the number of solutions of the equation $g(x) = 0$ in the interval

$(1, 8]$ and $\beta = \displaystyle\lim_{x \to 1+} \dfrac{g(x)}{x-1}$. Then the value of $\alpha + \beta$ is equal to _____.

Answer (5)

SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.

- Based on each paragraph, there are **TWO (02)** questions.

- The answer to each question is a **NUMERICAL VALUE**.

- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.

- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

  Full Marks : +3 If **ONLY** the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

PARAGRAPH I

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$ and

$Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$.

Let $n(A)$ denote the number of elements in a Set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

14. If $n(X) = {}^mC_6$, then the value of m is _____.

Answer (20)

PARAGRAPH I

Let S = {1, 2, 3, 4, 5, 6} and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii.   For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let Y = {$R \in X$: The range of R has exactly one element} and

Z = {$R \in X$: R is a function from S to S}.

Let $n(A)$ denote the number of elements in a Set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

15.   If the value of $n(Y) + n(Z)$ is k^2, then $| k |$ is _____.

Answer (36)

PARAGRAPH II

Let $f : \left[0, \dfrac{\pi}{2}\right] \to [0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g : \left[0, \dfrac{\pi}{2}\right] \to [0,\infty]$ be the function defined by

$g(x) = \sqrt{\dfrac{\pi x}{2} - x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

16.   The value of $2\displaystyle\int_{0}^{\frac{\pi}{2}} f(x)g(x)\,dx - \displaystyle\int_{0}^{\frac{\pi}{2}} g(x)\,dx$ us _____.

Answer (0)

PARAGRAPH II

Let $f : \left[0, \dfrac{\pi}{2}\right] \to [0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g : \left[0, \dfrac{\pi}{2}\right] \to [0,\infty]$ be the function defined by

$g(x) = \sqrt{\dfrac{\pi x}{2} - x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

17.   The value of $\dfrac{16}{\pi^3}\displaystyle\int_{0}^{\frac{\pi}{2}} f(x)g(x)\,dx$ is _____.

Answer (0.25)