MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\dfrac{\sqrt{3}\,\text{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is

(1) 12 (2) 16

(3) 64 (4) 32

Answer (3)

Sol. $\therefore\ \sqrt{3}\,\text{cosec}20^\circ - \sec 20^\circ$

$= \dfrac{\sqrt{3}}{\sin 20^\circ} - \dfrac{1}{\cos 20^\circ}$

$= 4\cdot\dfrac{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{2\sin 20^\circ\cdot\cos 20^\circ}$

$= 4\cdot\dfrac{\sin 40^\circ}{\sin 40^\circ}$

$= 4$

and $\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 60^\circ\cdot\cos 80^\circ$

$= \cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ\cdot\cos 60^\circ$

$= \dfrac{1}{4}\cos 60^\circ\cdot\cos 60^\circ$

$\left\{\cos\left(\dfrac{\pi}{3}-\theta\right)\cdot\cos\theta\cdot\cos\left(\dfrac{\pi}{3}+\theta\right) = \dfrac{1}{4}\cos 3\theta\right\}$

$= \dfrac{1}{16}$

$\therefore\ \dfrac{\sqrt{3}\,\text{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ\cdot\cos 60^\circ} = \dfrac{4}{\frac{1}{16}}$

$= 64$

2. The number of solution for $x \in R$, $x|x-4| + |x-1| - 2 = 0$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (1)

Sol. The number of real roots of

$x|x-4| + |x-1| - 2 = 0$

1 4

If $x \geq 4 \Rightarrow x(x-4) + x - 1 - 2 = 0$

$x^2 - 3x - 3 = 0$

$\Rightarrow\ x = \dfrac{3 \pm \sqrt{21}}{2}$ $\Rightarrow$ both roots less than 4 $\Rightarrow$ so solution

If $1 \leq x < 4$

$\Rightarrow x(4-x) + x - 1 - 2 = 0$

$\Rightarrow x^2 - 5x + 3 = 0$

$\Rightarrow\ x = \dfrac{5 \pm \sqrt{13}}{2}$, both are not in interval

If $x \leq 1$

$\Rightarrow x(4-x) + 1 - x - 2 = 0$

$x^2 - 3x + 1 = 0$

$\Rightarrow\ x = \dfrac{3 \pm \sqrt{5}}{2}$

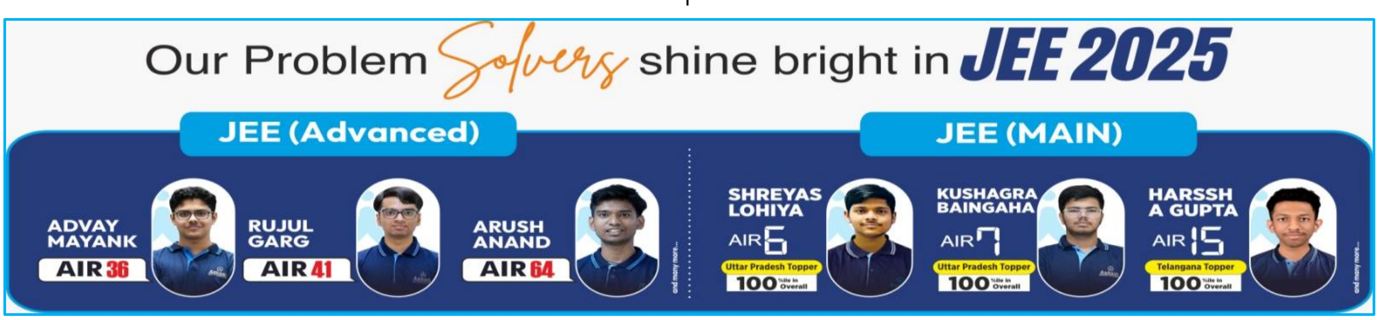
$\Rightarrow$ only one solution $x = \dfrac{3-\sqrt{5}}{2}$

3. Consider 10 data such that their mean is 10 and variance is 2. If one of the data α is removed and new data entry β is inserted. Now new mean is 10.1 and new variance is 1.99 then $(\alpha + \beta)$ is equal to

(1) 10 (2) 20

(3) 1 (4) 2

Answer (2)



Sol. $\Sigma x_i = 100$

$$\frac{\Sigma x_i^2}{10} - (10)^2 = 2$$

$$\Rightarrow \Sigma x_i^2 = 1020$$

$$\mu' = \frac{\Sigma(x_i) - \alpha + \beta}{10} \Rightarrow 100 - \alpha + \beta = 101$$

$$\Rightarrow \beta - \alpha = 1$$

$$\sigma' = \left(\frac{\Sigma x_i^2 - \alpha^2 + \beta^2}{10}\right) - \left(\frac{101}{10}\right)^2 = \frac{199}{100}$$

$$\Rightarrow \frac{1020 - \alpha^2 + \beta^2}{10} = \frac{199}{100} + \left(\frac{101}{100}\right)^2$$

$$= \frac{10400}{100} = 104$$

$$\Rightarrow 1020 - \alpha^2 + \beta^2 = 1040$$

$$\Rightarrow \beta^2 - \alpha^2 = 20$$

$$\beta - \alpha = 1$$

$$\Rightarrow (\beta + \alpha)(\beta - \alpha) = 20$$

$$\Rightarrow \alpha + \beta = 20$$

4. If $F(t) = \int \frac{1-\sin(\ln t)}{1-\cos(\ln t)} dt$ and $F(e^{\pi/2}) = -e^{\pi/2}$ then $F(e^{\pi/4})$ is

(1) $\left(-1-\sqrt{2}\right)e^{\pi/4}$

(2) $\left(1-\sqrt{2}\right)e^{\pi/4}$

(3) $\left(1+\sqrt{2}\right)e^{\pi/4}$

(4) $\left(-2-\sqrt{2}\right)e^{\pi/4}$

Answer (1)

Sol. $\int \frac{1-\sin(\ln t)}{1-\cos(\ln t)} dt$

Let $\ln t = x$

$$t = e^x$$

$$dt = e^x dx$$

$$\int e^x \frac{(1-\sin x)}{1-\cos x} dt$$

$$\Rightarrow \int e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$$

$$\Rightarrow \int e^x \left(\frac{1}{2} - \operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$$

$$\Rightarrow \int e^x \left[\underbrace{-\cot\frac{x}{2}}_{f(x)} + \underbrace{\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}}_{f'(x)}\right]$$

$$\Rightarrow -e^x \cot\frac{x}{2} + c$$

$$f(t) = -t\cot\left(\frac{\ln t}{2}\right) + c$$

$$f(e^{\pi/2}) = -e^{\pi/2} + c = -e^{-\pi/2}$$

$$\Rightarrow c = 0$$

$$f(e^{\pi/4}) = -e^{\pi/4}\cot\left(\frac{\pi}{8}\right)$$

$$= -e^{\pi/4}\left[\sqrt{2}+1\right]$$

5. Consider a sequence 729, 81, 9, 1, ...

Let P_n = product of first n terms of the given sequence

and $\sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = \frac{3^\alpha - 1}{2 \times 3^\beta}$.

Then the value of $\alpha + \beta$ is

(1) 73 (2) 75

(3) 76 (4) 81

Answer (1)

Sol. $3^6, 3^4, 3^2, 3^0, ...$

$P_n = 3^{6+4+2+\ldots\, n \text{ terms}}$

$$= 3^{\frac{n}{2}[2\times6+(n-1)(-2)]} = 3^{n(6-n+1)} = 3^{n(7-n)}$$

$$\Rightarrow \sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = \sum_{n=1}^{40}3^{7-n} = 3^7 \times \frac{1}{3}\left(\frac{1-\frac{1}{3^{40}}}{1-\frac{1}{3}}\right)$$

$$= 3^7\left(\frac{3^{40}-1}{2\times3^{40}}\right) = \frac{3^{40}-1}{2\cdot3^{33}}$$

$\Rightarrow \quad \alpha + \beta = 73$

6. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in $A.P.$ and given that $a_2 - a_1 = -\frac{3}{4}$ and $a_1 + a_2 + \ldots + a_n = \frac{525}{2}$ and $a_n = \frac{1}{4}a_1$. Then $\sum_{i=1}^{17} a_i$ is equal to

(1) 276 (2) 238

(3) 189 (4) 258

Answer (2)

Sol. $\because a_1, a_2, a_3, \ldots\ldots a_n$ are in $A.P.$

Given that $a_2 - a_1 = -\frac{3}{4}$ = common difference (d).

$\therefore d = -\frac{3}{4}$

and also given that $a_n = \frac{1}{4}a_1$

$$\therefore S_n = \frac{n}{2}(a_1 + a_n) = \frac{525}{2}$$

$$\therefore n\left(a_1 + \frac{1}{4}a_1\right) = 525$$

$\therefore a_1 n = 420$ ...(i)

Now $\frac{n}{2}\left\{2a_1 + (n-1)\left(-\frac{3}{4}\right)\right\} = \frac{525}{2}$

$8 \times 420 - 3n^2 + 3n = 2100$

$\therefore 3n^2 - 3n - 1260 = 0$

$\therefore n^2 - n - 420 = 0$

$(n - 21)(n + 20) = 0$

$\therefore n = 21$

$\therefore a_1 = 20$

$$\sum_{i=1}^{17} a_i = \frac{17}{2}\left\{40 + 16 \times -\frac{3}{4}\right\} = 238$$

7. Number of matrices A of order 3 × 2 such that all of its elements are from the set {–2, –1, 0, 1, 2} such that trace of AA^T is 5, is equal to

(1) 120 (2) 312

(3) 192 (4) 126

Answer (2)

Sol. $AA^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$= \begin{bmatrix} a^2+d^2 & - & - \\ - & b^2+e^2 & - \\ - & - & c^2+f^2 \end{bmatrix}$$

$\Rightarrow$ sum of diagonal (trace) = 5

$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 5$

where $a, b, c, d, e, f \in \{-2, -1, 0, 1, 2\}$

Case A 5 of them square is 1

$\Rightarrow (^6C_5) \times (2^5) = 6 \times 32 = 192$

Case B one of them square 4 and another one is square is 1

$\Rightarrow$ {4, 1, 0, 0, 0, 0} are possible as square

$\Rightarrow {}^6C_4 \times (2!) \cdot (2\cdot2) = 15 \times 8 = 12$

$\Rightarrow$ number of such matrices

= 192 + 120 = 312

8. Out of 100 bulbs, 10 are defective and 90 are non-defective. If the probability of finding 7 defective bulbs out of 8 draws, with replacement, is $\frac{K}{10^8}$, then the value of K is

(1) 69 (2) 72

(3) 75 (4) 96

Answer 2)

Sol. Probability a bulb is defective on any draw

$$p = \frac{10}{100} = \frac{1}{10}$$

$$\Rightarrow q = \frac{9}{10}, n = 8$$

$$\Rightarrow P(X = 7) = {}^8C_7 \left(\frac{1}{10}\right)^7 \left(\frac{9}{10}\right)^1 = \frac{72}{10^8}$$

$\Rightarrow K = 72$

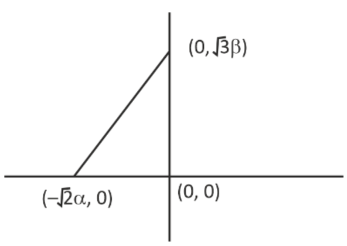
9. Let a circle passes through points $A(-\sqrt{2}\alpha, 0)$, $B(0, \sqrt{3}\beta)$ and O (0, 0) such that its radius is 4. Then the radius of locus of centroid of triangle OAB is

(1) $\frac{2}{3}$ (2) $\frac{8}{3}$

(3) $\frac{4}{3}$ (4) $\frac{11}{3}$

Answer (2)

Sol.



$$\Rightarrow \text{radius} = \frac{\sqrt{(\sqrt{3}\beta)^2 + \left(-\sqrt{2}\alpha\right)}}{2} = 4$$

$$\Rightarrow 3\beta^2 + 2\alpha^2 = 64$$

Let the centroid of the triangle is

$$(h, k) \Rightarrow h = \frac{-\sqrt{2}\alpha}{3} \text{ and } k = \frac{-\sqrt{3}\beta}{3}$$

$$\Rightarrow \alpha = \frac{-3h}{\sqrt{2}}, \beta = \frac{3k}{\sqrt{3}}$$

$$\Rightarrow 2\left(\frac{9h^2}{2}\right) + 3\left(\frac{9k^2}{3}\right) = 64$$

$$\Rightarrow 9h^2 + 9k^2 = 64$$

$$\Rightarrow \text{Locus is } x^2 + y^2 = \frac{64}{9}$$

10. Let $\cot\theta = \frac{5}{12}$ and $\theta \in \left(\pi, \frac{3\pi}{2}\right)$.

Then the value of $\cos 7\theta\left(\sin\frac{13\theta}{2} + \cos\frac{13\theta}{2}\right)$ + $\sin 7\theta\left(\sin\frac{13\theta}{2} - \cos\frac{13\theta}{2}\right)$ is

(1) $-\frac{1}{\sqrt{13}}$ (2) $\frac{1}{\sqrt{13}}$

(3) $-\frac{5}{\sqrt{13}}$ (4) $\frac{5}{\sqrt{13}}$

Answer (3)

Sol. $\because \cot\theta = \frac{5}{12}, \quad \theta \in \left(\pi, \frac{3\pi}{2}\right)$

$$\Rightarrow \sin\theta = -\frac{12}{13}, \quad \cos\theta = -\frac{5}{13}$$

$$\text{Let } P(\theta) = \cos 7\theta\left(\sin\frac{13\theta}{2} + \cos\frac{13\theta}{2}\right)$$

$$+\sin 7\theta\left(\sin\frac{13\theta}{2} - \cos\frac{13\theta}{2}\right)$$

$$= \left(\sin\frac{13\theta}{2}\cos 7\theta - \cos\frac{13\theta}{2}\sin 7\theta\right) + \left(\cos 7\theta\cos\frac{13\theta}{2} + \sin 7\theta\sin\frac{13\theta}{2}\right)$$

$$= \sin\left(\frac{13\theta}{2} - 7\theta\right) + \cos\left(7\theta - \frac{13\theta}{2}\right)$$

$$= -\sin\frac{\theta}{2} + \cos\frac{\theta}{2}$$

$$\left(P(\theta)\right)^2 = 1 - \sin\theta = 1 + \frac{12}{13} = \frac{25}{13}$$

$$\Rightarrow P(\theta) = \pm\frac{5}{\sqrt{13}}$$

$$\theta \in \left(\pi, \frac{3\pi}{2}\right),\ \frac{\theta}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$$

$$\sin\frac{\theta}{2} > 0,\ \cos\frac{\theta}{2} < 0$$

$$\Rightarrow \cos\frac{\theta}{2} - \sin\frac{\theta}{2} < 0$$

$$\Rightarrow P(\theta) = -\frac{5}{\sqrt{13}}$$

11. A line passing through point P (1, 1, 1), which is perpendicular to $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$ and $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$. Let the line intersect the $y - z$ plane at point Q. Another line parallel to L and passing through S(1, 0, –1) intersect another plane at point R. Then the square of area of parallelogram $PQRS$ is

(1) 11 (2) 12

(3) 13 (4) 6

Answer (4)

Sol. Line passing through P(1, 1, 1) and perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$ will have DC

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$= -\hat{i} + \hat{j} + 3\hat{k}$

$L: \frac{x-1}{-1} = \frac{y-1}{1} = \frac{z-1}{3} = \lambda$

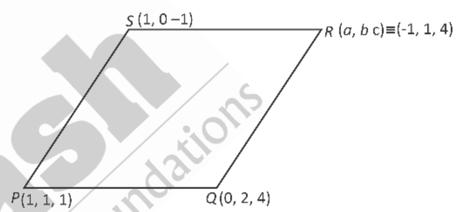
any point $(1 - \lambda, \lambda + 1, 3\lambda + 1)$

$\therefore$ L intersect $y - z$ Plane $\Rightarrow 1 - \lambda = 0$

$\Rightarrow \lambda = 1$

$\Rightarrow Q\ (0, 2, 4)$

$L_1 = \frac{x-1}{-1} = \frac{y}{1} = \frac{z+1}{3}$



Now $\frac{1}{2} = \frac{a+1}{2} \Rightarrow a = -1$

$\frac{2}{2} = \frac{b+1}{2} \Rightarrow b = 1$

$\frac{3}{2} = \frac{c+1}{2} \Rightarrow c = 2$

Area = $|PQ \times PS|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -3 \\ 0 & 1 & 2 \end{vmatrix} = |\hat{i} - 2\hat{j} + \hat{k}|$$

$\Rightarrow$ Area^2 = 1 + 4 + 1

3 vertices are sufficient to get area of parallelogram

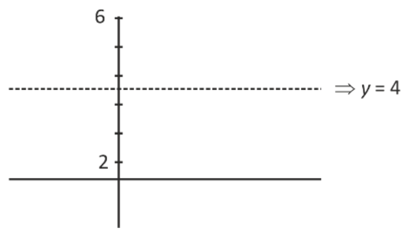
12. Let $\left|\frac{z-6i}{z-2i}\right|=1, \left|\frac{z-8+2i}{z+2i}\right|=\frac{3}{5}$ then if ω satisfy both equation then find $\Sigma|\omega^2|$.

(1) 398 (2) 385

(3) 413 (4) 433

Answer (2)

Sol. $|z-6i| = |z-2i|$



$z = x + iy$

$5|(x-8)+(y+2)i| = 3|(x+0)^2+(y+2)^2|$

$\Rightarrow 25(x-8)^2 + 25(y+2)^2$

$= 9(x^2) + 9(y+2)^2$

$\Rightarrow 25x^2 - 16 \times 25x + 25 \times 64 + 25y^2 + 100y + 100$

$= 9x^2 + 9y^2 + 36y + 36$

$\Rightarrow 16x^2 + 16y^2 - 400x + 64y + 166y = 0$

$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$

This circle intersects lines $y = 4$

at $x^2 + 16 - 25x + 16 + 104 = 0$

$x^2 - 25x + 136 = 0 \Rightarrow x = 8, 17$

$\Rightarrow z$ can be (17, 4) and (8, 4)

$\Rightarrow \sum|z|^2 = \left(\sqrt{8^2+4^2}\right)^2 + \left(\sqrt{4^2+17^2}\right)^2$

$= 64 + 16 + 16 + 289 = 385$

13. Let A_1 be the area enclosed by $y = x^2 + 2$, y-axis and $x + y = 8$ and

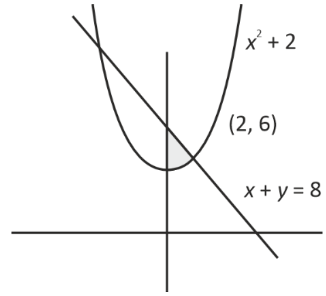
Let A_2 be the area enclosed by $y = x^2 + 2$, $y^2 = x$, $x = 2$ and y-axis, then the value of $A_1 - A_2$ is

(1) $\frac{4+8\sqrt{2}}{3}$ (2) $\frac{2+4\sqrt{2}}{3}$

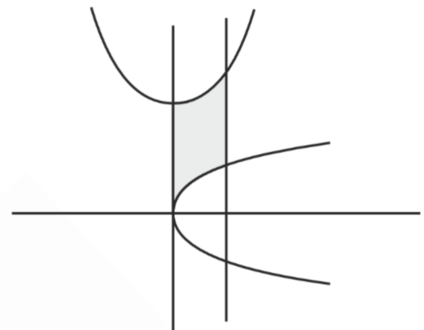
(3) $\frac{8+2\sqrt{2}}{3}$ (4) $\frac{8-2\sqrt{2}}{3}$

Answer (2)

Sol.



$$A_1 = \int_0^2 ((8-x)-(x^2+2))dx = \frac{22}{3}$$



$x = 2$

$$A_2 = \int_0^2 ((x^2+2)-(\sqrt{x}))dx$$

$$= \left.\frac{x^3}{3}+2x-\frac{2x^{3/2}}{3}\right]_0^2$$

$$= \frac{8}{3}+4-\frac{2(2)^{3/2}}{3}$$

$$= \frac{20}{3}-\frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{22}{3}-\frac{20}{3}+\frac{4\sqrt{2}}{3} = \frac{2+4\sqrt{2}}{3}$$

14. If $f(x) = \frac{e^x(\tan x - x)+\ln(\sec x+\tan x)-x}{\tan x - x}$, $x \neq 0$. If $f(x)$ is continuous at $x = 0$, then $f(0)$ is equal to

(1) $\frac{3}{2}$ (2) 1

(3) $\frac{1}{2}$ (4) 2

Answer (1)

Sol. $f(x)=e^x+\ln\dfrac{(\sec x+\tan x)-x}{\tan x-x}$

since $\lim\limits_{x\to 0}\dfrac{\ln(\sec x+\tan x)-x}{\tan x-x}$ $\quad \lim\limits_{x\to 0}\dfrac{\sec x-1}{\sec^2 x-1}=\dfrac{1}{2}$

Using L'Hospital

$\Rightarrow \lim\limits_{x\to 0} f(x)=f(0)=1+\dfrac{1}{2}=\dfrac{3}{2}$

15. $E_1:\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$E_2:\dfrac{x^2}{A^2}+\dfrac{y^2}{B^2}=1$

Let eccentricity of both E_1 and E_2 be $\dfrac{4}{5}$, $\quad 2l_1^2=9l_2$

where l_1 and l_2 are the length of latus rectum of E_1 and E_2 respectively. Distance between the foci of E_1 be 8. Then distance between foci of ellipse E_2 is

(1) $\dfrac{32}{5}$ (2) $\dfrac{16}{5}$

(3) $\dfrac{8}{5}$ (4) $\dfrac{4}{5}$

Answer (1)

Sol. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ $\qquad \dfrac{x^2}{A^2}+\dfrac{y^2}{B^2}=1$

$l_1^2=1-\dfrac{b^2}{a^2}$ $\qquad l_2^2=1-\dfrac{B^2}{A^2}$

$\dfrac{16}{25}=1-\dfrac{b^2}{a^2}$ $\qquad \dfrac{16}{25}=1-\dfrac{B^2}{A^2}$

$\boxed{\dfrac{b^2}{a^2}=\dfrac{9}{25}}$ ... (1) $\qquad \boxed{\dfrac{B^2}{A^2}=\dfrac{9}{25}}$ ... (2)

Now $2l_1^2=9l_2$

$2\left(\dfrac{2b^2}{a}\right)^2=9\left(\dfrac{2B^2}{A}\right)$

$8\dfrac{b^4}{a^2}=18\dfrac{B^2}{A}$

$\boxed{\dfrac{b^4}{a^2}=\dfrac{9}{4}\dfrac{B^4}{A}}$

Also given: $2ae=8$

$2\times\dfrac{4}{5}a=8$

$\boxed{a=5}$

$\Rightarrow \boxed{b=3}$

Now $\dfrac{81}{25}=\dfrac{9}{4}\dfrac{B^2}{A}$

$\dfrac{36}{25}A=B^2$

Sub in (2)

$\dfrac{36}{25}\dfrac{A}{A^2}=\dfrac{9}{25}$

$\boxed{A=4}$

Now $2Ae$

$=2\times 4\times\dfrac{4}{5}$

$=\dfrac{32}{5}$

16. Find the number of numbers greater than 5000 and less than 9000, formed by using numbers 0, 1, 2, 5, 9 with repetition allowed and divisible by 3.

(1) 31 (2) 42

(3) 48 (4) 52

Answer (2)

Sol. As number is more than 5000 and less than 9000 then thousand place must be 5.

5	a	b	c

For (a, b, c) = (0, 0, 1) $\to$ 3 ways

(0, 1, 9) $\to$ 6 ways

(0, 2, 5) $\to$ 6 ways

(0, 2, 2) $\to$ 3 ways

(0, 5, 5) $\to$ 3 ways

(1, 1, 2) $\to$ 3 ways

(1, 1, 5) $\to$ 3 ways

$(1, 9, 9) \rightarrow$ 3 ways

$(2, 2, 9) \rightarrow$ 3 ways

$(2, 5, 9) \rightarrow$ 6 ways

$(5, 5, 9) \rightarrow$ 3 ways

42

$\therefore$ Total 42 numbers are possible.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $S = \frac{1}{25!} + \frac{1}{23!3!} + \frac{1}{21!\,5!} + \ldots\ldots$ upto 13 terms. Then $13\,S = \frac{2^{\alpha}}{\beta!}$, then $\alpha + \beta$ is

Answer (49)

Sol. $\frac{1}{25!} + \frac{1}{23!3!} + \frac{1}{21!5!} + \ldots\ldots$ till 13 term = S

$$26!S = \frac{26!}{25!1!} + \frac{26!}{23!3!} + \frac{26!}{21!5!} + \ldots\ldots$$

$$= {}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \ldots\ldots + {}^{26}C_{25}$$

$$26!\,S = 2^{25}$$

$$S = \frac{2^{25}}{26!}$$

$$13\,S = 13 \times \frac{2^{25}}{26 \times 25!}$$

$$= \frac{2^{24}}{25!} \Rightarrow \alpha = 24 \quad \beta = 25$$

$$\alpha + \beta = 49$$

22.

23.

24.

25.

❑ ❑ ❑